MAST90105 Lab and Workshop 12 Solutions

The Lab and Workshop this week covers problems arising from Module 8.2 to the end of the course.

1 Lab

1. A company that manufactures brackets for an auto maker regularly selects brackets from the production line and performs a torque test. The goal is for the mean torque to equal 125. Let $X \sim N(\mu, \sigma^2)$ be the torque and suppose we take a random sample of size $n = 15$ to test $H_0 : \mu = 125$ against a two sided alternative. Suppose the following data are observed:

   $$\begin{array}{cccccccc} 128 & 149 & 136 & 114 & 126 & 142 & 124 & 136 \\ 122 & 118 & 122 & 129 & 118 & 122 & 129 & \end{array}$$

   Use the t.test command to test the hypotheses and construct a 95% confidence interval for μ.

   - *We do not reject H_0 at the 5% significance level and a 95% confidence interval is (122.3,133.0) (1dp) - see R output.*

```
x <- c(128, 149, 136, 114, 126, 142, 124, 136, 122,
    118, 122, 129, 118, 122, 129)
t.test(x, mu = 125, alternative = "two.sided")

##
##  One Sample t-test
##
## data:  x
## t = 1.0762, df = 14, p-value = 0.3
## alternative hypothesis: true mean is not equal to 125
## 95 percent confidence interval:
##  122.3522 132.9811
## sample estimates:
## mean of x
##  127.6667
```

2. Let X be the weight in grams of a Low Fat Strawberry Kudo and Y ther weight of a Low Fat Blueberry Kudo. Assume $X \sim N(\mu_X, \sigma_X^2)$ and $Y \sim N(\mu_Y, \sigma_Y^2)$. A random sample of 9 observations on X yielded
   21.7, 21.0, 21.2, 20.7, 20.4, 21.9, 20.2, 21.6, 20.6
   and a random sample of 13 observations on Y yielded
   21.5, 20.5, 20.3, 21.6, 21.7, 21.3, 23.0, 21.3, 18.9, 20.0, 20.4, 20.8, 20.3

a. Test $H_0 : \mu_X = \mu_Y$ against a two-sided alternative. You may choose the significance level.

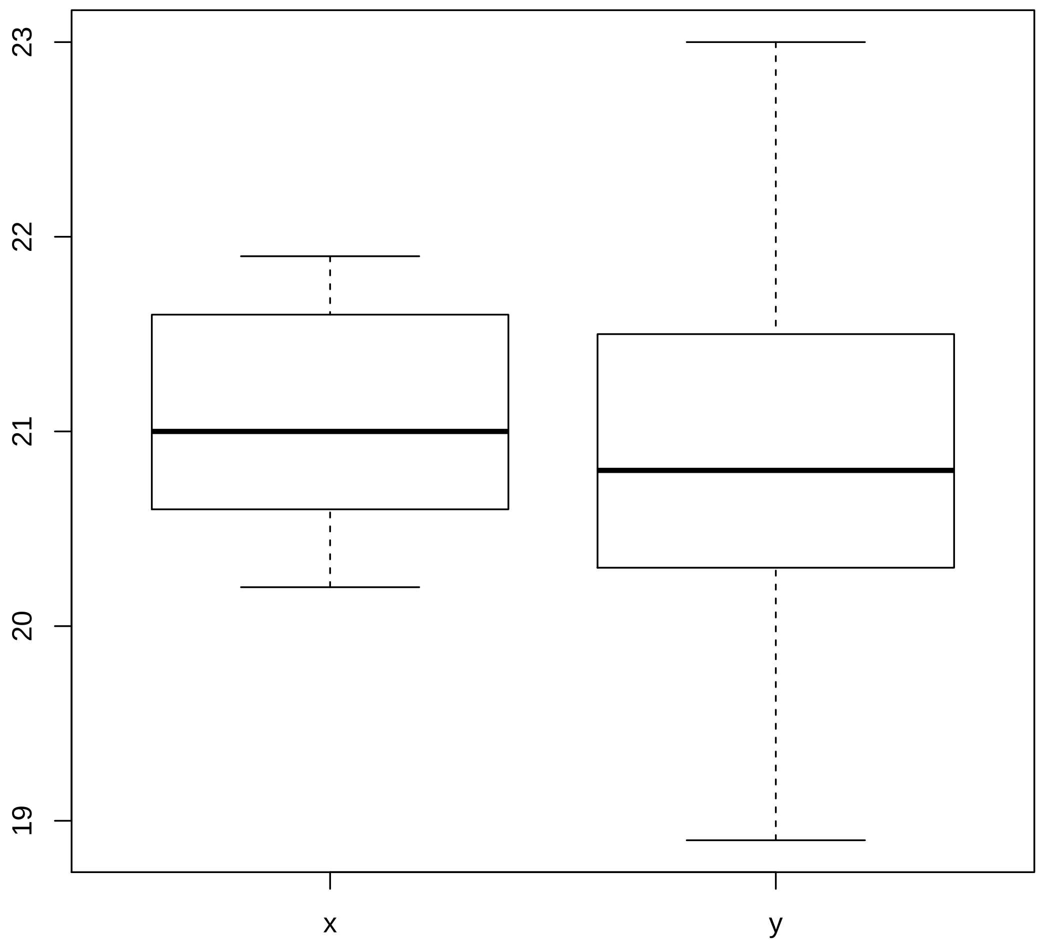
   - *We do not reject H_0 at the 10% (or any lower) significance level - see R output.*

```
x <- c(21.7, 21, 21.2, 20.7, 20.4, 21.9, 20.2, 21.6,
    20.6)
y <- c(21.5, 20.5, 20.3, 21.6, 21.7, 21.3, 23, 21.3,
    18.9, 20, 20.4, 20.8, 20.3)
t.test(x, y, alternative = "two.sided")

##
##  Welch Two Sample t-test
##
## data:  x and y
## t = 0.40909, df = 19.738, p-value = 0.6869
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.5786875  0.8607388
## sample estimates:
## mean of x mean of y
##  21.03333  20.89231
```

b. Construct a box plot to support your conclusions.

- *The boxplot needs a combination of samples to display them side by side - see R output.*

```
combin <- c(x, y)
sample <- c(rep("x", length(x)), rep("y", length(y)))
boxplot(combin ~ sample)
```



3. Let $X_1, \ldots, X_{12}$ be a random sample from an exponential distribution with mean θ. Then $f(x;\theta) = \theta^{-1}e^{-x/\theta}, \quad 0 < x < \infty$. We shall test the null hypothesis $H_0 : \theta = 9$ against the alternative $H_1 : \theta \neq 9$.

   a. Recall that the moment generating function (m.g.f) of an exponential random variable with mean θ is $M(t) = (1-\theta t)^{-1}$ and that of a $\chi^2(r)$ random variable is $M(t) = (1-2t)^{-r/2}$. Find the moment generating function of $(2/\theta)\sum_{=1}^{12} X_i$ and hence deduce that $(2/\theta)\sum_{i=1}^{12} X_i \sim \chi^2(24)$.

      - *The expectation of product of independent random variables is the product of the expectations. So the moment generating function of* $(2/\theta)\sum_{=1}^{12} X_i, M_{12}(t)$, *is given by*

$$M_{12}(t) = E(\prod_{i=1}^{12} \exp(1 - 2/\theta X_i t) = \prod_{i=1}^{12}(1-2t) = (1-2t)^{-24/2}.$$

This is that of a $\chi^2(24)$ random variable as required.

b. Find constants a and b so that, under H_0,

$$P\left[\frac{2}{9}\sum_{i-1}^{12} X_i \leq a\right] = 0.05, \text{ and } P\left[\frac{2}{9}\sum_{i-1}^{12} X_i \geq b\right] = 0.05$$

- *The question originally had 2 typos. and an omission in it - sorry! These have been corrected here (otherwise, the next part would not work).*
- $P\left[\frac{2}{9}\sum_{i-1}^{12} X_i \leq a\right] = P\left[\chi^2_{24} \leq a\right]$
- $P\left[\frac{1}{9}\sum_{i-1}^{12} X_i \geq b\right] = P\left[\chi^2_{24} \geq b\right]$
- *See R output below.*

```
(a <- qchisq(0.05, 24))

## [1] 13.84843

(b <- qchisq(0.95, 24))

## [1] 36.41503
```

c. Show that a critical region of size $\alpha = 0.10$ is

$$C = \left\{(x_1, \ldots, x_{12}) : \sum_{i=1}^{12} x_i \leq \frac{9a}{2} \text{ or } \sum_{i=1}^{12} x_i \geq \frac{9b}{2}\right\}.$$

- *Under* $H_0, P(C) = P(\frac{2}{9}\sum_{i=1}^{12} X_i \leq a) + P(\frac{2}{9}\sum_{i=1}^{12} X_i \geq b) = 0.1$

d. Define the power function for this test.

- *The power function is the probability of the critical region as a function of the mean of the exponential distribution.*
- *With mean θ, the random variable X_i/θ has exponential distribution mean 1.*
- *Hence* $\frac{2}{\theta}\sum_{i=1}^{12} X_i \sim \chi^2_{24}$.
- *Taking account of the definition of the critical region, the power function is given by* $K(\theta) = P(\chi^2_{24} \leq \frac{9a}{\theta}) + P(\chi^2_{24} \geq \frac{9b}{\theta})$.
- *The R output to set up the power function is shown below.*

```
K <- function(theta) {
    pchisq(9 * a/theta, 24) + 1 - pchisq(9 * b/theta,
        24)
}
# check that power at theta=9 is the 10%
# significance level
K(9)

## [1] 0.1
```

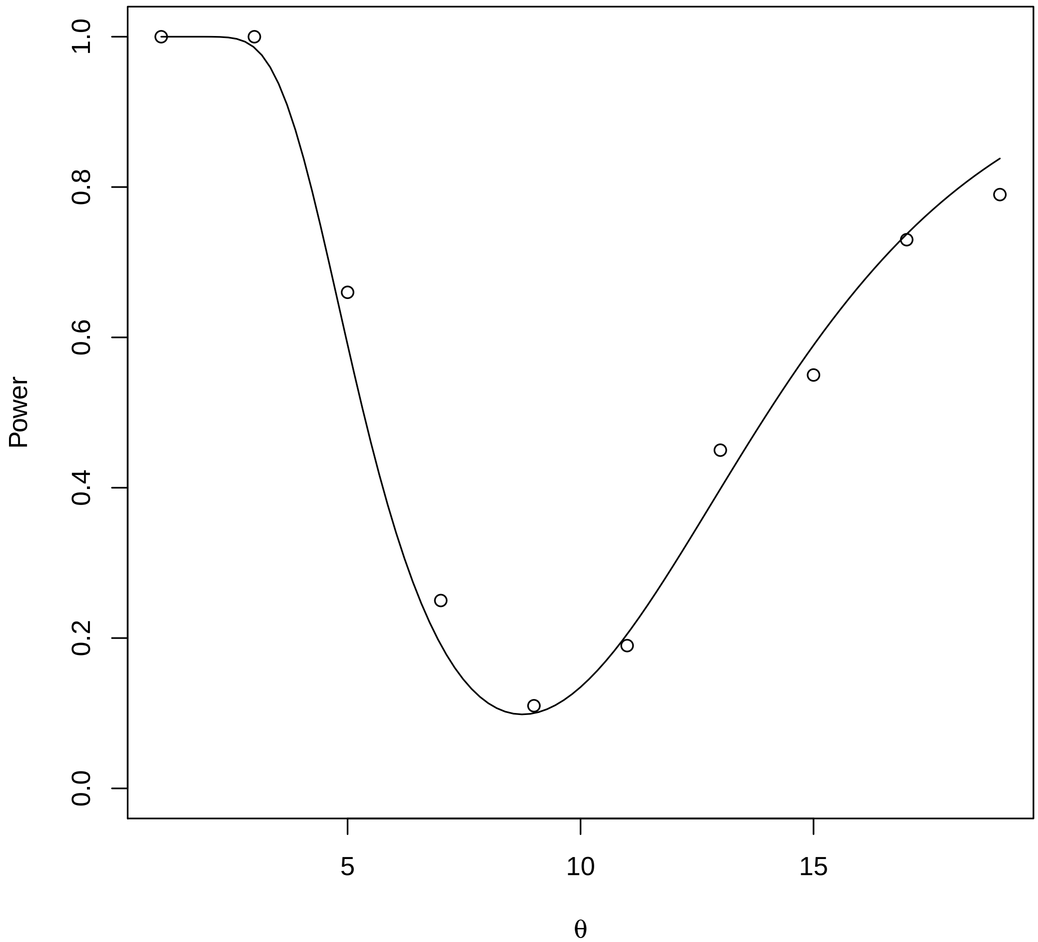
e. For $\theta = 1, 3, 5, \ldots, 19$, generate 100 random samples of size 12 from an exponential distribution with mean θ. For each θ determine the number of times H_0 was rejected.

```
# crit gives TRUE if x in critical region
crit <- function(theta) {
    x <- theta * rexp(12)
    as.vector(sum(x) <= 9 * a/2 | sum(x) >= 9 * b/2)
}
theta <- seq(1, 19, 2)
p <- rep(0, 10)
# loop to give empirical power for each theta =
# 1,3, ... ,19
for (i in 1:10) {
    t <- as.matrix(rep(theta[i], 100))
    p[i] <- sum(apply(t, 1, crit))/100  # trick as in Lab 10, Qn 1
}
```

f. Show that the theoretical and empirical power functions are the same by plotting them on the same figure.

- *The power function is the probability of the critical region as a function of the mean of the exponential distribution.*
- *With mean θ, the random variable X_i/θ has exponential distribution mean 1.*
- *Hence $\frac{2}{\theta}\sum_{i=1}^{12} X_i \sim \chi^2_{24}$.*
- *Taking account of the definition of the critical region, the power function is given by $K(\theta) = P(\chi^2_{24} \leq \frac{9a}{\theta}) + P(\chi^2_{24} \geq \frac{9b}{\theta})$.*
- *The R output to set up the power function is shown below.*

```
plot(theta, p, xlab = expression(theta), ylab = "Power",
    ylim = range(0, 1))
curve(K, from = 1, to = 19, add = TRUE)
```



4. Let X be the weight (in grams) of a grape flavoured jolly rancher. Denote the median of X by m. We shall test $H_0 : m = 5.900$ against $H_1 : m > 5.900$. A random sample of size $n = 25$ yielded the data:
5.625, 5.665, 5.697, 5.837, 5.863, 5.870, 5.878, 5.884, 5.908, 5.967, 6.019, 6.020, 6.029, 6.032, 6.037, 6.045, 6.049, 6.050, 6.079, 6.116, 6.159, 6.186, 6.199, 6.307, 6.387 Test the hypotheses using

   a. The sign test.

      - *We cannot reject H_0 at the 5% significance level using the sign test - see R output.*

```
X <- c(5.625, 5.665, 5.697, 5.837, 5.863, 5.87, 5.878,
    5.884, 5.908, 5.967, 6.019, 6.02, 6.029, 6.032,
    6.037, 6.045, 6.049, 6.05, 6.079, 6.116, 6.159,
    6.186, 6.199, 6.307, 6.387)
```

```r
Z <- X - 5.9
sum(sign(Z) > 0)

## [1] 17

1 - pbinom(16, 25, 0.5)

## [1] 0.05387607
```

b. The Wilcoxon test. (wilcox.test)

- *Using the rank sum test we reject H_0 at the 5% significance level - see R output.*

```r
wilcox.test(X, mu = 5.9, alternative = "greater")

##
##  Wilcoxon signed rank test
##
## data:  X
## V = 248, p-value = 0.01014
## alternative hypothesis: true location is greater than 5.9
```

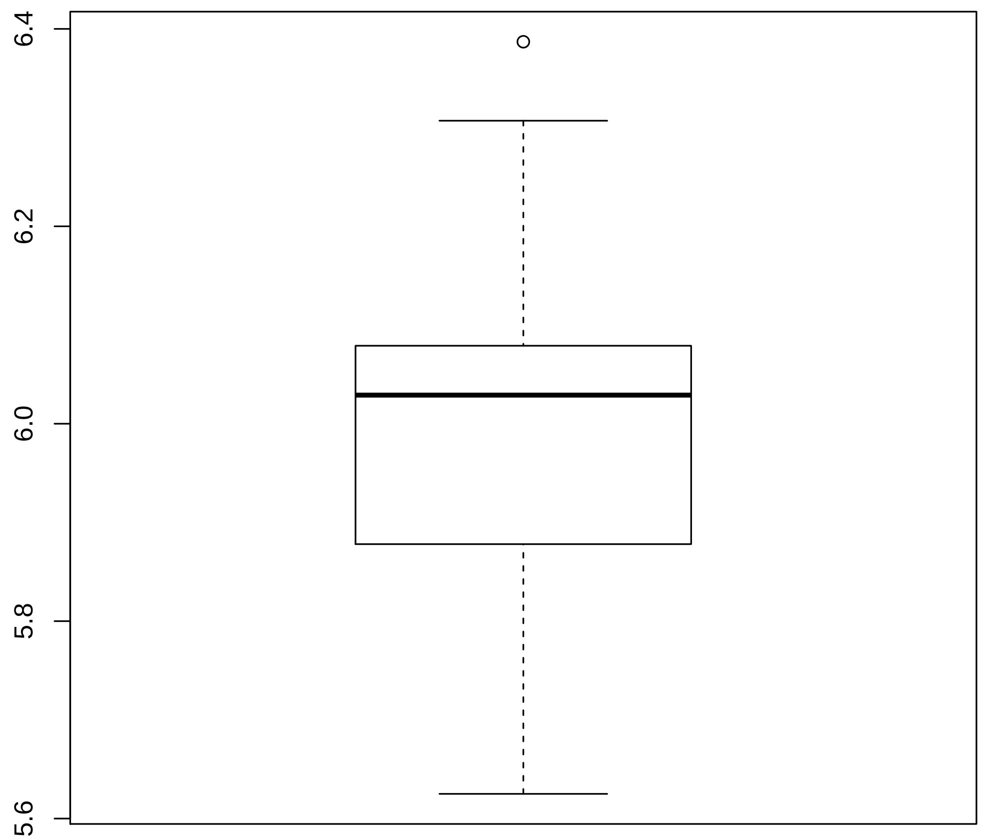
c. The t test.

- *We also reject H_0 using the t-test at the 5% significance level - see R output.*

```r
t.test(X, mu = 5.9, alternative = "greater")

##
##  One Sample t-test
##
## data:  X
## t = 2.6077, df = 24, p-value = 0.007716
## alternative hypothesis: true mean is greater than 5.9
## 95 percent confidence interval:
##  5.933127      Inf
## sample estimates:
## mean of x
##   5.99632
```

d. Construct a box plot of the data.

```r
boxplot(X)
```



e. Compare your results and discuss.

- *At the 5% significance level both the rank sum test and the t-test reject the null hypothesis whereas the sign test does not. The boxplot (and a stem and leaf plot) suggests the underlying population may be close to symmetrical so the rank sum test is appropriate. A more detailed examination of the assumptions may be appropriate.*

5. Let X equal the amount of butterfat in pounds produced by 90 cows during a 305-day milk production period following their first calf. Test the hypothesis that the distribution of X is $N(\mu, \sigma^2)$ using the following data:
486 537 513 583 453 510 570 500 458 555 618 327 350 643 500 497 421 505 637 599 392 574 492 635 460 696 593 422 499 524 539 339 472 427 532 470 417 437 388 481 537 489 418 434 466 464 544 475 608 444 573 611 586 613 645 540 494 532 691 478 513 583 457 612 628 516 452 501 453 643 541 439 627 619 617 394 607 502 395 470 531 526 496 561 491 380 345 274 672 509 The data is available as `Butterfat.xls` in theLMS and in the MAST90105 folder in the Lab.

a. Compute the sample mean $\bar{x}$ and standard deviation s_x.

```
library(readxl)
Butterfat <- read_excel(
"L:/MAST90105MethodsofMathematicalStatistics/Butterfat.xls"
)
X <- Butterfat$Butterfat
(mu <- mean(X))
(s <- sd(X))
```

```
## [1] 511.6333
## [1] 87.5768
```

-

b. Use the commands

```
b <- c(0, seq(374, 624, 50), 1000)
(T <- table(cut(X, breaks = b)))

##
##     (0,374]   (374,424]   (424,474]   (474,524]
##           5           9          16          22
##   (524,574]   (574,624] (624,1e+03]
##          15          13          10

O <- as.numeric(T)
```

to compute observed frequencies in the given cells.

c. Compute expected frequencies using

```
p <- rep(0, 7)
p[1] <- pnorm(b[2], mu, s) - pnorm(b[1], mu, s)
# ...
p[7] <- pnorm(b[8], mu, s) - pnorm(b[8], mu, s)
```

To speed this up, you can use

```
for (k in 1:7) {
    p[k] = pnorm(b[k + 1], mu, s) - pnorm(b[k], mu,
        s)
}
E <- p * length(X)
C <- sum((O - E)^2/E)
d1 <- length(T)
qchisq(0.95, d1 - 3)
```

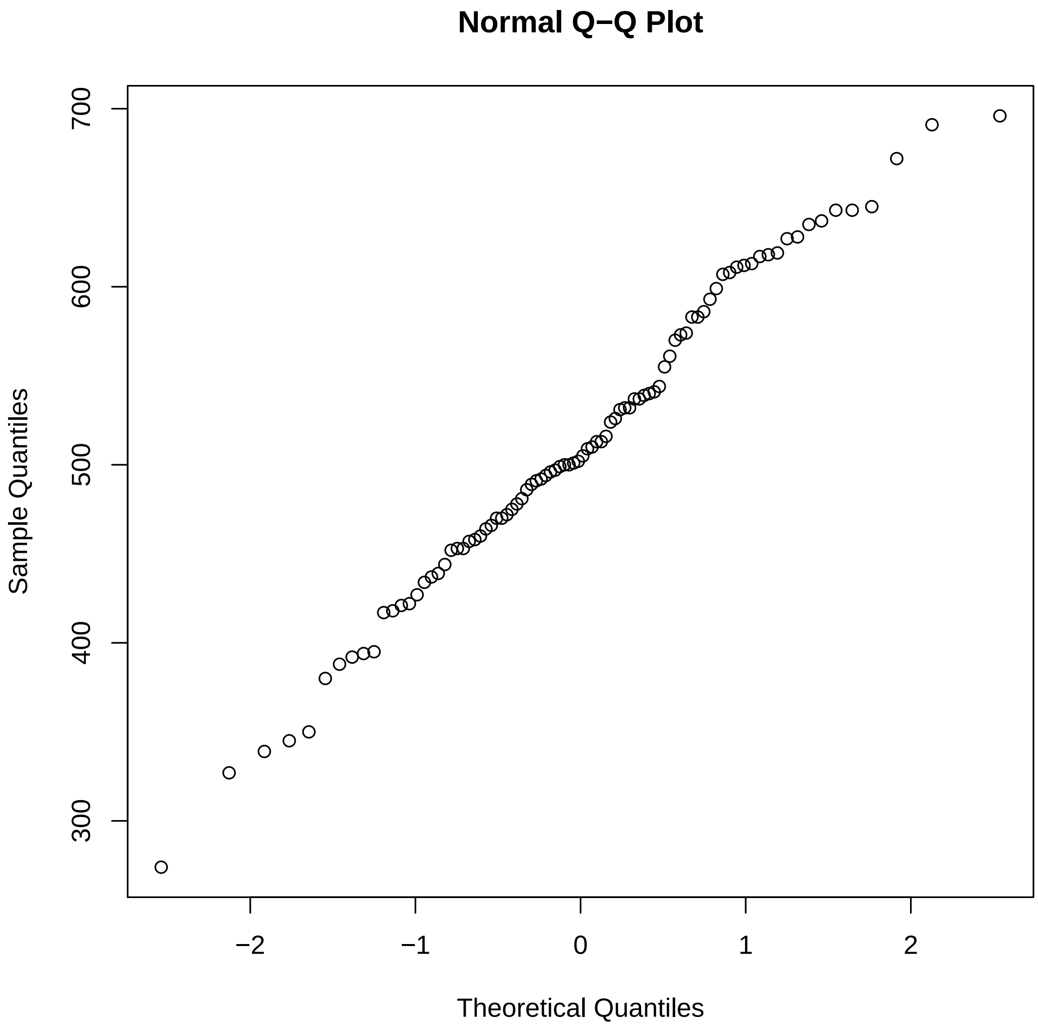
```
## [1] 9.487729

1 - pchisq(C, d1 - 3)

## [1] 0.9076048

1 - pchisq(C, d1 - 3)  #p-value

## [1] 0.9076048

cbind(O, E)

##       O         E
## [1,]  5  5.222232
## [2,]  9  9.042687
## [3,] 16 15.768114
## [4,] 22 20.020264
## [5,] 15 18.509602
## [6,] 13 12.460972
## [7,] 10  8.976128
```

d. If you conduct the chisquare test using
remember to adjust your degrees of freedom.
   - *We cannot reject H_0 at the 5% significance level using the chisquare test - see R output.*
   - *The degrees of freedom must be lowered by 2 and the p-value is lowered but not anywhere close to the critical 5%.*
   - *So no evidence against a normal fit from this test.*

```
(test <- chisq.test(O, p = p))

##
##  Chi-squared test for given probabilities
##
## data:  O
## X-squared = 1.0144, df = 6, p-value = 0.9851

stat <- test$statistic
df <- test$parameter - 2
1 - pchisq(stat, df)

## X-squared
## 0.9076048
```

e. Use the `qqnorm` command to illustrate your result.
   - *The qqplot for the normal quantiles shows a roughly linear fit affirming that the normal distribution assumption is not inconsistent with this data.*

```
qqnorm(X)
```



6. The Mathematica notebook, Lab12.nb, is available on the LMS and also on the server. It contains instructions on exploring the distribution of the Wilcoxon signed rank statistic. At the end of working through it you will have computed the exact p-value that was obtained in lectures, seen how to get the distribution symbolically (and thus exactly) as well as computing and plotting the distribution, including getting an expression for the moment generating function as the sample size varies.

   - *This question requires you to work through the questions in the notebook which are mostly about observation and understanding the commands.*
   - *The question about the values of the Wilcoxon signed rank statistic is resolved by induction.*
   - *For sample size 2, the possible values are* $\{-1-2, 1-2, 2-1, 2+1\} = \{-3, -1, 1, 3\}$.

- *Suppose the proposition "the Wilcoxon signed rank statistic takes values* $\{-n(n+1)/2, 0, \cdots, 0, n(n+1)/2\}$ *" is true for* n*.*
- *If* n *is even then the induction hypothesis makes all the values odd because* $2m(2m+1)/2$ *is odd.*
- *The values of the signed rank statistic for* $n+1$ *are odd plus an odd number, either* $n+1$ *or* $-n-1$*, so they are indeed* $\{-n(n+1)/2, 0, \cdots, 0, n(n+1)/2\}$ *which are all the even numbers between the maximum and minimum possible values.*
- *The argument is similar if* n *is odd.*

7. Develop a function to simulate the distribution of the Wilcoxon two sample statistic based on sample sizes n, m by drawing random samples, using the R command `sample`.

```
f <- function(x) {
    sum(sample(x[1] + x[2], size = x[2]))
}
W <- function(x, r) {
    t <- matrix(rep(x, r), byrow = TRUE, nrow = r,
        ncol = 2)
    apply(t, 1, f)
}
# This can be tested out on the sample sizes of 8
# and 8, used in the cinammon packet filling
# example as follows.  The code produces 10,000
# values of the W statistic It checks the mean and
# variance of the simulated samples It then finds
# the emprical probability that W is at least 87
w <- W(c(8, 8), 10000)
# compare to theoretical values
c(mean(w), 8 * 17/2)

## [1] 67.9984 68.0000

c(sd(w), sqrt(8 * 8 * 17/12))

## [1] 9.605916 9.521905

# empirical p-value compared to normal
# approximation
c(sum(w >= 87)/10000, 1 - pnorm((87 - 4 * 17)/sqrt(64 *
    17/12)))

## [1] 0.02720000 0.02299968
```

```
# try again with a larger number of repititions
w <- W(c(8, 8), 1e+06)
# compare to theoretical values
c(mean(w), 8 * 17/2)

## [1] 67.99821 68.00000

c(sd(w), sqrt(8 * 8 * 17/12))

## [1] 9.516501 9.521905

# empirical p-value
c(sum(w >= 87)/1e+06, 1 - pnorm((87 - 4 * 17)/sqrt(64 *
    17/12)))

## [1] 0.02476700 0.02299968
```

2 Workshop

8. Vitamin B_6 is one of the vitamins in a multiple vitamin pill manufactured by a pharmaceutical company. The pills are produced with a mean of 50 milligrams of vitamin B_6 per pill. The company believes there is a deterioration of 1 milligram per month, so that after 3 months they expect that $\mu = 47$. A consumer group suspects that $\mu < 47$ after 3 months.

    a. Define a critical region to test $H_0 : \mu = 47$ against $H_1 : \mu < 47$ at the $\alpha = 0.05$ significance level based on a random sample of of size $n = 20$. ($t_{0.05}(19) = 1.729$, $t_{0.05}(20) = 1.724$, $t_{0.025}(19) = 2.093$, $t_{0.025}(20) = 2.086$).
    b. If the 20 pills yielded a mean of $\bar{x} = 46.94$ with standard deviation of $s = 0.15$, what is your conclusion?
    c. What is the approximate p-value of this test?

    - a. *Critical region is:*

    $$t = \frac{\bar{x} - 47}{s/\sqrt{20}} < -t_{0.05}(19) = -1.729$$

      b. $t = (46.94 - 47)/0.15/\sqrt{20} = -1.789$. *This is less than -1.729 so we reject* H_0.

      c. *The p-value is between 0.025 and 0.05.*

9. Let X be the forced vital capacity (FVC) in liters for a female college student. Assume that $X \sim N(\mu, \sigma^2)$ approximately. Suppose it is known that $\mu = 3.4$ litres. A volleyball coach claims the FVC of volleyball players is greater than 3.4. She plans to test this using a random sample of size $n = 9$.

   a. Define the null hypothesis.

   b. Define the alternative hypothesis.

   c. Define a critical region for which $\alpha = 0.05$. lllustrate this on a figure. ($t_{0.025}(8) = 2.306$, $t_{0.05}(8) = 1.859$, $t_{0.01}(8) = 2.896$)

   d. Calculate the value of the test statistic if $\bar{x} = 3.556$ and $s = 0.167$.

   e. What is your conclusion?

   f. What is the approximate p-value of this test?

   - *a.* $H_0 : \mu = 3.4$

     b. $H_1 : \mu > 3.4$

     c. $t = (\bar{x} - 3.4)/(s/3) > 1.859$

     d. $t = (3.556 - 3.4)/(0.167/3) = 2.802$

     e. Reject H_0.

     f. $2.306 < 2.802 < 2.896$ *so the p-value is between 0.01 and 0.025.*

10. Among the data collected for the World Health Organisation air quality monitoring project is a measure of suspended particles in $\mu g/m^3$. Let X and Y equal the concentration of suspended particles in the city centres of Melbourne and Houston. Using $n = 13$ observations of X and $m = 16$ observations of Y, we shall test $H_0 : \mu_X = \mu_Y$ against $H_1 : \mu_X < \mu_Y$.

    a. Define the test statistic and the critical region assuming the variances are equal. Let $\alpha = 0.05$

    b. If $\bar{x} = 72.9$, $s_x = 25.6$, $\bar{y} = 81.7$ and $s_y = 28.3$, calculate the value of the test statistic and state your conclusion.
    ($t_{0.025}(27) = 2.052$, $t_{0.05}(27) = 1.703$, ($t_{0.1}(27) = 1.314$, ($t_{0.25}(27) = 0.684$)

    c. Give limits for the p-value of this test.

    - *a. Critical region is given by*

      $$t = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{12s_x^2 + 15s_y^2}{27}\left(\frac{1}{13} + \frac{1}{16}\right)}} \leq t_{0.05}(27) = -1.703$$

      b. Observed value is

      $$t = \frac{72.9 - 81.7}{\sqrt{\frac{12 \times 25.6^2 + 15 \times 28.3^2}{27}\left(\frac{1}{13} + \frac{1}{16}\right)}} = -0.869 > -1.703$$

      so we cannot reject H_0.

c. $0.10 <$ *p-value* < 0.25.

11. It is claimed that the median weight m of certain loads of candy is 40,000 pounds.

a. Use the following data and the Wilcoxon test statistic at an approximate significance level of $\alpha = 0.05$ to test the null hypothesis $H_0 : m = 40,000$ against $H_1 : m < 40,000$.

41195, 39485, 41229, 36840, 38050, 40890, 38345, 34930, 39245, 31031, 40780, 38050, 30906

It may help to complete the following table. Ties are assigned the average rank.

	X	$X - m$	Rank	Sign
1	41195			
2	39485			
3	41229			
4	36840			
5	38050			
6	40890			
7	38345			
8	34930			
9	39245			
10	31031			
11	40780			
12	38050			
13	30906			

b. What is the approximate p-value?

```
qnorm(seq(0.9, 0.975, 0.025))

## [1] 1.281552 1.439531 1.644854 1.959964
```

- *The table is*

	X	X-m	Rank	Sign
1	41195.00	1195.00	5.00	1.00
2	39485.00	−515.00	1.00	−1.00
3	41229.00	1229.00	6.00	1.00
4	36840.00	−3160.00	10.00	−1.00
5	38050.00	−1950.00	8.50	−1.00
6	40890.00	890.00	4.00	1.00
7	38345.00	−1655.00	7.00	−1.00
8	34930.00	−5070.00	11.00	−1.00
9	39245.00	−755.00	2.00	−1.00
10	31031.00	−8969.00	12.00	−1.00
11	40780.00	780.00	3.00	1.00
12	38050.00	−1950.00	8.50	−1.00
13	30906.00	−9094.00	13.00	−1.00

- *So*

$$W = 5.0-1.0+6.0-10.0-8.5+4.0-7.0-11.0-2.0-12.0+3.0-8.5-13.0 = -55.$$

- *Recall*

$$\mu_W = 0, \quad Var(W) = \frac{n(n+1)(2n+1)}{6} = 819.$$

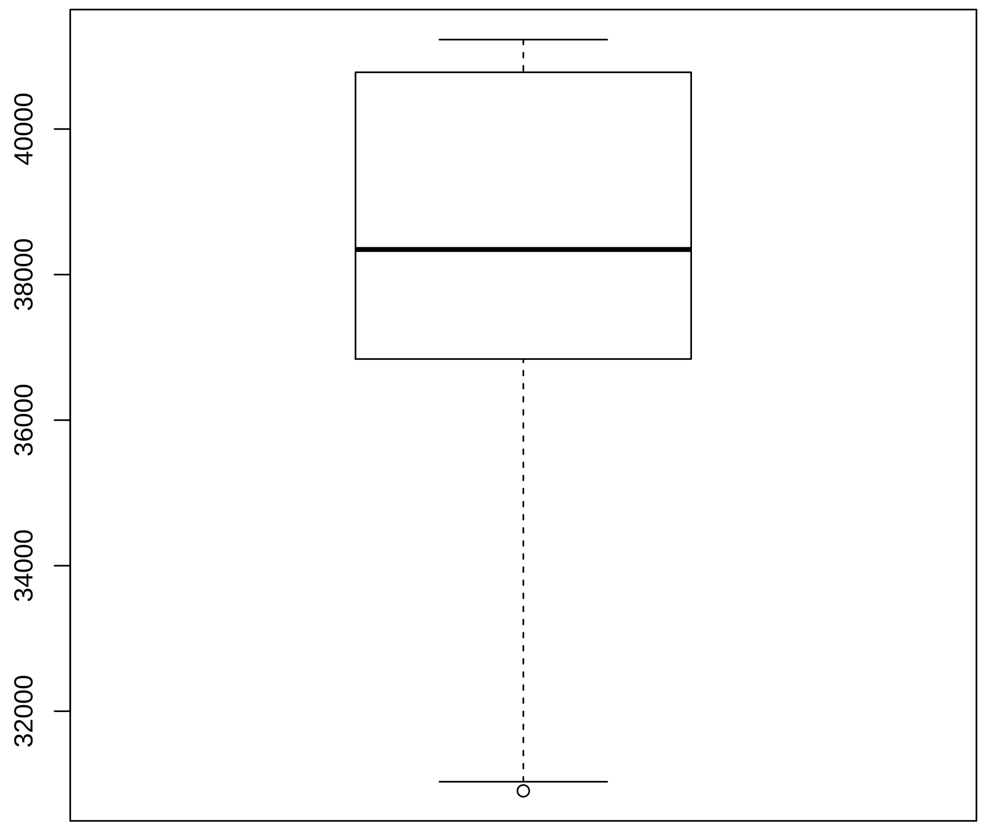
- *So*

$$Z = \frac{-55}{\sqrt{819}} = -1.9218.$$

- *This is less than -1.645 so at the 5% level of significance we reject H_0.*
- *The approximate p-value is between 0.025 and 0.025. The R output gives the approximate p-value as 0.0273.*
- *An exact evaluation using the Mathematica Lab12.nb shows the p-value is 0.0286865.*

- *There is some evidence of asymmetry of the distribution which is skewed to the left. This is seen from the boxplot and also from the different mean and median.*

```
x <- c(41195, 39485, 41229, 36840, 38050, 40890, 38345,
    34930, 39245, 31031, 40780, 38050, 30906)
pnorm(-55/sqrt(819))

## [1] 0.02731197

boxplot(x)
```




```
c(mean(x), median(x))

## [1] 37767.38 38345.00
```

c. Use the sign test to test the same hypothesis.

```
pbinom(6:12, 13, 0.5)

## [1] 0.5000000 0.7094727 0.8665771 0.9538574
## [5] 0.9887695 0.9982910 0.9998779
```

- *There are 9 negative signs.*

$$P(Y \geq 9 \mid p = 0.5) = 1 - 0.8666 = 0.1334$$

- *So we cannot reject the null hypothesis even at the 10% level.*

d. Compare the results of the two tests.

- *The null hypothesis is rejected using the rank sum test but cannot be rejected using the sign test.*
- *The difference could in part be rejection of the hypothesis of symmetry but the Wilcoxon signed rank test is more powerful than the sign test if the symmetry hypothesis holds, so it could also be due to the difference in power.*

12. A 1-pound bag of candy-coated chocolate covered peanuts contained 224 pieces of candy coloured brown, orange, green and yellow. Test the null hypothesis that the machine filling these bags treats the four colours of candy equally likely. That is test

$$H_0 : p_B = p_O = p_G = p_Y = \frac{1}{4}.$$

The observed values were 42 brown, 64 orange, 53 green, and 65 yellow. You may select the significance level or give an appropriate p-value.
($\chi^2_{0.025}(3) = 9.348$, $\chi^2_{0.05}(3) = 7.815$, $\chi^2_{0.10}(3) = 6.251$.

- *Observed and expected frequencies are:*

	B	*O*	*G*	*Y*	*Total*
O	*42*	*64*	*53*	*65*	*224*
E	*56*	*56*	*56*	*56*	*224*

- *So*

$$\chi^2 = \frac{(42-56)^2}{56} + \cdots + \frac{(65-56)^2}{56} = 6.25 < 7.815$$

- *Hence we cannot reject H_0 at the 5% level of significance.*

13. In a biology laboratory the mating of two red eye fruit flies yielded $n = 432$ offspring among which 254 were red-eyed, 69 were brown-eyed, 87 were scarlet-eyed, and 22 were white-eyed. Use these data to test, with $\alpha = 0.05$, the hypothesis that the ratio among the offspring would be 9:3:3:1 respectively.
($\chi^2_{0.025}(3) = 9.348$, $\chi^2_{0.05}(3) = 7.815$, $\chi^2_{0.10}(3) = 6.251$).

- *Observed end expected frequencies are:*

	red	*brown*	*scarlet*	*white*
O	*254.00*	*69.00*	*87.00*	*22.00*
Prob.	*0.56*	*0.19*	*0.19*	*0.06*
E	*243.00*	*81.00*	*81.00*	*27.00*

- $\chi^2 = 3.646$ *with 3 d.f. so we do not reject* H_0 *at the 5% level of significance.*

14. We wish to determine if two groups of nurses distribute their time in six different categories about the same way. That is, the hypothesis under consideration is H_0 : $p_{i1} = p_{i2}$, $i = 1 \ldots, 6$. To test this, nurses are observed at random throughout several days, each observation resulting in a mark in one of the six categories. The summary data is given in the following frequency table

	Category						
	1	2	3	4	5	6	Total
Group I	95	36	71	21	45	32	300
Group II	53	26	43	18	32	28	200

Use a chi-square test with $\alpha = 0.05$.

```
qchisq(seq(0.9, 0.975, 0.025), 5)

## [1]  9.236357 10.008315 11.070498 12.832502
```

- *R output below.*
- *Note that the appropriate hypotheses are that the two distributions for Groups 1 and 2 are the same.*
- *The chi-square test gives the right p-value since no extra parameters have been estimated.*
- *We cannot reject the null hypothesis at the 5% level and conclude there is not sufficient evidence that the two groups distribute their time differently.*

```
observed <- matrix(c(95, 36, 71, 21, 45, 32, 53, 26,
    43, 18, 32, 28), byrow = T, nrow = 2)
c1 <- chisq.test(observed, correct = F)
c1$observed

##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]   95   36   71   21   45   32
## [2,]   53   26   43   18   32   28

c1$expected

##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,] 88.8 37.2 68.4 23.4 46.2   36
## [2,] 59.2 24.8 45.6 15.6 30.8   24

c1$p.value

## [1] 0.6645015
```

15. A random sample of 1000 individuals from a rural area had 620 in favour of the election of a certain candidate, whilst a random sample of 1000 individuals from an urban area had 550 in favour of the same candidate. At the 5% level, test the hypothesis that area and opinion about the candidate are independent.

    - *R commands and output are below but this can be done by hand easily.*
    - *Since the p-value is smaller 5%, we reject the hypothesis that area and opinion are independent.*
    - *Note that the same p-value arises from a two-sample proportion test that the probabilities of favouring the candidates are the same for the two areas. This can be seen numerically from the following R commands and output.*
    - *The two tests are for* different *hypotheses but the square of the Z has the same value algebraically (it is a good exercise to check this out) and have the same p-values.*

```
observed <- array(c(620, 380, 550, 450), dim = c(2,
    2))
(c1 <- chisq.test(observed, correct = F))

##
##  Pearson's Chi-squared test
##
```

```
## data:  observed
## X-squared = 10.092, df = 1, p-value =
## 0.001489

# manual check of chisquare
chisqman <- (620 - 585)^2/585 + (550 - 585)^2/585 +
    (380 - 415)^2/415 + (450 - 415)^2/415
chisqman

## [1] 10.09165

p1 = 620/1000
p2 = 550/1000
p = (620 + 550)/2000
z = (p1 - p2)/(p * (1 - p) * (1/1000 + 1/1000))^0.5
z^2

## [1] 10.09165
```