Solutions to Sample Exam 1 MAST90105

1. You ask your neighbour to water 3 sickly plants in your front yard while you are on vacation. Each of the plants will be alive with probability 0.85 if watered; it will be alive with probability 0.2 if not watered. You are 90 percent certain that your neighbour will remember to water the plants. [7]

   (a) What is the probability that you will find at least two of the three plants alive when you return home?

   (b) What is the expected number and variance of the number of plants that will be alive when you return home?

   - *Let A be the event that your neighbour remembers to do the watering and X be the random variable that gives the number of plants alive when you return home. Conditional on $A, X \sim Bin(3, 0.85)$ and conditional on $A^c, X \sim Bin(3, 0.2)$*
   - *Required probability is* $P(X \geq 2) = P(X = 2) + P(X = 3) = (P(X = 2|A)P(A) + P(X = 2|A^c)P(A^c)) + (P(X = 3|A)P(A) + P(X = 3|A^c)P(A^c)) = (3 \times 0.85^2 \times 0.15 \times 0.9 + 3 \times 0.2^2 \times 0.8 \times 0.1) + (0.85^3 \times 0.9 + 0.2^3 \times 0.1) =$ *0.855725*
   - $E(X) = \sum_{x=0}^{3} xP(X = x) = \sum_{x=0}^{3} x(P(X = x|A)P(A) + P(X = x|A^c)P(A^c) = P(A)sum_{x=0}^{3}xP(X = x|A) + P(A^c)sum_{x=0}^{3}xP(X = x|A^c) = 0.9 \times 3 \times 0.85 + 0.1 \times 3 \times 0.2 = 2.355$
   - *For any rv Y,* $\quad E(Y^2) = Var(Y) + (E(Y))^2$
   - $E(X^2) = \sum_{x=0}^{3} x^2P(X = x) = \sum_{x=0}^{3} x^2(P(X = x|A)P(A) + P(X = x|A^c)P(A^c)) = P(A)sum_{x=0}^{3}x^2P(X = x|A) + P(A^c)sum_{x=0}^{3}x^2P(X = x|A^c) = 0.9 \times (3 * 0.85 * 0.15 + (3 * 0.85)^2) + 0.1 \times (3 * 0.2 * 0.8 + (3 * 0.2)^2) = 6.2805$
   - $Var(X) = E(X^2) - (E(X))^2 = 6.2805 - 2.355^2 = 0.734475$

2. A certain cancer is found in one person in 5000. If a person does have the disease, in 92% of the cases the diagnostic procedure will show that he or she actually has it. If a person does not have the disease, the diagnostic procedure in one out of 500 cases gives a false positive result. [6]

   (a) Determine the probability that a person with a positive test result has the cancer.

   - $P(positive\ test) = \frac{1}{5000} \times 0.92 + \frac{4999}{5000} \times \frac{1}{500} = 0.0021836.$
   - *Thus* $P(cancer \mid positive\ test) = \frac{\frac{1}{5000} \times 0.92}{0.0021836} = \frac{0.000184}{0.0021836} = 0.08426.$

   (b) Four people are found to have positive test results. Let X be the number of people in these four that actually have the cancer. Determine the probability $P(X = 1)$.

   (c) What is $E(X)$ and $Var(X)$

   - $X \stackrel{d}{=} Bin(4, 0.08426).$

- *Thus* $P(X=1) = 4 \times 0.08426 \times (1-0.08426)^3 = 0.2588$.

3. Let X be a discrete random variable with pmf (probability mass function)

$$f(x) = \frac{k}{2^x}, \quad x = 0, 1, 2$$

[7]

(a) Find the value of k

- $f(0) + f(1) + f(2) = k + \frac{k}{2} + \frac{k}{4} = \frac{7k}{4} = 1$ *so* $k = \frac{4}{7}$

(b) Find the mgf (moment generating function) of X and hence find $E(X)$ and $E(X^2)$

- *Moment generating function,* $M(t)$ *is given by* $M(t) = E(e^{tX}) = \frac{4}{7}(1 + \frac{e^t}{2} + \frac{e^{2t}}{4})$
- $E(X) = M'(0) = \frac{4}{7}(\frac{e^0}{2} + 2\frac{e^{2\times 0}}{4}) = \frac{4}{7}($ *or* $= 0.5714286)$
- $E(X^2) = M''(0) = \frac{4}{7}(\frac{e^0}{2} + 4\frac{e^{2\times 0}}{4}) = \frac{6}{7}($ *or* $= 0.8571429)$

(c) Find $Var(X)$

- $Var(X) = E(X^2) - E(X)^2 = \frac{42}{49} - \frac{16}{49} = \frac{26}{49}($ *or* $= 0.5306122)$

4. In an annual charity drive, 30% of a population of 1500 make contributions. Suppose, after the drive, 15 people are selected at random and without replacement from the population to be given a statistical survey. Let X be the number of people in the survey who have made contributions. [6]

(a) What is the name and the associated parameter values of the exact distribution of X? Also determine the probability $P(X \geq 2)$.

- $X \stackrel{d}{=} hypergeometric(N_1 = 450, N_2 = 1050, n = 15)$.
- $P(X \geq 2) = 1 - P(X=0) - P(X=1) = 1 - \frac{\binom{450}{0}\binom{1050}{15} + \binom{450}{1}\binom{1050}{14}}{\binom{1500}{15}} = 1 - 0.004606159 - 0.03001117 = 0.9653827$.

(b) Since the survey sample size is much smaller than the population size, X can be approximated by a binomial distribution. Use this approximation to give an approximate value of $P(X \geq 2)$.

- $P(X \geq 2) \approx 1 - \binom{15}{0} \cdot 0.3^0 \cdot 0.7^{15} - \binom{15}{1} \cdot 0.3^1 \cdot 0.7^{14} = 1 - 0.004747562 - 0.03052004 = 0.9647324$.

5. Let X be a continuous random variable with pdf (probability density function)

$$f(x) = \frac{1}{30}(x+6), \quad -4 < x < 2$$

[12]

(a) Find $E(X)$ and $Var(X)$

(b) Find the cdf (cumulative distribution function) of X.

- *For* $-4 < x < 2$, $P(X \leq x) = \int_{-4}^{x} \frac{1}{30}(t+6)dt = \frac{1}{60}(t+6)^2 \mid_{-4}^{x} = \frac{(x+6)^2-4}{60}$.
- *Hence the cdf of* X *is* $F(x) = \begin{cases} 0 & \text{if } x \leq -4 \\ \frac{(x+6)^2-4}{60} & \text{if } -4 < x < 2 \\ 1 & \text{if } x \geq 2 \end{cases}$

(c) Find the 25-th percentile of X.

- *Solve* $\frac{1}{4} = F(\pi_{0.25}) = \frac{1}{60}(\pi_{0.25}+6)^2 - \frac{4}{60}$, *and it follows* $\pi_{0.25} = \sqrt{19} - 6 = -1.6411$.

(d) Consider the transformation $Y = 3X + 2$ of X.

i. Derive the cdf of Y.

- *Let* G *be the cdf of* Y. *The values of* Y *will be* y *such that* $-10 = -4 \times 3 + 2 < y < 2 \times 3 + 2 = 8$ *For* $-10 \leq y \leq 8$,
$G(y) = P(Y \leq y) = P(3X+2 \leq y) = P(X \leq \frac{y-2}{3})$
$= \int_{-4}^{\frac{y-2}{3}} \frac{1}{30}(x+6)\,dx = \frac{1}{60}[(\frac{y-2}{3}+6)^2 - 4]$.
- *So the cdf of* Y *is*

$$G(y) = \begin{cases} 0, & y < -10, \\ \frac{1}{60}[(\frac{y-2}{3}+6)^2 - 4], & -10 \leq y < 8 \\ 1, & y \geq 16. \end{cases}$$

ii. Find the pdf of Y.

- *Let* g *be the pdf of* Y. *Then*

$$g(y) = G'(y) = \begin{cases} 0, & y < -10, \\ \frac{1}{30}(\frac{y-2}{3}+6), & -10 \leq y < 8, \\ 0, & y \geq 8. \end{cases}$$

6. A certain rare blood type can be found in only 0.05% of people. Let X be the number of people, in a population of 3000, that have this blood type. Note that X would follow a binomial distribution. [6]

(a) Find $P(X \leq 1)$.

- $P(X \leq 1) = P(X=0) + P(X=1) = 0.9995^{3000} + 3000 \times 0.0005 \times 0.9995^{2999} = 0.2230465 + 0.3347371 = 0.5577836$.

(b) A binomial distribution $b(n,p)$ can be approximated by a Poisson($\lambda = np$) distribution if p is small and n large. Use this result to approximate the probability in part (a) by a Poisson probability.

- $X \overset{d}{\approx} Poisson(\lambda = 1.5)$, *thus* $P(X \leq 1) \approx e^{-1.5} + 1.5e^{-1.5} = 0.5578254.$

(c) The probability in part (a) may also be approximated by a normal probability based on the central limit theorem. Give a normal approximation (using continuity correction) to $P(X \leq 1)$.

- *By CLT,* $X \overset{d}{\approx} N(\mu = 1.5, \sigma^2 = 1.49925).$
- *So* $P(X \leq 1) \approx P\Big(Z \leq \frac{1+0.5-1.5}{\sqrt{1.49925}}\Big) = P(Z \leq 0) = \Phi(0) = 0.5.$

7. Police are to conduct random breath testing on drivers on a busy road one Friday evening. Suppose 2% of the drivers drink and drive at the time. Let X be the number of drivers that police need to test to capture the first case of drinking and driving. Let Y be the number of drivers tested to find 3 such cases. [11]

(a) Name the probability distribution and specify the value of any parameter(s) for each of the two random variables X and Y.

- $X \overset{d}{=} Geometric(p = 0.02), \quad Y \overset{d}{=} Negative\ binomial(r = 3, p = 0.02).$

(b) What is the probability that at least 10 drivers are to be tested to capture the first drinking and driving case?

- $P(X \geq 10) = \sum_{k=10}^{\infty} 0.02 \times 0.98^{k-1} = 0.98^9 = 0.8337.$

(c) What is the probability that exactly 30 drivers are to be tested to capture 3 drinking and driving cases?

- $P(Y = 30) = \binom{29}{2} \times 0.02^3 \times 0.98^{27} = 0.001882435.$

(d) On average, how many drivers do police need to test to find 3 cases of drinking and driving?

- $E(Y) = \frac{r}{p} = \frac{3}{0.02} = 150.$

(e) Find $P(Y > 50)$.

- *Let* $Z \overset{d}{=} b(50, 0.02)$. *Then using the relation between* $b(n,p)$ *and* $Nb(r,p)$,
- $P(Y > 50) = P(Z \leq 2) = 0.98^{50} + 50 \cdot 0.02 \cdot 0.98^{49} + \binom{50}{2} \cdot 0.02^2 \cdot 0.98^{48} = 0.3641697 + 0.3716017 + 0.1858009 = 0.9215723.$

8. An insurance company employs four telephone operators who receive customers' calls from different regions, so the calls occur independently of each other. The number of calls received by each operator during follows a Poisson process with rate of occurrence parameter $\lambda = 2$ per minute. Let X_1, X_2, X_3 and X_4 be respectively the number of calls received by each of the four operators during a 1-minute period. [21]

(a) Calculate the probability $P(X_1 = 0)$.

- $P(X_1 = 0) = e^{-2} = 0.1353.$

(b) Let $Y = X_1 + X_2 + X_3 + X_4$ be the total number of calls received by the four operators during the 1-minute period. Name the probability distribution of Y and specify its parameter(s) value(s).

- $M_Y(t) = \prod_{i=1}^{4} M_{X_i}(t) = e^{8(e^t-1)}.$

- *So* $Y \stackrel{d}{=} \text{Poisson}(8)$.

(c) Continuing (b), find the conditional probability $P(X_1 = 1|Y = 4)$.

- $$P(X_1 = 1|Y = 4) = \frac{P(X_1 = 1, Y = 4)}{P(Y = 4)}$$
$$= \frac{P(X_1 = 1)P(X_2 + X_3 + X_4 = 3)}{P(Y = 4)} = \frac{\frac{2^1 e^{-2}}{1!}\frac{6^3 e^{-6}}{3!}}{\frac{8^4 e^{-8}}{4!}}$$
$$= \frac{4!}{1!3!}\left(\frac{2}{8}\right)^1\left(\frac{6}{8}\right)^3 = 4 \cdot 0.25^1 \cdot 0.75^3 = \frac{27}{64} = 0.4219.$$

(d) Let Z be the number of operators who do not receive any calls during the given 1-minute period. What probability distribution does Z have? Calculate the probability $P(Z = 2)$.

- $Z \stackrel{d}{=} b(4, e^{-2})$.
- $P(Z = 2) = \binom{4}{2}(e^{-2})^2(1 - e^{-2})^2 = 0.08216$.

(e) If T_1, T_2, T_3, T_4 are the random variables giving the time in minutes till the first call of each of the operators arrives, derive the cdf (cumulative distribution function) of T_1.

- *Let* $X_1(t)$ *be the number of calls received by the operator in* t *minutes, so that* $X_1(t) \sim Poisson(2t)$.
- *The event* $[T_1 > t] = [X_1(t) = 0]$.
- *Hence* $F_{T_1}(t) - 1 - P(T_1 > t) = \begin{cases} 1 - e^{-2t} & t > 0 \\ 0 & t \le 0 \end{cases}$.

(f) Derive the pdf (probability density function) of T_1

- *For any* $t > 0$, *let* f_1 *be the required pdf of* T_1.
- $f_1(t) = \begin{cases} F_1'(t) = 2e^{-2t} & t > 0 \\ 0 & t \le 0 \end{cases}$.

(g) Derive the mgf (moment generating function) of T_1 and hence $E(T_1)$

- *For any* $u < 2$, *the moment generating function, of* $M_1(u)$ *is*
$$\begin{aligned} M_(u) &= E(e^{-uT_1}) \\ &= \int_0^\infty e^{ut} 2e^{-2t}\,dt \\ &= \frac{2}{2-u}\int_0^\infty (2-u)e^{-(2-u)t}\,dt \\ &= \frac{2}{2-u} \end{aligned}$$
since the integral is that of the pdf of an exponential distribution with rate parameter $\frac{2}{2-u}$.
- $E(T_1) = M_1'(0) = \frac{1}{2}$.

(h) Derive the mgf of $T_1 + T_2 + T_3 + T_4$.

- *Let M_4 be the mgf of $T_1 + T_2 + T_3 + T_4$.*
- *Since $T_1, T_2 T_3, T_4$ are independent each with the same mgf $M1$,*

$$M_4(u) = (M_1(u))^4 = \left(\frac{2}{2-u}\right)^4.$$

(i) What is the probability that $T_1 + T_2 + T_3 + T_4 > 2$?

- *$T_1 + T_2 + T_3 + T_4$ has a Gamma distribution with index 4 and rate 2, so it has the same distribution as the 4th event in a Poisson process, $N(t)$, of rate 2.*
- *So $N(2) \sim Poisson(4)$.*
- *Hence*

$$P(T_1 + T_2 + T_3 + T_4 > 2) = P(N(2) \leq 3) = \sum_{x=0}^{3} \frac{e^{-4}4^x}{x!} = 0.4334701.$$

9. The error when using a surveying instrument to measure a distance is normally distributed with mean 0 and variance 1 mm. Measurements of the height, width and depth of the building are taken with this measuring device. [10]

(a) What is the probability that the error in (say) height exceeds an acceptable level of 2 mm?

- *If e_H is the (signed) error in height, then $e_H \sim N(0,1)$.*
- *The required probability is $P(e_H > 2) = 02275013$.*

(b) Derive the mgf (moment generating function) of the square on the error of one of these measurements?

What is the name of this distribution?

- *The mgf at $t < 0.5$ is $\int_{-\infty}^{\infty} e^{tz^2} \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}}\, dz = (1-2t)^{-\frac{1}{2}}$.*
- *This is the $\chi^2(1)$ distribution.*

(c) Derive the mgf of the sum of the squares of the three errors. What is the name of this distribution?

- *The mgf of the sum of squares of the three errors evaluated at $t < 0.5$ is $(1-2t)^{-\frac{3}{2}}$ since it is the product of three the same moment generating functions for the three squares of errors.*
- *This is the $\chi^2(3)$ distribution.*

(d) The total error is defined as the square root of sum of the squares of the three errors. What is the probability that this exceeds 6 mm, that is three times the acceptable level in one measurement? Comment.

- *Suppose the three errors are E_H, E_W, E_L and $E = \sqrt{E_H^2 + E_L^2 + E_W^2}$ is the total error.*
- *The required probability is $P(E > 6) = P(E^2 > 36) < 0.000001$.*
- *Independence means that the total error is very unlikely to be more than three times an unlikely error for one measurement.*

10. Let X_1, X_2, X_3 be independent Binomial$(n = 4, p = \frac{1}{2})$ random variables. Define $Y_1 = X_1 + X_3$ and $Y_2 = X_2 + X_3$. [10]

(a) Find the value of $\mathrm{Cov}(Y_1, Y_2)$.

- $\mathrm{Cov}(Y_1, Y_2) = E((X_1 + X_3)(X_2 + X_3)) - E(X_1 + X_3)E(X_2 + X_3) = E(X_1X_2)+E(X_1X_3)+E(X_3X_2)+E(X_3^2)-[E(X_1)E(X_2)+E(X_1)E(X_3)+E(X_3)E(X_2) + (E(X_3))^2] = 0+0+0+Var(X_3) = np(1-p) = 1.$

(b) Define

$$Z_1 = \begin{cases} 1 & \text{if } Y_1 = 0, \\ 0 & \text{otherwise;} \end{cases} \quad \text{and} \quad Z_2 = \begin{cases} 1 & \text{if } Y_2 = 0, \\ 0 & \text{otherwise.} \end{cases}$$

i. Find the marginal pmf (probability mass function) of Z_1 and Z_2 respectively.

- *Both Z_1 and Z_2 are Bernoulli r.v.s with* $P(Z_1 = 1) = P(Z_2 = 1) = P(X_1 = X_3 = 0) = P(X_2 = X_3 = 0) = \frac{1}{2^8}$.

ii. Find the joint pmf (joint probability mass function) of (Z_1, Z_2).

- $P(Z_1 = 1, Z_2 = 1) = P(X_1 = X_2 = X_3 = 0) = \frac{1}{2^{12}}$.
- $P(Z_1 = 1, Z_2 = 0) = P(X_1 = X_3 = 0, X_2 > 0) = \frac{1}{2^8}(1 - \frac{1}{2^4}) = \frac{15}{2^{12}}$.
- $P(Z_1 = 0, Z_2 = 1) = P(X_1 > 0, X_2 = X_3 = 0) = (1 - \frac{1}{2^4})\frac{1}{2^8} = \frac{15}{2^{12}}$.
- $P(Z_1 = 0, Z_2 = 0) = 1 - \frac{1+15+15}{2^{12}} = \frac{4065}{2^{12}} = \frac{4065}{4096}$.

iii. Find the correlation coefficient between Z_1 and Z_2.

- $E(Z_1) = P(Z_1 = 1) = \frac{1}{2^8}$ *and* $Var(Z_1) = P(Z_1 = 1)P(Z_1 = 0) = \frac{255}{2^{16}}$.
- *Z_1 has the same distribution as Z_2. So* $E(Z_2) = \frac{1}{2^8}$ *and* $Var(Z_2) = \frac{255}{2^{16}}$.
- $E(Z_1Z_2) = P(Z_1 = Z_2 = 1) = \frac{1}{2^{12}}$. *So* $Cov(Z_1, Z_2) = \frac{1}{2^{12}} - \frac{1}{2^{16}}$.
- *Therefore,*

$$\rho(Z_1, Z_2) = \frac{Cov(Z_1, Z_2)}{\sqrt{Var(Z_1)}\sqrt{Var(Z_2)}} = \frac{\frac{1}{2^{12}} - \frac{1}{2^{16}}}{\frac{255}{2^{16}}} = \frac{15}{255} = \frac{1}{17} = 0.05882353.$$

11. Suppose X and Y are continuous random variables with the joint pdf (probability density function)

$$f(x, y) = \begin{cases} \frac{3}{2} & \text{if } 0 \le x \le 1 \text{ and } x^2 \le y \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

[4]

(a) Find the marginal pdf of Y.

- *The marginal of Y is* $f_2(y) = \int_0^{\sqrt{y}} \frac{3}{2}dx = \frac{3}{2}\sqrt{y}, \quad 0 \le y \le 1.$

(b) Find the conditional pdf of X given $Y = y$, $0 \le y \le 1$.

- $g(x|y) = \frac{f(x,y)}{f_2(y)} = \frac{\frac{3}{2}}{\frac{3}{2}\sqrt{y}} = \frac{1}{\sqrt{y}}, \quad 0 \le x \le \sqrt{y}; \ \ 0 \le y \le 1.$

(c) Are X and Y independent? Explain.

- *X and Y are not independent because $g(x|y)$ depends on y, or because the support of (X, Y) is not rectangular.*

Total marks = 100

End of the exam questions.
Formulas are on the next page.

Table XII: Discrete Distributions

Probability Distribution and Parameter Values	Probability Mass Function	Moment-Generating Function	Mean $E(X)$	Variance $\mathrm{Var}(X)$	Examples
Bernoulli $0 < p < 1$ $q = 1 - p$	$p^x q^{1-x}, \; x = 0, 1$	$q + pe^t$	p	pq	Experiment with two possible outcomes, say success and failure, $p = P(\text{success})$
Binomial $n = 1, 2, 3, \ldots$ $0 < p < 1$	$\binom{n}{x} p^x q^{n-x}$, $x = 0, 1, \ldots, n$	$(q + pe^t)^n$	np	npq	Number of successes in a sequence of n Bernoulli trials, $p = P(\text{success})$
Geometric $0 < p < 1$ $q = 1 - p$	$q^{x-1}p$, $x = 1, 2, \ldots$	$\dfrac{pe^t}{1 - qe^t}$	$\dfrac{1}{p}$	$\dfrac{q}{p^2}$	The number of trials to obtain the first success in a sequence of Bernoulli trials
Hypergeometric $x \le n, x \le N_1$ $n - x \le N_2$ $N = N_1 + N_2$ $N_1 > 0, \; N_2 > 0$	$\dfrac{\binom{N_1}{x}\binom{N_2}{n-x}}{\binom{N}{n}}$		$n\left(\dfrac{N_1}{N}\right)$	$n\left(\dfrac{N_1}{N}\right)\left(\dfrac{N_2}{N}\right)\left(\dfrac{N-n}{N-1}\right)$	Selecting r objects at random without replacement from a set composed of two types of objects
Negative Binomial $r = 1, 2, 3, \ldots$ $0 < p < 1$	$\binom{x-1}{r-1} p^r q^{x-r}$, $x = r, r+1, \ldots$	$\dfrac{(pe^t)^r}{(1 - qe^t)^r}$	$\dfrac{r}{p}$	$\dfrac{rq}{p^2}$	The number of trials to obtain the rth success in a sequence of Bernoulli trials
Poisson $0 < \lambda$	$\dfrac{\lambda^x e^{-\lambda}}{x!}$, $x = 0, 1, \ldots$	$e^{\lambda(e^t - 1)}$	λ	λ	Number of events occurring in a unit interval, events are occurring randomly at a mean rate of λ per unit interval
Uniform $m > 0$	$\dfrac{1}{m}, \; x = 1, 2, \ldots, m$		$\dfrac{m+1}{2}$	$\dfrac{m^2 - 1}{12}$	Select an integer randomly from $1, 2, \ldots, m$

Table XIII: Continuous Distributions

Probability Distribution and Parameter Values	Probability Density Function	Moment-Generating Function	Mean $E(X)$	Variance $\mathrm{Var}(X)$	Examples
Beta $0 < \alpha$ $0 < \beta$	$\dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$, $0 < x < 1$		$\dfrac{\alpha}{\alpha+\beta}$	$\dfrac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}$	$X = X_1/(X_1 + X_2)$, where X_1 and X_2 have independent gamma distributions with same θ
Chi-square $r = 1, 2, \ldots$	$\dfrac{x^{r/2-1}e^{-x/2}}{\Gamma(r/2)2^{r/2}}$, $0 < x < \infty$	$\dfrac{1}{(1-2t)^{r/2}}, \; t < \dfrac{1}{2}$	r	$2r$	Gamma distribution, $\theta = 2$, $\alpha = r/2$; sum of squares of r independent $N(0, 1)$ random variables
Exponential $0 < \theta$	$\dfrac{1}{\theta} e^{-x/\theta}, \; 0 \le x < \infty$	$\dfrac{1}{1 - \theta t}, \; t < \dfrac{1}{\theta}$	θ	θ^2	Waiting time to first arrival when observing a Poisson process with a mean rate of arrivals equal to $\lambda = 1/\theta$
Gamma $0 < \alpha$ $0 < \theta$	$\dfrac{x^{\alpha-1}e^{-x/\theta}}{\Gamma(\alpha)\theta^\alpha}$, $0 < x < \infty$	$\dfrac{1}{(1-\theta t)^\alpha}, \; t < \dfrac{1}{\theta}$	$\alpha\theta$	$\alpha\theta^2$	Waiting time to αth arrival when observing a Poisson process with a mean rate of arrivals equal to $\lambda = 1/\theta$
Normal $-\infty < \mu < \infty$ $0 < \sigma$	$\dfrac{e^{-(x-\mu)^2/2\sigma^2}}{\sigma\sqrt{2\pi}}$, $-\infty < x < \infty$	$e^{\mu t + \sigma^2 t^2/2}$	μ	σ^2	Errors in measurements; heights of children; breaking strengths
Uniform $-\infty < a < b < \infty$	$\dfrac{1}{b-a}, \; a \le x \le b$	$\dfrac{e^{tb} - e^{ta}}{t(b-a)}, \; t \ne 0$; $1, \; t = 0$	$\dfrac{a+b}{2}$	$\dfrac{(b-a)^2}{12}$	Select a point at random from the interval $[a, b]$

Appendix C Tables