

## Sample Lab Test

Name : \_\_\_\_\_ Student Number : \_\_\_\_\_

This is an open-book test. You are permitted to use any notes, computer or calculator. Please write your answers in the boxes provided. **The total marks are 40.**

**Use Mathematica and/or R to complete the questions, as appropriate. Simplify results when possible.**

### Question 1.

(*Banach Matchbox Problem*) A mathematician carries two matchboxes, one in his right pocket and one in his left pocket. Whenever he wants a match, he selects a pocket at random and takes a match from the box in that pocket. Suppose each box contains 100 matches initially. Let  $X$  be the number of matches left in a box when the mathematician for the first time discovers that the other box is empty. It can be shown that the pmf of  $X$  is

$$P(X = x) = \binom{200 - x}{100} 2^{x-200}, \quad x = 0, 1, 2, \dots, 100.$$

Complete the following tasks and keep 4 significant digits after the decimal point in your answers.

- (a). Find the probability  $P(X \geq 10)$ .

[3]

Answer:

- (b). Find the mean  $E(X)$ .

[3]

Answer:

- (c). Find  $E(X^2)$ .

[3]

Answer:

- (d). Find  $E[(X + 1)^{-2}]$ .

[3]

Answer:

### Question 2.

Let a continuous random variable  $X$  have the following pdf

$$f(x) = \frac{2}{9}(x + 1)(2 - x), \quad -1 < x < 2.$$

- (a). Find the cdf  $F(x)$  of  $X$ .

[2]

Answer:

(b). Find the probability  $P(-2 < X < \frac{9}{5})$ .

[2]

Answer:

(c). Find the mean  $E(X)$ .

[2]

Answer:

(d). Find the mgf  $M(t) = E[\exp(tX)]$ .

[2]

Answer:

(e). Find the third moment  $E(X^3)$ .

[2]

Answer:

(f). Let  $Y = X^2$ .

(i) What are the possible values of  $Y$ ?

[2]

Answer:

(ii) Find the pdf  $g(y)$  of  $Y$ .

[2]

Answer:

### Question 3.

The R dataset named "trees" has three variables "Girth", "Height" and "Volume" of 31 black cherry trees in inches, feet and cubic feet respectively. The girth is the diameter of the tree measured at 4 ft 6 in above the ground. Give answers correct to 4 significant figures.

- (a). Do boxplots of the three variables and use these to briefly describe the three distributions. [2]

Answer:

- (b). Find the minimum, quartiles, median and maximum for Volume. [2]

Answer:

- (c). Find the mean and sd of Height. [2]

Answer:

- (d). Plot Volume versus Height. Comment on the plot. [2]

Answer:

- (e). Find the intercept and slope of the line of best fit for Volume versus Height. [2]

Answer:

- (f). Write down the commands to add the line of best fit to the plot of Volume versus Height and comment on the fit. [2]

Answer:

- (g). Give commands to check on the normality of Volume and Height and comment on the results. [2]

Answer:

Total marks = 40