MAST90105 Lab and Workshop 2 Solutions

1 Lab

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1.2 Coding Example

1.2.1 Sampling Without Replacement

In Module 1 Section 1.2 we did the computer coding example to illustrate sampling without replacement. Carry out the Mathematica commands necessary to get the calculations given in the Lecture.

- ***Solution:***

  ***Binomial*[10000, 10]**

  2743355077591282538231819720749000

  ***Binomial*[30, 1] * *Binomial*[10000 − 30, 9]/*Binomial*[10000, 10]**

  $\frac{2143751464028247883152007617}{73351740042547661450048655635}$

  ***N*[%, 18]**

  0.0292256388571663780

  ***Binomial*[30, 0] * *Binomial*[10000 − 30, 10]/*Binomial*[10000, 10]**

  $\frac{1016852777770732245908435612997}{1047882000607823735000695080500}$

  ***N*[%]**

  0.970389

  Note the use of the % to be the result of the last command

Lookup the documentation on the commands, PDF and HypergeometricDistribution to find out Mathematica can calculate the probability that one line of code could be improved using these commands. Looking up the documentation on PDF, find a command that will enable you to plot the probabilities of $0, 1, \cdots 10$ improvable lines of code in the sample.

- ***Solution:***

  PDF - type this in and right and click up the i button to see documentation.
  HypergeometricDistribution - type this in and right and click up the i button to see documentation.
  Notice that the arguments in the HypergeometricDistribution are in the order sample size, followed by number of lines of code that could be improved in total and lastly total number of lines of code.

  ***PDF*[*HypergeometricDistribution*[10, 30, 10000], 1]**

  $\frac{2143751464028247883152007617}{73351740042547661450048655635}$

  $\boldsymbol{N\left[\frac{2143751464028247883152007617}{73351740042547661450048655635}\right]}$

  0.0292256

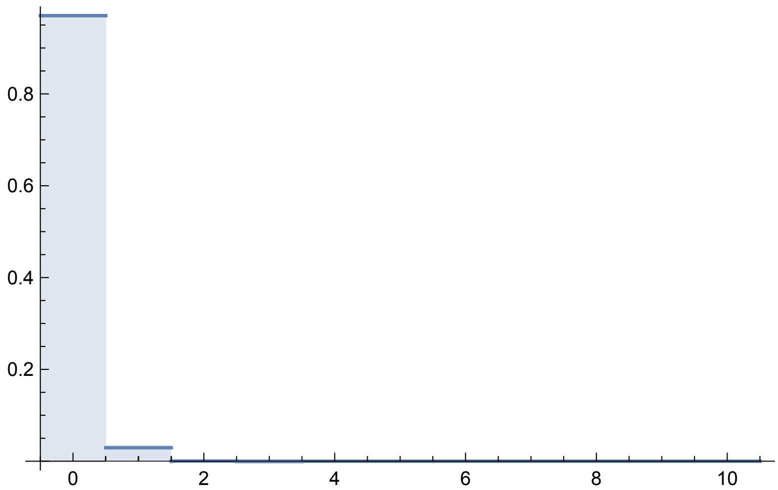
$N\left[\frac{5455259554236573653}{23730591032609929084}\right]$

0.229883

Next, use the command DiscretePlot to Plot the Hypergeometric Probabilities.

***DiscretePlot**[**PDF**[**HypergeometricDistribution**[10, 30, 10000], k], {k, 0, 10},*

***PlotRange** → **Full**, **ExtentSize** → 1]*



The ExtentSize command has the effect of giving a shaded histogram rather than a line with dots graph. It was used so that you don't miss the fact that most of the probability is with 0!

Now suppose that the total number of lines of code from which the sample of size 10 is taken is (a) 1000, (b) 100 or (c) 50, whilst the total number of improvable lines of code remains the same at 30. Alter your command to permit, the probabilities for each of (a), (b) and (c) to be plotted on the one plot. Comment on the differences between the probabilities.

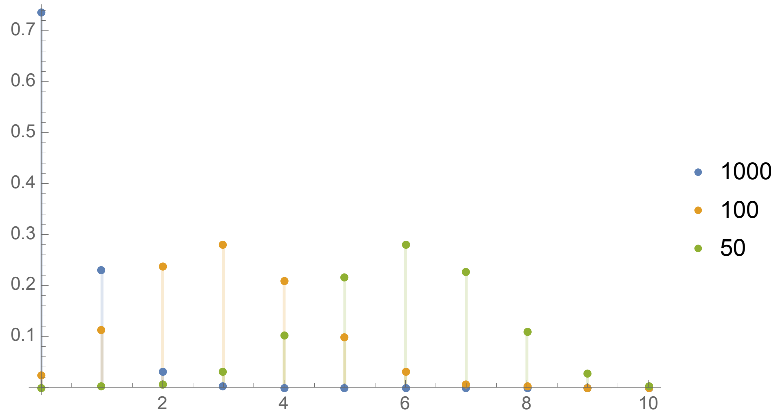
- ***Solution:***

***Table**[**PDF**[**HypergeometricDistribution**[10, 30, N], k], {N, {1000, 100, 50}}]*

$$\left\{ \begin{cases} \frac{\text{Binomial}[30,k]\,\text{Binomial}[970,10-k]}{263409560461970212832400} & 0 \leq k \leq 10 \\ 0 & \text{True} \end{cases}, \begin{cases} \frac{\text{Binomial}[30,k]\,\text{Binomial}[70,10-k]}{17310309456440} & 0 \leq k \leq 10 \\ 0 & \text{True} \end{cases}, \ldots \right\}$$

Notice that Mathematica puts in True to mean any value of k that has not been mentioned above.

***DiscretePlot**[**Evaluate**[%], {k, 0, 10}, **PlotRange** → **Full**, **PlotLegends** → {1000, 100, 50}]*



The use of Evaluate tells Mathematica to compute the symbolic table just defined into a numerical one, and this produces the three plots in different colours.
The PlotLegends option gives the labelling with colours - note that 1000,100 and 50 are in the same order as in the Table command.
The probabilities for N=1,000 are like those for N=10,000 except that there is now a probability of about 0.2 for 1 line of code to be improved.
For N=100, the probabilities increase up to 3 and then decrease.
For N=50, the probabilities increase up to 6 and the decrease. So the probability mass functions shift to the right as N decreases towards the fixed total number of lines of code that could ebe improved.

1.2.2 Bayes Theorem - Sampling Without Replacement

Follow the method and calculations in Module 1 Section 1.3 to work out the answer to the probability that, given an observed number of 5 lines of code that could be improved in the 20 sampled, the total number of lines of code that could be improved out of 1,000 was $0, 1, 2, \cdots, 1000$. You'll need to use operations on lists, the Sum and Table commands.

Plot these probabilities.

What do you think about whether the code meets the standard on the basis of 5 lines of code in the sample of 20 that could be improved?

- ***Solution:***

The ***prior*** *probabilities are the assumed probabilities from the past. Join is a command to Join together two lists. The prior probabilities add to one as they must since exactly one of the 1001 possibilities for the total lines of code that can be improved must be the truth. The output of this command is a list with 1001 elements.*

$\boldsymbol{prior = Join[Table[0.9/101, 101], Table[0.1/900, 900]]}$

$\{0.00891089, 0.00891089, \cdots, 0.00891089, 0.000111111, 0.000111111, \cdots, 0.000111111\}$

The ***joint*** *probabilities are the probabilities for each of the 1001 possibilities for the total lines of code to be improved together with observing 5 lines of code that could be improved in the random sample of size 20.*
Evaluate is used to get numerical answers for each of the 1001 probabilities. Observe that the first 5 and the last 15 are zero as noted in class.

$\boldsymbol{joint = prior * Evaluate[Table[PDF[HypergeometricDistribution[20, i, 1000], 5], \{i, 0, 1000\}]]}$

$\{0., 0., 0., 0., 0., 1.67454\times10^{-11}, \cdots, 2.50336\times10^{-33}, 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.\}$

Bayes Theorem says that the conditional probabilities for each of the 1001 possibilities for the total lines of code that could be improved are the ratio of the joint probabilities just calculated with their sum. The Mathematica command Total computes the sum of a list.

conditional = joint/ Total[joint]

$\{0., 0., 0., 0., 0., 1.49742\times10^{-9}, \cdots, 2.23858\times10^{-31}, 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.\}$

The probability from Lectures that we said was small but did not compute, namely the probability that there were five lines of code that could be improved in total if our random sample of 20 gave five lines of code that could be improved is the 6th element of the conditional list. Double brackets are used for extracting elements from a list.

conditional[[6]]

1.49743×10^{-9}

The conditional probabilities increase over the total lines of code that could be improved ranging from 0 to 100 (this makes sense because we observed 25% that could be improved in the sample, but our assumption is that it is 90% likely that only 10% of the total could be improved), but then drops sharply because there is much less prior probability for each of the numbers between 101 and 1000 for the total lines of code that could be improved. The following command extracts this.

Take[conditional, {99, 102}]

$\{0.0230815, 0.0239049, 0.0247442, 0.000319198\}$

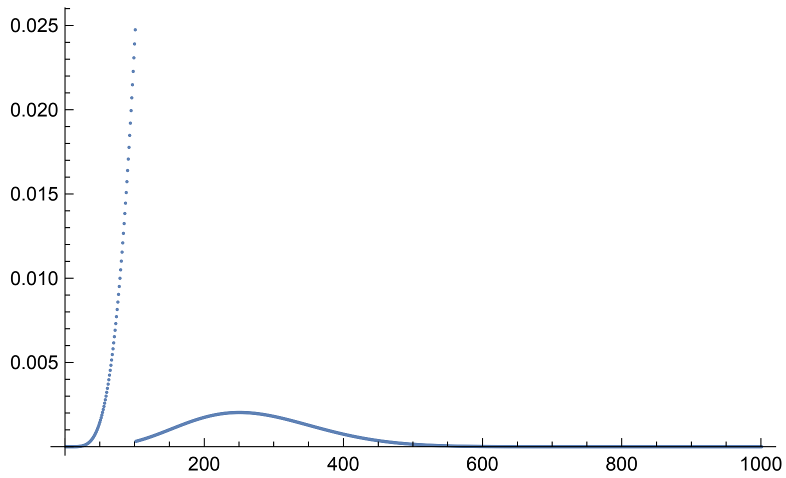
The maximum probability is for 100 lines of code that could be improved but this reflects prior beliefs considerably. There is quite a lot of probability between 101 and 1000. We can get this as follows.

Total[Take[conditional, {1, 101}]]

0.533034

So with our strong prior belief of 90% of code meeting the standard, there is just slightly more than 50% chance that the code meets the standard on the basis of the observation of 25% in the random sample of 20. A manager might reasonably conclude either way with these prior beliefs, but would need to acknowledge the prior beliefs in any report up the chain. Further, caution and the data by itself would suggest reporting in a different way - for example that the data showed 25% could be improved, but this was a small sample and based on past experience that 90% met the standard it was quite possible that the code did meet the standard.

ListPlot[conditional, PlotRange → Full]

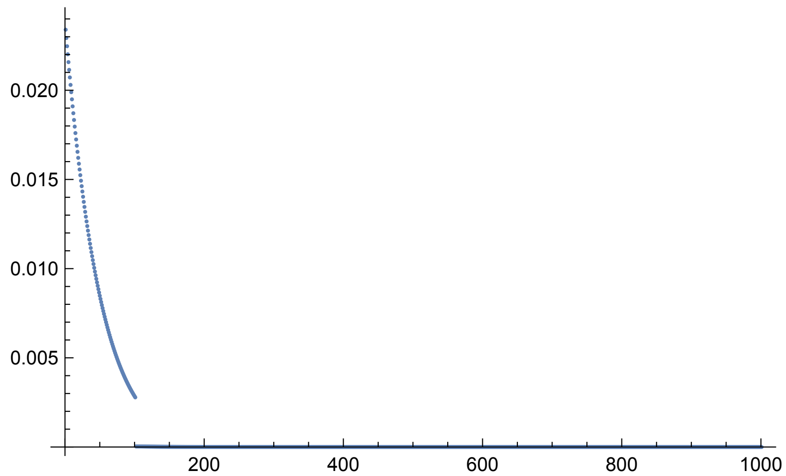


What happens if the observed number was (a) 0 or (1) ? Comment.

- ***Solution:***

  We can repeat this with 0 lines of code in the random sample that could be improved as follows.

```
conditional =
prior * Evaluate[Table[PDF[HypergeometricDistribution[20, i, 1000], 0], {i, 0, 1000}]]/
Total[prior * Evaluate[Table[PDF[HypergeometricDistribution[20, i, 1000], 0], {i, 0, 1000}]]];
ListPlot[conditional, PlotRange → Full]
```



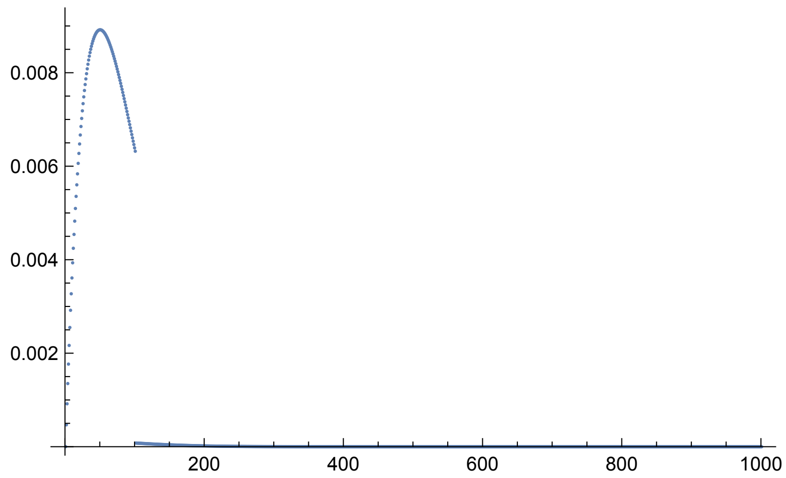
So there is now a very strong chance that the code meets the standard on the basis of 0 lines of code in the sample and the peak probability is 0 lines of code that could be improved. In fact:

```
Total[Take[conditional, {1, 101}]]
```

0.998545

there is only 0.15% chance that it does not meet the standard. With 1 line of code in the sample that could be improved:

```
conditional =
prior * Evaluate[Table[PDF[HypergeometricDistribution[20, i, 1000], 1], {i, 0, 1000}]]/
Total[prior * Evaluate[Table[PDF[HypergeometricDistribution[20, i, 1000], 0], {i, 0, 1000}]]];
ListPlot[conditional, PlotRange → Full]
```



We now see that the peak probability has moved to less than one hundred based on our strong prior :

***Take*[*conditional*, {99, 102}]**

{0.00646369, 0.0063921, 0.00632051, 0.000077919}

And the probability that the code meets the standard on the basis of the observation of one line of code that could be improved is:

***Total*[*Take*[*conditional*, {1, 101}]]**

0.717343

only 71%. Although all the probabilities between 101 and 1000 are individually small they accumulate to a reasonable amount.

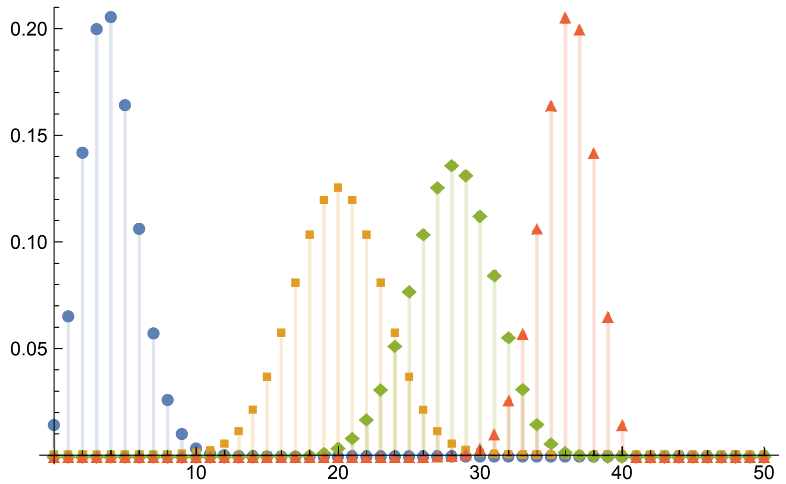
1.3 Binomial Probabilities

Work out the Mathematica Commands to produce the Binomial plots given in Module 2 Section 1.7

- ***Solution:***

***Table*[*PDF*[*BinomialDistribution*[40, *p*], *k*], {*p*, {0.1, 0.5, 0.7, 0.9}}];**

***DiscretePlot*[*Evaluate*[%], {*k*, 0, 50}, *PlotMarkers* → *Automatic*, *PlotRange* → *Full*]**



Note that in a previous version of the notes in class the probability of 0 had been omitted - it requires both the minimum and maximum for k to get it included.

2 Workshop

1. Let $P(A) = 0.4$ and $P(A \cup B) = 0.6$.

   a. What is the value of $P(B)$ if A and B are mutually exclusive?

   - ***Solution:*** *If A and B are mutually exclusive, then $P(A \cup B) = P(A) + P(B)$. Since $P(A) = 0.4$ and $P(A \cup B) = 0.6$, it follows that $P(B) = 0.2$.*

   b. What is the value of $P(B)$ if A and B are independent?

   - ***Solution:*** *If A and B are independent, then $P(A \cap B) = P(A)P(B)$. So $0.6 = P(A \cup B) = P(A) + P(B) - P(A)P(B) = 0.4 + P(B) - 0.4P(B)$. It follows that $P(B) = 1/3$.*

2. A box contains 5 green balls, 3 black balls, and 7 red balls. Two balls are selected at random without replacement from the box. What is the probability that:

   a. both balls are red?

   - ***Solution:*** *Method 1 (taking the sample space to be subsets of size 2): $P(\text{both are red}) = P(R_1 \cap R_2) = \frac{\binom{7}{2}}{\binom{5+3+7}{2}} = \frac{1}{5}$. Method 2: By the multiplication rule,*
     $P(R_1 \cap R_2) = P(R_1)P(R_2|R_1) = \frac{7}{5+3+7} \times \frac{7-1}{5+3+7-1} = \frac{1}{5}$.

   b. both balls are of the same colour?

   - ***Solution:*** *Method 1 (again using the same sample space of subsets of size 2): $P(\text{both are same color}) = P(G_1 \cap G_2) + P(B_1 \cap B_2) + P(R_1 \cap R_2) = \frac{\binom{5}{2}}{\binom{5+3+7}{2}} + \frac{\binom{3}{2}}{\binom{5+3+7}{2}} + \frac{\binom{7}{2}}{\binom{5+3+7}{2}} = \frac{34}{105}$.*
     Method 2: $P(\text{both are same color}) = P(G_1)P(G_2|G_1) + P(B_1)P(B_2|B_1) + P(R_1)P(R_2|R_1) = \frac{5}{5+3+7} \times \frac{5-1}{5+3+7-1} + \frac{3}{5+3+7} \times \frac{3-1}{5+3+7-1} + \frac{7}{5+3+7} \times \frac{7-1}{5+3+7-1} = \frac{2}{21} + \frac{1}{35} + \frac{1}{5} = \frac{34}{105}$.

   c. one ball is red and the other is black?

   - *Method 1(again the sample space is subsets of size 2): $P(R_1 \cap B_2 \cup B_1 \cap R_2) = \frac{\binom{5}{0}\binom{3}{1}\binom{7}{1}}{\binom{5+3+7}{2}} = \frac{1}{5}$.*
   - *Method 2: $P(R_1 \cap B_2 \cup B_1 \cap R_2) = P(R_1)P(B_2|R_1) + P(B_1)P(R_2|B_1) = \frac{7}{5+3+7} \times \frac{3}{5+3+7-1} + \frac{3}{5+3+7} \times \frac{7}{5+3+7-1} = \frac{1}{5}$.*

   Try to find the above probabilities using two ways: one through a classical probability model and counting formulas, and the other through *conditional probabilities/multiplication rule.*

3. The probability that a marksman hits a target is 0.9 on any given shot, and repeated shots are independent. He has two pistols; one contains two bullets and the other contains only one bullet. He selects a pistol at random and shoots at the target until the pistol is empty. What is the probability of hitting the target exactly one time?

   - ***Solution:*** *Denote $A_1 = \{\text{choose the first pistol}\}$, $A_2 = \{\text{choose the second pistol}\}$, and*
     $B = \{\text{hit the target exactly one time}\}$.
     Then by the law of total probability and multiplication rule
     $P(B) = P(A_1)P(B|A_1) + P(A_2)P(B|A_2)$
     $= 0.5 \times (0.9 \times 0.1 + 0.1 \times 0.9) + 0.5 \times 0.9 = 0.54$.

4. Let A_1 and A_2 be the events that a person is left eye dominant and right eye dominant, respectively. When a person folds his/her hands, let B_1 and B_2 be the events that their left thumb and right thumb, respectively, are on top. A survey in one statistics class yielded the following table.

	B_1	B_2	Totals
A_1	5	7	12
A_2	14	9	23
Totals	19	16	35

If a student is selected randomly, find the following probabilities:

a. $P(A_1 \cap B_1)$,
   - ***Solution:*** $P(A_1 \cap B_1) = \frac{5}{35} = \frac{1}{7}$.

b. $P(A_1 \cup B_1)$,
   - ***Solution:*** $P(A_1 \cup B_1) = P(A_1) + P(B_1) - P(A_1 \cap B_1) = \frac{12}{35} + \frac{19}{35} - \frac{5}{35} = \frac{26}{35}$.

c. $P(A_1|B_1)$,
   - ***Solution:*** $P(A_1|B_1) = \frac{5}{19}$.

d. $P(B_2|A_2)$.
   - ***Solution:*** $P(B_2 \cap A_2) = \frac{9}{23}$.

5. Each of three football players will attempt to kick a field goal from the 25-yard line. Let A_i denote the event that the field goal is made by player i, $i = 1, 2, 3$. Assume that A_1, A_2 and A_3 are mutually independent and that $P(A_1) = 0.5$, $P(A_2) = 0.7$, $P(A_3) = 0.6$.

a. Compute the probability that exactly one player is successful.
   - ***Solution:*** $P(\textit{Exactly one kicks a goal})$
   $= P(A_1 \cap A_2^c \cap A_3^c) + P(A_1^c \cap A_2 \cap A_3^c + P(A_1^c \cap A_2^c \cap A_3)$
   $= 0.5(1 - 0.7)(1 - 0.6) + (1 - 0.5)0.7(1 - 0.6) + (1 - 0.5)(1 - 0.7)0.6 = 0.29.$

b. Compute the probability that exactly two players make a field goal (i.e., one misses).
   - ***Solution:*** $P(\textit{Exactly two make a goal})$
   $= P(A_1 \cap A_2 \cap A_3^c) + P(A_1^c \cap A_2 \cap A_3) + P(A_1 \cap A_2^c \cap A_3)$
   $= 0.5 \times 0.7 \times (1 - 0.6) + (1 - 0.5) \times 0.7 \times 0.6 + 0.5 \times (1 - 0.7) \times 0.6 = 0.44.$

6. Lie detectors are controversial instruments, barred from use as evidence in many courts. Nonetheless, many employers use lie detector screening as part of their hiring process in the hope that they can avoid hiring people who might be dishonest. There has been some research, but no agreement, about the reliability of polygraph tests. Based on this research, suppose that a polygraph can detect 65% of lies, but incorrectly identifies 15% of true statements as lies.

   A certain company believes that 95% of its job applicants are trustworthy. The company gives everyone a polygraph test, asking "Have you ever stolen anything from your place of work?" Naturally, all the applicants answer "No", but the polygraph identifies some of those answers as lies, making the person ineligible for a job. What is the probability that a job applicant rejected under suspicion of dishonesty was actually trustworthy?

- ***Solution:*** *Let* $A = \{An\ applicant\ chosen\ at\ random\ is\ trustworthy\}$, $B = \{detected\ by\ polygraph\ as\ lies\}$.
  Then $P(A) = 0.95$, $P(B|A) = 0.15$ *and* $P(B|A') = 0.65$.
  And by Bayes's theorem
  $P(trustworthy|detected\ as\ lying) = P(A|B)$
  $= \frac{P(A)P(B|A)}{P(A)P(B|A)+P(A')P(B|A')} = \frac{0.95\times0.15}{0.95\times0.15+0.05\times0.65} = 0.8143.$

7. *Game Show Paradox*Suppose you're on a game show, and you're given the choice of three doors. Behind one door is a car, behind the others, goats. You pick a door, say number 1, and the host, who knows what's behind the doors, opens another door, say number3, which has a goat. He says to you, "Do you want to pick door number 2?" Is it to your advantage to switch your choice of doors?

   - ***Solution:*** *A lot has been written about this question on the internet and a lot of it either does not follow the rules of probability or is not clear about the assumptions. Let* $C1, C2, C3$ *be the events that the car is behind doors 1,2 or 3 respectively. It is reasonable to assume that, at the time the door to hide the car is being chosen,* $P(C1) = P(C2,) = P(C3) = 1/3$ *since otherwise there is a bias in the game and nothing is mentioned about this in the question. It is also reasonable to assume that the game show host knows which door the car is behind. If the car is behind door 1, then the game show host has a choice of which door to choose to show. Let* $S2, S3$ *be the events that the game show host chooses door 2 or door 3. If the car is behind door 3, then he has to show door 2 because otherwise he would be showing the car, so we can assume* $P(S3|C3) = 0\&P(S3|C2) = 1$. *On the other hand, we are not told anything about the mechanism for the game show host choosing between door 2 and door 3 when there is a choice. So lets write* $P(S3|C1) = p, 0 < p \leq 1$. *We can now apply Bayes' theorem to get the probability that the car is behind door 2 given he has shown you the goat behind door 2 as*

     $$\begin{aligned} P(C2|S3) &= \frac{P(S3|C2) \times P(C2)}{P(S3|C1) \times P(C1) + P(S3|C2) \times P(C2) + P(S3|C3) \times P(C3)} \\ &= \frac{1 \times 1/3}{p \times 1/3 + 1 \times 1/3 + 0 \times 1/3} = \frac{1}{1+p}. \end{aligned}$$

     If $p = 1/2$, *i.e the game show host flips a coin to decide which door to reveal when there are two choices, then* $P(C2|S3) = 2/3$ *and you should change. Although this may surprise you, it should not because there was always a probability of 1/3 that your were right in picking door 1 and this has not changed. What has changed is that the game show host narrowed the field down on the 2/3 chance that it was door 2 or door 3. On the other hand, if the game show host loves door 3 for some reason and* $p = 1$, *then* $P(C2|S3) = 1/2$ *and it does not matter whether you change or not.*

8. Suppose events A and B satisfy $P(A \cap B) = P(A)P(B)$. Show that they are independent by calculating all the relevant probabilities.

   - ***Solution:*** *We need* $P(A \cap B^c) = P(A)P(B^c)$. *Since* $A \cap B^c, A \cap B$ *are disjoint and have union* A, *rule (c) for probability gives* $P(A) = P(A \cap B) + P(A \cap B^c)$. *But* $P(A \cap B) = P(A)P(B)$, *so* $P(A \cap B^c) = P(A) - P(A)P(B) = P(A)(1 - P(B)) = P(A)P(B^c)$, *as required. Similarly,* $P(B \cap A^c) = P(B)P(A^c)$. *Finally,*

$P(A^c \cap B^c) = P((A \cup B)^c) = 1 - P(A \cup B) = 1 - (P(A) + P(B) - P(A \cap B)) = 1 - (P(A) + P(B) - P(A)P(B) = (1 - P(A))(1 - P(B)) = P(A^c)P(B^c).$

9. Suppose that the genes for eye colour for a certain male fruit fly are (R, W) and the genes for eye colour for the mating female fruit fly are (R, W), where R and W represent red and white, respectively. Their offspring receive one gene for eye colour from each parent.

   a. Define the sample space for the genes for eye colour for the offspring.

      - $S = \{(R_m, R_f), (R_m, W_f), (W_m, R_f), (W_m, W_f)\}$.

   b. Assume that each of the four possible outcomes has equal probability. If an offspring ends up with either two red genes or one red and one white gene for eye colour, its eyes will look red. Given that an offspring's eyes look red, what is the conditional probability that it has two red genes for eye colour?

      - $P((R_m, R_f)|\{(R_m, R_f), (R_m, W_f), (W_m, R_f)\}) = 1/3$.

10. (*Q1.4-9*) An urn contains four coloured balls: two orange and two blue. Two balls are selected at random without replacement, and you are told that at least one of them is orange. What is the probability that the other ball is also orange?

    - $P(\textit{at least one orange}) = 1 - P(\textit{no orange}) = 1 - \frac{2}{4} \times \frac{1}{3} = \frac{5}{6}$. *So*
    - $P(\textit{both orange}|\textit{at least one orange}) = \frac{P(\textit{both orange})}{P(\textit{at least one orange})} = \frac{\frac{2}{4}\times\frac{1}{3}}{\frac{5}{6}} = \frac{1}{5}$.

11. A life insurance company issues standard, preferred, and ultra-preferred policies. Of the company's policyholders of a certain age, 60% are standard with a probability of 0.01 of dying in the next year, 30% preferred with a probability of 0.008 of dying in the next year, and 10% are ultra-preferred with a probability of 0.007 of dying in the next year. A policyholder of that age dies in the next year. What are the conditional probabilities of the deceased being standard, preferred, and ultra-preferred?

    - $P(\textit{standard}) = P(S) = 0.6$, $P(P) = 0.3$, $P(UP) = 0.1$. *And* $P(D|S) = 0.01$, $P(D|P) = 0.008$, $P(D|UP) = 0.007$.
    - *Thus by the law of total probability and multiplication rule* $P(D) = P(S)P(D|S) + P(P)P(D|P) + P(UP)P(D|UP) = 0.6 \times 0.01 + 0.3 \times 0.008 + 0.1 \times 0.007 = 0.0091$.
    - *Hence by Bayes's theorem*
    - $P(S|D) = \frac{P(S)P(D|S))}{P(D)} = \frac{0.6\times0.01}{0.0091} = 0.6593407$.
    - $P(P|D) = \frac{P(P)P(D|P))}{P(D)} = \frac{0.3\times0.008}{0.0091} = 0.2637363$.
    - $P(UP|D) = \frac{P(UP)P(D|UP))}{P(D)} = \frac{0.1\times0.007}{0.0091} = 0.07692308$.

12. Let a chip be taken at random from a bowl that contains 6 white chips, 3 red chips, and 1 blue chip. Let the random variable $X = 1$ if the outcome is a white chip; let $X = 5$ if the outcome is a red chip; and let $X = 10$ if the outcome is a blue chip.

    a. Find the pmf of X. (Namely, find the possible values of X and then the probability for each such possible value.)

       -

x	1	5	10
$f(x)$	$\frac{6}{10}$	$\frac{3}{10}$	$\frac{1}{10}$

b. Find the expectation of X.

   - $E(X) = \sum_{x \in S_X} xf(x) = 1 \times \frac{6}{10} + 5 \times \frac{3}{10} + 10 \times \frac{1}{10} = 3.1.$

13. Let $f(x) = \frac{x}{c}$, $x = 1, 2, 3, 4$. Find the value of c so that $f(x)$ satisfies the conditions of being a pmf for a random variable X.

    - *First* $f(x) > 0$ *is known for any* $x \in S_X = \{1, 2, 3, 4\}$. *In order that* $f(x)$ *is a pmf, it has to satisfy that* $f(1) + f(2) + f(3) + f(4) = 1$, *i,.e.* $\frac{1+2+3+4}{c} = 1$. *Therefore,* $c = 10$.

14. Let $f(x) = (1/4)^{|x|}(1/2)^{1-|x|}$ for $x = -1, 0, 1$ and $f(x) = 0$ for other x values. Is $f(x)$ a pmf? If yes, re-express the pmf by a table.

    - *It follows from the formula that* $f(-1) = 1/4$, $f(0) = 1/2$, $f(1) = 1/4$ *and* $f(x) = 0$ *for all other* x *values. The sample space* $S_X = \{-1, 0, 1\}$; $f(x) > 0$ *for any* $x \in S_X$ *and the total probability* $f(-1) + f(0) + f(1) = 1$. *Therefore,* $f(x)$ *is a pmf and can be expressed by the following table:*

x	-1	*0*	*1*
$f(x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$