

MAST90105 Methods of Mathematical Statistics

Assignment 1, Semester 1 2019**Due date: Sunday March 17, end of day.**

- Your assignment should show all working and reasoning. Marks will be given for method as well as for correct answers. You may use R or Mathematica for calculations but must include the code that you input to get the results.
 - Assignments count for 20% of your assessment.
 - Please submit a scanned or other electronic copy of your work via the Learning Management System - see [this link for instructions](#)
 - Late assignments will only be accepted under exceptional circumstances and must be handed personally to me, with a written explanation for submitting late and/or a medical certificate. A late penalty will be imposed.
 - **PLEASE DO NOT FORGET TO COMPLETE THE PLAGIARISM DECLARATION IN THE LEARNING MANAGEMENT SYSTEM BEFORE YOU SUBMIT YOUR FIRST ASSIGNMENT - ONE SUBMISSION COVERS ALL ASSIGNMENTS.**
1. Five fair coins are tossed (i.e., the probability of tail and head equals 0.5 for each coin) and the number of tails, T is counted. Find the conditional probability that $T \geq 1$ given that at least one coin shows head.
 2. *Birthday paradox.*
 - (a) Consider a group of 3 students. Each student has a birthday that can be any one of the days numbered 1, 2, 3, ..., 365. What is the probability that none of them have the same birthday with each other?
 - (b) Consider a group of 23 students. Each student has a birthday that can be any one of the days numbered 1, 2, 3, ..., 365.
 - i. What is the probability that none of them have the same birthday with each other?
 - ii. What is the probability that some of them have the same birthday with each other? Is this probability greater than 0.5?
 3. It is known from experience that in a certain industry 60 percent of all labor-management disputes are over wages, 15 percent are over working conditions, and 25 percent are over fringe issues. Also, 45 percent of the disputes over wages are resolved without strikes, 70 percent of the disputes over working conditions are resolved without strikes, and 40 percent of the disputes over fringe issues are resolved without strikes. What is the probability that a labour-management dispute in this industry will be resolved without a strike?

4. In a certain community, 8 percent of all adults have Type 2 diabetes. If a health service in this community correctly diagnoses 95 percent of adults with diabetes as having the disease and incorrectly diagnoses 2 percent adults without diabetes as having the disease, find the probabilities that
 - (a) the community health service will diagnose an adult as having diabetes;
 - (b) an adult diagnosed by the health service as having diabetes actually has the disease.
5. A bag contains 5 coins, one of which has a head on both sides, another one has a tail on both sides while the other 3 coins are normal (with equal probability of tail and head). A coin is chosen at random from the bag and tossed 2 times. The number of heads obtained is a random variable, say X .
 - (a) What are the possible values of X ? Also tabulate the pmf of X .
(*Hint:* The coin chosen is either normal (a) or with head (b) or tail (c) on both sides. Find the conditional probability of X in each of these three cases (a), (b) and (c). Then use the law of total probability and multiplication rule to find the pmf of X .)
 - (b) Calculate $E(X)$ and $\text{Var}(X)$.
6. Among the 16 applicants for a job, 10 have university degrees. A sample of 3 applicants are to be randomly chosen for interviews. Let X be the number of applicants in the sample who have university degrees.
 - (a) Give the name to the distribution of X if it has a name. Also specify the values of all parameters involved in this distribution.
 - (b) Find the probability that exactly 1 applicant in the sample has a university degree.
 - (c) Find the probability that at most 1 applicant in the sample has a university degree.
7. A moment-generating function of X is given by $M(t) = e^t/(2 - e^t)$.
 - (a) Find the values of the mean, μ , and variance, σ^2 , for X .
 - (b) Calculate $\Pr\{X \geq 4\}$.