CS3110: Solutions for Assignment 8 (NP-Completeness) July 31 2017

Q1. A) Interval Scheduling $\leq_P$ Vertex Cover?
Yes, Interval Scheduling is P, so it's NP, so it is reducible to Vertex Cover (All NP problems are reducible to Vertex Cover)
B) Independent Set $\leq_P$ Interval Scheduling?
Unknown, if yes, P would be NP!

Q2)

1. AD: Shortest Path: Given G(V,E), s and t in V and an integer k, is there a simple path from s to t with at most k edges?
2. AD: Longest Path(G,s,t,k): Given G(V,E), s and t in V and an integer k, is there a simple path from s to t with at least k edges?
3. True: It has a polynomial solution (assuming no negative cycle), hence it is in P
4. True: It is NP too, as $P \subset NP$
5. Unknown: We don't know, if it is incomplete, then it would imply P=NP
6. If $P \neq NP$, then it is not because NP-hard is at least as hard as NP, If P=NP, then True because $NP \subset$ NP-Hard

Regarding BD:
Let's first prove it is NP-complete:
1- It is NP because we can have a simple polynomial certifier: Given a path, we check if it is simple (does not visit any node more than once), and the length is greater than or equal to k
2- We will prove Hamiltonian Cycle $\leq_p$ Longest Path
It is very easy to get a Hamiltonian Cycle if we have a Longest Path solver, we simply call *LongestPath(G,s,t,*|V|-1*)* for every (*s*,*t*)

7. Unknown,
8. True, we just proved it
9. True, proved
10. True