WS2122 MDO Exam Solution

EXAM February 21st, 2022

Multidisciplinary Design Optimization

Prof. Dr. Markus Zimmermann
Technische Universität München
Lehrstuhl für Produktentwicklung und Leichtbau

The problems can be solved in any order.

The contents of the exam are based on the lecture „Multidisciplinary Design Optimization“ as it was held in the summer terms 2021.

Sign the first page with your name and matriculation number! During the exam you may, without exception, only write on the exam papers distributed. All exam papers are to be returned in full for marking. Copying of the exam, in part or in full, in any shape or form will be considered cheating.

Surname:	
First name:	
Matriculation number:	

SOLUTION

Problem	1	2	3	4	5	6	
Points	7	13.5	13	11	9	11.5	$\sum = $ 65
Obtained							

Problem 1: General Questions

7

Tick the right answers.

	true	false
After a succesful topology optimization, shape and size optimization are often carried out to optimize the structure further.	✓	
Non-convex objective functions only possess one global optimum.		✓
The number of constraint functions determines the dimension of the design space.		✓
For a constrained optimization problem, the partial derivatives of an interior optimum are zero.	✓	
Barrier functions cannot be used for equality constraints.	✓	
The update scheme of Genetic Algorithms is inspired by a bird swarm analogy.		✓
Sampling methods can be used for black-box functions.	✓	

Problem 2:
Fundamentals of Optimization & Design Space Exploration

13.5

1. Engineering Problem:
   The revenue R of a wind farm is to be maximized for a certain wind load F_W. The wind farm consists of 6 windmills (shown above) with different towers. Among other requirements (not relevant here) the stresses σ_i, for $i = 1..6$, in the towers must not exceed the allowable stress σ_c (compression and tension). Furthermore, the average power output P has to be more than the required P_r.
   The height l_i, for $i = 1..6$, of each tower is to be chosen between $l_{lb} = 30$m and $l_{ub} = 80$m.

   Hint: For the following exercises you may use index notation where applicable.

   (a) State the quantities of interest for the given problem. (You do not need to compute them!)

2

$y_1 = R$ ✓

$y_{i+1} = |\sigma_i|$, $i = 1 \dots 6$ ✓ ($1/2$ f. σ, $1/2$ f. $||$)

$y_8 = P$ ✓

(b) State the constrained optimization problem. For this purpose, write down the objective function together with the constraints and bounds of the design space. 2.5

$$\begin{cases} f(\underline{l}) = -R \\ \min f(\underline{l}) \end{cases}$$

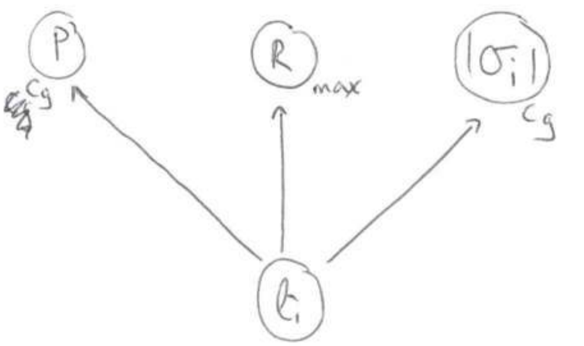
(auch: max R, $y_1 = -R$ $f(\underline{l}) = y_1$

subject to: $g_i(l_i) = |\sigma_i(l_i)| - \sigma_c \leq 0 \quad i = 1..6$

$g_7(\underline{l}) = P_r - P \leq 0$

$30m \leq l_i \leq 80m \quad i = 1..6$

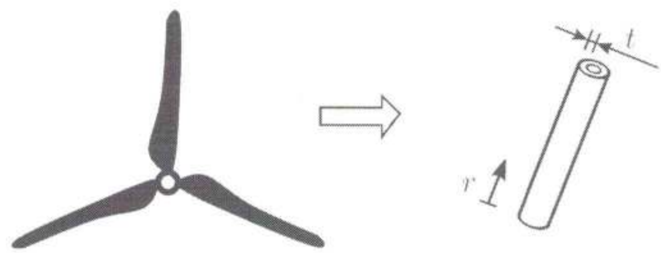
(c) Draw the dependency graph (compact notation) of the constrained optimization problem. (You may use index notation.) 1.5



(d) What does the Weierstrass theorem tell you about the solution to the engineering problem? What additional assumptions (if any) do you have to make? 2

Weierstrass: ensures extrema exists in compact set for continuous fcts

additional assumption: objective fct & constraint fcts. have to be sufficiently smooth (continuity assumed)



2. Engineering problem:
As a next step, the rotor blades of one windmill have to be optimized. Therefore, the rotor blade is modeled as a pipe (see sketch above). The thickness t has to be determined while minimizing the mass $m = \int_r \rho \cdot A(r,t)\,\mathrm{d}r$, where $A(r,t)$ is the (variable) cross-section.

(a) State the given optimization problem and name a possible solution of it! 1.5

min m(t) ✓

t = 0 ✓

(b) Give a possibility how you could extend the formulation and explain why this is necessary. 1.5

✓ 1) add constraint / bounds to DV

OR

1) Calculus of variations

✓ 2) bc. engineering problem does not give meaningful result

2) bc. no. of DV ∞

3. Pearson Correlation Coefficient:
The Pearson Correlation Coefficient ρ_{xy_1} quantifies to what degree the relation between two variables x, y_1 can be expressed as a linear function. By substituting $y_2 = f(x)$ the Pearson Correlation Coefficient $\rho_{y_2y_1}$ can be used to show a nonlinear dependency. Tick (✓) the boxes for which the Pearson Correlation Coefficient $\rho_{y_2y_1}$ is equal to 1 in an interval of $[-\infty; \infty]$. Cross (×) the boxes otherwise. 2.5

function y_1	$\frac{1}{6}(3x)^3 + 4.5x$	$\sin(5x)$	$(2x)^5 + (2x)^2$	e^{158x}	$(3x)^7$
substitution function y_2	$x^3 + x$	$\pi \sin(5x)$	$(2x)^5$	$100e^{58x}$	$(33x)^7$
	✓	✓	×	×	✓

(-1 Pkt wenn × × ☐ ☐ ×)

Problem 3: Basic Mathematics

13

1. Consider the objective function

$$f(\mathbf{x}) = 4x_1 + 3(x_3 - x_2^3) + 5x_1x_2$$

that is constrained by

$$g(\mathbf{x}) = (x_3 - 2x_2)^2 \leq 0 \quad \text{and}$$
$$h(\mathbf{x}) = x_2 - \sin(x_3) = 0\,.$$

(a) Write down the Lagrange function with the given expressions explicitly. 1

$$L(x_1, x_2, x_3, \lambda, \mu) = 4x_1 + 3(x_3 - x_2^3) + 5x_1x_2 + \mu(x_3 - 2x_2)^2 + \lambda(x_2 - \sin(x_3))$$ ✓

(b) Formulate the KKT conditions with the given expressions explicitly. (You do not need to solve the system of equations!) 2.5

$$\frac{\partial L}{\partial x} = 0, \quad h = 0, \quad g \leq 0, \quad \mu g = 0, \quad \mu \leq 0$$ ✓ (½ Pkt Abzug für jedes Fehlende/Falsche)

$$\frac{\partial L}{\partial x_1} = 4 + 5x_2$$ ✓

$$\frac{\partial L}{\partial x_2} = -9x_2^2 + 5x_1 - 4\mu(x_3 - 2x_2) + \lambda$$ ✓

$$\frac{\partial L}{\partial x_3} = 3 + 2\mu(x_3 - 2x_2) - \lambda\cos(x_3)$$ ✓

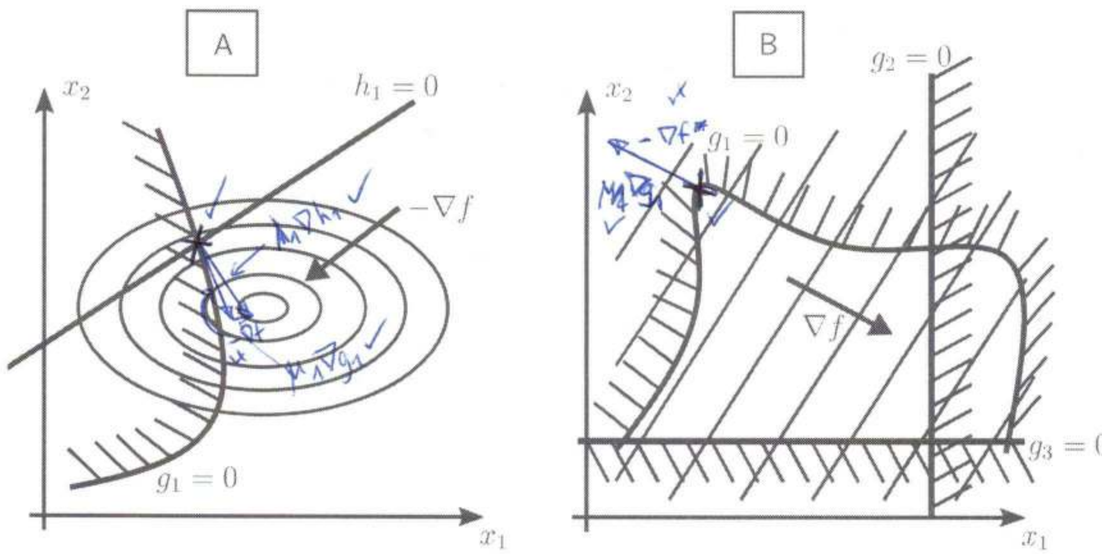
(c) Reduce the problem dimension explicitly with the equality constraint. 1.5

$$h(\underline{x}) = x_2 - \sin(x_3) = 0 \rightarrow x_2 = \sin x_3$$

$$f(x_1, x_3) = 4x_1 + 3(x_3 - (\sin^2 x_3)) + 5x_1 \sin x_3$$

$$g(x_3) = (x_3 - 2 \cdot \sin x_3)^2 \leq 0$$

2. Given are the following contour plots of two different minimization optimization problems.



(a) Mark the global optimum in each of the diagrams. **2**

(b) Draw the geometrical interpretation of the Karush-Kuhn-Tucker conditions at the global optimum in both figures. **4**

(c) State the sign of the following Lagrange multipliers. Please mark the right answer with a cross. **1**

	< 0	$= 0$	> 0
A: μ_1			X
B: μ_2		X	

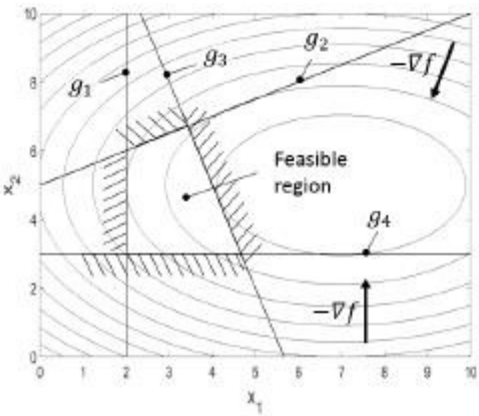
(d) The Karush-Kuhn-Tucker conditions for optimality for a convex optimization problem, are the following: **1**

- [ ] sufficient
- [ ] necessary
- [x] necessary and sufficient

Problem 4: Optimization Algorithms - Quadratic Programming

11

1. In the plot below, an optimization problem, with two design variables x_1 and x_2, a quadratic objective function $f(\boldsymbol{x})$ and the linear constraint functions $g_1(\boldsymbol{x})$, $g_2(\boldsymbol{x})$, $g_3(\boldsymbol{x})$ and $g_4(\boldsymbol{x})$, can be observed.



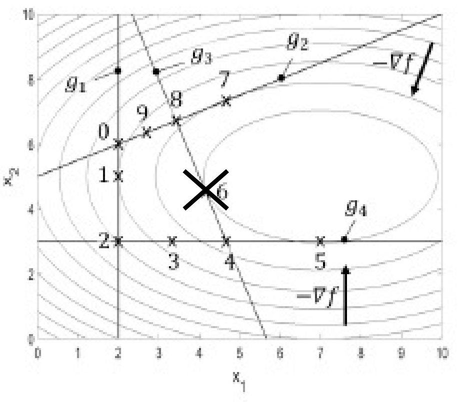
To solve this Quadratic Programming problem the active-set strategy of **the lecture** is utilized:

If	$(\boldsymbol{s}^{(k)} \neq 0$ and $g_j(\boldsymbol{x}^{(k)} + \boldsymbol{s}^{(k)}) \leq 0)$:	$\alpha^{(k)} = 1$,for $j \in J_g$
Elseif	$(\boldsymbol{s}^{(k)} \neq 0$ and $g_j(\boldsymbol{x}^{(k)} + \boldsymbol{s}^{(k)}) > 0)$:	Calculate step size $\alpha^{(k)}$
		Add most violated constraint to active set $J_{g,a}$.
Elseif	$(\boldsymbol{s}^{(k)} = 0$ and $min_{j \in J_{g,a}}(\mu_j^{(k+1)} < 0))$:	Remove constraint.
Elseif	$(\boldsymbol{s}^{(k)} = 0$ and $min_{j \in J_{g,a}}(\mu_j^{(k+1)} > 0))$:	Stop.

$\boldsymbol{x}^{(k+1)} = \boldsymbol{x}^{(k)} + \alpha \boldsymbol{s}^{(k)}$

with J_g as the set of all inequality constraints and $J_{g,a}$ as the set of **active** inequality constraints.

(a) For the start vector $x^{(0)} = [2, 6]^T$, tick [✓] the table for each point that is calculated during the quadratic programming optimization, cross [×] otherwise. 5



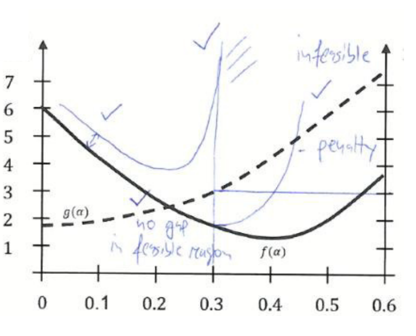
	x	Calculated point
0	$[2, 6]^T$	✓
1	$[2, 5]^T$	×
2	$[2, 3]^T$	×
3	$[3.33, 3]^T$	×
4	$[4.67, 3]^T$	×
5	$[7, 3]^T$	×
6	$[4.16, 4.53]^T$	✓
7	$[4.67, 7.33]^T$	✓
8	$[3.43, 6.71]^T$	✓
9	$[2.7, 6.35]^T$	×

(b) For the given Quadratic Programming problem, please mark the optimum in the diagramm. 1

(c) If the constraint $g_4(x)$ was removed from the optimization problem, would the course of the algorithm change? Tick [✓] the right answer! 1

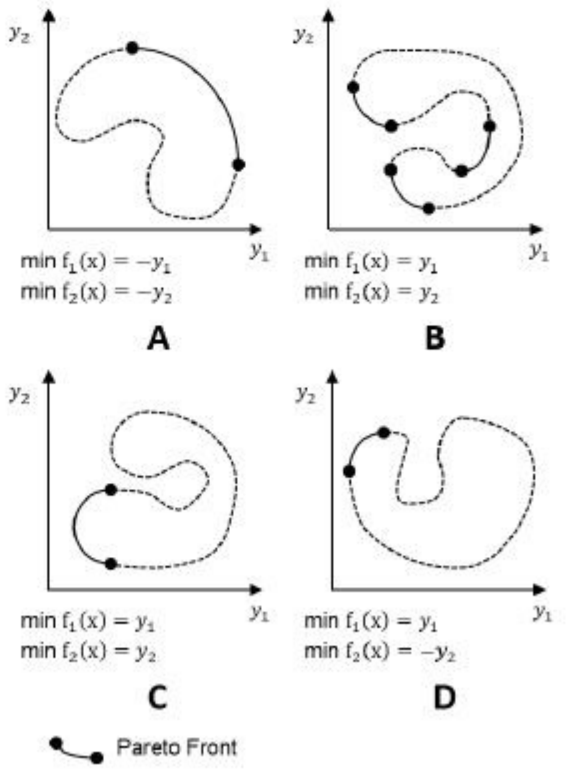
yes	no
☐	☑

2. The figure below shows a constraint optimization problem with objective function $f(x)$ and constraint function $g(x)$. To transform it to an unconstrained problem a penalty and a barrier formulation is to be applied. Please draw the meta objective function of each formulation (penalty/barrier) in the diagram and mark them clearly! 4



Problem 5: Many Objectives and Disciplines 9

1. Given below are four plots of feasible designs. Please tick in the table below whether the marked Pareto Front is correct or incorrect! 4



	A	B	C	D
Correct	√			
Incorrect		√	√	√

2. In the design of a steering system, requirements are given for the following quantities of interest.

Quantities of interest:

φ: steering wheel angle at the end stop
ω: maximum steering wheel angular speed while applying $10Nm$ steering wheel torque

Requirements:

$$\varphi_l \leq \varphi(\boldsymbol{x}) \leq \varphi_u$$
$$\omega(\boldsymbol{x}) \geq \omega_c$$

$\boldsymbol{x}$ are design variables.

a) Choose an appropriate type of objective meta function (linear, quadratic or worst-case) considering the requirements and explain your choice. 3

→ Worst-case ✓

a) 1, Linear meta objective fct does not work for two-sided requirements ✓

2, Quadratic meta objective fct penalizes also better (good) designs for one-sided requirements ✓

Or: → Quadratic ✓

1, ... ✓

b) 3, Worst-case meta objective fct is not smooth → numerical problems ✓

b) Formulate an objective meta function of the chosen type. 2

for a)

$$f(x) = \max \left\{ \frac{f(x) - f_u}{f_u - f_l}, \frac{f_l - f(x)}{f_u - f_l}, \frac{\omega_c - \omega(x)}{\omega_c} \right\}$$

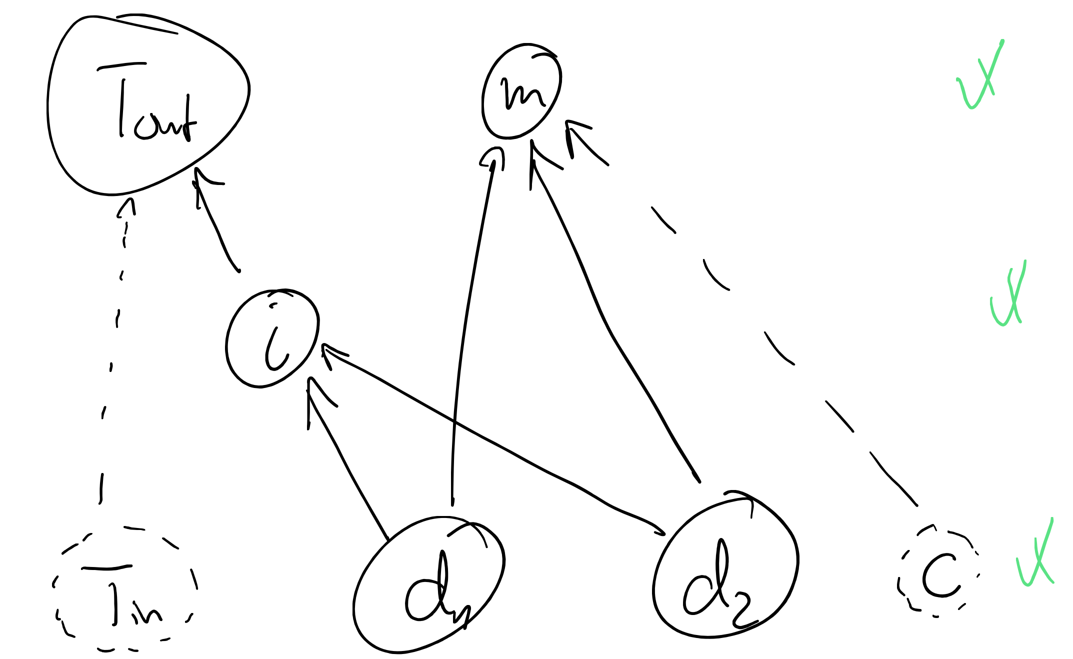
Problem 6: Systems Design and Decomposition

11.5

You have to design a transmission. For a given input torque $T_{in} = 10Nm$, the output torque has to be min $40Nm$ (T_{out}). Furthermore, the whole transmission should weigh max $2.5kg$ (m). You can estimate the weight by using a scaling factor $c = 0.0125\frac{kg}{mm}$, which calculates the weight of the gears depending on their diameters $d_{1,2}$.
In order to fulfill the requirements on the performance measures T_{out} and m, you have two design variables available: d_1 and d_2.
Both design variables are constrained due to manufacturing aspects: $d_{1,2} \geq 20mm$
You have the following mathematical models available:

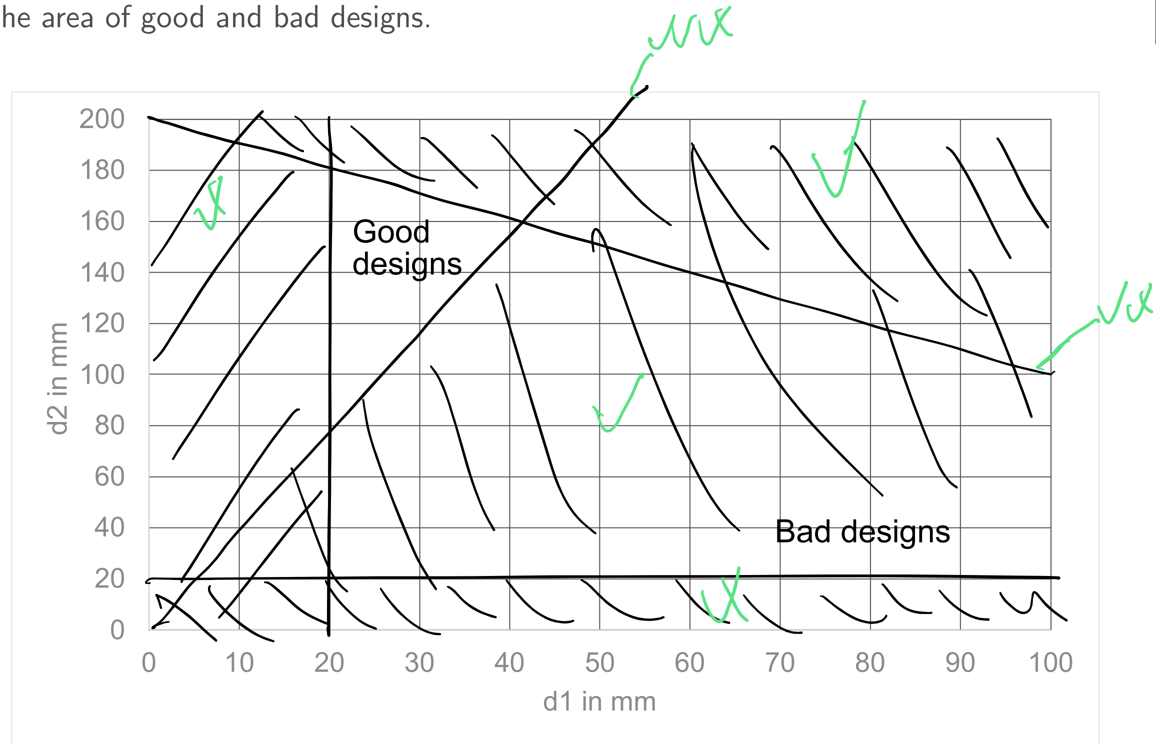
$$T_{out} = i\ T_{in}$$
$$i = \frac{d_2}{d_1}$$
$$m = c(d_1 + d_2)$$

(a) Draw the (Attribute) Dependency Graph/ Compact Notation for the problem stated above. 2.5



✓ if all arrows correct (− ½ P for each wrong arrow)
→ no minus points

(b) Sketch the limit lines for all requirements and constraints in the diagram below. Mark the area of good and bad designs. 7



(c) Compute meaningful requirements on d_1 and d_2 with concrete numerical values. Derive those quantitative requirements in a way that the design variables are independent of each other; that there are no point-based solutions and the overall system requirements are satisfied.
Sketch the derived requirements in the diagram above and write down the concrete numerical values in the solution template below. 2

Solution:

- Requirement on d_1: ✓
- Requirement on d_2: ✓

Multiple possible answers
For example:
° 20mm <= d1 <= 30mm
° 120mm <= d2 <= 160mm
would be correct.

Whatever you choose, remember to add it to the diagram above as well.