The Shape of Convergence in Growth Miracles: The Role of Human Capital *

Long Qian †

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Abstract

Economists have long studied the role that human capital plays in economic development. The hypothesis of Nelson and Phelps (1966) implies that higher education levels in an economy can facilitate technology diffusion and lead to faster convergence in technology. I incorporate the idea into a growth framework by developing a model of human capital investment, adding a role for human capital in the convergence of productivities towards the technology frontier. This introduces an externality through which individual education decisions affect aggregate productivity. I calibrate my model to the case of South Korea between 1960 and 2019. Like many growing countries, South Korea's experience exhibited convergence in output that was 'S Shaped'. My model matches the 'S Shaped' convergence trajectory well with the half-life of transition being 30-35 years and is consistent with the sharply rising education attainment observed in South Korea. More importantly, the quantitative exercises demonstrate that a significant extent of the externality is required to match the transition path of output in South Korea. If the externality is removed from the model, then one-third of the growth is not accounted for and thus it cannot quantitatively match South Korea's convergence pattern well.

Keywords: Convergence, Human capital, Technology diffusion

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†Arizona State University, email: lqian12@asu.edu

1 Introduction

The second half of the 20th century witnessed an explosive emergence of growth miracles. Some typical examples include Asian Tigers (South Korea, Taiwan, Hong Kong, and Singapore). While economists have recognized the emergence of convergence groups based on longer history after the mid-19th century (cf. Baumol (1986)), the very rapid income growth in Asian growth miracles is still remarkable. Along with the unprecedented rapid growth are four key observations that characterize the convergence of Asian growth miracles after WWII: (1) The takeoff initiates after more interactions with the rest of the world (such as greater levels of trade and foreign investment); (2) The productivity tends to converge towards the technology frontier (i.e. the U.S.); (3) The education attainment sharply rises and converges to almost the same level; (4) The convergence of output is 'S Shaped', i.e., the speed of convergence is slow in the beginning and end, and fast in the middle phase of the transition. The last fact contrasts with standard growth models that predict a monotonically declining growth rate after takeoff. This motivates an alternative view to examine the convergence path of Asian growth miracles.

In this paper, I reconcile the observations above by investigating the role of complementarity between human capital and convergence in productivity in Asian growth miracles. The idea is motivated by the seminal work by Nelson and Phelps (1966), which proposes that 'education speeds technological diffusion'. In the Nelson–Phelps hypothesis, technologies adopted by an economy might not be immediately appropriate to use. They require adaptation and the rate of this process depends on the education of people operating them. In addition, since Asian growth miracles are believed to benefit from adopting technology from the global frontier due to globalization and active opening policies after WWII, the productivity growth in those economies is primarily shaped by the rate of technology diffusion. At the aggregate level, therefore, the Nelson–Phelps hypothesis implies that the convergence in productivity in those economies should critically depend on the level of human capital. Indeed, the significantly positive correlation between education attainment of economies and their subsequent growth rates of productivity obtained from post-war cross-country data in Section 2.1 supports this view.

I develop an otherwise standard overlapping generation model of human capital investment and embed it in a growth framework. In the model, overlapping generations of agents choose years of schooling and expenditures on education quality at an early age to maximize individual lifetime earnings. The new element relative to the literature lies on the production side. I model an economy whose productivity is initially lower than the global technology frontier. The frontier is assumed to grow at an exogenous constant rate and serves as the locomotive of the growth of the follower economy. Consistent with the Nelson–Phelps hypothesis, I assume that the speed of convergence in productivity depends on the human capital stock and the distance to the frontier. This introduces an externality where the choice of education by households affects the growth rate of aggregate productivity. This interdependency between human capital accumulation and productivity growth is not present in previous growth models and will be examined in a quantitative framework.

The benchmark economy is then calibrated to reproduce the South Korean economy from 1960 to 2019. I assume the follower is initially a no-growth economy with productivity falling far behind the frontier. I characterize the initial condition by a series of low states (relative productivity, human capital, and physical capital stock) to match the Korean economy before the shock in 1960. The convergence occurs when the model economy is shocked by the opportunity of technology diffusion and starts catching up in productivity. It stops as the model economy hits the balanced growth path (BGP), in which it grows at the same constant rate as the frontier. I calibrate the model parameters associated with consumer decisions and production technology to result in moments that are consistent with the observation in 2019. In particular, I jointly calibrate the two parameters that are specific to this model, the externality of human capital and the catch-up speed parameter, to match the level and trajectory of the growth in output for the Korean economy from 1960 to 2019.

The whole transition dynamics in the path six decades are generated to simulate the growth path of the model economy. Analytically, the prospect of productivity growth in the future gives agents incentives to invest more in human capital when they expect higher growth in wage rates in the transition. The 'S Shaped' convergence path of the output emerges as a result of the lagging nature of the human capital investment. Since only the young generation is able to adjust the education when the shock hits, the spike in education will create a temporary shortage of human capital supplied to production as more of the time is devoted to schooling.

This gives rise to initially slow growth of output. As the young generation ages, the stock of human capital sharply rises, and output growth accelerates. This acceleration is strengthened as the growth of human capital, in turn, directly facilitates productivity growth because of the externality of human capital on productivity growth. The qualitative implications of the model are in line with convergence theories in the sense that it introduces a shock (the opportunity of technology diffusion) that is able to generate a transitory catch-up of a follower economy towards the frontier, followed by a BGP in which the follower grows at the same rate as the frontier.

The quantitative results indicate that the benchmark model can generate the magnitude of catch-up in output and education comparable to the data. The half-life for the transition of output is 30-35 years and is consistent with the data. More importantly, a significantly positive value for the extent of the externality of human capital on productivity growth is required to match the trajectory of the output observed. This suggests that it is critical to allow for the externality to match the distinct 'S Shaped' convergence path of output as observed in Korea. Taking others as fixed, when the externality is shut down, the model can only explain 67% of the growth in output in the past six decades for Korea. It can not quantitatively match the transition path of output well even when a counterfactually higher level of convergence speed is imposed such that the level of growth in output can be accounted for . Therefore, the model results underscore the essential role of taking into account the dependency of productivity growth on human capital by fitting the transition dynamics of the follower economy, which is an approach that is rarely taken in previous works.

Related Literature

This paper is related to and builds on broad literature in growth theory. The model developed in the paper speaks to the general consensus in the growth literature that differences in productivity play a critical role in income variation across countries (see, e.g., Hall and Jones (1999), Caselli (2005) and Jones (2016) for a review). The notable observation that most growth miracles significantly close their productivity gap relative to the frontier is also consistent with this view. The remaining debate is what distinct elements are in effect that drive the substantial catch-up in productivity for growth miracles after WWII.

It has long been recognized that the pace of productivity growth is not only determined by domestic technical changes, but also greatly shaped by the diffusion of technology. Parente and Prescott (1994) and Basu and Weil (1998) are examples of growth theories that recognize the critical role of technology diffusion in development.[1] Caselli and Coleman (2001) provides empirical evidence on this by conducting a case study of computers and underscores that educational attainment is a key determinant of computer-technology adoption. Comin and Hobijn (2004) explores the common patterns of technology diffusion using historical data. More recent works have contributed to the theory by developing models in which diffusion of technology, ideas and active imitations by poor countries can explain a considerable bulk of growth in productivity and income (Comin and Hobijn (2010), Alvarez et al. (2013), Perla and Tonetti (2014) , König et al. (2016), Buera and Lucas (2018), Buera and Oberfield (2020), Benhabib et al. (2021)). Indeed, the productivity in poor countries is determined to a greater extent by the flow of technical know-how from the technology frontier in the modern economy because only a few rich countries account for the majority of the world's creation of new technology, as noted in Keller (2004).

The empirical evidence and the model results of the paper suggest a critical role of human capital in income growth. The growth literature in the tradition of the neoclassical framework provides mixed evidence on this: while Mankiw et al. (1992) and Barro and Sala-i Martin (2003) demonstrate that a Solow model augmented with human capital can successfully explain cross-country income differences, Klenow and Rodriguez-Clare (1997) and Bils and Klenow (2000), among others, conclude that the role of human capital is limited in accounting for income growth. More recent works have attempted to address the issue of unmeasured labor quality and find more positive role of human capital in development (Schoellman (2012), Jones (2014), Cubas et al. (2016), Hendricks and Schoellman (2018)). This paper, in contrast, focuses on human capital externalities as emphasized in Lucas (1988).

The complementarity between technology diffusion and education has been noted in the early development literature. Welch (1970) and Schultz (1975) provide the insight that one benefit of education is the enhanced ability to deal with more advanced new technologies. Some empirical works have demonstrated this idea. Using plant-worker data, Doms et al.

[1] Parente and Prescott (1994) attributes the income disparity for growth miracles to the barriers to technology adoption, and the model is applied to explain the trajectory of output for Japan, South Korea, Taiwan, etc. relative to the U.S. after WWII. Those economies are empirically relevant to this paper.

(1997) and Dunne and Schmitz (1995) establish that plants with more advanced technology tend to hire workers with higher education and pay them higher wages. Bartel and Lichtenberg (1987) tests the hypothesis that educated workers have a comparative advantage in implementing new technologies, and the findings support this. Benhabib and Spiegel (1994, 2005) investigate the Nelson–Phelps hypothesis by explicitly including the complementarity in a model for growth accounting using cross-country data and finds a positive role of human capital in technology spillover from leaders to followers. Madsen (2014) implements an empirical strategy using long historical data from 1870 to 2009 and shows that educational attainment and its interaction with the distance to the frontier are significant determinants of productivity growth. This paper embeds those ideas in a growth model and demonstrates the importance of complementarity by examining the transition dynamics of the model economy.

The assumption that the rate of convergence in productivity also depends on the distance to the frontier is related to the other component of the Nelson-Phelps hypothesis, which postulates that the rate of technology diffusion should vary positively with the distance between the technology frontier and the current level of productivity for the follower. This is because a larger distance of the technology relative to the frontier will leave the follower more room to adopt the technology and a higher rate of growth. The inclusion of this relationship is a formalization of the catch-up hypothesis that was initially proposed by Gerschenkron (1962). Islam (2003) provides a survey of the convergence literature. Barro (1991) makes early contributions by presenting cross-country empirical evidence of the catch-up hypothesis, and Barro (2015) supplements this using panel data involving a large number of countries over a long-term period. This catch-up effect is widely applied to many endogenous growth models that feature technological convergence, like Sala-i Martin and Barro (1997). Recent works by Comin and Hobijn (2010, 2011) capture the idea by considering models where the cost of adopting technological vintages is decreasing in the distance to the technology frontier. The catch-up effect in my model is similar to Benhabib et al. (2014), which explores the distribution of productivity of heterogeneous economies through the lens of incentives to innovate and imitate. Like my model, it generates a convergence of follower economies towards the frontier when they fall far behind in productivity. Then they hit the BGP in which the followers grow at the same rate as the frontier. In their model, the technology frontier endogenously emerges from the innovator, while the technology frontier in my model is exogenous. This is intended to underscore the role of human capital in the convergence of productivity for the follower economy.

Finally, the paper is also related to the literature that emphasizes the role of trade in international technology diffusion and convergence. My paper does not directly model trade, but the shock introduced after 1960 in the model is motivated by the fact that the takeoff of Asian growth miracles occurred after they actively liberalized their trade regime after WWII and the surge in trade volumes. This suggests that international technology diffusion is critical in the convergence of productivity and income. Grossman and Helpman (1994) emphasizes that it is important to include international interdependence in the growth theory.[2] Coe and Helpman (1995) provides empirical evidence that trade serves as a transmission mechanism that links the productivity gains of an economy to the R&D of its trade partners.[3] Grossman and Helpman (1991b) builds a model to formalize this idea, and richer features are added to more recent growth models that emphasize the role of international trade in the process of development (Lucas (2009), Buera and Oberfield (2020), Perla et al. (2021)). This paper implicitly assumes that the factors related to higher levels of openness and resulting more interactions with the rest of the world trigger the transition in South Korea as well as other Asian growth miracles.

The paper is organized as follows. Section 2 exhibits motivating facts using post-war cross-country data, with an emphasis on Asian growth miracles. Section 3 sets up the model. In Section 4, I show the analytical solution of the model and solve for the balanced growth path (BGP). Section 5 reports the quantitative results of the benchmark economy when it is calibrated to Korea. The last section concludes.

2 Motivating Facts

This section presents motivating facts from post-war data that support the Nelson-Phelps hypothesis that education facilitates technology adoption and therefore serves as an engine

[2] See Grossman and Helpman (1991a) chapter 9 for a textbook review of the theory.

[3] See also Keller (2002, 2010) for surveys of studies that propose that trade, FDI, etc. are main channels through which technology transmits. Wacziarg and Welch (2008) and Chang et al. (2009) provide empirical evidence that countries implementing liberalization policies experience higher economic growth.

for convergence in productivity. I start with the cross-country evidence, which demonstrates the strong correlation between the human capital stock of economies and their growth rates of total factor productivity (TFP) relative to the frontier. Some typical Asian economies that feature a fast catch-up in productivity as well as output per capita, like Asian Tigers, are then examined to serve as a case study.

2.1 Empirical Evidence on Productivity Growth and Education

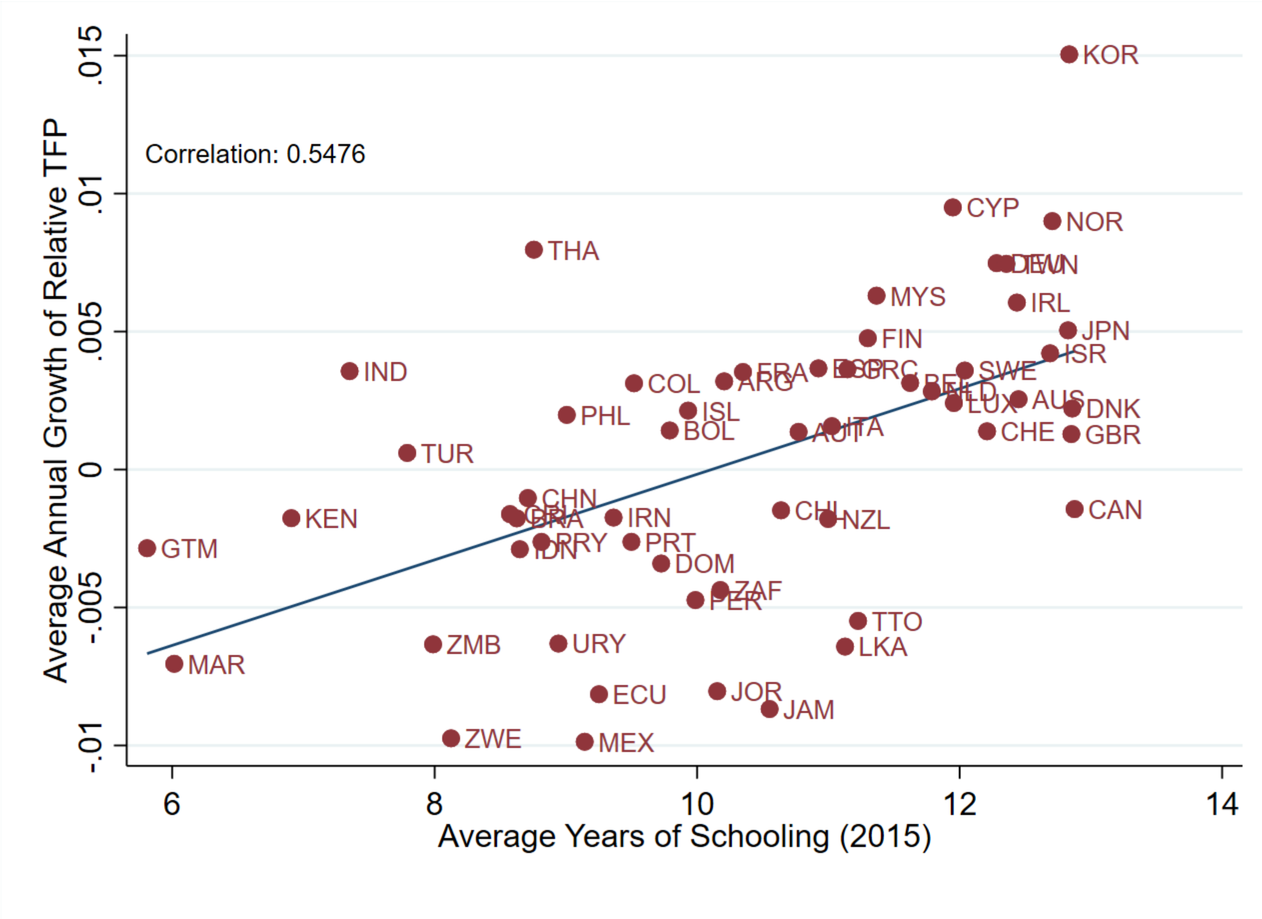
Figure 1 plots the average (geometric) annual growth of TFP relative to the frontier (U.S.) and average years of schooling for economies that have TFP data available from 1960 to 2019. A common episode is chosen for consistency of the comparison. The correlation coefficient between the two variables is about 0.55, which provides coarse cross-country evidence that there is a significant positive effect of the average years of schooling on the rate an economy closes its technology gap relative to the frontier (U.S). It is worth noting that well-known Asian growth miracles, like Japan and Asian Tigers, lie notably at the top right of the figure.[4] A more detailed case study of Asian economies will be conducted in the following subsection.

Figure 2 gives a more complete sense of how education and TFP growth are correlated by plotting the same variables for an extended sample of economies. More economies are included by allowing the first year of TFP that's available to be later than 1960. The average annual growth of relative TFP is then calculated using varying years with TFP data available. It turns out that the correlation increases (to 0.61) because of the inclusion of additional economies that lie at the bottom left and top right corners of the figure. Some former Soviet Union and Eastern European economies (Kazakhstan, Russia, Serbia, etc.) emerged as high education and high TFP growth economies after the 1990s. A cluster of African countries exhibits low growth in TFP and low levels of education, like Sierra Leone, Mozambique, and the Central African Republic.

The positive correlation between education and income growth has been established using post-war data (see, e.g., Barro (1991) and Hanushek and Woessmann (2016)). But debates

[4] Benhabib and Spiegel (2005) plots the average growth of TFP from 1960 to 1995 against the initial average years of schooling in 1960 and finds that all the Asian economies mentioned above (including Thailand) lie notably at the top right of the plot.

Figure 1: Average Annual Growth of Relative TFP and Average Years of Schooling



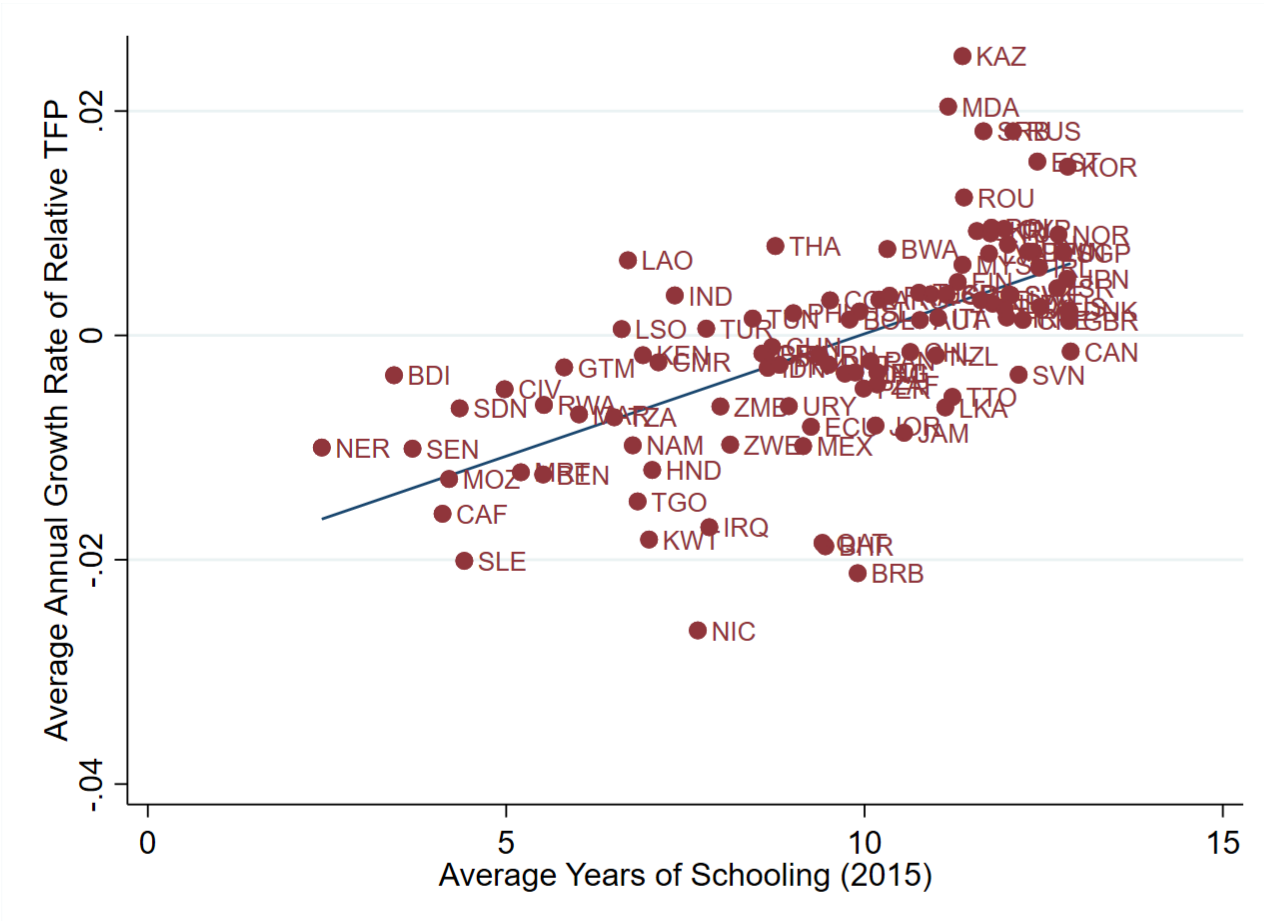
Note: The average years of schooling for each economy is obtained from Barro and Lee (2013). The average annual growth of TFP for each country is calculated from the geometric average of the growth rate of TFP level at current PPPs (USA=1) from 1960 to 2019 in the Penn World Table (PWT 10.0).

persist on the role of education in growth, primarily due to suspicion of reverse causality (Bils and Klenow (2000)). This paper revisits the issue through the lens of complementarity between technology and education.

2.2 Post-war Growth Miracles: The Case of Asia

In this section, I conduct a case study of Asian growth miracles that experience takeoff in terms of various welfare indicators after WWII. The economies I examine here include Asian Tigers (South Korea, Singapore, Hong Kong, and Taiwan) because of their common development patterns, specifically in education, GDP per capita, productivity, and featuring

Figure 2: Average Annual Growth of Relative TFP and Average Years of Schooling (Extended Sample)

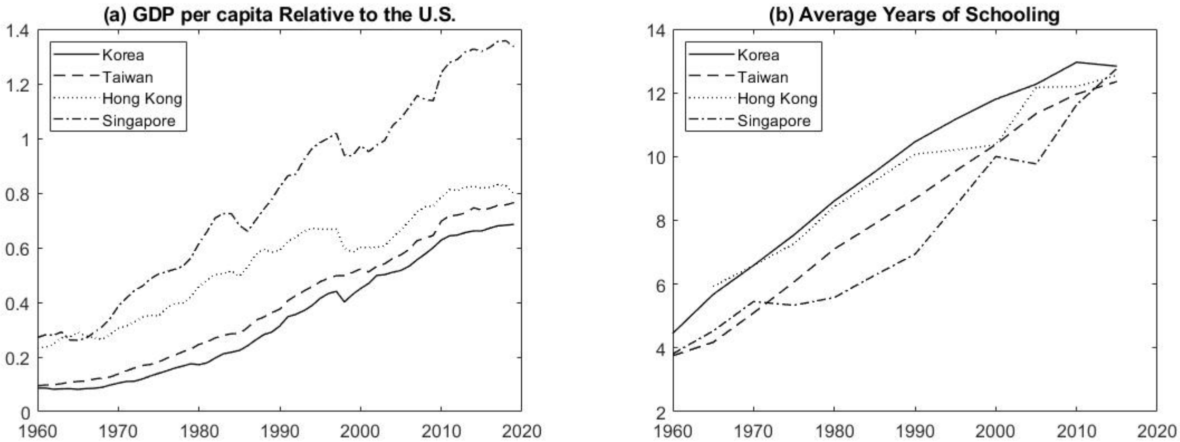


Note: The average years of schooling and average annual growth of TFP are obtained from the same source as in Figure 1. This figure includes all the economies that have TFP levels at current PPPs available and calculates the average annual growth of TFP for each country from the geometric average of the growth rate of TFP level at current PPPs for whatever years that have the TFP data available.

export-oriented economies. Those factors amount to the four aspects of development for those economies that I will exhibit below.

The most important observation, as indicated in panel (a) of Figure 3 is that these economies significantly converge in the output towards the U.S. after WWII, and the path of the convergence is 'S Shaped', i.e., the growth rate was initially slow (in the 1960-70s), then faster (in the 1980-90s), and then slow (after 2000) (see Table 1). This pattern is observed in most economies that exhibit strong catch-up post World War II, like Asian Tigers. Little attention, however, has been paid in the growth literature to explicitly allow the transition dynamics of the output to discipline the model. As a result, most studies would end up with

Figure 3: Relative GDP per capita and Average Years of Schooling in Asian Tigers



Note: The average years of schooling for each economy is from Barro and Lee (2013). The relative GDP per capita is calculated by GDP per capita in the designated economy divided by GDP per capita in the U.S. from the Penn World Table (PWT 10.0).

a monotonically declining speed of convergence in transition.[5] In contrast, this 'S Shaped' convergence of output serves as a key empirical observation that should be accounted for in the model developed later.

Panel (b) documents the evolution of education attainment and indicates a strong catch-up in human capital accumulation along with the takeoff in economic welfare. It is also worth noting that the education attainment in these economies converges almost to the same level around 13 years, and the transition path does not vary significantly.

The other two aspects are productivity growth and trade volume, which are exhibited in Figure 11 in Appendix A. Panel (b) of Figure 11 plots the export share as a percentage of GDP for Asian Tigers and demonstrates the increasing importance of trade in these economies after WWII. The fact that the takeoff in GDP per capita took place after the surge in trade volume suggests that embracing the global market triggers the growth of the economy, and this is in line with the view that globalization after WWII contributes to the emergence of convergence clubs like Asian Tigers.[6] Panel (a) suggests that there is a significant catch-up in

[5]See Solow (1956) for an example of neoclassical growth theory and Sala-i Martin and Barro (1997) for endogenous growth theory.

[6]Note that trade will not be explicitly modeled in this paper, but just serves as an underlying driving force of the transition.

Table 1: Average Annual Growth of GDP per capita Relative to the U.S.

	1960-1970	1970-1997	1997-2019
Korea	0.019	0.053	0.038
Taiwan	0.038	0.049	0.037
Hong Kong	0.026	0.030	0.015
Singapore	0.035	0.038	0.023

Note: The growth rates are calculated from the geometric average of annual growth of GDP per capita relative to the U.S. for designated periods.

productivity after 1960 when trade liberalization brought Asian Tigers more interactions with the rest of the world, which is in line with the view that the productivity in emerging economies is greatly shaped by the diffusion of technology. The observation that a considerable chunk of the growth in GDP per capita can be attributed to productivity growth is also consistent with the consensus in growth literature.

In what follows, I develop a model to account for the above observations. Specifically, motivated by the fact that the takeoff initiates after more interactions with the rest of the world, I model an economy that is shocked by the opportunity of technology diffusion, which triggers the catch-up in productivity, and human capital can facilitate productivity growth. Then, the key empirical observations as mentioned above, namely, the convergence in productivity, education attainment, as well as the distinct 'S Shaped' trajectory of growth in output, can emerge from the model results.

3 The Model

In this section, I develop an overlapping generation model in which a representative young cohort chooses years of schooling and expenditure on education quality that determine the permanent human capital once at a time to maximize individual lifetime earnings. The key ingredient added to the framework of human capital investment lies in the production side. In recognition of the hypothesis discussed above, productivity growth depends on the human capital stock and the technology distance to the frontier. The role of the externality of human capital on productivity growth is examined for a follower economy catching up with the frontier. I will show in the quantitative results that in a calibrated example, this model delivers 'S Shaped' convergence in the output that matches well with the data.

Demographic Structure I consider an economy populated by overlapping generations of people who live for four periods, a period as a child, two periods as an adult, and a period as a retiree (throughout they will be denoted by superscript 1, 2, 3, 4 separately). So a model period is set to 18 years. τ is used to index the generation (generation τ is composed of those who are children at date $t = \tau$). Since I abstract from heterogeneity across agents of the same cohort, I assume a representative agent for each generation. In sum, in each period, the economy comprises four representative cohorts: a young cohort, two cohorts of adults, and a cohort of retirees.

Human Capital Investment The set-up for the human capital investment builds on Erosa et al. (2010). I assume that investments in human capital take place only in the first period of life and involve children's time as well as expenditures on education quality following the recent works on human capital investment.[7] Specifically, the human capital is produced according to the following production function :

$$h = (s^{\eta} e^{1-\eta})^{\xi}, \tag{1}$$

where $s \in [0, 1]$ is the schooling time and can be interpreted as the fraction of time in the first period that is devoted to human capital investments, and $e > 0$ measures expenditures in education quality and is assumed to be in terms of labor. The parameter η is the share of schooling in human capital production, and ξ determines returns to scale.

Since the model economy is calibrated to match the data of human capital investment in an economy (South Korea) where public education is prevalent, it is essential to reflect the sizable share of public expenditure on education. This is modeled as an education subsidy to expenditures on education quality. It is assumed for simplicity that the subsidy is funded by

[7] See e.g., Erosa et al. (2010), Restuccia and Vandenbroucke (2013) and Manuelli and Seshadri (2014).

a lump-sum tax only on the young generation.[8]

Preferences and the Household Problem The preferences of an agent of generation τ are defined over life-time sequences of consumption, $\{c_t\}_{t=\tau}^{\tau+3}$, and the per-period utility is standard logarithm. The timing of individual decisions is described as follows. A newly born child of generation τ chooses schooling time s and expenditures in education quality e in the first period of life, which determine her permanent human capital h.

After schooling, the rest of the $(1-s)$ units of time in her first period as well as the two whole periods as adults are devoted to work to earn wages. I abstract from borrowing constraints and assume complete markets, so agents can make unconstrained life-cycle borrowing/saving decisions (a) to smooth lifetime consumption. Agents earn from the savings or pay the debt at the net interest rate (the rental price of capital minus depreciation) in the next period.[9] A retiree does nothing but consumes whatever is left in the retiring age. Under the perfect-foresight assumption, all decisions are made by agents when they are born once at a time, taking true prices in the future as given. Formally, the household problem for the cohort τ can be described as the following sequential problem:

$$\max_{s_\tau, e_\tau, \{c_t\}_{t=\tau}^{\tau+3}} \sum_{t=\tau}^{\tau+3} \beta^{t-\tau} log(c_t) \tag{2}$$

$$s.t. \quad c_\tau + a_\tau = h_\tau(\phi_1(1-s_\tau)w_\tau - (1-b)e_\tau w_\tau - T_\tau \tag{3}$$

$$c_{\tau+1} + a_{\tau+1} = (1+R_{\tau+1})a_\tau + h_\tau \phi_2 w_{\tau+1} \tag{4}$$

$$c_{\tau+2} + a_{\tau+2} = (1+R_{\tau+2})a_{\tau+1} + h_\tau \phi_3 w_{\tau+2} \tag{5}$$

$$c_{\tau+3} = (1+R_{\tau+3})a_{\tau+2} \tag{6}$$

$$h_\tau = (s_\tau^\eta e_\tau^{1-\eta})^\xi \tag{7}$$

$$s_\tau \in [0,1] \tag{8}$$

where ϕ_1, ϕ_2, ϕ_3 are life-cycle productivity parameters, and R is the interest rate. The left-hand sides of (3)-(6) are consumption plus borrowing or saving, while the right-hand sides are

[8] This assumption is without loss of generality because it is assumed below that the credit market is complete.

[9] Throughout, it is assumed that the net asset is supplied at the end of the period and sums up to the capital supply in the next period, so the return to the asset is the interest rate in the next period. This assumption ensures a smoother transition of the interest rate, as shown in Figure 4.

the sum of labor and capital income. The expenditures on education quality cost after subsidy $(1-b)e_\tau w_\tau$ and taxes T_τ are subtracted from the income of children.[10]

Production Technology I assume there is only one production sector that produces the only good in the economy for consumption and is used as the numeraire. The production technology for the competitive consumption good is a standard neoclassical technology that uses physical capital K and human capital H as inputs, with labor augmenting technology progress z. The production function is as follows:

$$Y_t(K_t, H_t) = K_t^\theta (z_t H_t)^{1-\theta}, \tag{9}$$

where z_t is the labor productivity in period t. The evolution of z_t obeys the following law of motion:

$$\frac{z_t - z_{t-1}}{z_{t-1}} = \underbrace{c}_{\text{constant}} \cdot \underbrace{g(H_t^s)}_{\text{externality of human capital}} \cdot \underbrace{\frac{F_{t-1} - z_{t-1}}{z_{t-1}}}_{\text{cath-up effect}} \tag{10}$$

This specification of productivity growth is motivated by Nelson and Phelps (1966) to capture two important mechanisms in a reduced form.[11] The first is a standard catch-up effect. Since F is the technology of the frontier growing at a constant exponential rate ($\frac{F_t - F_{t-1}}{F_{t-1}} = \lambda$), the term $\frac{F_{t-1} - z_{t-1}}{z_{t-1}}$ measures the relative gap of the follower productivity to the frontier. The larger the gap is in the last period, the greater the catch-up effect is, and so is the productivity growth in the current period. c is the parameter that governs the catch-up speed.

What is new here relative to the literature in recognition of the idea in Nelson and Phelps (1966) is that the technology diffusion could be made easier the higher the human capital stock is for a country. In the context of a catch-up framework, this translates to higher productivity growth as an economy accumulates more human capital. Therefore, a strictly increasing function g is added to complement the catch-up effect, so the growth rate of labor

[10] The price for expenditures on education quality e is the wage rate because expenditures on education quality are assumed to be in terms of labor. This is in line with the idea that the price of services depends on the wage rate in a model with education service sector, as in Erosa et al. (2010).

[11] Benhabib and Spiegel (1994, 2005) empirically test the role of the externality of human capital on productivity growth based on the specification very close to the one used here.

productivity is increasing in the amount of human capital supplied to this economy in the current period, H_t^s.[12] This introduces an externality where the human capital investment decisions by households affect the growth rate of aggregate productivity. The set-up augments standard growth models viewing human capital as just a conventional input that directly enters the neoclassical production process by taking the key idea into account that human capital in an economy can also facilitate the catch-up of technology.

To close the production side of the economy, a representative firm, taking the rental rate of physical and human capital r, w as given, runs the technology and maximizes the profit as follows:

$$\max_{K_t, H_t} \quad K_t^{\theta}(z_t H_t)^{1-\theta} - r_t K_t - w_t H_t \tag{11}$$

The aggregate physical capital depreciates at the rate δ and follows a standard law of motion:

$$K_{t+1} = (1-\delta)K_t + I_t, \tag{12}$$

where I_t is the aggregate investment in terms of the consumption good.

Definition of Equilibrium The equilibrium of the model consists of, at any date t, prices $\{w_t, r_t, R_t\}$, decision rules of household, $\{c_t^1, c_t^2, c_t^3, c_t^4, s_t^1, e_t^1, a_t^1, a_t^2, a_t^3, h_t^1, h_t^2, h_t^3, \}$, an associated human capital supply$\{H_t^s\}$, a technology frontier $\{F_t\}$, labor-augmenting productivity $\{z_t\}$, decision rules of the firm $\{K_t, H_t\}$ and a tax scheme by the government $\{T_t\}$ such that:

(1) Facing the prices $\{w_t, R_t\}$, the household's allocations $\{c_t^1, c_t^2, c_t^3, c_t^4, s_t^1, e_t^1, a_t^1, a_t^2, a_t^3, h_t^1, h_t^2, h_t^3, \}$ solve the problem (2)-(8) .

(2) Taking the frontier technology F_t and prices $\{w_t, r_t\}$ as given, the firm chooses physical and human capital $\{K_t, H_t\}$ that solve problem (11).

(3) The prices $\{w_t, r_t, R_t\}$ are such that the labor, capital and goods market clear:

$$H_t + e_t^1 = H_t^s = \phi_1(1 - s_t^1)h_t^1 + \phi_2 h_t^2 + \phi_3 h_t^3$$

[12] The dependency of productivity growth on human capital in the current period instead of the last can be justified by the fact that both productivity and human capital are stock variables that can be interpreted as the average over a certain period.

$$K_t = a^1_{t-1} + a^2_{t-1} + a^3_{t-1}$$

$$C_t + I_t + e^1_t w_t = Y_t$$

and the interest rate of asset is the rental price of capital minus depreciation: $R_t = r_t - \delta$.
(4) The technology frontier grows exogenously at a constant exponential rate λ ($\frac{F_{t+1}-F_t}{F_t} = \lambda$), with $F_0 = 1$ given. And the labor productivity evolves according to (10) given an initial condition z_0.
(5) The government runs a balanced budget:

$$T_t = bw_t e^1_t$$

4 Solution of the Balanced Growth Path

In this section, I solve for the BGP of the model, in which, at the aggregate level, the model economy grows at a constant exponential rate when some aggregate variables end up being constant following the transition. The constant growth rate is exactly the exogenous growth rate of frontier technology. Therefore, the frontier technology acts as the locomotive for the followers and determines their pace of growth in the BGP, in line with the results in Benhabib et al. (2014).

4.1 Solving for the Household Problem

To solve for the BGP of the model, it is essential to characterize the decisions of human capital investment for workers first, because the law of motion of labor productivity in (10) suggests that a steady state may not exist unless H^s_t is a constant. Ignoring the schooling constraint $s \in [0, 1]$ for now and assuming complete markets, the agent of generation τ maximizes lifetime utility subject to lifetime budget constraint:

$$\max_{s_\tau, e_\tau, \{c_t\}_{t=\tau}^{\tau+3}} \sum_{t=\tau}^{\tau+3} \beta^{t-\tau} log(c_t) \tag{13}$$

$$s.t.\quad c_\tau + \frac{c_{\tau+1}}{(1+R_{\tau+1})} + \frac{c_{\tau+2}}{(1+R_{\tau+1})(1+R_{\tau+2})} + \frac{c_{\tau+3}}{(1+R_{\tau+1})(1+R_{\tau+2})(1+R_{\tau+3})}$$
$$= h_\tau(\phi_1(1-s_\tau)w_\tau + \phi_2\frac{w_{\tau+1}}{1+R_{\tau+1}} + \phi_3\frac{w_{\tau+2}}{(1+R_{\tau+1})(1+R_{\tau+2})}) - (1-b)e_\tau w_\tau - T_\tau \quad (14)$$

$$h_\tau = (s_\tau^\eta e_\tau^{1-\eta})^\xi \quad (15)$$

Assuming an interior solution for s_τ^y, the FOCs for the decisions of schooling and expenditures on education quality for the young generation are:

$$s_\tau : \xi(s_\tau^\eta e_\tau^{1-\eta})^{\xi-1} e_\tau^{1-\eta}\eta s_\tau^{\eta-1}(\phi_1(1-s_\tau)w_\tau + \phi_2\frac{w_{\tau+1}}{1+R_{\tau+1}} + \phi_3\frac{w_{\tau+2}}{(1+R_{\tau+1})(1+R_{\tau+2})}) = h_\tau\phi_1 w_\tau \quad (16)$$

$$e_\tau : \xi(s_\tau^\eta e_\tau^{1-\eta})^{\xi-1}(\phi_1(1-s_\tau)w_\tau + \phi_2\frac{w_{\tau+1}}{1+R_{\tau+1}} + \phi_3\frac{w_{\tau+2}}{(1+R_{\tau+1})(1+R_{\tau+2})}) = w_\tau \quad (17)$$

Condition (16) and (17) simply equate the marginal benefits of schooling and expenditures on quality to associated marginal costs. The complete credit market assumption implies efficient consumption smoothing decisions designated by standard Euler equations: $u'(c_\tau^1) = \beta(1+R_{\tau+1})u'(c_{\tau+1}^2)$, $u'(c_{\tau+1}^2) = \beta(1+R_{\tau+2})u'(c_{\tau+2}^3)$ and $u'(c_{\tau+2}^3) = \beta(1+R_{\tau+3})u'(c_{\tau+3}^4)$. With log utility ($u(c) = log(c)$), the consumption decisions satisfy $c_{\tau+1}^2 = \beta(1+R_{\tau+1})c_\tau^1$, $c_{\tau+2}^3 = \beta(1+R_{\tau+2})c_{\tau+1}^2$ and $c_{\tau+3}^4 = \beta(1+R_{\tau+3})c_{\tau+2}^3$.

One can verify that the human capital investment decisions can also be formulated as choosing schooling time (s) and expenditures on education quality (e) to maximize the present value of the lifetime earnings:

$$\max_{s,e}\quad h[\phi_1(1-s)w_\tau + \phi_2\frac{w_{\tau+1}}{1+R_{\tau+1}} + \phi_3\frac{w_{\tau+2}}{(1+R_{\tau+1})(1+R_{\tau+2})}] - (1-b)w_\tau e - T_\tau \quad (18)$$

$$s.t.\quad h = (s^\eta e^{1-\eta})^\xi \quad (19)$$

4.2 Characterization of the BGP

To characterize the BGP of the model with the household problem solved, consider the evolution of labor productivity in (10). It is crucial to define $x = \frac{z}{F}$ to be the productivity relative to the frontier, with the following law of motion:

$$x_t = (\frac{g(H_t^s)c(1 - x_{t-1}) + 1}{1 + \lambda})x_{t-1} \tag{20}$$

Taking H^s as fixed, if the steady state for the system of relative productivity in (20) does exist under appropriate parameterization, x has a globally stable steady state that satisfies:

$$x = \frac{g(H_t^s)c - \lambda}{g(H_t^s)c} \tag{21}$$

Then in the BGP, z will grow at the same rate as F. Also, K/Y has to be a constant in the BGP, i.e., the aggregate capital K grows at the same rate as the output Y. Since z grows at an exogenous rate, the economy would converge to a path on which K grows at the same rate as z (as in Solow (1956)). It can be verified that since the growth decomposition of the output is as follows:

$$g_Y = \theta g_k + (1 - \theta)g_z$$

when human capital in the economy is constant, $g_Y = \theta\lambda + (1-\theta)\lambda = \lambda$ and K/Y is a constant in the BGP. In addition, the rental rate of capital r will also be a constant along the BGP.

It remains to verify that H^s is a constant in the BGP specified above. To do this, I solve the household problem assuming the economy is in the BGP. Since the productivity and aggregate capital will grow in the BGP, the wage rate w_t is not a constant but changes over time, growing at the constant exponential rate of λ. It is observed that if I divide the FOCs (16) and (17) by w_τ, then the schooling and education quality expenditure decisions of agents depend only on the growth rate of the wage, but not the level. This is because the marginal benefits and costs of human capital investment decisions are all proportional to the wage rates. Then applying the condition in the BGP that $\frac{w_{t+1}}{w_t} = 1 + \lambda$, $\forall t$, the growth of wage rates drop out and boil down to a constant. Because of the overlapping generation nature of the model, the young cohort of any generation will face a constant growth of wage rate over the life-cycle once the economy hits the BGP. Therefore, the schooling and education quality expenditure decisions are independent of the wage and any other changes in the level of aggregate variables. Since the rental rate of capital is also constant in the BGP, the schooling time and expenditures on education quality will be constant for all generations in the BGP.

Then, under appropriate parameterization, a unique solution exists for human capital investment s and e in the BGP, which determines a constant human capital stock in the economy. The constant human capital stock H^s in the BGP in turn rationalizes a constant x and thus a constant growth of z, as shown in (21).

4.3 Model Solution and the BGP

The solution of the model consists of, at any period, prices $\{w, r, R\}$, household decisions $\{s^1, s^2, s^3, e^1, e^2, e^3, c^1, c^2, c^3, c^4\}$, firm decisions $\{K, H\}$, a tax scheme by the government $\{T\}$ and aggregate variables $\{x, z, F, H^s, I, Y\}$ that can be solved using the full dynamics of the system in Appendix B.1.

Define $\tilde{a} = \frac{a}{F}$ as the de-trended variable that is normalized by F at the same period. The solution for the BGP of the model consists of variables $\{x, s, e, H, H^s, r, R\}$ that are constant in the BGP and variables $\{w, K, Y, C, I, T\}$ that are growing at the constant exponential rate λ, such that their de-trended counterparts $\{\tilde{w}, \tilde{K}, \tilde{Y}, \tilde{C}, \tilde{I}, \tilde{T}\}$ are constant in the BGP. The details of the BGP conditions can be found in Appendix B.2.

5 Quantitative Results

This section reports the quantitative results for the benchmark model defined in Section 3. The benchmark economy is calibrated to a typical Asian growth miracle that exhibits strong catch-up in economic welfare, South Korea. The transition dynamics are then examined to simulate the convergence path of the model economy when it is shocked by a permanent opportunity of technology diffusion. The quantitative results suggest a critical role of including the externality of human capital on productivity growth in matching the transition dynamics of output for the follower in the model with the empirical counterpart.

5.1 Calibration

The model is calibrated to match the Korean economy from 1960 to 2019, with the U.S. economy serving as the technology frontier in the quantitative study. The motivation for this strategy is that Korea has experienced a significant catch-up in economic welfare post World War II and is typically recognized as a successful example of convergence clubs after it experienced a surge in trade and initiated a series of opening policies starting from the 1960s.[13]

The parameters to set include those related to household decisions, human capital production, goods production technology, and those that govern the path for the catch-up in output. The functional form for $g(H_t^s)$ that disciplines the dependency of catch-up in productivity oh human capital is set to have constant elasticity: $g(H_t^s) = (H_t^s)^\gamma$, so the growth rate of productivity in (10) becomes

$$\frac{z_t - z_{t-1}}{z_{t-1}} = c(H_t^s)^\gamma \frac{F_{t-1} - z_{t-1}}{z_{t-1}}, \tag{22}$$

where γ is interpreted as the extent of the externality of human capital on productivity growth. Therefore, there are twelve parameters to set in the model: $\{\beta, \phi_1, \phi_2, \phi_3, b, \eta, \xi, \delta, \theta, \lambda, \gamma, c\}$.

I follow the literature (e.g., Erosa et al. (2010)) and set a standard value for the discount factor. This gives an annual individual discount factor of 0.9646, which gives rise to $\beta = 0.9646^{18}$ in the model with 18 years as a period. An annual depreciation rate of 6% is chosen, so $\delta = 0.67$. This depreciation rate results in an investment share of the output of 20% in the new BGP and is in line with what is observed in the data.

I set life-cycle productivity $\phi_1 = 1$ (normalization), $\phi_2 = 2$ and $\phi_3 = 1.8$ using estimates of experience premium from Mincer regression in Heckman et al. (2006). The value for the schooling time share in human capital production $\eta = 0.6$ is chosen from Erosa et al. (2010).

For a given value of η, the returns to scale on human capital production (ξ) is set to match the average years of schooling in Korea in 2015 (Barro and Lee (2013)). The value for the

[13] Connolly and Yi (2015) documented that one major area of reform after Park Chung Hee seized power in 1961 was trade policy. In the early 1960s, Korea eliminated tariffs for imported inputs and capital goods used to produce goods for export. In the next two decades, Korea engaged in broader trade liberalization policies (e.g., Advance Notice of Tariff Reduction in 1984) that resulted in a gradual reduction of general tariff rates from about 40 percent to 13 percent (SaKong and Koh (2010)). From the 1990s, Korea started opening markets in services and FDI.

Table 2: Parameter Calibration and Targets

Parameter	Value	Target
Discount Factor (β)	.9646[18]	Erosa et al. (2010)
Share of Schooling Time (η)	0.6	Erosa et al. (2010)
Life Cycle Productivities (ϕ_1, ϕ_2,ϕ_3)	1, 2, 1.8	Estimated experience premium from Mincer regression[1]
Depreciation (δ)	0.67	Annual Rate of 6%; Investment share of 20%
Capital Share in Production (θ)	0.33	Capital Income Share
Rate of Education Subsidy (b)	0.2	GDP Share of Public Education Expenditure of 2%
Returns to Scale on Human Capital Production (ξ)	0.44	Average Years of Schooling in Korea (2015)[2]
Growth Rate of Frontier Technology (λ)	0.43	Average Growth of GDP per capita in the U.S of 2 %
Externality of Human Capital on Productivity (γ)	1.5	Transition for Relative Output
Catch-up Parameter (c)	0.3	Ratio of Relative Output (2019/1960)

[1] Heckman et al. (2006).

[2] Barro and Lee (2013).

rate of education subsidy (b) targets a GDP Share of public education Expenditure of roughly 2% in Korea (World Bank). The growth rate of frontier technology is set to match an annual growth rate of GDP per capita in the U.S. of 2%. The capital share (θ) is set to equal 0.33 according to the standard parameterization for the Cobb-Douglas production technology.

What is new relative to the literature here is setting values for two parameters that discipline the catch-up for the benchmark economy relative to the frontier. Specifically, the externality of human capital on the growth of productivity (γ) and the constant catch-up speed parameter (c) are chosen to result in the best match for the ratio of relative output per adult of Korea to the U.S. (2019/1960) as well as the trajectory of the whole transition from 1960 to 2019. Table 2 summarizes the results for parameter calibration and targets.

5.2 Quantitative Results for the Benchmark Economy

In this section, I exhibit key quantitative results of the model calibrated to the Korean economy and, in particular, examine the transition dynamics of the convergence for the model economy. In the quantitative experiment, the follower economy (South Korea in practice) is assumed to start with an initial low steady state, which is constructed to represent the Korean economy before the shock occurred in 1960. This initial condition is characterized by a series of states, including x, s, e, as well as $\tilde{K}$, that fall far below the values in the new steady state that will be specified below.[14] In the context of the model parameter, the low steady state can be reconciled by a zero speed of convergence ($c = 0$), which means there is no opportunity for technology diffusion, and the technology frontier is immaterial.

The economy starts transiting to a high steady state (BGP) after it is shocked by the opportunity of technology diffusion and able to converge towards the frontier in productivity.[15] This is embodied by a positive speed of convergence ($c > 0$). This is intended to capture the idea that greater interaction with the rest of the world, such as increasing trade volume, makes convergence possible. The calibrated parameters described above would then determine the BGP to which the economy would ultimately converge, as defined in Section 4.3.

Figure 4 depicts four aspects of the transition for the benchmark economy: the relative output, average years of schooling, relative productivity, and interest rate. When an economy starts from x_0, s_0, e_0 and $\tilde{K}_0$ that are way below the terminal steady state, it will 'catch up' in productivity, physical and human capital. Those factors amount to a convergence in output, which is the main target of the quantitative experiment.

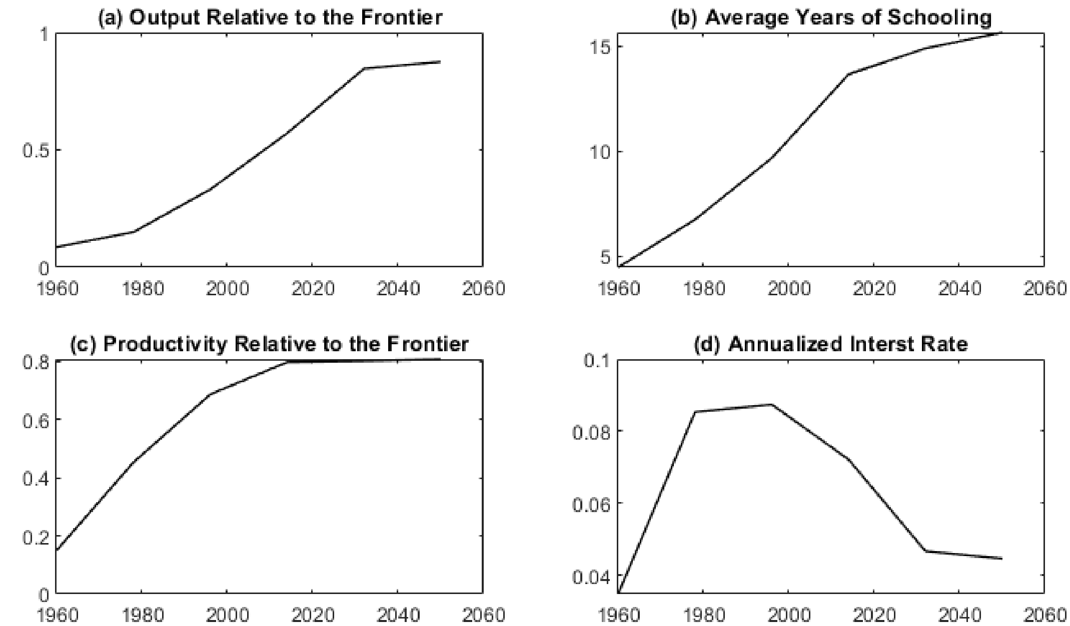
The path of the relative productivity in panel (c) features a monotonically declining rate of convergence along the transition. The growth rate of average years of schooling in the economy is mostly stable during the transition, mildly accelerating in the middle portion of the transition, as shown in panel (b).

There is, however, a 'S Shaped' convergence for the output of the follower relative to the frontier, as demonstrated in panel (a). This primarily results from the path of human capital supplied to production that is 'S Shaped' per se, as shown in Figure 5. The main intuition behind this result lies in the lagging nature of human capital investment. Since only the young cohort is able to adjust their education attainment when the shock comes, the spike in education attainment at the beginning of the transition leads to a temporary shortage of

[14] I assume the average years of schooling (s) in the initial year is the same as in the data in 1960 (4.5 years). $\tilde{K}$ in the initial year is set to be 1/9 of the level in the terminal steady state, close to the gap reported in the Penn World Table (PWT 10.0) between 2019 and 1960. The productivity relative to the frontier (x) starts at 0.15, and expenditures on education quality are assumed to be 1/30 as much in the terminal steady state. This parameterization results in output growth from 1960 to 2019 as observed in the data.

[15] This paper is agnostic about the exact form of the shock, but one can interpret it to be triggered by higher volume of trade, foreign direct investment and associated more interaction with of the world in technology, etc., as discussed in Section 1.

Figure 4: Transition Dynamics of the Benchmark Economy

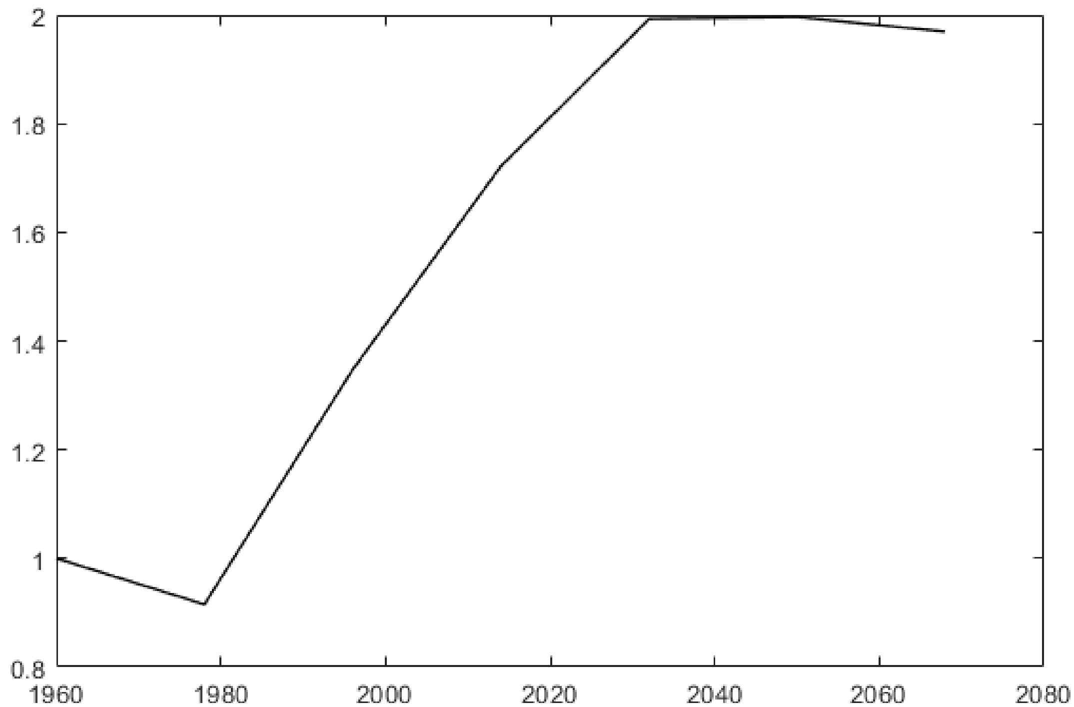


Note: The output relative to the frontier in panel (a) is normalized to start with what's in the data (PWT 10.0) in 1960. The average years of schooling in panel (b) in the initial year is the same as in the data (Barro and Lee (2013)) in 1960 (4.5 years). The initial capital stock is set such that the interest rate in the initial year in panel (d) is around 3.5%, which is a reasonable value.

human capital supplied to production because they spend more time in school. As a result, the supply of human capital mildly declines in the first model period. The human capital then rises sharply as the young cohort ages, which maps to faster growth in output in the middle phase of the transition. This 'S Shaped' convergence for the output is consistent with what's observed in the data and suggests that micro-founding the schooling decision has essential macroeconomic implications.

The transition dynamics of the output in the model also have important implications on the speed of convergence. The half-life for the transition of output is 30-35 years with realistic parameterization (capital share etc.) in this model, consistent with what's observed in the data, while it is typically less than 15 years in growth models that feature a monotonically declining growth rate of output like Solow (1956). The extended transition period in the model crucially depends on the inclusion of human capital investment, which converges at a slower rate than physical capital by nature. Last, the model fit for the transition of output with data

Figure 5: Human Capital Supplied to Production



Note: This figure plots the stock of human capital supplied to production, as defined by H in the model.

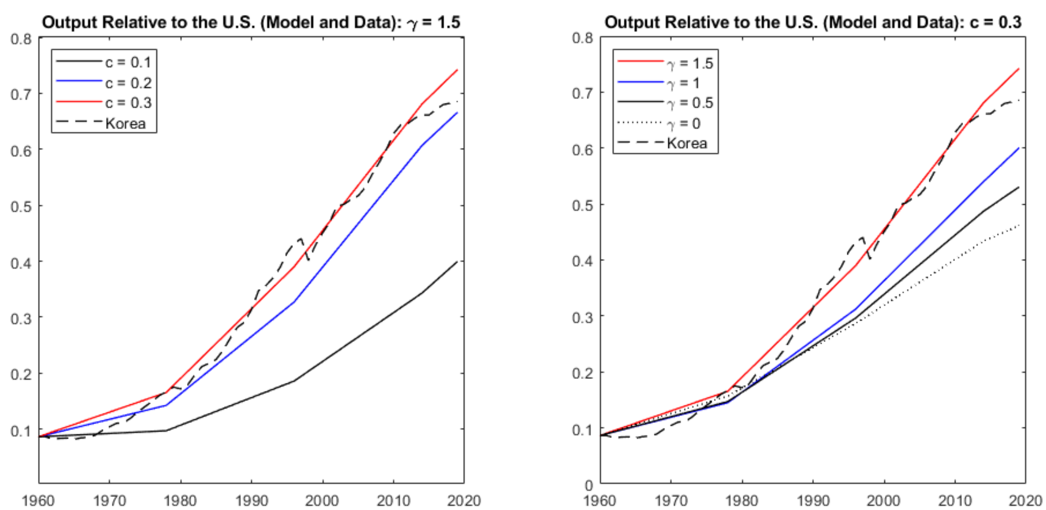
also critically depends on the externality of human capital on productivity growth, which is the essential new ingredient I add relative to the literature. I will leave the discussion of this to Section 5.3.

The other object of interest is the interest rate. Panel (d) of Figure 4 exhibits a hump-shaped interest rate along the transition, which reveals a relative scarcity of physical capital at the beginning of the transition when the positive shock just hits the economy and it faces an improvement in the prospect of investment. The excessive return on capital then diminishes as the economy transits and accumulates sufficient capital. The hump-shaped convergence pattern as well as the level of the interest rate are consistent with the observation in most emerging economies. To sum up, the broad implications of the model are in line with standard convergence theories. By introducing a shock (the opportunity of technology diffusion), the model is able to generate a transitory convergence of a follower economy towards the frontier.

5.3 The Externality of Human Capital on Productivity Growth

As discussed before, what is new in the model considered above relative to the literature is the inclusion of the dependency of productivity growth on human capital. The extent of the externality is governed by γ as shown in (22). The joint calibration in the benchmark gives rise to the combination of $\gamma = 1.5$ and $c = 0.3$ that allows the model to match perfectly with the data in the trajectory of relative output (see the red line in the left panel of Figure 6). This suggests a nontrivial role of the externality in disciplining the transition. Therefore, in what follows, I demonstrate the critical role the externality plays in generating the well-matched transition of the relative output of Korea to the U.S. with the data by comparing the benchmark to other cases where the two parameters (γ and c) are changed.

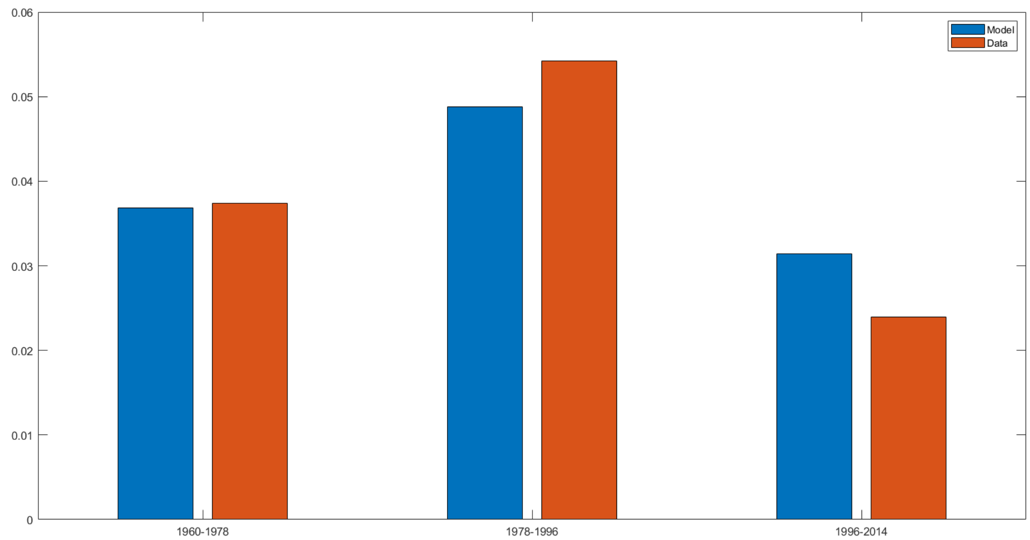
Figure 6: Model Fit of Output of Korea Relative to the U.S.



Note: The simulations with $\gamma = 1.5$ as in the benchmark and various relevant values of c are exhibited in the left panel. The right panel exhibits the case with $c = 0.3$ for various values of γ. The model results are compared to the relative GDP per capita data from Korea to the U.S. from the Penn World Table (PWT 10.0).

Figure 6 presents the model fit of data for the relative output of Korea to the U.S. with the various values of γ and c. For $\gamma = 1.5$ as in the benchmark, a series of relevant catch-up speed parameters c are examined for comparison in the left panel. The results suggest that for a fixed γ, the higher the c is, the more the seven-fold growth in relative output in the past six decades in Korea can be accounted for by the model. This is because a higher c can lead

Figure 7: Model Fit of Average Growth Rates of Relative Output

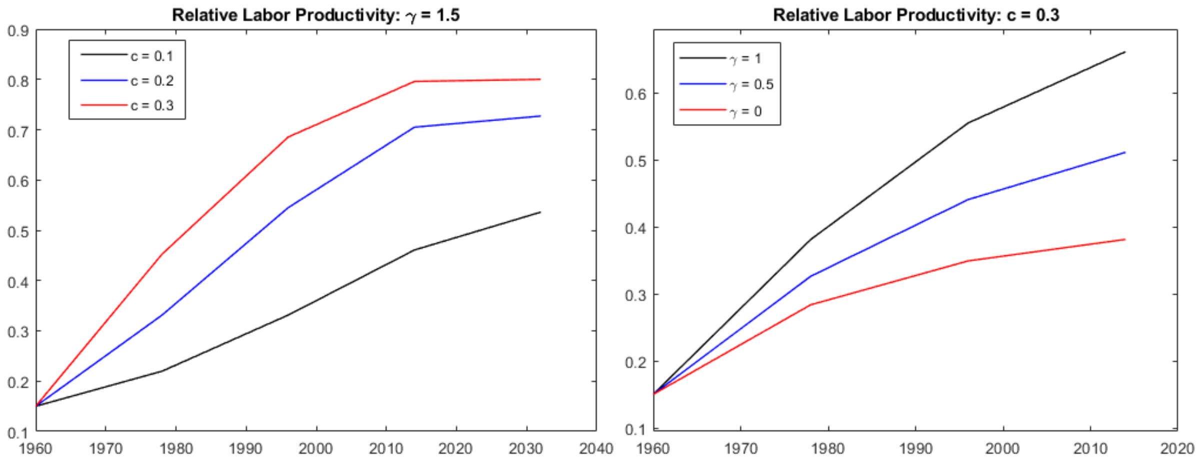


Note: ‘Model’ represents the (geometric) average growth rate of relative output in the benchmark economy to the frontier, which can be approximated by the growth rate of the benchmark economy minus 0.02 (the growth rate of the frontier); ‘Data’ is the empirical counterpart of the (geometric) growth rate of relative GDP per capita to the U.S., which can be can be approximated by the growth rate of Korea minus the growth rate of the U.S.

to faster convergence in relative productivity and a higher level in the BGP, as shown in the left panel of Figure 8. It maps into a faster convergence and higher steady-state level for the output. Therefore, an increase in c 'stretches' the trajectory and leads to higher output at each point of the transition, allowing the model to better explain the level of growth in output.

In the right panel, the simulated results with various levels of γ are considered for a fixed level of $c = 0.3$ as in the benchmark. Unlike the effects of c, it is observed that an increase in γ does not change the trajectory of the output much at the start of the transition. Then, the disparities widen as the economy approaches the end of the transition. This is because an increase in γ does not scale the productivity up in the beginning as much as an increase in c does, but affects it more later in the transition, as shown in the right panel of Figure 8. This result is closely related to the aforementioned lag in human capital investment, as shown in Figure 5. The intuition for this is that the effects of γ are not salient when human capital does not change a lot at the beginning. The effects scale up when human capital sharply rises later.

Figure 8: Productivity of the Benchmark Economy Relative to the Frontier



Note: Relative Productivity is defined in Section 4.2 as the productivity of the follower economy relative to the frontier technology. The simulations are the same as in Figure 6 and assumed to start from 0.15.

Therefore, in contrast to the impact of c, γ works to 'rotate' the later part of the transition and changes the curvature of the trajectory of the output.

The key takeaways from the experiments are that although an increasing c can help account for Korea's seven-fold growth in relative output in the past six decades, a nontrivial value of γ is essential to justify the trajectory of output in the data. It turns out that a combination of $\gamma = 1.5, c = 0.3$ is able to generate almost a perfect match of the model with data, both in level and trajectory of the transition. In particular, hump-shaped growth rates of output can result from it and match well with the data, as observed in Figure 7.[16] For lower levels of γ and c, either the level or the curvature is not matched well. This is how the whole transition dynamics is used to identify the key parameters of the model, and the calibration of γ and c in the benchmark economy is in this spirit.

The case of $\gamma = 0$ To make a sharper point on how the inclusion of the externality of productivity growth on human capital can help match the trajectory of the convergence in output, it would be interesting to consider the case where the externality if removed by setting

[16] An approximate relative output to the frontier can be obtained by subtracting 0.02 (the annualized growth rate of the frontier) from the growth rate of the follower economy in the model, and the difference between the growth rate GDP per capita in Korea and the U.S. provides the empirical counterpart.

Figure 9: Removing the Externality

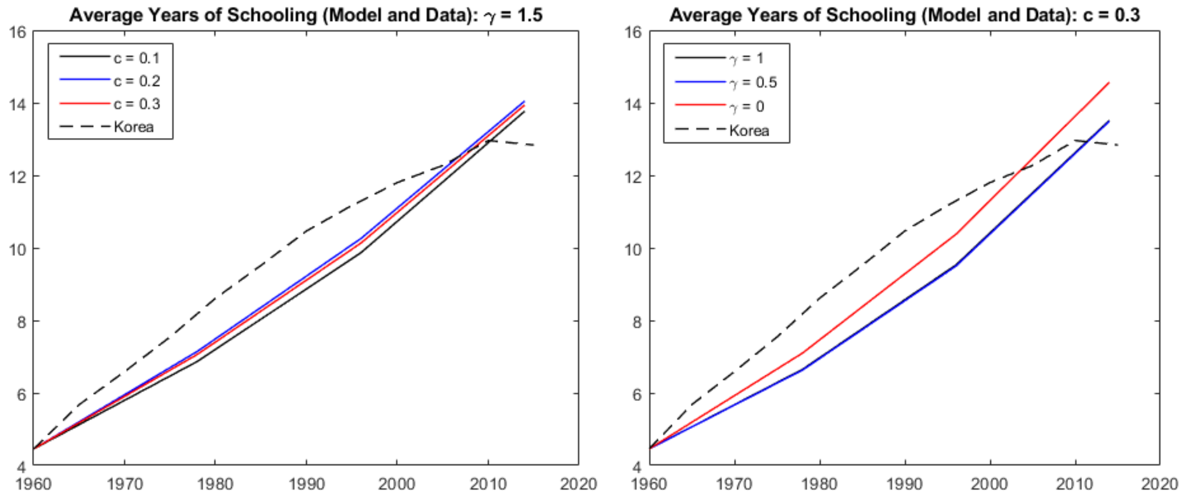
Note: The figure exhibits a counterfactual example where the externality of productivity growth on human capital is removed by setting $\gamma = 0$. An extreme value of $c = 0.98$ is used in this case. The left panel shows the model match in relative output with data as in Figure 6, and relative labor productivity as in Figure 8.

$\gamma = 0$. The dotted line in the right panel of Figure 6 exhibits this case when c is fixed at the benchmark value. It demonstrates that, ceteris paribus, the model can only account for 67% of the growth in the past six decades if the externality of human capital is removed.

As discussed above, the match in the level of the output growth can be affected by c, which is jointly calibrated with γ. Intuitively, allowing for a higher value of c can force the model to match the level of output growth. But problems immediately emerge if this is implemented. The left panel of Figure 9 shows that in the case where $\gamma = 0$, even though the model can almost account for the seven-fold growth in relative output in the past six decades with an extremely high value of $c = 1.2$, the curvature of the trajectory is poorly matched with data. Furthermore, with this value of c, the relative labor productivity converges almost in one period and suspiciously overshoots a lot, as shown in the right panel of Figure 9. This is obviously not a credible implication and demonstrates that imposing a higher value of c while removing the externality of human capital can not rationalize the observations from the data.

Education attainment and human capital The experiments above underscore the impact of the evolution of human capital on output growth, so it is useful to also examine the trajectory of education attainment. In contrast to the sensitive responses of the trajectories of output

Figure 10: Average Year of Schooling



Note: The data for the average years of schooling in Korea is obtained from Barro and Lee (2013). The simulations are the same as in Figure 6. The starting points are equal to the Korean data in 1960.

and productivity to the model parameters associated with the convergence in productivity, Figure 10 suggests that the transition path for the average years of schooling barely varies with those parameters. This lack of sensitivity is attributed to the fact that the decision for schooling only depends on the growth of wages. As a result, the schooling decision would not vary wildly as the economy scales up and down, leaving the growth rate of de-trended wages not changed much. Therefore, the path for the human capital shown in Figure 5 is rigid. This rigidity of human capital evolution is consistent with the observation in panel (b) of Figure 3 and suggests that the effect of human capital on output has to work through the interaction of human capital with other variables rather than the variation in human capital per se. This paper emphasizes the role of the externality of human capital on productivity growth in rationalizing the trajectory of output growth.

In sum, the experiments above demonstrate the critical role the externality of productivity growth on human capital plays in matching the trajectory of output in the model with data. While a higher value of c can 'stretch' the convergence path for output so that the seven-fold growth of the relative output can be accounted for, it is critical to take into account the externality of productivity growth on human capital to match the 'S Shaped' path of convergence for output while not imposing unreasonable speed of convergence for productivity.

It is important to exploit the endogenous force brought about by human capital, especially its externality on productivity growth, instead of allowing the constant c to be the dominant force.

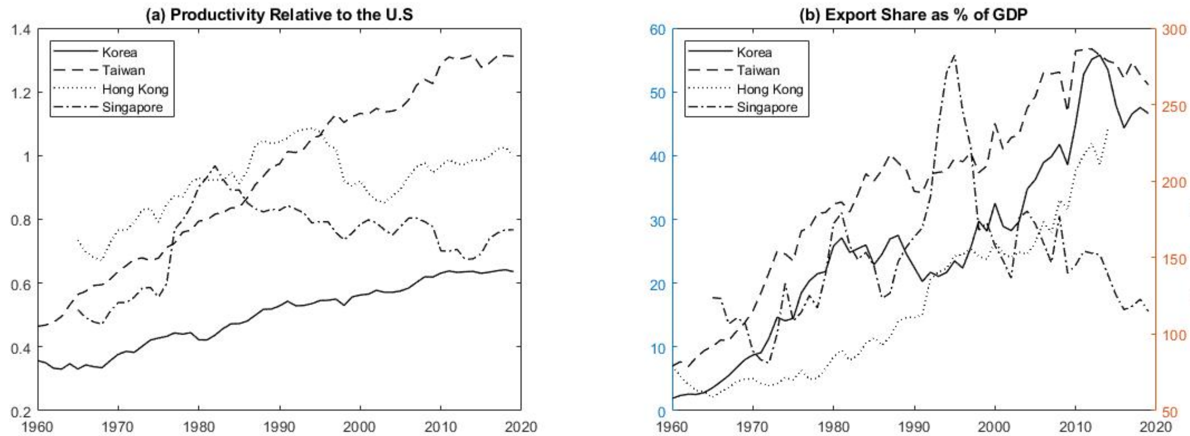
6 Conclusions

There has been a long-lasting debate on the quantitative importance of human capital in the convergence of income for emerging economies and it will continue to be a hot issue. Most existing works treat human capital as an ordinary input, but the Nelson-Phelps hypothesis suggests that this might cause misspecification and underestimate the role of human capital. This hypothesis is supported by the cross-country evidence of the positive correlation between productivity growth and education, as well as patterns of development for Asian growth miracles after WWII. In recognition of those facts, I build a standard growth model with individual human capital investment, featuring a dependency of convergence in productivity on human capital. The transition dynamics of the model are then examined to investigate the quantitative importance of the complementarity between technology diffusion and education, which is a unique approach that is rarely taken in the literature. The qualitative implications of the model are consistent with standard convergence theories and identify the opportunity of technology diffusion as a shock that is able to rationalize the transitory convergence of a follower economy towards the frontier. When the model is calibrated to a typical growth miracle (South Korea), the quantitative results suggest that it is critical to take into account the externality of human capital on productivity convergence. A significantly positive value for the extent of the externality of human capital on productivity growth is required to match the trajectory of convergence in output for the follower in the past six decades to the empirical counterpart. Once the externality is reduced or shut down in the experiments, the model can not quantitatively match the transition dynamics of the convergence in output for the Korean economy.

I leave for future work three extensions of the paper. First, identifying the externality of human capital has important policy implications on education subsidies. This is especially the case for an emerging economy actively adopting technology in a technologically progressive world. To investigate this further, I plan to introduce distortionary taxes instead of lump-sum taxes in the model to explore the welfare implications of education subsidies. Second, overlapping generations of agents in the model can be replaced by altruistic agents whose dynastic utility function depends on the utilities of all descendants (like in Becker and Barro (1988)). Then, optimal education subsidies can be obtained by solving the problem of the representative agent in the economy. Last, I assume in the model that productivity convergence depends on the aggregate stock of human capital in an economy. This is likely a simplification because the technology should be advanced by the research sector Romer (1990), which consists of researchers with higher education. Therefore, multiple levels of education can be modeled to address this, and it can also help us understand the evolution of different levels of education in the real world.

A Productivity Growth and Trade Volume in Asian Tigers

Figure 11: Productivity Growth and Trade Volume in Asian Tigers



Note: The source of data is Penn World Table (PWT 10.0). The productivity relative to the U.S. is obtained from TFP level at current PPPs (USA=1).

B Details of the Model Solution and BGP

B.1 Full Dynamics of the Model Solution

Dynamics for x:

$$x_t = (\frac{g(H_t^s)c(1 - x_{t-1}) + 1}{1 + \lambda})x_{t-1} \tag{23}$$

Dynamics for H:

To derive for H, first solve for decisions of schooling time and expenditures on education quality for the young cohort, s_t and e_t, as in (16) and (17):

$$s_t : \xi(s_t^\eta e_t^{1-\eta})^{\xi-1} e_t^{1-\eta} \eta s_t^{\eta-1}(\phi_1(1-s_t)w_\tau + \phi_2 \frac{w_{t+1}}{1 + R_{t+1}} + \phi_3 \frac{w_{t+2}}{(1 + R_{t+1})(1 + R_{t+2})}) = h_t \phi_1 w_t \tag{24}$$

$$e_t : \xi(s_t^\eta e_t^{1-\eta})^{\xi-1}(\phi_1(1 - s_t)w_t + \phi_2 \frac{w_{t+1}}{1 + R_{t+1}} + \phi_3 \frac{w_{t+2}}{(1 + R_{t+1})(1 + R_{t+2})}) = w_t \tag{25}$$

Denote the optimal decisions by s_t^1 and e_t^1. Similarly, the human capital investment decision of adults born at $t-1$ and $t-2$ can be obtained by inducting one and two period back for (24) and (25), and they are denoted by s_t^2, e_t^2, s_t^3, e_t^3. Therefore, the stock of human capital for the young and the old are $h_t^1 = (s_t^{1\eta} e_t^{1(1-\eta)})^\xi$, $h_t^2 = (s_t^{2\eta} e_t^{2(1-\eta)})^\xi$ and $h_t^3 = (s_t^{3\eta} e_t^{3(1-\eta)})^\xi$ separately. The total effective human capital supplied to production is equal to the demand H_t:

$$H_t + e_t^1 = \phi_1(1 - s_t^1)h_t^1 + \phi_2 h_t^2 + \phi_3 h_t^3 \tag{26}$$

Those measures of human capital will become a constant once the schooling choice is a constant for any generation in the BGP.

Dynamics for C:

Because of the complete markets assumption, the solution for the household problem suggests that the consumption path of a cohort is a series of efficient consumption smoothing decisions designated by standard Euler equations with log utility: $c_{t+1}^2 = \beta(1 + R_{t+1})c_t^1$, $c_{t+2}^3 = \beta(1+R_{t+2})c_{t+1}^2$ and $c_{t+3}^4 = \beta(1+R_{t+3})c_{t+2}^3$. Using this combined with individual budget constraint (14), the consumption of three cohorts becomes a function of lifetime earnings:

$$c_t^1 = \frac{h_t^1(\phi_1(1 - s_t^1)w_t + \phi_2 \frac{w_{t+1}}{1+R_{t+1}} + \phi_3 \frac{w_{t+2}}{(1+R_{t+1})(1+R_{t+2})}) - (1-b)e_t^1 w_t - T_t}{1 + \beta + \beta^2 + \beta^3} \tag{27}$$

$$c_t^2 = \beta(1 + R_t)\frac{h_t^2(\phi_1(1 - s_t^2)w_{t-1} + \phi_2 \frac{w_t}{1+R_t} + \phi_3 \frac{w_{t+1}}{(1+R_t)(1+R_{t+1})}) - (1-b)e_t^2 w_{t-1} - T_{t-1}}{1 + \beta + \beta^2 + \beta^3} \tag{28}$$

$$c_t^3 = \beta^2(1+R_{t-1})(1+R_t)\frac{h_t^3(\phi_1(1 - s_t^3)w_{t-2} + \phi_2 \frac{w_{t-1}}{1+R_{t-1}} + \phi_3 \frac{w_t}{(1+R_{t-1})(1+R_t)}) - (1-b)e_t^3 w_{t-2} - T_{t-2}}{1 + \beta + \beta^2 + \beta^3} \tag{29}$$

$$c_t^4 = \beta^3(1+R_{t-2})(1+R_{t-1})(1+R_t)\frac{h_t^4(\phi_1(1 - s_t^4)w_{t-3} + \phi_2 \frac{w_{t-2}}{1+R_{t-2}} + \phi_3 \frac{w_{t-3}}{(1+R_{t-3})(1+R_{t-2})}) - (1-b)e_t^4 w_{t-3} - T_{t-3}}{1 + \beta + \beta^2 + \beta^3} \tag{30}$$

The aggregate consumption at time t is the sum of them: $C_t = c_t^1 + c_t^2 + c_t^3 + c_t^4$.

Dynamics for K:

The dynamics for K is the standard law of motion

$$K_{t+1} = (1-\delta)K_t + I_t \tag{31}$$

All other variables can be expressed as:

$$z_t = x_t F_t \tag{32}$$

$$Y_t = K_t^{\theta}(z_t H_t)^{1-\theta} \tag{33}$$

$$w_t = \frac{\partial Y}{\partial H} = (1-\theta)K_t^{\theta} z_t^{1-\theta} H_t^{-\theta} \tag{34}$$

$$r_t = \frac{\partial Y}{\partial K} = \theta K_t^{\theta-1} z_t^{1-\theta} H_t^{1-\theta} \tag{35}$$

$$I_t = Y_t - C_t \tag{36}$$

$$T_t = b w_t e_t^1 \tag{37}$$

B.2 Solution for the BGP

Assume first all conditions needed for a BGP specified above hold except x, s, e and h (H^s, H). Equating x_t and x_{t+1} in (20) gives the steady state x, taking the supply of human capital H^s as given:

$$x = \frac{g(H^s)c - \lambda}{g(H^s)c} \tag{38}$$

Then, since all other variables are assumed to be in the BGP, it can be imposed that wages grow at a constant rate λ ($\frac{w_{t+1}}{w_t} = 1 + \lambda\,, \forall t$) and interest rates are constant $((R_t = R\,, \forall t))$.

The individual human capital investment decisions do not depend on time-varying variable, so become invariant across cohorts. They can be solved from the following equations:

$$s : \xi(s^{\eta}e^{1-\eta})^{\xi-1}e^{1-\eta}\eta s^{\eta-1}(\phi_1(1-s)+\phi_2\frac{1+\lambda}{1+R}+\phi_3\frac{(1+\lambda)^2}{(1+R)^2}) = h\phi_1 \tag{39}$$

$$e : \xi(s^{\eta}e^{1-\eta})^{\xi-1}(\phi_1(1-s)+\phi_2\frac{1+\lambda}{1+R}+\phi_3\frac{(1+\lambda)^2}{(1+R)^2}) = 1 \tag{40}$$

Denoting the optimal decisions by s and e, the total effective human capital supplied to the economy in the BGP is H^s:

$$H^s = (\phi_1(1-s)+\phi_2+\phi_3)(s^{\eta}e^{1-\eta})^{\xi}, \tag{41}$$

and the total effective human capital used in production is

$$H = H^s - e \tag{42}$$

Departing from those steady states, I maintain the assumption that K satisfies the BGP condition above and establish below all other variables satisfy the conditions assumed above. Recall that the production function is $Y = K^{\theta}(zH)^{1-\theta}$. Rewrite it in the de-trended form by dividing both sides by F:

$$\tilde{Y} = \tilde{K}^{\theta}(xH)^{1-\theta} \tag{43}$$

We know above x and H are constant in the BGP, so $\tilde{Y}$ is also constant given that $\tilde{K}$ is constant. Then consider the wage rate in the de-trended form:

$$\tilde{w} = \frac{\partial Y}{\partial H} = (1-\theta)\tilde{K}^{\theta}x^{1-\theta}H^{-\theta}, \tag{44}$$

which demonstrates that $\tilde{w}$ is constant in the BGP. Therefore, w will grow at the constant rate λ. This is an immediate result of constant human capital supply in the BGP. It remains to check the conditions from the household side (C and I). Recall from (27)-(30) that $C_t = c_t^1+c_t^2+c_t^3+c_t^4$

and note that each component of C_t grows at the constant rate λ and so is a constant in de-trended form:

$$\tilde{c}^1 = \frac{h^1(\phi_1(1-s^1)\tilde{w} + \phi_2\frac{\tilde{w}(1+\lambda)}{1+R} + \phi_3\frac{\tilde{w}(1+\lambda)^2}{(1+R)^2}) - (1-b)e^1\tilde{w} - \tilde{T}}{1+\beta+\beta^2+\beta^3} \tag{45}$$

$$\tilde{c}^2 = (\beta(1+R))\frac{h^2(\phi_1(1-s^2)\tilde{w} + \phi_2\frac{\tilde{w}(1+\lambda)}{1+R} + \phi_3\frac{\tilde{w}(1+\lambda)^2}{(1+R)^2}) - (1-b)e^2\tilde{w} - \tilde{T}}{1+\beta+\beta^2+\beta^3} \tag{46}$$

$$\tilde{c}^3 = (\beta(1+R))^2\frac{h^3(\phi_1(1-s^3)\tilde{w} + \phi_2\frac{\tilde{w}(1+\lambda)}{1+R} + \phi_3\frac{\tilde{w}(1+\lambda)^2}{(1+R)^2}) - (1-b)e^3\tilde{w} - \tilde{T}}{1+\beta+\beta^2+\beta^3} \tag{47}$$

$$\tilde{c}_4 = (\beta(1+R))^3\frac{h^4(\phi_1(1-s^4)\tilde{w} + \phi_2\frac{\tilde{w}(1+\lambda)}{1+R} + \phi_3\frac{\tilde{w}(1+\lambda)^2}{(1+R)^2}) - (1-b)e^4\tilde{w} - \tilde{T}}{1+\beta+\beta^2+\beta^3} \tag{48}$$

As a result, the aggregate C_t grows at λ: $C_{t+1} = (1+\lambda)C_t$, and the de-trended aggregate consumption $\tilde{C}$ is a constant that is equal to the sum of the de-trended components: $\tilde{C} = \tilde{c}_1 + \tilde{c}^2 + \tilde{c}^3 + \tilde{c}_4$. Then, since $\tilde{C}$ and $\tilde{Y}$ are constant as shown above, the de-trended aggregate resource constraint $\tilde{Y} = \tilde{C} + \tilde{I}$ implies that $\tilde{I}$ must be a constant, so I grows at the rate of λ in the BGP. Last, the aggregate tax can be related to the wage using the government budget balance condition:

$$\tilde{T} = b\tilde{w}e \tag{49}$$

Up until now I've shown that if $\tilde{K}$ is a constant in the BGP, the model has a solution for the BGP. In addition, aggregate variables that stay constant in the BGP $\{r, R, \tilde{w}, \tilde{Y}, \tilde{C}, \tilde{I}\}$ can be expressed as functions of the primitive $\tilde{K}$. Therefore, once we find the value of $\tilde{K}$ that the physical capital converges to, the model can be analytically characterized. Recall that the dynamics for K is:

$$K_{t+1} = (1-\delta)K_t + I_t \tag{50}$$

The law of motion for $\tilde{K}$ is therefore

$$\tilde{K}_{t+1}(1+\lambda) = (1-\delta)\tilde{K}_t + \tilde{I}_t, \tag{51}$$

which gives in the steady state:

$$\tilde{I} = (\lambda + \delta)\tilde{K} \tag{52}$$

$\tilde{I}$ can be substituted from the resource constraint $\tilde{I} = \tilde{Y} - \tilde{C}$. Substitute $\tilde{Y}$ and $\tilde{C}$ and it is obtained that:

$$\tilde{I} = \tilde{K}^{\theta}(xH)^{1-\theta} - \tilde{C} \tag{53}$$

Since x and H can be solved from (38) and (42) , and $\tilde{w}$, R are also functions of $\tilde{K}$, equation (52), combined with (53) gives a non-linear equation of $\tilde{K}$, so a steady state $\tilde{K}$ exists and all other steady state variables can be established accordingly. In sum, given the steady state $\tilde{K}$, the BGP of the model can be characterized by constant variables $\{s, e\}$ that can be solved from (39) and (40) and $\{x, h, H, H^s, r, R, \tilde{w}, \tilde{K}, \tilde{Y}, \tilde{C}, \tilde{I}, \tilde{T}\}$ that satisfy:

$$x = \frac{g(H^s)c - \lambda}{g(H^s)c} \tag{54}$$

$$h = (s^{\eta}e^{1-\eta})^{\xi} \tag{55}$$

$$H^s = (\phi_1(1-s) + \phi_2 + \phi_3)h \tag{56}$$

$$H = H^s - e \tag{57}$$

$$\tilde{w} = \frac{\partial Y}{\partial H} = (1-\theta)\tilde{K}^{\theta}x^{1-\theta}H^{-\theta} \tag{58}$$

$$r = \frac{\partial Y}{\partial K} = \theta\tilde{K}^{\theta-1}x^{1-\theta}H^{1-\theta} \tag{59}$$

$$R = r - \delta \tag{60}$$

$$\tilde{Y} = \tilde{K}^{\theta}(xH)^{1-\theta} \tag{61}$$

$$\tilde{C} = \frac{h(\phi_1(1-s)\tilde{w} + \phi_2\frac{\tilde{w}(1+\lambda)}{1+R} + \phi_3\frac{\tilde{w}(1+\lambda)^2}{(1+R)^2}) - (1-b)e\tilde{w} - \tilde{T}}{1+\beta+\beta^2+\beta^3}(1+\beta(1+R)+\beta^2(1+R)^2+\beta^3(1+R)^3) \tag{62}$$

$$\tilde{I} = (\lambda + \delta)\tilde{K} \tag{63}$$

$$\tilde{T} = b\tilde{w}s \tag{64}$$

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