Capital financing constraints, size-dependent distortions, and aggregate productivity

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Abstract

We document that firms in developing economies have *limited excess to external financing* and face *size-dependent distortions.* The former friction impedes the growth of small firms, while the latter restricts the operations of relatively large firms. This paper studies how these two frictions interact. It is well understood that in a setting in which firms have unlimited access to capital financing, size-dependent distortions necessarily reduce aggregate output. Our work demonstrates that the effects of size-dependent distortions crucially depend on firms' ability to access external capital. We show that the adverse effects associated with size-dependent distortions drastically reduce, and may even reverse, if firms face capital financing constraints. This occurs because the misallocation effects of capital financing constraints and size-dependent distortions may offset each other, and because the two frictions have opposing impacts along the extensive margin impacting the number and the composition of firms. Our quantitative analysis shows that the size-dependent distortions estimated using the World Bank Enterprise Survey data lead to up to 25 percent of output drop if they are implemented in an economy in which firms face unlimited excess to external financing, but have virtually no effect on aggregate output in the presence of capital financing constraints consistent with the levels of capital-output ratios for most low-income countries. These findings have implications for understanding the cross-country income differences, as well as for policy design.

Keywords: size-dependent distortions, capital financing constraints, misallocation, firm dynamics, cross-country income differences

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1 Introduction

The objective of this paper is to analyze the interaction between the two economic frictions - financing constraints and size-dependent distortions - that have been, in isolation, extensively discussed in economic literature. Each of these frictions is believed to be important for understanding cross-country productivity and income differences because, on its own, each of them causes misallocation of resources, thereby reducing aggregate productivity. At the same time, *both* of these frictions are commonly present in developing economies: it is well-documented that access to external financing is positively correlated with income per capita,[1] and there is some evidence demonstrating that, in less developed economies, large firms face more restrictions, either through direct taxes or via various indirect barriers.[2] We also independently document in this paper that more severe financing constraints and size-dependent distortions tend to coexist in developing economies. Thus, it is crucial to understand how these two frictions interact with each other and whether their interaction has any important policy implications.

Intuitively, size-dependent distortions - which directly or indirectly impose taxes on larger firms - may help offset the misallocation due to limited excess to external financing, both along the intensive and extensive margins. Along the intensive margin, conditional on firms' productivity level, financing constraints lead to misallocation of resources away from relatively small constrained firms toward relatively large unconstrained firms. In contrast, size-dependent distortions, by their nature, lead to misallocation of resources away from larger towards smaller firms. Thus, size-dependent distortions may help offset, at least to some extent, the misallocation caused by financing constraints by reallocating resources from larger to smaller firms within the same productivity class. At the same time, size-related distortions generate an additional adverse effect by misallocating resources away from more productive toward less productive firms, which does not help in alleviating the impacts of firm financing constraints.

Along the extensive margin, financing constraints discourage entry because their primary negative impact is on young constrained firms, while the older firms that build up assets over time become bigger than in the unconstrained economy. Size-dependent distortions, in contrast, may encourage entry by redistributing resources towards young smaller firms. This may help correct the extensive margin inefficiencies created by financing constraints.

In other words, size-dependent distortions may potentially help offset some of the inef-

[1] See, for example, Levine (2005), for extensive review.

[2] See, for example, Hsieh and Klenow (2009) and Bento and Restuccia (2017).

ficiencies created by firms' capital constraints, but whether this occurs, or how significant this effect is, ultimately becomes a quantitative question because it would likely depend on a number of factors, such as the extent of heterogeneity in firm productivity, the parameters of the process guiding its evolution and, of course, the severity of the financing constraints and the extent of size-dependent distortions.

To quantitatively explore the interaction between these two frictions, we incorporate capital financing constraints into an otherwise standard equilibrium firm dynamics model with endogenous entry and exit as in Hopenhayn (1992). We calibrate the model to match the firm dynamics data in the U.S., and then analyze how the impact of size-dependent distortions on the efficiency of resource allocation varies with the severity of capital financing constraints. We find that the presence of capital financing constraints considerably weakens the adverse effects of size-dependent distortions on output and, in some cases, the introduction of size-dependent distortion may result in an *increase* in output. For example, in an economy in which firms face a self-financing constraint that results in 50% and 33% drop in steady aggregate capital and output, respectively (and, as a result, reduces the capital-output ratio by 35 percent), introducing size-dependent distortions can increase output by up to 10 percent. For more severe financing constraints, the potential output gains from size-dependent distortions are even bigger. In contrast, in the absence of any financing constraints, size-dependent distortions necessarily decrease output.

To evaluate the empirical relevance of the mechanism studied in the paper, we use data from the World Bank Enterprise Survey (WBES) to estimate the extent of size-dependent distortions (namely, the revenue elasticity of the distortionary tax) across 147 countries. We then compute the effects of these distortions in the calibrated economy in the presence of a wide range of capital financing constraints that result in capital-output ratios consistent with the values reported in the Penn World Table, and compare these effects with their counterparts in the economy without financing constraints. We find that for the revenue elasticities of the distortionary tax below 0.3 (which is the case for 93 out of 147 countries in the WBES data), such distortionary tax has virtually no negative effect on output (or even increases output) if capital financing constraints are such that the resulting capital-output ratio is below 2 (which holds for the overwhelming majority of low- and middle-income countries in the Penn World Table data). For larger distortions, the negative effects on output are much smaller in the presence of empirically-relevant capital financing constraints than in the economy without any limits on capital financing. For instance, if the revenue elasticity of distortionary tax is below 0.4 (which is the case for 125 out of 147 countries in the WBES data) and capital financing constraints result in capital-output ratio below 2, the negative effect of distortions on output is at least five times smaller in the presence of the financing constraints than without them (up to 5 percent in capital-constrained economies economy versus 25 percent in the unconstrained economy).

These results suggest that accounting for the presence of firm financing constraints is key for understanding the role of size-dependent distortions in explaining the cross-country income differences, or for designing the tax policy reforms. In particular, they imply that because the extent of size-dependent distortions tends to be correlated with the severity of financing constraints, size-dependent distortions per se might be a less important vehicle of understanding the income differences across countries than previously thought. Second, our results point to a complementarity in policies aimed at reducing financing constraints or size-dependent distortions. Namely, they indicate that reducing size-dependent distortions may have little or no impact on aggregate output if these reforms are not accompanied by the policies improving firms' access to external financing.

Contribution to the Literature: First and foremost, this work contributes to the literature that studies in isolation the effects of each of the two frictions – firm financing constraints and size-dependent distortions – the interaction between which we analyze. Namely, there is extensive literature that analyzes the effects of firm financing constraints on aggregate outcomes. Banerjee and Duflo (2005) and Levine (2005) survey the literature documenting the evidence of credit constraints in poor countries and discuss why credit constraints may impede growth. Buera et al. (2011), Moll (2014), Midrigan and Xu (2014), Greenwood et al. (2013), Gorodnichenko and Schnitzer (2013), Caggese (2018) and Vereshchagina (2021), along with many others, quantify the impact of firm financing constraints on aggregate output and productivity and discuss various amplification channels of these effects. At the same time, numerous studies discussed the effects of size-dependent distortions and various features that amplify their effects. Restuccia and Rogerson (2008) and Guner et al. (2008) are among the first to quantify the factor misallocation effects of size-dependent distortions on output and productivity, while Bartelsman et al. (2013), Hsieh and Klenow (2014), Guner et al. (2018), Bruggemann (2021), and many others enrich the analysis by introducing additional margins and features. The objective of our work is to bridge these two streams of literature and analyze how the two specific frictions interact with each other. To our knowledge, this has not yet been done in previous works.

Our paper highlights that size-dependent distortions may be beneficial (or not as damaging as previously thought) because they help reduce the between-firms dispersion in marginal products of capital induced by the presence of capital financing constraints. Such benefits come at the extent of creating/magnifying a dispersion between the marginal products of labor triggered by the size-dependent taxes. Obviously, removing all the dispersion in all the marginal products, as it is done, for example, in a seminal paper by Hsieh and Klenow (2009), would result in output increase. However, removing only part of dispersion (e.g., by implementing a particular reform) can potentially impact the extent of the remaining dispersion. This paper describes one situation when this might occur: removing the dispersion in the marginal products of labor generated by size-dependent taxes may simultaneously magnify the dispersion of marginal products of capital arising due to capital financing constraints, thereby weakening the benefits of the former reform.

Our work is also related to a recent paper by Itskhoki and Moll (2019), which studies optimal Ramsey policies in a standard growth model with financial frictions. It suggests that an optimal policy intervention in emerging economies should involve pro-business policies that shift resources towards entrepreneurs. This helps suppress wages in early stages of the transition and could result in higher profits and faster wealth accumulation for entrepreneurs. Similar mechanism is emphasized in our paper: size-dependent distortions suppress wages and increase the profits of constrained firms, which, in turn, allows them to accumulate assets faster and overcome the financing constraints. Thus, size-dependent distortions play in our environment a role similar to pro-business policies in Itskhoki and Moll (2019), but rather than benefiting all the firms at the current stage of development, they only benefit a subset of firms who are most capital-constrained in our model, and these gains are partially offset by the losses incurred due to the effects of size-dependent distortions on the firms that are relatively less constrained.

More broadly, our work contributes to broader literature that emphasizes the importance of understanding the interaction between different features for policy design. For example, there is active research on the magnitude of top marginal income tax rates and it is found that large rates are required to maximize welfare (Conesa and Krueger (2006), Diamond and Saez (2011), Bakış et al. (2015), Badel et al. (2020), Bruggemann (2021), Kindermann and Krueger (2022)). It is interesting to note that most of the quantitative works emphasize the general equilibrium effects of changes in factor prices, a mechanism that is also important in our work. Another example of work studying the interaction between financing constraints and a policy is Brooks and Dovis (2020), which finds that gains from trade liberalizations could differ across forms of credit market frictions prevailing in an economy.

Paper organization: The paper is organized as follows. Section 2 provides empirical evidence showing that financing constraints and size-dependent distortions tend to coexist. Section 3 develops a simple illustrative example demonstrating why size-dependent distortions can help alleviate the inefficiencies created by capital financing constraints. Section 4 develops the full dynamic model and Section 5 calibrates it to match the features of the U.S. firm dynamics data. Section 6 analyzes how the effects of size-dependent distortions are impacted by the presence of capital financing constraints, explores which features may impact the extent of this interaction, and investigate quantitative implications in empirically-relevant environments. The last section concludes and discusses some avenues for related future research.

2 Motivating facts: financial frictions and size-dependent distortions across countries

It has been well documented that various measures of financial development are correlated with economic development. For example, the left panel of Figure 1 illustrates that the private credit as a share of GDP – which is one commonly used indicator of financial development[3] – is positively correlated with GDP per capita across countries, and the correlation coefficient is 0.49. Also, It is well understood that having limited excess to external financing is associated with lower capital-output ratio (Caselli (2005)). The right panel of Figure 1 demonstrates the positive correlation between capital-output ratio and GDP per capita, with correlation coefficient 0.58. Furthermore, in Figure 4 of Appendix A, we provide additional evidence on the financial development across income levels using firm-level data from the World Bank Enterprise Surveys (WBES), which suggests that firms in poor countries tend to be more constrained in getting access to external financing.

Additionally, some preceding studies have also argued that firms in less developed countries face bigger size-dependent distortions which, via different policies or features of economic environment, impose constraints, restrictions, or additional costs on relatively large firms.[4] In what follows, we construct a measure of size-dependent distortions using establishment-level data from the WBES that is consistent with how we model distortions

[3]See, for example, Arellano et al. (2012) or Gorodnichenko & Schnitzer(2013).
[4]See, for example, Hsieh and Klenow (2009) or Bento and Restuccia (2017).

Figure 1: Private-Credit-to-GDP Ratio and Capital-GDP Ratio vs. GDP per capita across countries

Note: The private credit-to-GDP ratio is obtained from the World Bank. For most countries we use data in 2018 before Covid-19, but for few countries for which 2018 is unavailable, we use the closest year before 2018. The capital-GDP ratio is the average level from 2001 to 2019. It is obtained from the Penn World Table (PWT 10.0), and we estimate the capital stock from the data following the procedure as in Greenwood et al. (2013). The full country level data can be found in Table 10 in the Appendix B.

in our main analysis in Sections 4 - 6. We show that this measure is also negatively correlated with income per capita across countries.

WBES is an ongoing project of the World Bank to collect standardized establishment-level information from more than 150 countries with specific emphasis on low- and middle-income countries. In each country, between 150 and more than 1,000 establishments have been surveyed for at least one year since 2002, and the surveys are intended to make these samples representative of the population of establishments with at least five employees. The data we use come from the newly combined dataset from 2006 to 2019. For establishments in countries with several years of data, we suppress the time dimension and treat them as cross-section. From this dataset, we use information on sales revenue, total wage bill, and industry classification for each establishment.

We measure distortion $\widehat{\tau}_i$ in country i by estimating the following regressions:

$$ln(LC_{ij}) = a_i + (1 - \widehat{\tau}_i)\, ln(SR_{ij}) + \beta\, Ind_{ij} + \mu_{ij}, \tag{1}$$

where LC_{ij} and SR_{ij} are the total wage bill and the sales revenue of establishment j in country i, Ind_{ij} is the industry dummy, a_i is the country-specific constant term, and μ_{ij} is the establishment-specific error term that is i.i.d across establishments.[5]

[5]We classify establishments in the survey into three coarse industries – manufacturing, services and 'other' – using the classification taken by the WBES data. This is done to account for potential differences

Intuitively, under homothetic production technology, labor share would be constant across firms in the absence of size-dependent distortions. If, however, labor share systematically varies across firms of different sizes, this might be indicative of systematic size-dependent distortions that can take the form of size-dependent taxes, restrictions, regulations, etc. To formally see this, suppose that the economy is populated by firms operating production technology

$$y(k,n) = z^{1-\gamma}\left(k^{\alpha}n^{1-\alpha}\right)^{\gamma}, \text{ where } \alpha, \gamma \in (0,1) \tag{2}$$

where z is the firm's productivity, k and n are, respectively, capital and labor inputs, and labor is hired in a competitive labor market at wage rate w. Firms may potentially differ in the levels of productivity or the levels of the capital they can use in production. To formalize the size-dependent distortions, suppose that the firm's revenue y is subject to the tax rate $T(y)$ given by

$$T(y) = 1 - \lambda y^{-\tau}, \text{ where } \lambda, \tau > 0. \tag{3}$$

This tax function has been proposed by Benabou (2002), and used in several recent studies analyzing the effects of size-dependent distortions.[6] It obviously implies that bigger firms pay higher tax rates, and the firm's after-tax revenue is $\lambda y^{1-\tau}$.

Then the labor choice of a firm with productivity z and capital k maximizes its after-tax profit, i.e.,

$$\max_{n} \quad \lambda[z^{1-\gamma}(k^{\alpha}n^{1-\alpha})^{\gamma}]^{1-\tau} - wn,$$

implying that

$$wn = \lambda\gamma(1-\tau)(1-\alpha)y^{1-\tau},$$

which, after taking logs, becomes

$$ln(wn) = (1-\tau)ln(y) + ln(\lambda\gamma(1-\tau)(1-\alpha)). \tag{4}$$

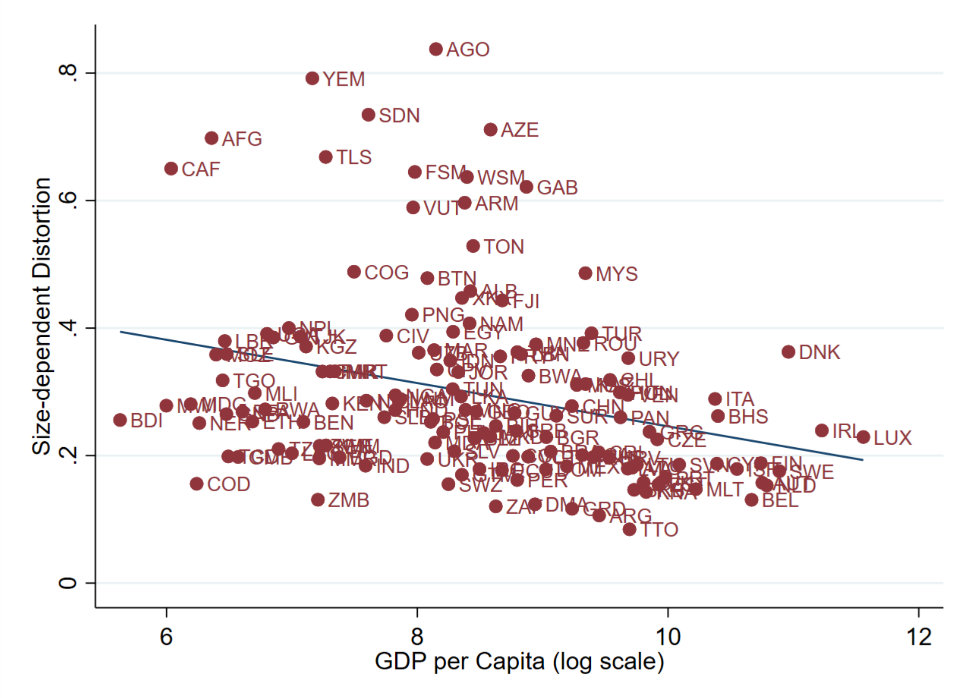
That is why, under tax and production function specifications (3) and (2), respectively, the distortion parameter τ faced by establishments in a given country can be estimated using the data on establishments' sales revenue and wage bill, as it is done in 1.[7] Note

in labor share across different sectors.

[6]See, for example, Guner et. al. (2018).

[7]In contrast to our specification of distortions, Hsieh and Klenow (2009) and Bento and Restuccia (2017) assume that the implicit tax varies with establishment productivity, not the establishment size,

Figure 2: Size-dependent Distortions vs. GDP per capita across countries



Note: Size-dependent distortions are estimated by (1) using establishment data from World Bank Enterprise Survey. The full country level data can be found in Table 10 in the Appendix B.

also that, given the production technology (2), the estimation of τ is mute on how capital is allocated across firms, i.e., it can be applied even if firms have limited access to external financing.

Figure 2 illustrates how the estimated values of $\widehat{\tau}_i$ vary with per capita income across countries. As can be seen, there is a negative relationship between our measure of size-dependent distortions and GDP per capita, and the correlation coefficient is -0.32.

The above evidence demonstrates that countries that have relatively high private credit-to-GDP and capital-GDP ratio also have relatively low measure of size-dependent distortions.[8] We further highlight this point in Table 1 by splitting the countries in our dataset into four income groups based on their income levels and reporting average private-credit-to-GDP ratio and average estimated $\widehat{\tau}$ for each group. As can be seen, there is an obvious negative relationship between the measures of financial development

and estimate the distortion accordingly. The advantages of specification (3) is that it does not require the policymaker to know or estimate the establishment's productivity. Also, the two forms of distortions are equivalent when there is no financing constraint because there is a one-to-one mapping between productivity and firm size in the unconstrained economy.

[8]The correlation between size-dependent distortions and private credit-to-GDP (capital-GDP ratio) is -0.16 (-0.17) in our full sample.

Table 1: Financial Development and Size-dependent Distortion across Income Levels

	High Income	Upper-Middle Income	Lower-Middle Income	Low Income
Private Credit/GDP	65.01	57.18	41.99	16.13
Capital/GDP	2.71	1.98	1.82	1.53
mean $\widehat{\tau}$	0.207	0.297	0.326	0.389
num. of countries	31	46	47	23

Note: The values for Private Credit/GDP, Capital/GDP and mean $\widehat{\tau}$ are given by the simple average for the countries in each income group. Countries are classified into the four income groups according to the definition by World Bank for the 2023 fiscal year, based on gross nation income per capita.

and size-dependent distortions.

Assuming that low credit-to-GDP and capital-GDP ratios are indicative of severity of financing frictions, the above observations suggest that the two frictions – limited excess to external financing and size-dependent distortions – tend to coexist. Thus, it is important to understand how these frictions may interact with each other, which we aim to do in this paper. In what follows, we first provide an illustrative example to highlight the mechanism through which firms' limited access to financing financing may interact with size-dependent distortions, and then develop a full firm dynamics model to quantitatively assess the magnitude of this interaction and its policy applications.

3 Illustrative example

The purpose of this example is to illustrate that size-dependent distortions can help mitigate the inefficiencies in the allocation of resources across firms arising due to the presence of firm financing constraints.

Consider an OLG-type model, where there is a mass one of identical firms born in every period. The firms live for three periods, and in each period have access to production technology (2). While labor is hired on the competitive market (at price w), capital must be accumulated within the firm. The firm's initial capital endowment (at age 1) is exogenous and equal to k_1. Profits in every period can be split between the dividends d_i, and capital that can be used in production in the next period k_{i+1}, $i = 1, 2, 3$. For analytical tractability, it is assumed that capital fully depreciates in production and there is no discounting. Thus, the firm's life-time decision problem is to choose capital levels k_2 and k_3 to maximize the firm's life-time value,

$$
\begin{aligned}
&\max_{\{d_i,k_{i+1}\}_{i=1}^3} \{d_1 + d_2 + d_3\} \\
&\qquad \text{s.t.} d_i \leq \pi(k_i) - k_{i+1}, i = 1,2,3 \\
&\qquad \quad d_i \geq 0, k_{i+1} \geq 0, i = 1,2,3 \\
&\qquad \quad k_1 > 0 \text{ is given} \\
&\qquad \quad \pi(k_i) = \max_n (k_i^\alpha n^{1-\alpha})^\gamma - wn.
\end{aligned}
\tag{5}
$$

In the stationary equilibrium, in every period the firms of different ages coexist and the equilibrium wage w is such that the labor market clears,

$$
n_1 + n_2 + n_3 = N^*,
$$

where N^* is the aggregate supply of labor.

It is straightforward to see that, as long as k_1 is sufficiently small, the firm's optimal choice is to pay no dividends in periods 1 and 2, reinvest all the profits in capital k_2 and k_3, and pay all the profit in period 3 out as dividends. Therefore,

$$
\begin{aligned}
k_{i+1} = \pi_i &= (1 - \gamma(1-\alpha))wn_i \\
&= (1 - \gamma(1-\alpha))w \left(\frac{z^{1-\gamma}(1-\alpha)\gamma}{w} k_i^{\alpha\gamma} \right)^{\frac{1}{1-(1-\alpha)\gamma}}, i = 1,2
\end{aligned}
\tag{6}
$$

where the expression in parenthesis in the second row is the optimal employment for the firm of age i (operating capital k_i). It then trivially follows that, as long as k_1 is sufficiently small, the firm's optimal capital path satisfies $k_3 > k_2 > k_1$.[9]

Crucially, this allocation of capital and labor across firms is inefficient. Denoting by K^* the total amount of capital used by firms in the equilibrium (e.g., $K^* = k_1 + k_2 + k_3$), the constrained (given exogenous k_1) efficient allocation of resources would maximize the

[9]To verify this, one can plot $k_{i+1}(k_i)$ along with the 45 degree line.

total output across three generations subject to feasibility constraints, i.e.,

$$\max_{k_2,k_3,n_1,n_2,n_3} \sum_{i=1}^{3} (k_i^{\alpha} n^{1-\alpha})^{\gamma}$$
$$\text{s.t. } k_1 + k_2 + k_3 = K^*$$
$$n_1 + n_2 + n_3 = N^*$$
$$k_1 \text{ is given.}$$

Obviously, the solution to this planning problem implies that $k_2 = k_3 = \frac{K^* - k_1}{2}$ and the marginal products of labor are equalized across the three types of firms.

This is a straightforward illustration of the inefficiency arising due to the firm's inability to access external financing. Obviously, this inefficiency can be corrected by designing a size-dependent tax, where a tax rate of 1 is imposed on all firm revenues above the level that the firm would produce by operating capital $(K^* - k_1)/2$ and choosing labor optimally.

More importantly, it can also be shown that the size-dependent distortion specified previously in (3) improves the allocation of capital.[10] It is straightforward to verify that, in the presence of such a distortions, the firm's optimal capital choice is

$$\begin{aligned} k_{i+1} = \pi_i &= (1 - \gamma(1-\alpha)(1-\tau)) w n_i \\ &= (1 - \gamma(1-\alpha)(1-\tau)) w \left(\frac{\lambda(1-\alpha)\gamma(1-\tau) z^{(1-\gamma)(1-\tau)}}{w} k_i^{\alpha\gamma(1-\tau)} \right)^{\frac{1}{1-(1-\alpha)\gamma(1-\tau)}}, \end{aligned} \tag{7}$$

Then,

$$\frac{k_3}{k_2} = \frac{n_2}{n_1} = \left(\frac{k_2}{k_1} \right)^{\frac{\alpha\gamma(1-\tau)}{1-(1-\alpha)\gamma(1-\tau)}}. \tag{8}$$

Suppose that λ is chosen in such a way that $k_1 + k_2 + k_3 = K^*$, i.e., the tax policy results in the reallocation of resources across firms, but does not affect the total amount of resources used in production. Then the left hand side of (8) can be re-written as $\frac{K^* - k_1 - k_2}{k_2}$, which is decreasing in k_2. The right hand side is increasing in k_2 but decreasing in τ. Thus, as τ increases, the equilibrium k_2 rises (and hence equilibrium k_3 falls). As τ approaches 1, k_3/k_2 also approaches 1, implying that the allocation of resources approaches the constrained efficient one (subject to fixed k_1).

[10] Assume that all the net tax revenues from such a distortionary tax system are distributed back to firm owners in the form of lump-sum taxes.

Intuitively, the firm's inability to access external capital results in misallocation of capital: young firms use too little capital but the old ones use too much of it, making the latter type of firms too big. Size-dependent distortions correct this inefficiency by discouraging old large firms from hiring workers. As a result, the labor demand falls and the young firms are able to generate more profit and accumulate capital faster.

Obviously, because the firms' productivity z is homogeneous, this example only underscores the 'benefits' of the size-dependent taxation that arise if firms have limited access to external financing. If the firms are heterogeneous in their productivity productivity levels, size-dependent distortions, in addition to creating reallocation of capital from older less constrained to younger more constrained firms, would also create a well-known misallocation of capital away from more productive towards less productive firms. In addition, in this example the amount of firms is exogenously fixed. In practice, however, it may adjust in response to size-dependent distortions or financing constraints. In the full model developed below, we allow for heterogeneity in firm productivity and for endogenous firm entry/exit, and analyze the relative costs vs. benefits associated with size-dependent distortions in the presence of capital financing constraints.

4 Model

We consider a modification of the industry dynamics model based on Hopenhayn (1992) in which the firms have limited access to external financing. Firms enter the industry with some initial asset level, accumulate assets over time, and exit the industry either because they are not sufficiently productive or because they draw an exogenous exit shock. They use capital and labor as inputs. Capital may be subject to external financing constraints. Labor is hired at a competitive market and financed by firms' revenues. The firm ownership shares are traded on a competitive stock market via a risk-neutral financial intermediary that aggregates all the risks. Thus, they are traded at the same price as a risk-free asset. We focus on a stationary equilibrium of the model, so aggregate variables are not indexed by time.

Consumers: A representative infinitely-lived household inelastically supplies one unit of labor in each period, acts as a representative shareholder, receives any government subsidies, and makes an intertemporal choice between consumption and saving given a risk-free interest rate r:

$$\max_{\{c_t, s_{t+1}\}_{t=0}^{+\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$
$$\text{s.t. } c_t + b_{t+1} + s_{t+1} \leq w + (1+r)(b_t + s_t) + T_t, \quad t \geq 0$$

where $\beta \in (0,1)$ is the time discount factor, b_t are the capital goods purchased in period $t-1$ that can be rented by firms in period t, s_t are the firm ownership shares that are purchased in period $t-1$ and pay out in period t, and T_t are the lump-sum subsidies paid by the government. Obviously, from the viewpoint of the consumer, savings in goods and in shares are perfect substitutes. Thus, only their total $b_t + s_t$ is pinned down by the consumer's decision problem, while their relative amounts are determined by the equilibrium conditions.

Note that the equilibrium interest rate in the stationary equilibrium must equal to $r = 1/\beta - 1$, and the consumer's utility function is immaterial for any aggregate quantities.

Incumbent firms: In the beginning of period t, the economy is populated by (an endogenous) mass of firms that are heterogeneous with respect to their productivity levels z_t and asset holdings a_t. Each firm faces a fixed production cost c_f, operates a production technology $f(z_t, k_t, n_t)$, where k_t and n_t are capital and labor inputs, respectively. In the end of the period, after production takes place, firms can decide whether to continue producing or exit in the next period. In addition, they face an exogenous exit probability ξ in every period. In case of exit, firm's assets are liquidated (with share χ of the assets' value lost in the process) and distributed to the shareholders. Firm productivity z_t evolves according to the Markov process described by $Q(z_{t+1}|z_t)$. Labor n_t is hired in a competitive market at wage rate w_t. Capital k_t can be rented at price r, depreciates at rate δ, but is subject to an capital financing constraint

$$k_t \leq \overline{k}(a_t), \tag{9}$$

where a_t is the firm's current assets. One common form of such a constraint is

$$k_t \leq \frac{1}{\phi} a_t, \tag{10}$$

with $\phi \in (0,1]$. Such a constraint can be rationalized by assuming that any capital rented in excess of own assets must be collateralized, and only a share of own assets and / or capital used in production can be pledged as collateral.[11]

To incorporate size-dependent distortions in this model, assume that the firms are subject to a tax described by (3). When $\tau = 0$, all firms face an output tax $(1-\lambda)$, but with $\tau > 0$, the tax rate is higher for larger firms. Thus, τ controls how dependent on size the distortions are. We assume that any net tax revenue collected by the government is distributed back to the households in the form of a lump-sum subsidy, implying that the equilibrium household consumption is still equal to the aggregate output (net of all the fixed costs).

The firms' future stream of expected dividends is discounted at a rate $\frac{1}{1+r}$ because the ownership shares are traded by the consumers on the competitive stock market and serve as a perfect substitute to risk-free savings in capital gods. As a result, the firm's decision problem can be set up recursively as follows:

$$\begin{aligned} V(a,z) = \max_{d\geq 0, a'\geq 0} & \; d + \frac{1}{1+r}\left[(1-\xi)\max\{\int V(a',z')\,\text{Q}(dz'|z), (1-\chi)a'\} + \xi(1-\chi)a'\right] \\ \text{s.t. } & \; d + a' \leq \pi(a,z) + (1+r)a, \\ & \; d \geq 0, \end{aligned} \tag{11}$$

where d is the dividends paid in the current period and $\pi(a,z)$ is the firm's current period profit. The latter is given by the maximization problem with the capital financing constraint:

$$\begin{aligned} \pi(a,z) = & \max_{k,n\leq 0} \lambda\left(f(z,k,n)\right)^{1-\tau} - wn - (r+\delta)k - c_f \\ & \text{s.t. } k \leq \overline{k}(a,z) \end{aligned} \tag{12}$$

Denote by $\{a'(a,z), d(a,z), k(a,z), n(a,z), x(a,z)\}$ the firm's optimal savings, dividend, capital, labor, and exit (upon production in the current period) policies, respectively, with $x(a,z) = 1$ if $\mathbb{E}_{z'}V(a'(a,z),z) \geq (1-\chi)a'(a,z)$, i.e. no voluntary exit takes place.

Free entry: In every period, a risk-neutral intermediary sets up a mass of (endogenous) μ new firms which requires c_n units of labor input (i.e., the set up cost is wc_n) and makes initial investment a_0. New firms draw productivity from initial distribution $F_0(z)$, i.i.d.

[11] For instance, if $\theta_a \in [0,1]$ of own assets and $\theta_k \in [0,1)$ of total capital input can be collateralized, the firm's borrowing constraint is $k_t - a_t \leq \theta_a a_t + \theta_k k_t$ implying that $k_t \leq \frac{1+\theta_a}{1-\theta_k} a_t$.

across firms Thus, the free entry condition is

$$\int V(a_0, z)\, F_0(z)\, dz \geq wc_n + a_0. \tag{13}$$

Notably, the free entry condition (13) differs from the one in a standard firm dynamics model in Hopenhayn (1992) because it includes firm's initial assets in the right hand side. At the same time, firms' assets are also embedded in the left hand side because they are paid out to shareholders either in form of dividends when the firm operates or as liquidation assets if the firm exits. Note also that if there are no capital financing constraints, firms have no incentives for asset accumulation and pay out a_0 together with dividends to the shareholders in the very first period. In this case, a_0 cancels out from the free entry condition and (13) collapses to the one used in Hopenhayn (1992).

Government: The government collects taxes and pays subsidies to firms in accordance with distortion rules (λ, τ), and, to balance the budget constraint, pays out any excess tax revenue in the form of lump-sum subsidies T to the consumers.

Equilibrium: Given the government policies (τ, λ) , the initial firm asset holding a_0, and the parameter characterizing capital financing constraints ϕ, the stationary competitive equilibrium of the model consists of prices $\{w, r\}$, the mass of entrants μ, the stationary distribution $\Gamma(a, z)$ of firms, firms' decision rules $\{a'(a, z), d(a, z), k(a, z), n(a, z), x(a, z)\}$, steady state levels of $\{c, b, s\}$, and aggregate tax liabilities $\{T\}$ such that:

1. Given prices and policies, each type of firm (a, z) follow the decision rules that solve the problem defined by (11) and (12).

2. The free-entry condition (13) is satisfied.

3. The stationary distribution $\Gamma(a, z)$ is consistent with firms optimal choices:

$$\Gamma(A, Z) = (1 - \xi) \int 1_{\{a'(a,z)\in A, x(a,z)=1\}} Q(Z, z) d\Gamma + \mu \int_Z 1_{\{a_0 \in A, z\in Z\}} dF_0, \tag{14}$$

   for any open (A, Z).

4. Given prices, dividends, and taxes, the steady state levels of consumption and savings $\{c, b, s\}$ are consistent with the consumer's optimal choice problem.

5. All markets in the economy clear. Namely, the capital supply is equal to the capital demand,

$$\int a\, d\Gamma + b = \int k(a,z)\, d\Gamma,$$

and the labor supply is equal to labor demand,

$$\int n(a,z)\, d\Gamma + \mu c_n = 1.$$

On the goods market, the amount of goods produced by the firm is equal to the sum of consumption and capital depreciation

$$\int \left(f(z, k(a,z), n(a,z)) - c_f \right)\, d\Gamma = c + \delta \int k(a,z)\, d\Gamma.$$

Finally, the payment on shares by the financial intermediary are equal to the firms' total dividends net of the costs associated with setting up new firms and the amount of firms' capital associated with capital destruction at the moment of firm closure,

$$rs = \int d(a,z)\, d\Gamma + \chi \int a'(a,z)(\xi x(a,z) + 1 - x(a,z))\, d\Gamma - \mu a_0 - w\mu c_n.$$

6. The government's budget constraint holds

$$T = \int f(z, k(a,z), n(a,z)) - \lambda f(z, k(a,z), n(a,z))^{1-\tau}\, d\Gamma$$

It is easy to verify that, as expected, in the equilibrium the market clearing condition on the goods market is implied by the other equilibrium conditions. We show this in the Appendix C.

5 Calibration

The objective of the quantitative analysis is to study how the effects of size-dependent distortions on aggregate output are impacted by the presence of capital financing constraints. For this purpose, the model is calibrated so that in the stationary equilibrium in the absence of capital financing constraints (i.e., $\phi = 0$) or distortions (i.e., $\tau = 0$ and $\lambda = 1$) it matches the relevant moments of the U.S. data. Then, we introduce capital financing constraints (by setting $\phi > 0$) and compare the effects of size-related distortions ($\tau > 0$) in the models without and with capital financing constraints.

With full access to capital financing, firms have no incentive to accumulate assets and return a_0 to shareholders upon entry, along with full profit, in the form of dividends. This implies that, as discussed above, a_0 appears additively in both sides of the free entry condition (13), cancels out and becomes irrelevant. Thus, the model collapses to a standard firm dynamics model as in Hopenhayn (1992). Also, the fraction of assets χ lost at liquidation plays no role, so it is not pinned down by the calibration procedure.

There are four types of parameters that need to be calibrated: firms' production technology ($f(z, k, n)$, δ, and c_f), dynamics of firms' productivity shocks ($Q(z'|z)$ and ξ), and parameters directly related to firms' entry (c_n and $F_0(z)$), and consumer's preferences (β and $u(c)$). The values of these parameters, along with the targeted moments are reported in Table 2. The top and bottom panels report the parameters that are calibrated individually and jointly, respectively.

The model time period corresponds to a year, so the model is calibrated so that interest rate $r = 0.04$ and normalized wage $w = 1$ clear the markets. This immediately implies that $\beta = 1/(1 + r) = 0.9615$ regardless of $u(c)$, and the model necessitates that $r = 0.04$ is the stationary equilibrium interest rate in all counterfactual experiments.[12] The depreciation rate is set at $\delta = 0.1$ (see Cummins and Violante (2002)).

We assume that the production technology is given by

$$f(z, k, n) = z^{1-\gamma} \left(k^{\alpha} n^{1-\alpha}\right)^{\gamma} \tag{15}$$

In the absence of capital financing constraints, the profit share of the firms (not counting the fixed costs which are relatively small) is equal to $1 - \gamma$. De Loecker et al. (2020) reports profit rates of 3-8 percent, so we set $\gamma = 0.94$. The labor share in the model is given by $\gamma(1 - \alpha)$, so, given the value of γ, $\alpha = 0.33$ is pinned down to match the labor share of 0.6 (BLS data).

To calibrate the rest of the parameters, we use the Business Dynamics Statistics (BDS) data by the Census Bureau averaged over 1998-2007. Given the values of γ and α, the entrant's productivity distribution $F_0(z)$ is chosen to match the distribution of age-0 firms in the BDS (reported in top panel of Table 3). The exogenous exit rate $\xi = 0.008$ is set to match the BDS exit rate for firms with 250 or more employees (because in the model large firms exit only if they draw the exogenous exit shock). Then, the transition matrix for productivity shocks $Q(z'|z)$ and the per-period fixed cost c_f are jointly set to match the features of the firm size distribution, firm volatility and firm exit.

[12] Since the focus is on the aggregate quantities in the stationary equilibrium, the shape of consumers' utility $u(c)$ is immaterial, as long as it satisfies Inada conditions.

Table 2: Calibrated parameter values and relevant targets from the data.

Parameter	Value	Targeted moment (source)
Parameters set outside the model:		
β, time discount	0.9615	Interest rate $r = 0.04$
γ, span-of-control in (15)	0.94	profit share = 0.06 (De Loecker et al. (2020))
α, capital share in in (15)	0.33	Labor cost share = 0.6 (BLS)
δ, capital depreciation	0.1	(Cummins and Violante (2002))
ξ, exog. exit rate	0.008	Exit rate of large firms = 0.008 (BDS)
Parameters calibrated jointly:		
a, growth in (16)	-0.0925	Average firm size = 22 workers (BDS)
ρ, persistency in (16)	1.01	Share of firms with 100+ employees = 0.02 (BDS)
σ, volatility in (16)	0.23	Job destruction rate = 0.1 (BDS)
c_f, fixed per-period cost	0.21	Average exit rate across all firms = 0.082 (BDS)
c_n, fixed entry cost	7.5	Free entry condition (13) holds with equality

Specifically, we assume that the law of motion for firm productivity is given by the AR(1) process as in Hopenhayn and Rogerson (1993):

$$\ln z_{t+1} = a + \rho \ln z_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma^2). \tag{16}$$

Here, parameter a guides how fast, on average, firms grow and hence, given that the choice of $F_0(z)$ ensures that the model matches the average size of the entrants, impacts the average size of the incumbents predicted by the model. Coefficient ρ impacts the persistency of firms' productivity and, hence, determines how heavy the right tail of the firm size distribution is. Parameter σ determines how volatile the firms are and, hence, impacts the job destruction and job creation rates. Finally, the fixed cost of operation c_f determines the cutoff of productivity below which the firms choose to exit and, hence, impacts the average firm exit rate. The calibrated parameters, along with the aforementioned moments in the BDS data are reported in Table 2. After the values of all these parameters are set, the value of the entry cost c_n is pinned down so that the free entry condition (13) is satisfied with equality.

The bottom three panels of Table 3 report the firm size distribution generated by the

Table 3: Firm Characteristics by Size (Model and Data)

Firm size distribution, entrants only					
Num. of workers	1-9	10-20	20-50	50-100	100+
Data	0.8775	0.0711	0.0365	0.0094	0.0055
Model	0.8551	0.0827	0.0467	0.0097	0.0058
Firm size distribution, all firms					
Num. of workers	1-9	10-20	20-50	50-100	100+
Data	0.7549	0.1239	0.0769	0.0240	0.0204
Model	0.7479	0.1243	0.0832	0.0227	0.0220
Employment shares by firm size					
Num. of workers	1-9	10-20	20-50	50-100	100+
Data	0.1145	0.0746	0.1041	0.0741	0.6327
Model	0.1291	0.0719	0.1117	0.0745	0.6129
Mean employment for firms employing					
Num. of workers	1-9	10-20	20-50	50-100	100+
Data	3.4465	13.6815	30.7603	70.1573	704.7472
Model	3.9151	13.1167	30.4589	74.4948	633.1481

model, along with its empirical counterpart. As can be seen, even though the calibration procedure targets only several moments of firm size distribution, it matches pretty well both the fractions of firms in different size bins, as well as their corresponding employment shares. Matching firm heterogenity is particularly important because the effects of size-dependent distortions are directly impacted by it. Not surprisingly, the model also predicts that the firm exit rates are high for small firms (about 11 percent for firms with 1 to 9 employees).

6 Results

In this section, we quantitatively explore the interaction of capital financing constraints and size-dependent distortions. Specifically, we compare the effects of introducing the size-dependent distortions in the benchmark model without capital financing constraints (referred to as 'the unconstrained economy' hereafter), and in a model in which capital financing constraints are present (referred to as 'constrained economy' hereafter). We show that the presence of capital financing constraints not only considerably weakens the adverse effects of size-dependent distortions on output, but may even result in output rising with the extent of size-dependent distortions. We explore the driving forces behind these results and analyze which factors play an important role in how size-dependent distortions and capital constraints interact. Finally, to assess the empirical relevance of this mechanism, we compare the effects of the size-dependent distortions estimated in Section 2 using the WBES data in the unconstrained benchmark model with the effects of the same distortions in the models characterized by empirically-relevant capital financing constraints.

6.1 The effects of size-dependent distortions on output

We analyze the steady state effects of the size-dependent distortions (τ, λ) described by the tax function (3). To make the results easier to interpret, we assume that the distortions are capital-neutral. Namely, for a given value of τ we choose λ in such a way that the amount of aggregate capital used in all the firms in the economy remains unchanged. As a result, the distortion may impact output only through the change in the allocation of factors of production across firms, as well as through the extensive margin effects, such as the change in the number of firms and/or their composition. For the capital financing constraint, we consider the linear specification (10) with $\phi \in [0, 1]$. When $\phi = 0$, there are no limits on firms' capital usage, and when $\phi = 1$, firms are self-financed, i.e. must rely on own assets to finance firms' capital input.

Table 4 reports the changes in output that occur as a result of introducing size-dependent distortion. The first row summarizes the changes in the unconstrained economy (i.e., $\phi = 0$), while rows 2 - 4 describe the effects in the economies in which capital financing constraints are present. They differ by the severity of the constraint, with $\phi = 0.33$, $\phi = 0.5$, and $\phi = 1$. As discussed below, in an undistorted economy, these constraints result in a drop in capital stock of 15 percent, 25 percent, and 50 percent, respectively, and a drop of output of 12 percent, 18 percent, and 33 percent, respectively.[13] In all the rows, the output in the absence of size-dependent distortions is normalized to 100, and

[13]In this series of exercises, a_0 is kept constant at a level at which capital-output ratio for $\phi = 1$ falls by 25 percent relative to the unconstrained benchmark. This gives $a_0 = 0.3$. The effects for a much wider range of (a_0, ϕ) is considered in Section 6.5 where we aim to reproduce the range of capital-output ratios observed in the data.

Table 4: The effects of size-dependent distortions (τ) on output depending on the presence and severity of firm financing constraints (ϕ)

τ	0	0.025	0.05	0.1	0.2	0.3	0.4	0.5
$\phi = 0$	100	96	94	92	88	82	74	62
$\phi = 0.33$	100	104	98	98	97	92	85	75
$\phi = 0.5$	100	106	104	99	99	94	87	76
$\phi = 1$	100	107	110	99	100	100	92	82

Note: This table reports the values of output (relative to output for a given ϕ in an undistorted economy with $\tau = 0$) for different values of τ (in the columns) and ϕ (in the rows).

the rest of the columns report the steady state output for varying τ relative to this value.

The main observation is that the effects of size-dependent distortions crucially depend on the presence, and severity, of firm financing constraints. First, while, as expected, distortions necessarily reduce aggregate output in the unconstrained economy, small size-dependent distortions actually lead to an *increase* in output when capital financing constraints are present. For example, imposing the tax scheme with $\tau = 0.05$ results in a 6 percent drop of output in the unconstrained economy, but generates a 10 percent output increase in the constrained economy with $\phi = 1$. Second, for higher levels of distortions τ for which output falls in both economies, the magnitude of the effect is very different. For example, for tax schemes with $\tau = 0.4$ and $\tau = 0.5$, the output in the unconstrained economy falls by 26 and 38 percent, respectively. In contrast, the same tax schemes result in only 8 and 18 percent, respectively, output decline in the constrained economy with $\phi = 1$. Third, the differences in the effects of size-dependent distortions in the unconstrained and constrained economies are significant even for relatively mild capital financing constraints. Namely, even for $\phi = 0.33$, small distortions increase output, and large distortions lead to a much smaller output drop compared to its counterpart in the unconstrained economy.

In what follows, we discuss the driving forces behind these results. To understand why size-dependent distortions have such different effects in the unconstrained and constrained environments, and, in particular, why they may result in an increase in output in the constrained economy, it is important to understand the inefficiencies that arise due to the presence of financing constraints. The next section zooms in on the driving forces behind these inefficiencies and assesses their relative contribution.

Table 5: Aggregate effects of the firm financing constraints in an undistorted economy ($\tau = 0$)

	ϕ	w	$E(z)$	μ_0	K	Y	std(MPK)
no constraints	0	1	100	100	100	100	0
mild constraint	0.33	0.86	229	38	85	88	0.09
medium constraint	0.5	0.80	365	33	75	82	0.13
severe constraint	1	0.65	448	34	50	67	0.17

Note: The Table reports how the presence and severity of firm financing constraints (measured by ϕ) impact steady state equilibrium wage w, average firm productivity $E(z)$, mass of entrants μ_0, capital input K, output Y, and the standard deviation of the marginal products of capital across firms in the absence of size-dependent distortions. The quantities in the top row normalized to 100, and the quantities in rows 2-4 are reported relative to their counterparts in the top row.

6.2 Aggregate effects of capital financing constraints

Table 5 compares key aggregate variables in the unconstrained and constrained economies in the absence of any size-dependent distortions. Aggregate quantities in the unconstrained economy are normalized to 100 for ease of comparison.

When a capital financing constraint (10) is introduced in an undistorted economy, for a given level of wage (i.e., in a partial equilibrium) firms naturally use less capital and make less profit. This reduces firms' expected life-time profit, encourages exit, and results in violation of the free-entry condition (13). Thus, to satisfy the free-entry condition, wage must fall, as reported in the third column. Despite this reduction in wage, capital financing constraints trigger exit by less productive firms that are constrained at the moment of origination, and the average productivity $E(z)$ increases, as reported in the fourth column.[14] At the same time, the decline in wage partly offsets the negative impact of the capital financing constraints on profit and induces the firms that are not constrained (e.g., those that managed to accumulate assets over time) to become bigger than in the unconstrained environment. This expansion of bigger firms, despite higher exit among smaller firms, implies that, for a given mass of entrants, the aggregate labor demand at the new lower wage is higher than the labor demand created by the same mass of entrants in the unconstrained economy. As a result, the mass of entrants falls to clear the labor market (as reported in column five), which leads to fewer firms operating in

[14]Capital financing constraints may reduce average productivity if the firm evolution of productivity is endogenous, as in Vereshchagina (2013) or if they encourage entry by less productive but not so financially constrained firm owners as in the span-of-control models by, for example, Buera et al. (2011).

the economy. In equilibrium, as seen in columns six and seven, respectively, aggregate capital use is suppressed and the aggregate output is reduced relative to the unconstrained economy. Also, since some of the firms are capital-constrained, and some are not, capital is not allocated efficiently across firms, as reflected in the positive standard deviation of marginal products of capital reported in the last column. In contrast, as long as $\tau = 0$, marginal products of labor are equalized across all the firms.

While a reduction in capital used in production and its inefficient allocation across firms necessarily lead to a decline in output, the changes along the extensive margin have, in general, ambiguous effects. On the one hand, an increase in average productivity has a positive effect on output. On the other hand, a reduction in the number of firms may lower it because of the decreasing returns to scale in the production technology. At the same time, the latter effect may be partly offset by the reallocation of labor from startup activities to actual production and by the reduction in the total fixed per period cost due to a decrease in the number of active firms.

To quantitatively assess the magnitude of these effects, Table 6 measures how much each of them contributes to the drop in output relative to the unconstrained and undistorted benchmark. This is done for different degrees of the capital constraint severity, with the results for the most severe constraint ($\phi = 1$) reported in the top row, and the results for the mild constraint ($\phi = 0.33$) reported in the bottom row. For each ϕ, the level of the output (when $\tau = 0$) is reported in column 2, measured as a percentage of the output in the unconstrained undistorted benchmark economy. The rest of the columns show by how much (as a percentage of the benchmark output) the output would rise if the corresponding margins are adjusted.[15] Columns 3 and 4 report by how much output would rise if capital were allocated efficiently across firms (i.e., the marginal products of capital were equalized). This is done in two steps: First, starting from the equilibrium allocation in the constrained economy we reallocate the capital efficiently within all the firms with the same level of productivity z (i.e., holding $\int k(a,z)\,\Gamma(da,z)$ constant for each z). For $\phi = 1$, this change alone would raise the output by 9.4 percent (relative to the unconstrained benchmark), suggesting that inefficient allocation of capital across firms of the same productivity is an important contributor to the output decline driven by firm financing constraints. At the second step, we reallocate the capital across *all* the firms by equalizing the marginal products of capital across them, while holding the aggregate amount of capital constant. For $\phi = 1$, this would only raise the output by additional 0.7 percent, suggesting that the across-z misallocation of capital is not a major

[15] The numbers in all the columns sum up to 100, subject to the rounding error.

Table 6: Decomposition of the output change (relative to the unconstrained undistorted benchmark) in the constrained economy for various ϕ

ϕ	output, %. of bench.	+eq.MPK within z	+eq.MPK across z	+adjust ext. margin		+adjust K
				$(E(Z))$	(M)	
1	67	+9.4	+0.7	+2		+21
				(-6.5)	(+9.5)	
0.50	82	+7.7	+0.6	+0.7		+9.1
				(-6.7)	(+8.1)	
0.33	88	+6.5	+0.5	-0.0		+5.1
				(-4.6)	(+4.9)	

Note: The second column reports the output value as a percentage of the unconstrained undistorted benchmark, and the rest of the columns report by how much it increases if capital is reallocated efficiently across firms of the same productivity (column three), across all the firms (column four), if all the extensive margin decisions are set to their benchmark values (column five), and if the capital is increased to its benchmark amount (column six). The values in parenthesis in column five are the changes in output (relative to column four) if only average firm productivity and only mass of firms are set to teir benchmark values.

driver of output loss in constrained economy.

When capital and labor are allocated efficiently across all firms, the output is still smaller than its benchmark counterpart because less capital is used in production and because extensive margin decisions are distorted. The last two columns assess the contribution of these effects. With marginal products of capital equalized across firms, the aggregate output can be expressed as

$$Y_{equalMPK} = (ME(z))^{1-\gamma} \left(K^{\alpha} N^{1-\alpha}\right)^{\gamma} - c_f M, \tag{17}$$

where $K = \int k(a,z)\, d\Gamma$ and $N = \int n(a,z)\, d\Gamma$ are the aggregate amounts of capital and labor used in production, $M = \int d\Gamma$ is the total mass of firms, and $E(z) = \frac{\int z\, d\Gamma(a,z)}{\int d\Gamma(a,z)}$ is the firm average productivity. Note also that N, M and $E(z)$ are related to each other because in equilibrium $1 - N$ workers are engaged in starting the firms, and the number of startups μ_0 is related to the mass of firms M and the firm's average productivity $E(z)$ via the endogenous firm exit decision. Column 5 reports the compound effect of replacing all extensive margin decisions in the constrained economy with their counterparts in the benchmark economy. As can be seen, these effects are rather modest (vary from 0 to 2 percent, depending on the severity of the financing constraint). In each row, the two values in parenthesis show by how much the output would change if only average productivity $E(z)$ were adjusted to its benchmark level, and only mass of firms M were adjusted to its benchmark value. The former effect is negative because the constrained economy features more selection. The latter effect is positive because, due to decreasing returns to scale in the production technology, the decrease in the number of firms generates bigger output losses than the savings accrued due to smaller aggregate fixed production costs and less labor engaged in startup activities. Overall, the two forces approximately offset each other, and the compound effect of all the changes in extensive margin decisions is small.

Finally, the last column reports by how much the output would adjust if, in addition to all the changes in the previous columns, the capital stock were to rise from its constrained value to the level in the benchmark unconstrained economy. Naturally, this number is bigger the more severe are the capital financing constraints (and thus is decreasing with ϕ).

This discussion suggests that a size-dependent distortion that has no impact on the aggregate capital stock might be beneficial because it could result in a more efficient allocation of capital within the firms of the same productivity – which, leaving the change in capital aside, is the biggest contributor to the output loss induced by the financing constraints for all values of ϕ. In addition, it might counter the extensive margin effects that have a slight negative effect on output. At the same time, size-dependent distortions are likely to lead to misallocation of capital across the firms with different productivities. In what follows, we analyze these effects.

6.3 The effects of size-dependent distortions on the inefficiencies arising due to firm financing constraints

Table 7 reports how the changes along the intensive and extensive margins induced by the firm financing constraints are impacted by the size-dependent distortions. It focuses on the constraint with $\phi = 1$, while Table 11 and 12 in the Appendix D report the results for $\phi = 0.5$ and $\phi = 0.33$. The structure of Table 7 is identical to the structure of Table 6, with the extent of the size-dependent distortions τ varying in rows.

First, recall that, by design, the size-dependent distortions in all the experiments are capital-neutral (because λ adjusts as τ changes to ensure that the aggregate capital input remains constant). Thus, as can be seen from the last column, the output change attributed solely to the change in aggregate capital used in production is the same across

Table 7: The effect of size-dependent distortions on output changes with $\phi = 1$

τ	output, %. of bench.	+eq.MPK within z	+eq.MPK across z	+adjust ext. margin		+adjust K
				$(E(Z))$	(M)	
0	67	+9.4	+0.7	+2		+21
				(-6.5)	(+9.5)	
0.05	73	+3.2	+2.4	+0.3		+21
				(-1.3)	(+1.6)	
0.10	66	+6.6	+2.1	+3.7		+21
				(+10)	(-4.9)	
0.20	66	+2.3	+3.8	+5.2		+21
				(+10.1)	(-3.5)	
0.30	66	+0.4	+4.3	+7.4		+21
				(+10.2)	(-1.3)	
0.40	61	+0.0	+4.3	+13.1		+21
				(+10.3)	(+4.4)	
0.50	55	+0.0	+4.4	+19.8		+21
				(+10.3)	(+11.1)	

Note: The second column reports the output value in the constrained economy as a percentage of the unconstrained undistorted benchmark, and the rest of the columns report by how much it increases if capital is reallocated efficiently across firms of the same productivity (column three), across all the firms (column four), if all the extensive margin decisions are set to their benchmark values (column five), and if the capital is increased to its benchmark amount (column six). The values in parenthesis in column five are the changes in output (relative to column four) if only average firm productivity and only mass of firms are set to their benchmark values.

all the experiment. Hence, size-dependent distortions τ may affect output via their impact on capital allocation within and across productivity classes or via their impact on the extensive margin decisions.

Examining the values in each column, one can observe that size-dependent distortions help to reduce the misallocation of capital within firms of the same productivity level. This can be seen from a reduction in output loss reported in column 3. This occurs because higher values of τ hurt large firms and benefit small ones, which allows small firms to faster accumulate the optimal amount of capital, as outlined in Section 3. At the same time, as well understood in the misallocation literature, size-dependent distortions result in a less efficient allocation of capital across firms of different productivities: large more productive firms face higher tax rates and use 'too little' capital, and the opposite is true for small less productive firms. Therefore, the misallocation across firms of different productivities increases as τ rises, which is reflected in higher output loss values in column 4.

Turning to the effects of size-dependent distortions along the extensive margin, it is well-understood that they reduce the average firm productivity and increase the number of operating firms because they are relatively beneficial for small less productive firms (some of which choose to stay in business in the presence of distortions but would have exited otherwise) and because they discourage hiring by larger more productive firms. Thus, size-dependent distortions offset the impacts of capital financing constraints which, as discussed above, increase the average productivity of the surviving firms but reduce the mass of operating firms. The benefits of more operating firms outweigh the loss due to reduced productivity for small magnitude of size-dependent distortions in the case of $\phi = 1$. Thus, as reported in the fifth column, the loss due to extensive margin effect reduces as τ increases from $\tau = 0$ to $\tau = 0.05$.[16] However, as τ keeps growing, the losses associated with reduced productivity and too many firms being started (resulting in higher fixed entry and operational costs) dominate, and extensive margin effects become a major contributor to output losses.

Table 8 sheds additional light on the equilibrium channels via which size-dependent distortions help to allocate capital across firms more efficiently. To ensure that the free-entry condition is satisfied and the aggregate amount of capital input remains constant across all experiments, the distorted economies are characterized by higher λ and/or smaller wage w than the undistorted economy (column 3 and 4). As a result, smaller constrained firms (whose benefit from changes in λ and w outweigh the losses due to higher τ) obtain higher profits which allow them to accumulate capital faster. As a result, conditional on drawing constant productivity level over time, they are able to achieve optimal capital input sooner than in the undistorted economy (see the last column). As reported in column 5, the standard deviation of the marginal products of capital across firms is generally lower in distorted economies than in the undistorted one, especially for the higher values of τ. This is closely related to the fact that firms are able to achieve

[16]For lower extent of capital financing constriants ($\phi = 0.33$), the opposite is true as shown in Table 12 of Appendix D.

Table 8: Aggregate effects of size-dependent distortions in the economy with self-financing constraints ($\phi = 1$)

	output	λ	w	$std(MPK)$	cor. coeff. (MPK, MPN)	cor. coeff. (LS, CS)	periods constrained
$\tau = 0$	100	1	0.65	0.17	–	–	41
$\tau = 0.05$	110	1	0.52	0.26	-0.91	-0.80	39
$\tau = 0.1$	99	1.18	0.63	0.12	-0.72	-0.71	27
$\tau = 0.2$	100	1.14	0.59	0.16	-0.97	-0.85	18
$\tau = 0.3$	100	1.07	0.54	0.08	-0.84	-0.61	10
$\tau = 0.4$	92	1.04	0.54	0.04	-0.50	-0.31	6
$\tau = 0.5$	82	1.06	0.56	0.03	-0.36	-0.10	5

Note: The output in column 2 is normalized relative to its value for $\tau = 0$. Policy parameter λ is chosen so that size-dependent distortions are capital-neutral, and w is the equilibrium wage rate. Column 5 reports the standard deviation of marginal products of capital across firms. Columns 6 and 7 report, respectively, the correlation coefficients between the marginal products of capital and labor across firms, and the correlation coefficients between the labor and capital shares across firms. The last column measures the average time firms are contained. This is obtained from the weighted average of periods needed to reach the optimal scale of operation given the productivity, assuming firms stay at the same productivity level.

optimal scale of operation sooner and thus are more concentrated in asset distribution. The relationship between the standard deviation and τ, however, is not monotone because of the adjustments in firm selection (as τ grows, more of the less productive firms choose to remain in business) and the misallocation of factors of production across firms of different productivities created by size-dependent distortions.

Columns 6 and 7 in Table 8 report two other equilibrium moments that might be relevant for policy design – the correlation coefficients between marginal products of capital and labor, and between the labor and capital shares. As can be seen, for the values of τ for which these coefficients are sufficiently close to -1, the corresponding level of size-dependent distortions does not lead to significant output drops. In contrast, when such correlation weakens, size-dependent distortions are associated with bigger output losses. Intuitively, size-dependent distortions might be beneficial (or not very harmful) when they act in the "opposite direction" to the distortions induced by the capital constraints. Capital financing constraints reallocate capital from smaller to larger firms but do not distort the allocation of labor (given the capital input). Size-dependent distortions reallocate factors of production from large to small firms. Thus, high negative correlation between labor and capital shares signifies that the distortions created by the capital financing constraints are still large, and there is a scope for the size-dependent distortions to 'correct' the capital misallocation. When this correlation approaches zero (or becomes positive), the direct loss associated with size-dependent distortions outweighs the benefit of mitigating the capital constraints. Thus, the correlation between the marginal product of capital and labor (or between the capital and labor shares) can serve as an indicator for policy-makers of whether the size-dependent distortions are excessive and should be reduced.

To sum up, size-dependent distortions in the constrained economy improve the efficiency of capital allocation within firms of the same productivity – which is the biggest (other than capital reduction) source of inefficiency associated with firm financing constraints in an undistorted economy, and help offset the adverse extensive margin effects arising due to the presence of firm financing constraints. This occurs because, on the one hand, constrained firms (who tend to be small and benefit the most from the size-dependent distortions) are able to boost their asset accumulation and overcome the capital financing constraints faster; and, on the other hand, large unconstrained firms (who in the presence of capital constraints become excessively big) scale back due to size-dependent distortions, which leads to more entry in equilibrium. As a result, introducing size-dependent distortions in a financially constrained economy may increase output. However, excessive size-dependent distortions create substantial adverse effects along the extensive margin, lead to capital misallocation away from most productive firms and induce higher output losses.

6.4 Sensitivity analysis

In this section, we perform sensitivity analysis, focusing on the parameters which are likely to be different in the developing economies or the calibrated values of which are likely to be considerably different in the presence of firm financing constraints.

Lower persistence on productivity shocks In the model, higher level of persistence are associated with heavier tails of firm size distribution. In the calibrated version of the model, in which there are no capital financing constraints, persistence is set at a relatively high value of $\rho = 1.01$ to match the heavy tails of the firm size distribution. Capital financing constraints increase the persistency of firms' employment because next period employment is positively related to not only the next period's productivity but also to the amount of profit the firm generates in the current period, which is also increasing in current productivity and current asset. It has also been well understood (see, e.g., Moll (2014)) that high levels of persistency in firm productivity help alleviating the effects of firm financing constraints because they allow high productivity firms to accumulate assets relatively fast, which helps alleviate the misallocation of capital across firms. In addition, empirical studies (e.g., Casey C. Maue and Emerick (2020)) have documented that productivity in agricultural firms, which are more prevalent in the developing countries, exhibit lower persistence and higher uncertainty.

In this experiment, we set $\rho = 0.95$ and recalibrate the rest of the parameters to match the same moments (except for the tails of the size distribution).[17] The top panel of Table 9 reports the aggregate effects of capital financing constraints, analogous to the ones reported in Tables 4 (in the right panel of Table 9) and 5 (in the left panel of Table 9) for the same levels of (a_0, ϕ) as in Tables 4 and 5. Setting $\phi = 0.33$, 0.5, and 1 results in capital decreasing by 27, 36, and 55 percentage, and output decreasing by 23, 32, 48 percentage, respectively.[18]

Comparison of the results in Tables 4 and 9, yields the observation that when persistency of firm productivity is relatively low, size-dependent distortions generate bigger improvements in output, and widen the gap between the effects in the unconstrained and constrained economies. For instance, a distortion of $\tau = 0.2$ leads to a 13 percent drop in output in the unconstrained economy, but to 36 percent *increase* in output in an economy in which forms must rely on self-financing ($\phi = 1$). These results suggest that if the model is calibrated to match the firm dynamics in a developing economy where capital financing effects are likely to be substantial, persistency is likely to be smaller than in our benchmark exercise, and the benefits of size-dependent distortions are likely to be bigger.

Low growth in firm productivity Next, we perform comparative statics with respect to the parameter that effects firms' growth over lifecycle, a in the AR(1) process of firm productivity shocks (16). First, as is well-documented, firms in the poorer countries tend to grow slower than in the rich countries (see, for example, Hsieh and Klenow (2014)). Second, if the capital financing constraints are present, some of the firm growth is likely to be driven by the growth in firms' capital stock, not the firms' productivity. Thus, we recalibrate the model so that it generates less lifecycle growth (with firms growing from

[17]The recalibrated parameter values are $a = 0.17$, $\sigma = 0.19$, $c_f = 1.4$ and $c_n = 2.35$.

[18]Note that, consistent with findings in previous literature, lower persistency indeed magnifies the impact of capital financing constraints on output. This is evident by comparing the effects of setting $\phi = 1$: in the high-persistency environment it reduces capital and output by 50 and 33 percent, respectively; but in the low-persistency environment it reduces capital and output by 55 and 48 percent, respectively.

Table 9: Sensitivity analysis: the effects of firm capital financing constraints on aggregate variables in the absence of distortions , and the effects of size-dependent distortions on output depending on the presence and severity of firm financing constraints for the parameter changes described in Section 6.4

	aggregate effects of capital constraints under $\tau = 0$				output changes, $Y_{\tau>0}/Y_{\tau=0}$ τ						
	K	Y	$E(z)$	μ_0	0	0.05	0.1	0.2	0.3	0.4	0.5
Lower persistency in productivity shocks											
$\phi = 0$	100	100	100	100	100	99	97	87	73	53	23
$\phi = 0.33$	77	73	82	106	100	111	114	107	93	73	47
$\phi = 0.5$	68	64	63	83	100	116	121	117	101	81	50
$\phi = 1$	51	45	46	67	100	123	134	136	117	96	52
Lower firm growth over lifecycle											
$\phi = 0$	100	100	100	100	100	94	93	87	81	71	57
$\phi = 0.33$	87	83	165	57	100	97	97	96	90	82	70
$\phi = 0.5$	81	75	258	46	100	104	99	99	94	85	74
$\phi = 1$	66	54	347	34	100	110	102	102	99	92	79
More uncertainty in the evolution of firm productivity											
$\phi = 0$	100	100	100	100	100	94	93	88	82	72	59
$\phi = 0.33$	86	82	176	50	100	102	100	97	91	84	72
$\phi = 0.5$	79	72	241	49	100	104	102	101	94	85	75
$\phi = 1$	63	48	227	56	100	110	109	106	104	95	82

Note: The left panel reports the effects of firm financing constraints with corresponding ϕ (in different rows) on aggregate capital stock, output, average firm productivity, and the mass of entrants. The right panel reports the values of output (relative to output for a given ϕ in an undistorted economy with $\tau = 0$) for different values of τ (in the columns) and ϕ (in the rows).

the average size of 6 at the moment of entry to the average size of 12 for the incumbent, instead of 22 in the benchmark model), the same level of persistence, and matches the same exit and job destruction rates.[19]

The second panel of Table 9 reports that, given the level of capital financing constraints, size-dependent distortions have a slightly more negative impact on output in

[19]The parameters that changed values are $a = -0.974$, $\sigma = 0.16$, $c_f = 0.2$ and $c_n = 5.5$.

this version of the model than under the benchmark parameterization. This is due to the fact that, with smaller productivity growth, firms are able to accumulate assets faster to achieve their optimal scale of operation, implying that there is less misallocation of capital created by the presence of capital financing constraints.

Higher uncertainty in the evolution of firm productivity Finally, we analyze how the model's predictions change if there is more uncertainty in the evolution of firm productivity shocks. It has been documented that firms in the poorer countries face more idiosyncratic risk (e.g., Moscoso Boedo (2018)), which is also consistent with poorer countries having higher job flow rates (e.g. Donovan et al. (2022)). So we recalibrate the model to generate higher job destruction rate of 0.15 (instead of 0.10 in the benchmark model) while the rest of the moments are unchanged.[20] The bottom right panel of Table 9 reports that, conditional on the level of ϕ, size-dependent distortions have slightly higher positive effect on output. This is because higher uncertainty in the evolution of firm productivity shocks, like lower persistence, undermines the benefits of self-financing in the presence of capital financing constraints and amplifies the extent of misallocation associated with limited excess to external financing.

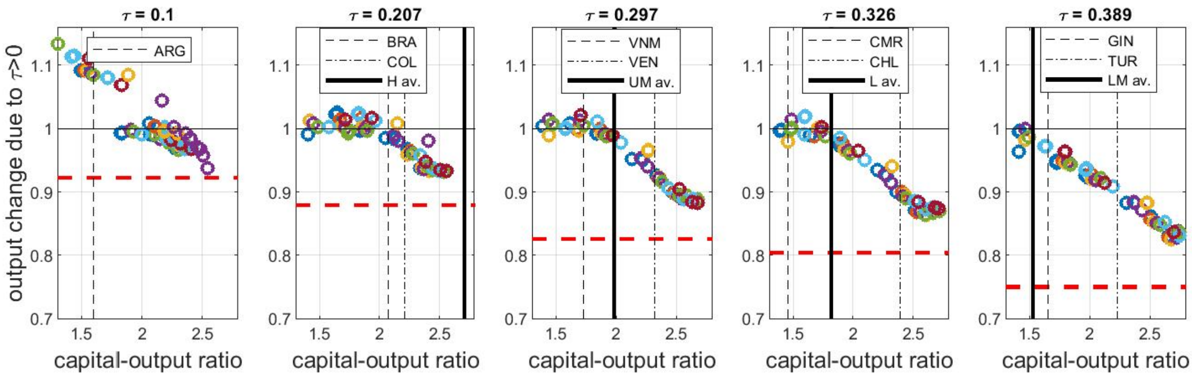
6.5 The effects of size-dependent distortions for a range of empirically-relevant firm financing constraints

In the above analysis, we have described in details the effects, as well as the driving forces behind them, for a few *ad hoc* capital financing constraints characterized by particular combinations of (a_0, ϕ). In this section, we extend this analysis for a wide range of capital financing constraints consistent with the relevant moments from the data in order to assess the quantitative importance of the mechanism described in the paper for empirically-relevant levels of size-dependent distortions.

It is well understood that having limited excess to external financing hampers the aggregate amount of capital used in production and leads to lower capital-output ratio. For example, Caselli (2005) finds that the capital-output ratio in the 20 percent of poorest countries is 1.85 times lower than in the 20 percent of richest countries.[21] Based on our measure of capital-output ratio derived using the Penn World Table data detailed in Table 10 of Appendix B, we find that for most countries capital-output ratios fall between 1.2 and 2.7. In our model, the same level of capital-output ratio can be achieved, given other

[20]The new corresponding parameter values are $a = -0.13$, $\sigma = 0.34$, $fc = 0.24$ and $c_n = 7.3$.
[21]See Khan et al. (2009) for a concise summary.

Figure 3: The relationship between the effects of size-dependent distortions τ on output and the equilibrium capital-output ratio



Note: The plots show the changes in output due to size-dependent distortions (with τ varying across the plots and output under $\tau = 0$ normalized to 1) vs. equilibrium capital-output ratios. The red horizontal dashed lines show the effects of respective τ on output in the unconstrained economy. Each dot corresponds to an equilibrium of an economy with one combination of (a_0, ϕ). The vertical lines mark the levels of capital-output ratios for the countries (labeled in the plots) whose size-dependent distortion τ estimated in Section 2 fall within 10 percent of the value of τ for which the plot is produced. The capital-output levels for these countries are constructed based on the Penn World Table data following Greenwood et al. (2013).

parameter values, for a range of (a_0, ϕ) which determine the firms' initial asset endowment (a_0) and the ability to access capital (ϕ). So we simulate the model for a range of (a_0, ϕ) and compare the effects of size-dependent distortions in these constrained economies with the effects of the same size-dependent distortions in the unconstrained economy.

Figure 3 illustrates the simulation results for this experiment. Different plots show the results for different levels of size-dependent distortion τ. The values of τ in the right four plots are the average levels of distortions for all income groups of countries (H, UM, LM and L) reported in Table 1 using the WBES data. Each circle corresponds to a combination of equilibrium capital-output ratio (on the horizontal axis) and output relative to its value in the absence of distortions (on the vertical axis) for one particular combination of (a_0, ϕ) under the corresponding level of distortions. The red horizontal dashed lines show the output for corresponding τ (relative to its value when $\tau = 0$) in the economy without capital financing constraints. As expected, the drop in output due to size-dependent distortions increases with τ: namely, in the benchmark unconstrained economy, the output would fall by 8, 12, 18, 20 and 25 percent going from the left to the right plot.[22]

[22]Since the size-dependent distortions, by design, are capital-neutral, capital-to-output ratio increases with τ in the unconstrained economy, from 2.4 in the benchmark model to 3.3 in the model with $\tau = 0.389$.

As can be seen from the plots in Figure 3, size-dependent distortions have much smaller effects on output in the presence of capital financing constraints. Naturally these differences are bigger the more severe the constraints (which is reflected in lower equilibrium capital-output ratios). As reported in Table 10 in Appendix B, the values of capital-output ratio fall below 2 for roughly half of the countries in the sample. Figure 3 shows that for the capital financing constraints consistent with such values, distortions of $\tau = 0.3$ or below have virtually no adverse effect on output. According to our estimates of τ based on the WBES data reported in Table 10 in the Appendix 2, 93 out of 147 countries in the sample have τ of 0.3 or below. This suggests that these seemingly large distortions might actually have no negative impact on the output of these countries because of the capital financing constraints present in them.

The dash-and-dot vertical lines in the right four plots mark the capital-output ratios for several countries whose estimated distortion levels (reported in Table 10 in Appendix 2) fall within 10 percent of the values of τ for which these plots are produced. These countries are Brazil and Colombia (in the plot for $\tau = 0.207$), Vietnam and Venezuela (in the plot for $\tau = 0.297$), Cameroon and Chile (in the plot for $\tau = 0.326$), Guinea and Turkey (in the plot for $\tau = 0.389$). For most of those countries, their estimated size-dependent distortions change the output (up or down) by less than 5 percent (relative to the benchmark undistorted economy) if firms face capital constraints consistent with the capital-output ratios for these economies. This result also holds for the average capital-output ratios observed in countries in the income groups of UM, LM and L for the values of τ consistent with the income groups. Recall that if these constraints are not accounted for, the model predicts that output falls by 12 - 25 percent due to these distortions. For instance, the size-dependent distortions observed in Guinea lead to the output drop of less than 2 percent if there are financing constraints consistent with Guinea's capital-output ratio. In contrast, if such constraints are not accounted for, the predicted drop in output is 25 percent – twelve times as large.

The left panel of Figure 3 illustrates that relatively small distortions are beneficial for a wide range of capital financing constraints. Specifically, it shows that $\tau = 0.1$ can lead to a significant output increase if the capital constraint lowers the capital-output ratio to the levels observed in low-middle and low-income countries. For example, for the level of capital-output ratio equal to that of Argentina (1.6, as reported in Table 10), such a level of size-dependent distortions (consistent with the estimated level of τ for Argentina) increases the output by about 10% relative to the undistorted economy.

To sum up, the results presented in this section suggest that, due to the presence of capital financing constraints, empirically-relevant levels of size-dependent distortions might not be as damaging for output as previous studies have suggested. This is especially the case for developing countries for which we tend to observe the coexistence of the two friction. Moreover, in some cases, such distortions might even be beneficial.

7 Final Remarks

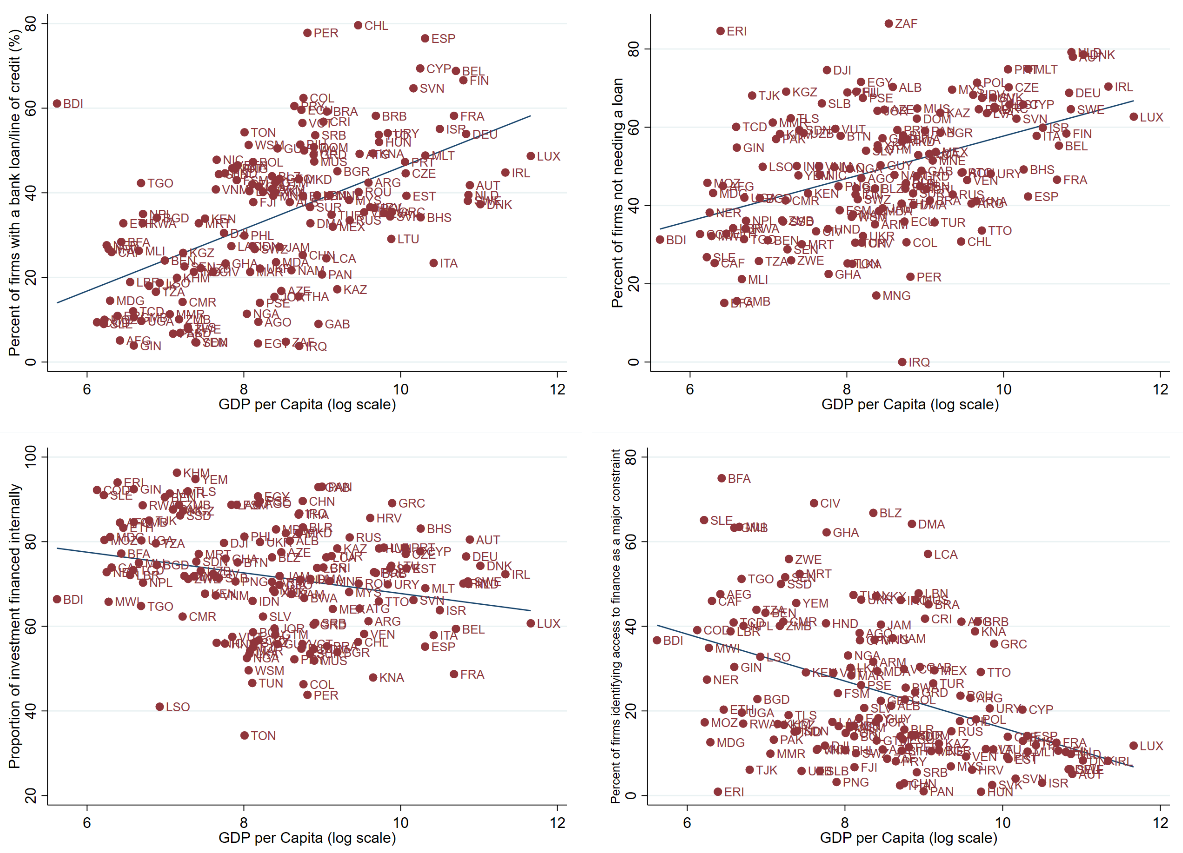
This paper demonstrated that the effects of size-dependent distortions crucially depend on the ease of accessing production capital. In particular, when firms have limited access to capital, size-dependent distortions can help improve the allocation of resources, both along the intensive and the extensive margins. The numerical analysis shows that empirically-relevant levels of capital constraints drastically diminish the adverse affects of empirically-relevant size-dependent distortions, and such distortions may even result in a positive impact on aggregate output. Therefore, our findings suggest that it is necessary to take into account the presence of both frictions in understanding the cross-country income differences or designing policies. Specifically, the benefits of removing or mitigating one of the frictions might be considerably overstated if the presence of the other friction is not taken into account.

Our model builds on a standard firm dynamics model (as in Hopenhayn (1992)) with firm capital constraints. One feature of such a model is the exogenous evolution of firm productivity. Previous studies (e.g., Bhattacharyaa et al. (2013), Bento and Restuccia (2017) or Vereshchagina (2021)) have illustrated that both size-dependent distortions and capital financing constraints can impact firms' productivity investment which may lead to large additional effects on the aggregate impacts of these frictions / distortions. It would be interesting to investigate how endogenizing firm productivity growth could impact the effects of the interaction between size-dependent distortions and capital financing constraints.

Our numerical analysis calibrates the model to the US and analyzes the interaction between empirically-relevant capital constraints and size-dependent distortions. An alternative approach would be to calibrate the model to a developing economy, in which both size-dependent distortions and capital constraints are significant, and assess the effects arising due to potential removal of size-dependent distortions. Such analysis could shed light on the extent of complementarities in policies that are designed to ease size-dependent frictions and relax excess to capital financing.

A Additional Evidence on Financial Development across Countries

Figure 4: Various Financial Indicators vs. GDP per capita across countries



Note: The figure plots various indicators reflecting financial development across countries from World Bank Enterprise Survey (WBES). The top left panel plots percentage of firms with a bank loan/line of credit; The top right panel plots the percentage of firms claiming that they are not needing a loan; The bottom left panel plots the proportion of investment that is financed internally in a firm; The bottom right panel plots the percentage of firms identifying access to finance as a major constraint.

B Full Data for the Empirical Evidence

Table 10: Financial Development and Size-dependent Distortions (Full Sample)

Country	Income Level	Distortion	Private Credit/GDP (%)	Capital/GDP
AFG	L	0.698 (0.059)	3.51	
ALB	UM	0.458 (0.042)	32.99	1.996
AGO	LM	0.838 (0.041)	14.94	3.132
ATG	H	0.181 (0.073)	43.10	2.884
ARG	UM	0.106 (0.013)	15.96	1.600
ARM	UM	0.597 (0.031)	55.50	1.286
AUT	H	0.157 (0.022)	84.32	3.253
AZE	UM	0.711 (0.030)	20.77	1.062
BHS	H	0.262 (0.051)	51.46	3.333
BGD	LM	0.197 (0.008)	46.94	1.789
BRB	H	0.146 (0.043)	80.89	2.353
BEL	H	0.131 (0.024)	68.09	3.446
BLZ	LM	0.227 (0.032)	54.09	1.745
BEN	LM	0.253 (0.039)	16.75	1.410
BTN	LM	0.478 (0.029)	58.09	3.257
BOL	LM	0.253 (0.022)	67.85	1.175
BIH	UM	0.247 (0.038)	57.50	1.350
BWA	UM	0.325 (0.033)	34.98	2.690
BRA	UM	0.206 (0.017)	60.22	2.079
BGR	UM	0.229 (0.016)	50.34	1.759
BFA	L	0.269 (0.058)	27.15	1.415
BDI	UM	0.256 (0.029)	18.03	1.174
KHM	LM	0.216 (0.041)	99.57	1.911
CMR	LM	0.332 (0.036)	14.68	1.586
CPV	LM	0.335 (0.065)	60.17	
CAF	L	0.650 (0.102)	12.34	1.636
TCD	L	0.199 (0.031)	9.76	1.549
CHL	H	0.319 (0.013)	116.91	2.228
CHN	UM	0.278 (0.012)	157.81	2.822
COL	UM	0.200 (0.010)	49.57	2.212

COG	LM	0.488 (0.077)	14.05	2.076
CRI	UM	0.205 (0.026)	60.30	1.745
CIV	LM	0.225 (0.022)	54.63	0.836
HRV	H	0.388 (0.023)	136.55	1.974
CYP	H	0.196 (0.049)	51.34	3.627
CZE	H	0.187 (0.043)	19.44	2.544
DNK	H	0.124 (0.016)	5.86	3.033
DMA	UM	0.178 (0.086)	161.26	1.654
DOM	UM	0.156 (0.033)	47.53	2.085
COD	L	0.363 (0.024)	27.21	2.680
ECU	UM	0.177 (0.019)	38.31	2.358
EGY	LM	0.394 (0.007)	25.55	0.869
SLV	LM	0.207 (0.015)	53.51	1.015
ERI	L	0.505 (0.096)	17.56	
EST	H	0.155 (0.025)	62.22	1.983
SWZ	LM	0.155 (0.036)	20.89	1.607
ETH	L	0.253 (0.021)		1.101
FJI	UM	0.443 (0.124)	92.80	2.157
FIN	H	0.188 (0.018)	94.25	3.444
GAB	UM	0.622 (0.052)	13.08	1.732
GMB	L	0.198 (0.039)	7.05	1.427
GEO	UM	0.269 (0.028)	62.65	1.025
GRC	H	0.237 (0.018)	91.74	3.381
GRD	UM	0.117 (0.062)	50.65	2.623
GTM	UM	0.170 (0.016)	35.11	1.468
GIN	L	0.385 (0.035)	9.65	1.458
GNB	L	0.265 (0.087)	14.31	1.024
GUY	UM	0.267 (0.049)	36.57	2.262
HND	LM	0.271 (0.024)	62.75	2.154
HUN	H	0.299 (0.022)	32.39	2.484
IND	LM	0.184 (0.006)	50.37	1.996
IDN	LM	0.349 (0.009)	38.81	2.104
IRQ	UM	0.179 (0.037)	8.58	1.887
IRL	H	0.239 (0.032)	40.83	2.342

ISR	H	0.179 (0.027)	65.95	2.834
ITA	H	0.289 (0.026)	76.72	3.298
JAM	UM	0.236 (0.019)	42.59	2.393
JOR	UM	0.331 (0.025)	76.39	2.165
KAZ	UM	0.312 (0.017)	25.93	1.369
KEN	LM	0.282 (0.015)	31.20	1.184
XKX	L	0.447 (0.040)	44.46	
KGZ	LM	0.371 (0.038)	23.37	1.272
LAO	LM	0.286 (0.026)	20.92	1.852
LVA	H	0.180 (0.025)	36.61	1.769
LBN	UM	0.359 (0.027)	105.97	2.298
LSO	LM	0.204 (0.030)	20.46	2.396
LBR	L	0.380 (0.033)	16.98	2.470
LTU	H	0.186 (0.025)	40.37	1.395
LUX	H	0.229 (0.081)	105.78	3.462
MDG	L	0.281 (0.025)	12.97	1.069
MWI	L	0.278 (0.035)	10.47	1.921
MYS	UM	0.486 (0.023)	120.28	2.636
MLI	L	0.298 (0.026)	25.59	1.176
MLT	H	0.147 (0.034)	74.63	2.758
MRT	LM	0.332 (0.032)	22.40	2.207
MUS	UM	0.311 (0.050)	78.25	2.119
MEX	UM	0.183 (0.010)	34.55	2.166
FSM	LM	0.645 (0.079)	19.82	
MDA	UM	0.220 (0.031)	23.22	
MNG	LM	0.272 (0.022)	55.41	2.157
MNE	UM	0.375 (0.057)	49.56	1.735
MAR	LM	0.366 (0.027)	85.14	2.871
MOZ	L	0.358 (0.019)	22.39	2.022
MMR	LM	0.196 (0.019)	28.32	1.208
NAM	UM	0.407 (0.072)	65.84	2.302
NPL	LM	0.400 (0.016)	76.32	2.122
NLD	H	0.153 (0.022)	105.49	2.637
NIC	LM	0.286 (0.023)	39.86	2.741

NER	L	0.251 (0.061)	10.23	1.770
NGA	LM	0.295 (0.026)	10.25	2.988
MKD	UM	0.230 (0.036)		1.645
PAK	LM	0.332 (0.016)	18.83	1.174
PAN	UM	0.260 (0.034)	87.04	1.936
PNG	LM	0.421 (0.076)	18.74	
PRY	UM	0.356 (0.026)	43.94	2.020
PER	UM	0.162 (0.011)	43.91	1.939
PHL	LM	0.237 (0.011)	47.56	1.896
POL	H	0.299 (0.031)	52.53	1.846
PRT	H	0.165 (0.018)	96.99	3.181
ROU	UM	0.377 (0.029)	25.73	2.149
RUS	UM	0.198 (0.011)	51.24	1.210
RWA	L	0.273 (0.026)	21.39	0.922
WSM	LM	0.637 (0.109)	87.22	
SEN	LM	0.214 (0.017)	29.55	1.622
SRB	UM	0.239 (0.031)	41.40	1.584
SLE	L	0.359 (0.040)	5.70	0.772
SVK	H	0.158 (0.020)	62.02	2.309
SVN	H	0.186 (0.026)	43.32	2.962
SLB	LM	0.260 (0.078)	31.49	
ZAF	UM	0.120 (0.012)	117.01	2.044
LKA	LM	0.293 (0.026)	50.17	1.978
KNA	H	0.143 (0.073)	49.08	3.613
LCA	UM	0.201 (0.044)	55.79	1.990
VCT	UM	0.198 (0.043)	48.43	2.161
SDN	L	0.735 (0.228)	9.35	1.317
SUR	UM	0.262 (0.045)	24.28	3.246
SWE	H	0.176 (0.016)	131.87	3.130
TJK	LM	0.386 (0.038)	11.94	0.461
TZA	LM	0.210 (0.018)	12.70	2.112
THA	UM	0.362 (0.018)	144.13	2.581
TLS	LM	0.678 (0.055)	73.88	
TGO	L	0.318 (0.043)	27.48	2.098

TON	UM	0.529 (0.057)	39.65	
TTO	H	0.084 (0.038)	39.27	1.687
TUN	LM	0.304 (0.024)	81.71	2.025
TUR	UM	0.392 (0.015)	67.41	2.391
UGA	L	0.391 (0.022)	13.64	1.546
UKR	LM	0.195 (0.014)	34.51	1.981
URY	H	0.353 (0.022)	25.27	2.388
UZB	LM	0.361 (0.019)	22.78	0.802
VUT	LM	0.589 (0.167)	61.49	
VEN	H	0.295 (0.068)	29.90	2.320
VNM	LM	0.289 (0.015)	133.14	1.734
PSE	LM	0.257 (0.030)	44.84	
YEM	L	0.792 (0.037)	5.64	1.252
ZMB	LM	0.131 (0.011)	14.97	1.830
ZWE	LM	0.215 (0.020)	10.99	

Note: This table provides data mentioned in Section 2 for the full sample. The size-dependent distortions in column 3 is identified from regression (1) for each country, with the standard errors reported in the parentheses. The private credit-to-GDP ratio is obtained from the World Bank. For most countries we use data in 2018 before Covid-19, but for few countries for which 2018 is unavailable, we use the closest year before 2018. The capital-GDP ratio is the average level from 2001 to 2019. It is obtained from the Penn World Table (PWT 10.0), and we estimate the capital stock from the data following the procedure as in Greenwood et al. (2013).

C Equilibrium conditions

In what follows, we show that the equilibrium the market clearing condition on the goods market is implied by the other equilibrium conditions.

Aggregating the profits across firms gives:

$$\int \pi(a,z)\,d\Gamma = \int \lambda\left(f(z,k,n)\right)^{1-\tau}\,d\Gamma - w\int n(a,z)\,d\Gamma - (r+\delta)\int k(a,z)\,d\Gamma - c_f. \quad (18)$$

Then, since c_f is a constant, apply the government's budget constraint and the left hand side of the goods market condition becomes:

$$\int \left(f(z,k,n) - c_f\right)\,d\Gamma = \int \lambda f(z,k,n)^{1-\tau}\,d\Gamma + T - c_f \quad (19)$$

Plug (18) in (19) and we get

$$\int \left(f(z,k,n) - c_f\right)\,d\Gamma = \int \pi(a,z)\,d\Gamma + w\int n(a,z)\,d\Gamma + (r+\delta)\int k(a,z)\,d\Gamma + T \quad (20)$$

Aggregating the constraint in (11) and forcing the stationary condition $\int a\,d\Gamma = \int a'\,d\Gamma - \chi\int a'(a,z)(\xi x(a,z) + 1 - x(a,z))\,d\Gamma + \mu a_0$, we get

$$\int \pi(a,z)\,d\Gamma = \int d(a,z)\,d\Gamma + \chi\int a'(a,z)(\xi x(a,z)+1-x(a,z))\,d\Gamma - \mu a_0 - r\int a\,d\Gamma \quad (21)$$

Plugging (21) to (20) results in:

$$\begin{aligned}\int \left(f(z,k,n) - c_f\right)\,d\Gamma = &\int d(a,z)\,d\Gamma + \chi\int a'(a,z)(\xi x(a,z) + 1 - x(a,z))\,d\Gamma - \mu a_0 \\ &- r\int a\,d\Gamma + w\int n(a,z)\,d\Gamma + (r+\delta)\int k(a,z)\,d\Gamma + T\end{aligned} \quad (22)$$

Apply the capital market clearing condition and the above equation becomes

$$\int \left(f(z,k,n) - c_f\right)\, d\Gamma = \int d(a,z)\, d\Gamma + \chi \int a'(a,z)(\xi x(a,z) + 1 - x(a,z))\, d\Gamma - \mu a_0 \\ + w \int n(a,z)\, d\Gamma + rb + \delta \int k(a,z)\, d\Gamma + T \tag{23}$$

The budget constraint of the representative consumer in the stationary equilibrium implies

$$c = rb + rs + w + T \tag{24}$$

Plugging (24) to (23) leads to:

$$\int \left(f(z,k,n) - c_f\right)\, d\Gamma = \int d(a,z)\, d\Gamma + \chi \int a'(a,z)(\xi x(a,z) + 1 - x(a,z))\, d\Gamma - \mu a_0 \\ + c - rs - w + w \int n(a,z)\, d\Gamma + \delta \int k(a,z)\, d\Gamma \tag{25}$$

Then, insert the equilibrium condition on the financial intermediary and (25) becomes

$$\left(f(z,k,n) - c_f\right)\, d\Gamma = c - w + w\mu c_n + w \int n(a,z)\, d\Gamma + \delta \int k(a,z)\, d\Gamma, \tag{26}$$

and the goods market clearing condition follows from applying the labor market clearing condition to (26):

$$\left(f(z,k,n) - c_f\right)\, d\Gamma = c + \delta \int k(a,z)\, d\Gamma. \tag{27}$$

D The effect of size-dependent distortions on output changes for more ϕ's

Table 11: The effect of size-dependent distortions on output changes with $\phi = 0.5$

τ	output, %. of bench.	+eq.MPK within z	+eq.MPK across z	+adjust ext. margin		+adjust K
				$(E(Z))$	(M)	
0	82	+7.7	+0.6	+0.7		+9
				(-6.8)	(+8.1)	
0.05	85	+2.1	+3.8	-0.2		+9
				(+3.8)	(-3.9)	
0.10	81	+3.1	+3.4	+3.4		+9
				(+11.6)	(-6.7)	
0.20	81	+0.2	+4.7	+9.3		+9
				(+11.7)	(-4.9)	
0.30	77	+0.1	+4.9	+9		+9
				(+11.8)	(-1)	
0.40	71	+0.1	+5	14.3		+9
				(+11.9)	(+4.3)	
0.50	62	+0	+5.1	+23.8		+9
				(+11.8)	(+13.8)	

Note: The structure of this table is identical to the structure of Table 7 for $\phi = 0.5$.

Table 12: The effect of size-dependent distortions on output changes with $\phi = 0.33$

τ	output, %. of bench.	+eq.MPK within z	+eq.MPK across z	+adjust ext. margin		+adjust K
				$(E(Z))$	(M)	
0	88	+6.5	+0.6	+0		+5
				(-4.6)	(+4.9)	
0.05	87	+3.2	+2.1	+2.7		+5
				(+12)	(-7.7)	
0.10	86	+1.1	+4	+3		+5
				(+12)	(-7.4)	
0.20	85	+0.1	+4.9	+5.1		+5
				(+12.3)	(-5.3)	
0.30	80	+0.1	+5.1	+9.2		+5
				(+12.4)	(-1.2)	
0.40	74	+0	+5.2	15.1		+5
				(+12.3)	(+4.7)	
0.50	66	+0	+5.3	+23.9		+5
				(+12.3)	(+13.5)	

Note: The structure of this table is identical to the structure of Table 7 for $\phi = 0.33$.

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