

Technical Change, Task Reallocation and Wage Inequality

Long Qian^{*}

Arizona State University

Oct, 2022

Abstract

This paper empirically investigates wage inequality within the group of skilled workers in the recent four decades in the U.S. using CPS data and finds evidence that the trend of wage growth of the top and bottom 10th percentile of skilled workers significantly diverged starting from 2000. Using a task-based framework of occupation, I find that the changing trend of wage inequality was entirely driven by one category of occupation, namely the non-routine analytic occupation. Then, I consider in a model task reallocation between two broad task categories, namely, routine and abstract task, induced by an ongoing investment-specific technical change. In my model, the labor in the routine task is replaced by cheaper machines due to investment-specific technical change, then workers that are less productive in the abstract task enter abstract occupations. As a result, the wage inequality in the abstract task widens because of the reallocation of less productive workers from the routine task to the abstract task, i.e., the “composition effect”. In addition, since economic agents tend to postpone the investment in machines after the ongoing investment-specific technical change takes place for a while, the expansion path of wage inequality is not linear but features an acceleration of wage dispersion in the middle of the technical change. The quantitative results suggest that the model is able to provide a well-matched timing and magnitude of the non-linear expansion path in wage inequality that is observed in the data.

^{*}This paper is for my third-year project in Arizona State University. I’m greatly indebted to my advisor, Professor Bart Hobijn, for his motivation and instructive advising on this project. I’m also grateful for the useful comments from Professor Gustavo Ventura, Professor Natalia Kovrijnykh, and my cohort Siyu Shi. All errors are on my own.

1 Introduction

The past several decades have witnessed prominent changes in the U.S. labor market. Wage inequality has greatly increased over this period, which resulted in numerous literature investigating what might cause an increase in wage inequality. Part of this rise in inequality can be attributed to expanding wage differentials between observable groups, such as education and experience (see e.g., [Katz et al. \(1999\)](#)). However, measurable characteristics can not fully explain the striking surge in wage inequality. [Juhn et al. \(1993\)](#) and [Lemieux \(2006\)](#) find that the bulk of the increase in U.S. wage inequality is residual, i.e., due to unobserved attributes of workers belonging to the same educational or demographic group.

Since [Tinbergen \(1974, 1975\)](#)'s work, the paradigm has been that the increase in wage dispersion is driven by the evolution of technology. For example, [Katz and Murphy \(1992\)](#) and [Autor et al. \(2008\)](#), among others, provide a successful framework in accounting for how skill-biased technical change induces higher demand for skilled labor and increases the returns to skills (as measured by skill premium). In spirit of this, [Krusell et al. \(2000\)](#) show that an increase in skill premium could be induced by skill-capital complementarity, while [Acemoglu \(2002\)](#) proposes a framework where skill premium could rise as a result of an endogenous response to the increase in the supply of skilled workers. Another strand of literature (e.g., [Aghion et al. \(2002\)](#)) utilizes Schumpeterian Growth Theory to explain within-group inequality.¹

This paper intends to fill in the gap in the literature in two directions. First, it documents some new features of wage inequality in the four decades, especially its changing trend after 2000, focusing on how it differs across occupations within the group of skilled workers (those who hold a bachelor's degree). Second, building on the conventional view that wage inequality is affected by the change in technology, I link the changing structure of within-group wage inequality to a task reallocation model and quantitatively evaluate whether the mobility of heterogeneous workers across tasks could explain the empirical finding.

In this paper, I empirically investigate wage inequality within the group of skilled workers using a task-based framework formalized by [Autor et al. \(2003\)](#) (ALM hereafter) and find that the trend of wage growth of the top and bottom 10th percentile of skilled workers significantly

¹See [Aghion \(2002\)](#) for a simple example.

diverged after 2000. Following their procedure of classification, the occupations in CPS data based on consistent three-digit Census Occupation Code (COC) are sorted into five categories of tasks: Non-routine Analytic, Non-routine Interactive, Routine Cognitive, Routine Manual, and Non-routine manual. Interestingly, I find an appealing fact that the acceleration of the growth of wage differential for the whole group of skilled workers is exclusively driven by that within the non-routine analytic task, which suggests that a model based on tasks might be successful in explaining the behavior of wage inequality.

I then link it to a task reallocation model borrowed from [vom Lehn \(2020\)](#) to quantitatively evaluate how such a model would predict within-group wage inequality in the recent four decades. In his work, the author investigates how the task reallocation induced by replacement of labor in the routine task by cheaper machines shape the labor market polarization, in terms of employment share and average wage of different tasks. I follow his approach, which utilizes the basic theoretical framework brought about by ALM, but extend it by focusing on its implication on wage inequality within the non-routine analytic task.² To investigate this neatly, I simplify the structure in [vom Lehn \(2020\)](#) by assuming that workers are heterogeneous in one dimension, namely the labor efficiency in the abstract task, and choose to work either on the abstract or routine task based on the comparative advantage.³

The key mechanism of this model is that workers in the routine task can be substituted by capital equipment or machines, which is subject to an ongoing investment specific technical change featured by a decrease in the price of investment goods. As in [vom Lehn \(2020\)](#), cheaper investment goods would result in a reallocation of labor in the routine task, which is replaced by capital equipment, to the abstract task. Since the marginal entrants to the abstract task have lower labor efficiency units in that task than the incumbents, wage inequality is mechanically widened due to the change of composition of workers within the abstract task. I call this mechanism the “composition effect”. The finding in [Lemieux \(2006\)](#), that a large fraction of the growth in residual wage inequality can be attributed to composition effects, is remarkably consistent with the prediction of the model.⁴

²To the best of my knowledge, no previous work has linked the task-based framework to the study of within-group inequality.

³[vom Lehn \(2020\)](#) assumes heterogeneity in both general skills and sector-specific labor efficiencies. The simpler structure suffices here because I focus only on wage inequality within the non-routine analytic task.

⁴This paper shows that most of the growth in residual wage inequality from 1973 to 2003 is due to composition

The quantitative results suggest the model is able to predict the empirical fact that the growth of wage inequality accelerated from 2000 onward. To be specific, a sustained investment specific technical change could give rise to a non-linear expansion path of wage inequality within the abstract task throughout the transition dynamics. This is because when economic agents perfectly anticipate a sustained technical change, which will make the investment goods cheaper as the process proceeds, they would postpone the investment to the future when the price is even lower. As a result, the acceleration in investment occurs after the technical change takes place for a while, and this determines that the reallocation of labor and the growth rate of wage inequality will accelerate in the middle of the technical change. I call this pattern of agents’ decision the “lagging effect”. What’s remarkable is that the quantitative results indicate that the model provides a similar timing and magnitude of the change of the trend in wage inequality despite its parsimonious set-up.

There are many advantages for the task-based framework compared with a dichotomy of skilled and unskilled workers.⁵ First, when focusing on the group of skilled workers and exploring the wage distribution by tasks/occupations in the data, I indeed find heterogeneous behaviors of wage inequality across tasks/occupations. This suggests that investigating the wage structure solely from the perspective of workers would omit the nuanced but important fact that workers with similar observable characteristics might perform differently when working on distinct tasks. I will explore more on this in Section 2 where I provide a comprehensive overview of wage inequality in the U.S. labor market through the lens of tasks/occupations. Furthermore, treating the skill of workers as a direct input for production and ignoring the context of what types of tasks they work on are conceptually misleading.⁶

effects linked to the secular increase in experience and education. Despite working through different channels, the two factors associated with higher within-group wage inequality are comparable to the labor efficiency in the abstract task that I explore in this paper in the sense that they all mechanically amplify the wage dispersion by changing the composition of the population.

⁵See [Acemoglu and Autor \(2011\)](#) for a detailed discussion. Here I just present the two most relevant reasons for doing so.

⁶In [Sattinger \(1993\)](#), the author questions the one-dimensional skill measure in an interesting way: “What would the economy be like without an assignment problem? With only a single, observable skill, a worker would be able to get the same wage no matter which job he or she took. Professors at universities could be replaced by a sufficient number of high school graduates, presumably all lecturing at the same time. This

Second, the task-based framework better matches theories because it is relatively simpler to incorporate an investment specific technical change process, which is easier to identify empirically than the standard skill-biased technical change embodied by a relative (labor augmenting) total factor productivity (TFP thereafter) change. [Greenwood et al. \(1997\)](#) find that investment specific technical change accounts for the bulk of output growth in the second half the 20th century. [Krusell et al. \(2000\)](#) utilize the data of capital equipment prices and conclude that capital-skill complementarity could induce an increase in wage inequality favoring skilled workers when capital equipment prices drop. In contrast, the evidence on whether TFP change is skill-biased is less direct, because ultimately it is not directly identified but derived from residuals. For example, [Goldin and Katz \(1998\)](#) and [Acemoglu \(2002\)](#) argue that the evidence suggests that technical change in the nineteenth century often replaced rather than complemented skilled artisans and is therefore not skill-biased. So the main challenge is that there is no general consensus on how a TFP change might affect the labor market.⁷

The paper is organized as follows. Section 2 presents empirical evidence on the features of within-group wage inequality in the recent four decades. In section 3, I develop a model of task reallocation induced by an investment-specific technical change and characterize the main equilibrium conditions. Section 4 and 5 link the model back to the data and evaluate quantitatively to what extent an investment-specific technical change can generate the patterns of wage inequality as observed. In section 6, I briefly discuss how one can build on the parsimonious framework in this paper to better fit into the richer empirical evidence. Section 7 concludes.

2 Empirical Facts

This section empirically investigates the main features of the wage structure in the U.S labor market and how it has changed over the recent four decades. The main finding is that the trend of the growth of wage dispersion for skilled workers significantly accelerated from

very article could have been written by anybody, perhaps in less time.”

⁷[Aghion \(2002\)](#) constructs a framework where growth is driven by a sequence of quality-improving innovations embodied by increases in TFP to show how Schumpeterian Growth Theory could explain wage inequality, but there is little empirical evidence that supports the theories (e.g., [Aghion et al. \(2002\)](#) and [Violante \(2002\)](#)).

2000 onward. I then link the results to a task-based framework and find that the changing trend of wage inequality was entirely driven by one category of occupation, namely the non-routine analytic task.

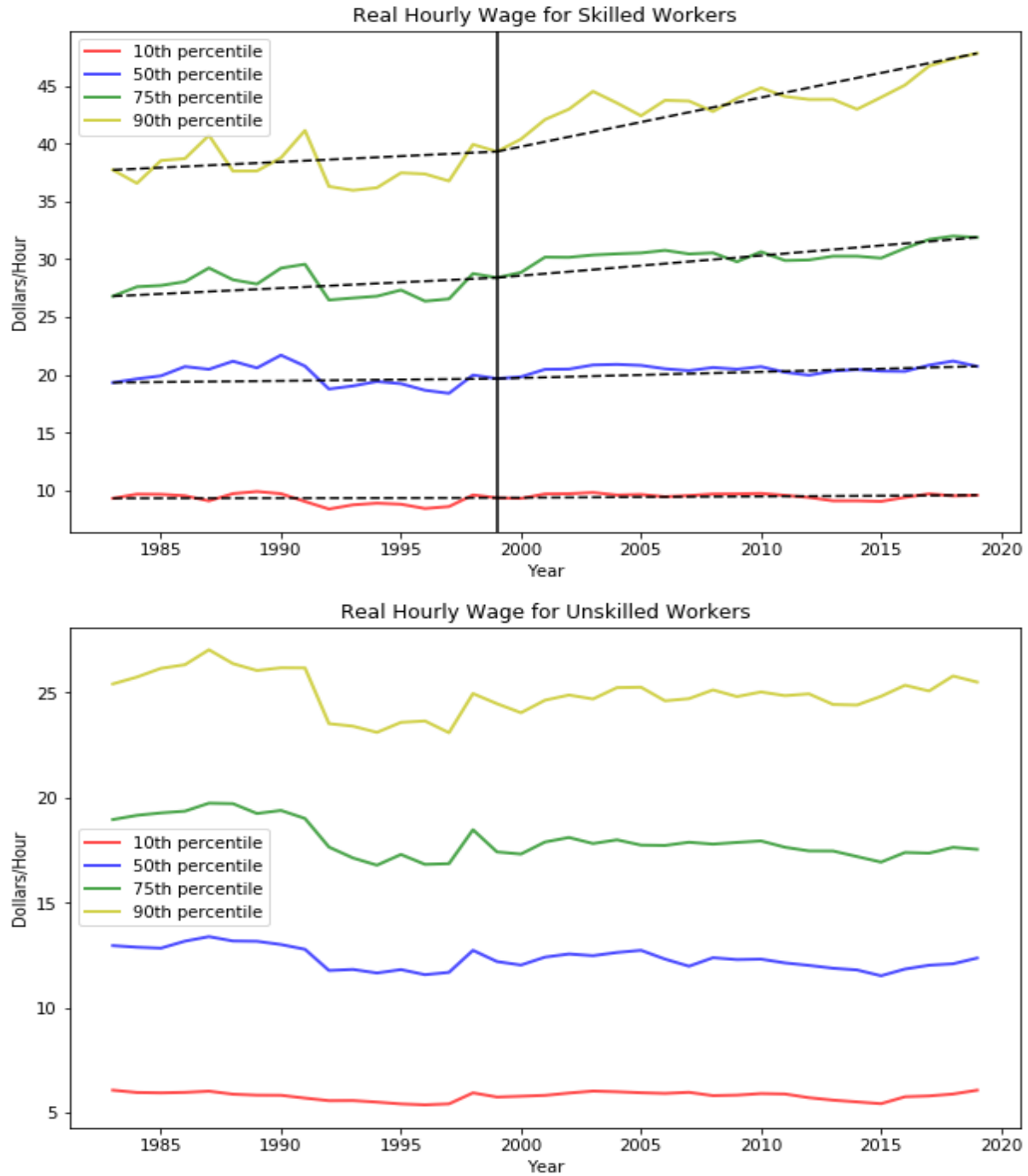
2.1 Observations

There is well-documented evidence that wage inequality both between- and within- observable groups substantially increased over the last few decades. In this section, I summarize several salient features of those two types of wage inequality observed in the recent 40 years, focusing on how they differ when workers are sorted based on their education and occupations. Whereas some were discussed extensively in previous works, one particular fact, that the trend of the growth of wage inequality became significantly steeper from 2000 up until now, has rarely been explored before.

First and foremost, the trend of wage growth for skilled workers varies a lot across percentile groups before and after 2000. To be specific, as shown in the top panel of Figure 1, the growth rate of real wages for above-median college degree holders significantly accelerated around 2000 relative to their lower-tail peers. Table 1 reemphasizes the point by confirming that the gaps in real wage growth between percentiles were not large during 1983-1999, but were substantially widened afterward. For example, one can find in particular that the real wage growth of the top 10th percentile of skilled workers significantly accelerated starting from 2000 compared to their peers, as reflected by the larger difference between the average annual growth before and after 2000. However, the divergence is not present for the group of unskilled workers, within which the growth rates of wages are relatively invariant across percentile groups. This observation is consistent with the finding in [Autor et al. \(2008\)](#) that the bulk of within-group inequality occurs above the median of the wage distribution in recent decades, given that presumably college degree holders represent the upper tail of the wage distribution. One subtle difference of my observation from what they found is essentially the notable acceleration of divergence within the educated group rather than the whole population after 2000.

Second, despite the sharp increase in the college wage premium, the bottom half of skilled workers barely benefited from it. Table 1 makes it clear that the growth of real wage for the first decile of college degree holders was almost zero in the past four decades, and it is

Figure 1: Real hourly wage for skilled and unskilled workers



Note: Calculation based on CPS ASEC data, 1983-2019. Both panels show various percentiles of the real hourly wage for workers with (skilled) or without (unskilled) a bachelor's degree. Real hourly wages are calculated using annual wage and salary divided by usual hours worked per year. I adjust for inflation using Consumer Price Index adjustment factors which convert dollar amounts to constant 1999 dollars (CPI99 in IPUMS), so all values are expressed as 1999 dollars. The sample comprises non-farm wage and salary workers who are 25-64 years old. The dotted lines in the top panel give the slope of wage growth during periods before and after 1999 (vertical line).

Table 1: Annual % Change in Real Wage

	Skilled Worker			Unskilled Worker		
	1983-2019	1983-1999	1999-2019	1983-2019	1983-1999	1999-2019
10th percentile	0.08	0.03	0.12	0.00	-0.34	0.28
50th percentile	0.20	0.11	0.27	-0.13	-0.37	0.07
75th percentile	0.53	0.37	0.61	-0.21	-0.51	0.04
90th percentile	0.74	0.26	1.08	0.01	-0.23	0.21

Note: Annual % change is calculated by the cumulative % change over the corresponding period divided by the number of years.

probably surprising to note that even the median college degree holders experienced negligible real wage increase compared to their upper-tail peers. This observation, combined with the last one, suggests that in contrast to the conventional view that the increase in the college wage premium benefits most of the college degree holders, it is just the top half of them that benefits. Thus, wage dispersion has been increasingly enlarged since the beginning of the 21st century.

Lastly, unskilled workers saw a mild decline in real wages in the recent 40 years. Although there is a general agreement on this fact in many works based on various sources of dataset,⁸ the fact is still relevant and noteworthy in the context of wage inequality because it serves as a natural contrast to the first fact related to within-group inequality. The divergence of wage growth across percentile groups that is salient for the skilled group is not present for the unskilled group at all, which indicates a tied connection between the widening of within-group wage inequality and the specific group of skilled workers.

2.2 Empirical Implementation

In this section, I implement an empirical strategy to quantify the magnitude of the change in the trend of wage inequality as observed. Besides, using the task-based framework by ALM, I run the same regression for the sub-samples of workers in five occupations to explore which

⁸See [Autor et al. \(2008\)](#) and [Acemoglu and Autor \(2011\)](#) for investigations based on earlier and recent March CPS data for example.

occupations drive the changing trend. One remarkable finding is that the changing trend of wage inequality was entirely driven by one category of task, namely the non-routine analytic task.

The occupations based on consistent three-digit Census Occupation Code (COC) in CPS data are classified as five categories of task groups following the criterion in ALM and [David and Dorn \(2013\)](#). They are non-routine analytic, non-routine interactive, routine cognitive, routine manual, and non-routine manual tasks. They are constructed using the rank of certain measures of task intensity by the Dictionary of Occupational Titles (DOT). Occupations with the highest rank among five measures of task intensity are attributed to corresponding categories.⁹

I then run the following regression for skilled workers as well as the sub-samples of workers in each of the five occupations defined above:

$$WI^i = \beta_0^i + \beta_1^i Year + \beta_2^i \mathbb{1}(Post2000) + \beta_3^i \mathbb{1}(Post2000) \times Year + \epsilon^i \quad (1)$$

where i denotes different (sub) groups of workers and WI^i is a set of measures of wage differentials for the whole group and five sub-groups obtained by taking log differences between wage percentiles in each group; $Year$ captures the time trend of wage differential; and $\mathbb{1}(Post2000)$ is an indicator of whether the observation is on or after 2000 (1 if it is and 0 otherwise). Therefore, the coefficients of interest are β_2^i and β_3^i , which reflect the magnitude of the shift in wage inequality and the change of the slopes (trend) after 2000 separately.

The results shown in the first column of Table 2 confirm the observation that the top 10 percent of skilled workers gained greatly over their peers after 2000. For example, the growth of the 90/10 percentile differential widens about 0.4% per year on average after 2000 relative to before. This is a sizable change of wage structure because on average, the differential grows about 0.2% each year (β_1), which means the magnitude of the widening amounts to two times the previous trend of the growth of wage differential. In some cases, this change of trend is augmented with a one-time upward shift after 2000. Other measures of upper-tail wage inequality also significantly increased after 2000, while measures of lower-tail wage inequality (50-10) did not.

⁹See Appendix B, ALM and [David and Dorn \(2013\)](#) for details.

Table 2: Changes in Wage Inequality after 2000

	(1)		(2)		(3)	
	Skilled		Non-Routine Analytic		Non-Routine Interactive	
90-10	0.004***	0.002	0.010***	0.110***	0.002	-0.021
90-50	0.003*	0.044**	0.004	0.094***	0.000	-0.003
75-10	0.002*	-0.001	0.010***	0.066***	0.002	-0.021
75-50	0.001	0.020**	0.003**	0.049***	0.000	-0.006
50-10	0.001	-0.022	0.006***	0.016	0.002	-0.018
	(4)		(5)		(6)	
	Routine Cognitive		Routine Manual		Non-Routine Manual	
90-10	-0.006	-0.087	0.000	-0.001	-0.018***	-0.200***
90-50	-0.002	-0.030	0.002	0.025	-0.010***	-0.107***
75-10	-0.006	-0.033	0.000	-0.023	-0.014**	-0.156**
75-50	-0.002	-0.024	0.002	0.003	-0.006*	-0.064**
50-10	-0.004	-0.057	-0.001	-0.025	-0.008	-0.092

Note: This table displays the results of the regression (equation (1)) for the whole sample of skilled workers (column 1), as well as the sub-samples of workers in the five occupations (columns (2) - (6)). The values on the left and right in each column are estimates of β_3 and β_2 separately. ***, **, * indicate the estimator is significant in 1%, 5% 10% level.

I then investigate which category of occupation (task) drives the changing trend of wage inequality for the overall skilled workers by implementing the same regression for the five sub-groups of workers defined above, from which we might have a sense of what kinds of skilled workers gained more after 2000 and why. Interestingly, the classification of occupation under the task-based framework shows an informative picture of differential behaviors of wage inequality across occupation categories, although the interpretation is slightly different from ALM which focuses only on the reallocation of task input demand rather than wage inequality.

The results in columns (2) - (6) demonstrate it is essentially just one occupation (task) that contributes to the increases in the growth of overall within-group inequality, namely the non-routine analytic task. It is appealing to find that the non-routine analytic task not only underwent a widening of upper-tail wage inequality in trend after 2000, but also the magnitude was more than twice as much compared to the whole skilled group. The change of trend (slope) was also augmented with a sizable one-time upward shift. In contrast, none of the rest of the occupations showed a significant shift in the trend of the wage structure after 2000. If any, there was a notable contraction of wage inequality for the non-routine manual jobs. Put together, the evidence suggests that the change of the trend of upper-tail wage differential within the skilled group is exclusively driven by that within non-routine analytic jobs, which also more than offset the opposite force brought about by the contraction of wage inequality within non-routine manual jobs.

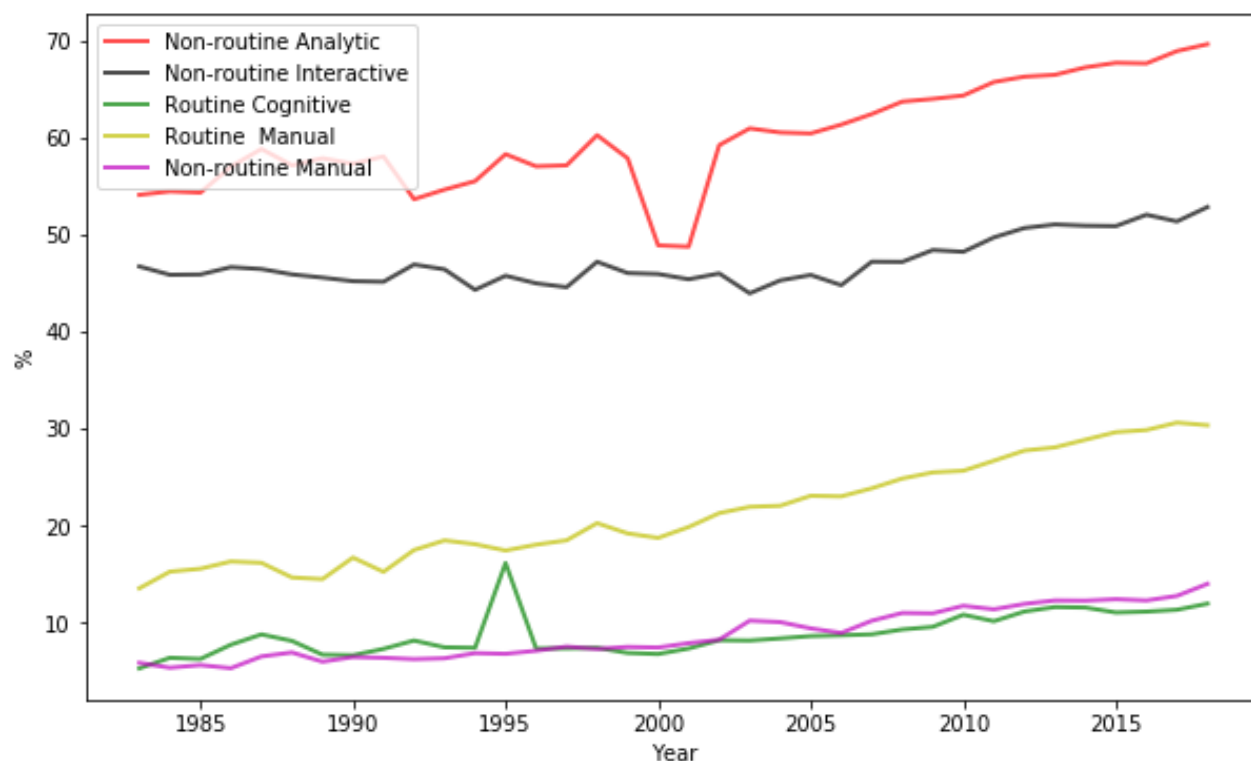
2.3 Educational Distribution

Because my empirical analysis mainly focuses on wage inequality within skilled workers who have at least a bachelor's degree, I am interested in to what extent does this relate to the overall inequality. It turns out the answer greatly depends on which category of occupation we are looking at. I also investigate which occupations employ the most workers with college degrees. Understanding those helps shed a light on what might drive the changing structure of wage inequality within the skilled group.

One appealing finding is that the bulk of workers consists of skilled workers in the analytic task, which is exactly the task that underwent a changing trend of wage inequality. This suggests a close link between the skill intensity of a task and its structure of wage inequality.

Figure 2 shows a clear hierarchical pattern of the share of skilled workers for the five occupation categories. More than half of the workers doing non-routine analytic jobs are college degree holder and the task skill intensity kept increasing in the recent four decades. For non-routine interactive jobs, this number is a bit lower. For the rest of the three categories, the skill intensity is greatly lower, although routine manual jobs are well above the other two.

Figure 2: Percentage of College Degree Holder

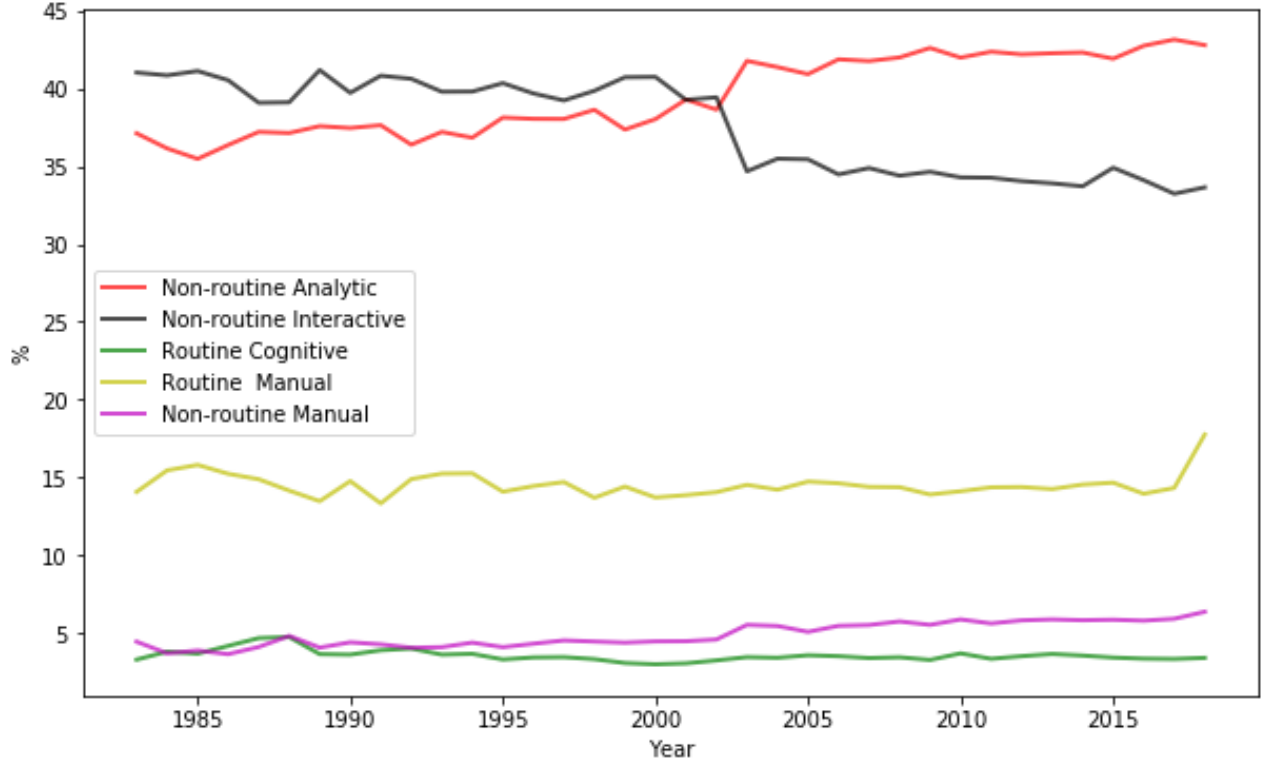


Note: The values are shares of workers with at least a bachelor's degree in each category of occupation.

The data on the destinations of skilled workers conveys a similar big picture as Figure 2 with slightly more information. Figure 3 explores the educational distribution from the other direction, which is what percentage of skilled workers goes to the five categories of occupations separately. It shows most (more than 80 percent) of the college degree holders work in non-routine cognitive occupations. It is notably interesting that the percentage of skilled workers working in non-routine analytic jobs surpassed interactive jobs just around 2000, coinciding with the timing of the change of the wage structure within non-routine cognitive jobs.¹⁰

¹⁰This observation should not be viewed as direct evidence in explaining the changing structure of wage

Figure 3: Destinations of College Degree Holders



Note: The values show what percentage of skilled workers (with at least a bachelor's degree) goes to each category of occupation, so the five numbers in each year add up to one.

3 A Model of Task Reallocation and Wage Inequality

To interpret the empirical evidence that the trend of wage dispersion became steeper and the changing trend was entirely driven by the non-routine analytic task, one starting point is the standard implications of the task-based framework as in ALM, which suggest a composition effect as the main driving force of the change of wage structure. The idea is that machines are more likely to substitute labor in the routine task, then when the price of capital (investment) drops, labor in the routine task will be replaced by machines and crowded out to other tasks.¹¹

inequality, because part of this might be due to the change of occupation classification by CPS. I use the occupation classification on the 1990 basis, but the criteria have been changed in the 2010 basis. Further exploration of this will be left to future research.

¹¹In ALM, the sole source of technological upgrading is embodied by the drop in the price of computers, which is argued to favor the non-routine cognitive task against the routine task (see their Table 1). vom Lehn (2020) makes a more explicit assumption that machines can only substitute labor in the routine task.

Since workers are heterogeneous in task-specific labor efficiencies, the reallocation of labor will result in a changing composition of workers and affect the demand and supply of labor within each task.

In this section, I investigate how the composition effect would affect within-group wage inequality and whether it could broadly fit into the empirical evidence by developing a discrete time general equilibrium model building on [vom Lehn \(2020\)](#) that features labor reallocation across tasks. Instead of three task inputs (abstract, routine, manual) in his work, I consider only two tasks: abstract and routine. One can understand the abstract task as a model counterpart of the non-routine analytic task defined in Section 2 and the routine task as a composite of the rest of the tasks. In this model, workers are heterogeneous in one dimension, namely the amount of labor efficiency in the abstract task, and the task reallocation is driven by the substitution of labor in the routine task by machines embodied by capital. The capital stock accumulates because of ongoing investment-specific technical change, as emphasized in the work of [Greenwood et al. \(1997\)](#).¹² The goal of abstracting from more task inputs and using the simplest possible framework is to focus on the implications of wage inequality in the non-routine analytic task when it is linked to the task reallocation model. This is the main theoretical innovation and contribution of the paper. A richer model could be developed later to better explain the empirical facts we found in the last section.

3.1 Environment: Firms and Production

There is a representative firm operating a production technology for the final output. The production function is a standard nested CES production function using two task inputs - abstract and routine task:

$$Y_t = [\mu_a N_{at}^{\frac{\gamma_a-1}{\gamma_a}} + (1 - \mu_a)[(1 - \mu_r)K_t^{\frac{\gamma_r-1}{\gamma_r}} + \mu_r N_{rt}^{\frac{\gamma_r-1}{\gamma_r}}]^{\frac{\gamma_r(\gamma_a-1)}{\gamma_a(\gamma_r-1)}}^{\frac{\gamma_a}{\gamma_a-1}} \quad (2)$$

where the subscript a (r) refers to the abstract (routine) task. N_{at} (N_{rt}) is efficiency units of labor in the abstract (routine) task and K_t is machines embodied by capital. The parameters

¹²The paper concludes that investment-specific technical change accounts for around 60 percent of overall output growth, while conventional neutral TFP change contributing the rest. Thus, the set-up with machines replacing routine labor is not only suitable for the task-based framework, but also capture the main driving force of technological change.

include γ_a (γ_r), which is the corresponding elasticity of substitution between routine and abstract tasks (routine labor and machines), and μ_a (μ_r), which is the relative weight of the abstract (routine) labor in production. The terms in the first square bracket indicate that the output of the abstract task is produced simply using the total efficiency units of labor in that task, while for the routine task, the output could be produced by either machines K or labor. That investment-specific technological change can potentially substitute for routine labor and is the key mechanism of shifts in employment across tasks capture the fundamental task replacement arguments put forth by ALM.

The problem for the firm is:

$$\max_{K_t, N_{rt}, N_{at}} Y_t - r_t K_t - w_{rt} N_{rt} - w_{at} N_{at} \quad (3)$$

3.2 Environment: Workers

To simplify the household problem, I assume a representative family set-up, in which the household owns the capital and is comprised of a continuum of workers of measure one. Each worker supplies one unit of labor inelastically. For Preferences, I assume a standard logarithmic specification for households' instantaneous utility with discount factor β :

$$u(c) = \log(c_t) \quad (4)$$

The household accumulates capital by the following law of motion:

$$K_{t+1} = (1 - \delta)K_t + q_t I_t \quad (5)$$

where q_t is the relative price of consumption to investment goods and included in the equation of capital evolution following Greenwood et.al (1997).¹³ An increase in q_t represents increased investment-specific technological progress and is the main driving force of the model. Its movement is assumed to be exogenously determined for now.

In addition, workers are heterogeneous in efficiency units of labor in the abstract task, which is represented by a single index z . This assumption is key in generating a dispersed

¹³Note that although it is interpreted as the price, q_t is an exogenous technological parameter but not an endogenous variable determined in the equilibrium.

wage distribution in the abstract task that can be mapped back to the data and evaluated quantitatively. I assume that the distribution of z is simply standard normal. A worker with z provides $\phi_a(z)$ efficiency units of labor in the production of abstract task, where $\phi_a(z) = e^{\alpha_a z}$, so that the distribution of efficiency units is log normal. Therefore, wages for a worker with skill level z in the abstract occupation are given by the product of the common skill price, w_{at} , and the heterogeneous efficiency units in the abstract occupation, $\phi_a(z)$. For simplicity, I assume that the abstract skill level does not affect workers' productivity in the routine task ($\phi_r(z) = 1$), so wages in that task are homogeneous. Abstracting from the heterogeneity of workers in the routine task simplifies the structure but will not affect the main focus of the model.

Thus, the representative household problem is given by:

$$\max_{C_t(z), O_t(z), K_{t+1}} \sum_{t=0}^{\infty} \beta^t \int_z \log(C_t(z)) d\Phi(z) \quad (6)$$

$$s.t. \quad C_t + I_t = r_t K_t + \int_z W_t(z) d\Phi(z) \quad \text{and} \quad K_{t+1} = (1 - \delta)K_t + q_t I_t$$

where K_t is the household's total capital holdings, $O_t(z)$ is the occupational choice (abstract or routine task) for each individual and $W_t(z)$, the wage for an individual with z , is obtained by $\sum_j \mathbb{1}(O_t(z) = j) w_{jt} \phi_j(z)$.

The representative family set-up implies that consumption will be equalized across all workers, which greatly simplifies the model. Since consumption inequality and its labor market implications are not the main focus here, this parsimonious framework would not affect the central point of interest, which is wage inequality.

3.3 Definition of Equilibrium

The competitive equilibrium for the economy consists of a set of prices (r_t, w_{at}, w_{mt}) and allocations $(C_t(z), O_t(z), K_t, N_{rt}, N_{at})$, such that:

- (1) Taking prices (r_t, w_{at}, w_{mt}) as given, workers choose $(C_t(z), O_t(z), K_{t+1})$ to solve (6) starting from any initial conditions K_0 for all $t \geq 1$
- (2) Taking (r_t, w_{at}, w_{mt}) as given, the firm chooses (K_t, N_{rt}, N_{at}) to solve (2);

(3) Labor and output markets clear:

$$N_{jt} = \int_z \mathbb{1}(O_t(z) = j) \phi_j(z) d\Phi(z) \quad j = a, r \quad (7)$$

and resource constraint:

$$Y_t = C_t + I_t = C_t + \frac{1}{q_t}(K_{t+1} - (1 - \delta)K_t) \quad (8)$$

3.4 Characterization of Equilibrium

Since the full path of q_t is deterministic and its movement is the primary driving force for the incentives of investment, it is useful to relate the rental rate of capital r_t to q_t . Therefore, it is convenient to rewrite the Euler equation for the model in the following way:

$$r_{t+1} = \frac{1}{q_t} \left(\frac{C_{t+1}}{\beta C_t} - \frac{1 - \delta}{\frac{q_{t+1}}{q_t}} \right) \quad (9)$$

A worker's problem can then be characterized by the Euler equation (9). Note that the rental rate of capital next period r_{t+1} is not only a decreasing function of q_t , but also an increasing function of the growth rate of q . The first result holds simply because an increase in q_t leads to higher investment and thus induces a higher supply of capital in this period, which has a downward pressure on the rental rate next period. The growth rate of q_t also affects the rental rate as a high growth rate will lower the relative capital gains on undepreciated capital, and thus offsets declines in the rental rate.

Then, I characterize the assignment of workers into tasks based on a comparative advantage structure. The main feature of the equilibrium is that the assignment of labor takes a form of the threshold value. Formally, there exists a unique z^* such that workers with $z > z^*$ choose to work in the abstract task while workers with $z < z^*$ choose to work in the routine task.¹⁴

The key driving force of the model is that as a technical change takes place, the prices for investment goods drop (q increases). Cheaper investment goods induce the firm to replace the labor in the routine task by machines. Workers who become obsolete in the routine task

¹⁴To see this, suppose workers with $z = z_1$ choose to work in the abstract task, it must be $\phi_a(z_1)w_{at} \geq w_{rt}$.

Then for any $z > z_1$, $\phi_a(z)w_{at} > \phi_a(z_1)w_{at} \geq w_{rt}$, so workers with $z > z_1$ also choose to work in the abstract task. A similar argument applies to workers who work in the routine task.

have to relocate to the abstract task. From the threshold argument above we know that the marginal entrants to the abstract task are lower types of z compared to the incumbents. This drives the threshold value z^* down and changes the composition of the workers in terms of z in the abstract task. This mechanism of labor assignment and mobility based on comparative advantage is closely related to the classic work by Roy (1951) and many modern treatments of the Roy model.¹⁵

Finally, the prices (r_t , w_{at} , w_{mt}) can be obtained in terms of aggregate variables using the first order conditions by the firm's profit maximization problem (3):

$$w_{at} = \mu_a \left(\frac{Y_t}{N_{at}} \right)^{\frac{1}{\gamma_a}} \quad (10)$$

$$w_{rt} = (1 - \mu_a) \mu_r Y_t^{\frac{1}{\gamma_a}} R_t^{\frac{\gamma_a - \gamma_r}{\gamma_a(\gamma_r - 1)}} \left(\frac{1}{N_{rt}} \right)^{\frac{1}{\gamma_r}} \quad (11)$$

$$r_t = (1 - \mu_a)(1 - \mu_r) Y_t^{\frac{1}{\gamma_a}} R_t^{\frac{\gamma_a - \gamma_r}{\gamma_a(\gamma_r - 1)}} \left(\frac{1}{K_t} \right)^{\frac{1}{\gamma_r}} \quad (12)$$

4 Model Solution and Calibration

This section links the model to the data by constructing the path of the sole exogenous driving force of the model, q_t . In this section, I describe the measurement of q_t , the calibration of all model parameters, with which I briefly discuss how the model can be solved.

¹⁵The closest benchmark framework relating comparative advantage and the assignment of skills to tasks goes back to ALM. But the precedent of this approach is an assignment model introduced by citettinbergen1974substitution, and further developed in Sattinger (1975, 1993) and Acemoglu and Zilibotti (2001) (see Acemoglu and Autor (2011) for a detailed description of this strand of literature). Young (2014) finds empirical evidence that workers self-select into the industries they have an advantage on and thus expansions of certain sectors will draw in individuals with less comparative advantage from competing sectors, leaving the average productivity in expanding sectors to decline. This mechanism of the changing composition of workers in terms of sector-specific productivity is also closely related to the one in this paper.

4.1 Exogenous Forcing Process and Model Solution

To solve and evaluate the model quantitatively, I construct the entire path of q_t following vom Lehn (2020), which used the deflator for investment goods as constructed following Gordon (1990). As in Greenwood et al. (1997), q_t is measured as the price of consumption divided by the price of equipment and software investment. I assume that the time series starts in a steady state with q_1 normalized to 1 in the year 1983. The data is available until 2015, after which I assume the growth rate of q_t linearly decays to zero in a few years.¹⁶ The entire path of q_t and its growth is plotted in Figure 4.

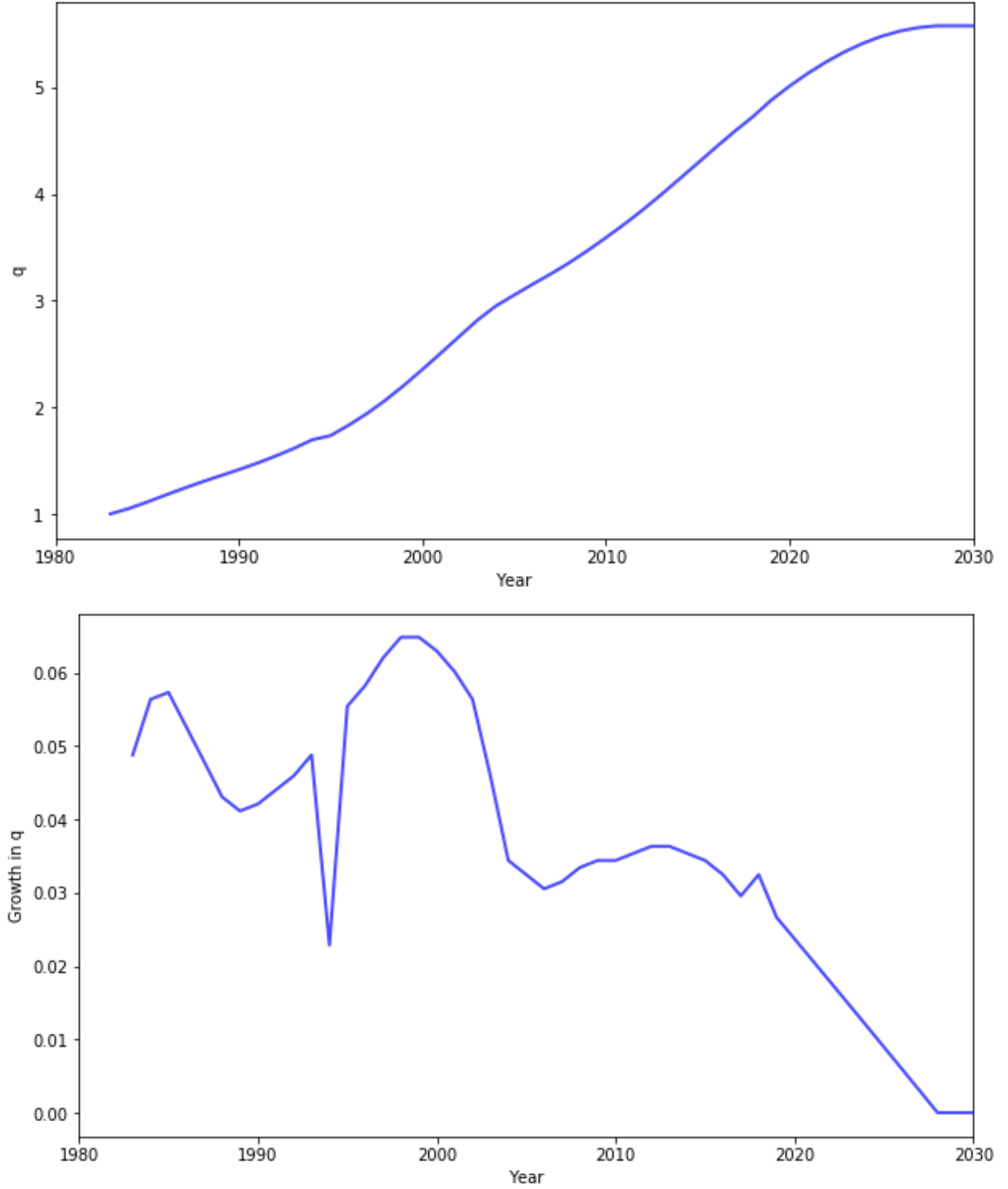
With the values of q_t , I can first obtain any steady-state variables using the Euler equation (9). I then calibrate the parameters of the model so that the model solution matches certain empirical counterparts. Lastly, the transition path is solved using a standard shooting algorithm given that we know the initial and terminal steady states. The details of solving the model are in Section 5 and Appendix A.

4.2 Calibration

The parameters to calibrate are the elasticities of substitution (γ_a, γ_r), share parameters in the production function (μ_a, μ_r), the scaling parameter of productivity function in the abstract task (α_a), capital depreciation rate (δ) and discount factor (β). Among those parameters, the discount factor and depreciation rates can be set externally following a standard value as estimated in Cummins and Violante (2002). This gives $\beta = 0.96$ and $\delta = 0.1$. Besides, since the structure of production is almost identical to the one in vom Lehn (2020), except that manual jobs are aggregated in the routine task, I use his calibration results for the elasticities of substitution. So we have $\gamma_a = 0.43$ and $\gamma_r = 1.38$. $\gamma_r > 1 > \gamma_a$ implies that capital is a substitute for the labor input in the routine task, while the routine task and abstract task are a gross complement. The values of elasticities capture the key qualitative attributes of tasks that are necessary for the desired task reallocation and are therefore reasonable results for calibration.

¹⁶vom Lehn (2020) constructed a longer time series of q_t by assuming a 100 years decaying period. Making the path shorter and simpler helps circumvent technical issues in solving the transition path but does not change the key results on wage inequality.

Figure 4: Time Path and Growth Rate of q_t



Note: The time series starts in a steady state with q_1 normalized to 1. The data of q_t calculated based on the price index of investment goods constructed in [Gordon \(1990\)](#) is available until 2015, after which it is assumed that the growth rate of q_t linearly decays to zero in 12 years. The growth of q in period t is given by $\log(q_{t+1}) - \log(q_t)$.

I calibrate the share parameters in the production function (μ_a, μ_r) in the way that the labor share of output and the fraction of labor income in routine and abstract jobs in the initial steady state match their empirical counterparts in 1983. α_a is calibrated such that the log difference between the 90th percentile and 10th percentile of the wage is matched to the data in 1983. Table 3 summarizes parameter values for the calibration.

Table 3: Calibrated Values for Model Parameters

Parameter	Value
γ_a	0.43
γ_r	1.38
μ_a	0.87
μ_r	0.52
α_a	0.77
β	0.96
δ	0.10

Note: This table reports the values for calibrated model parameters. They are elasticities of substitution (γ_a, γ_r), share parameters in the production function (μ_a, μ_r), the scaling parameter of productivity function in the abstract task (α_a), capital depreciation rate (δ) and discount factor (β).

5 Results

In this section, I present results to the model solution by showing how the steady states and transition dynamics would evolve when q changes exogenously. The results not only establish the intuition that labor in the routine task will relocate to the abstract task when an ongoing investment specific technical change takes place, and wage inequality in the abstract task widens as a result of the composition effect, but also remarkably predict a non-linear expansion path of wage dispersion in the abstract task which is comparable to the empirical evidence. The last subsection discusses why such a parsimonious framework could generate the desired transition path.

5.1 Steady States

Before showing the full solution to the model, I provide a rough idea of how the movement of q_t could shape wage inequality by presenting the solution of the model for two steady states and characterize the implications for wage inequality. The two steady states are initial steady state, in which q is normalized to 1 and is already in a steady state, and terminal steady state, in which q is equal to its terminal value q_T and stays there afterward.

All steady state variables can be easily obtained by utilizing properties of truncated log-normal distribution and solving equations (9) - (12) in closed form.¹⁷ The results of two steady states with $q_1 = 1$ and $q_T = 5.6$ are displayed in Table 4 below.

Before looking directly at the implications of investment-specific technical change and task reallocation for wage inequality, we can see that the results suggest one obvious mechanism that would affect wage inequality within the abstract task, namely the composition effect, which results from the reallocation of labor with low z from the routine task to the abstract task. This could be made clear by the different threshold values of z^* in two steady states as shown in the first row of Table 4. The transition of the economy induced by increased investment-specific technical change features departing from a high level of z^* to a low level, which implies more workers in the abstract task and fewer workers in the routine task. This is because increases in q stimulate a higher investment level and capital stock, which replaces the labor in the routine task. As a result, an increasing amount of labor in the routine task relocates to the abstract task, and since the marginal entrants are lower types of z compared to the incumbents, the range of z in the abstract task is expanded and the wage inequality could be mechanically widened due to the composition effect.

As shown in the second row of the table, there should be a price effect because the transition of the economy is also accompanied by an increase in the wage rate in the abstract task.¹⁸ Since the total wage of a worker is the product of wage rate and efficiency units, the increase of wage rate amplifies the variance of total wage within the abstract task holding the distribution of z fixed. However, since we focus on the relative change of wage inequality within the abstract task throughout the transition path, this price effect would drop out and not affect the measure

¹⁷The procedure is briefly discussed in Appendix A.

¹⁸I leave the effects of price on overall wage inequality to future work. See Section 6 for a brief discussion.

of (relative) wage inequality.

Table 4: Comparison of Steady State Variables

	$q = 1$	$q = 5.6$
z^*	-0.54	-1.07
\bar{w}_a	1.27	2.73
\bar{w}_r	0.42	0.37
K	1.19	5.17

Note: Values are for initial and terminal steady states separately. z^* , \bar{w}_a , \bar{w}_r , K represent the threshold value of z , average wages for abstract and routine tasks, and capital stock.

Table 5: Comparison of Wage inequalities

	$q = 1$	$q = 5.6$
Var	0.32	0.46
90-10	1.39	1.70
90-50	0.93	1.09
75-10	0.91	1.15
75-50	0.44	0.53
50-10	0.46	0.62

Note: Variances are calculated using log wages within the abstract task. The rest are log differences between corresponding percentiles of wage. So for example 1.39 means the ratio of 90th percentile 10th percentile of wages is $e^{1.39}$. The percentage change of the ratio is approximately the difference of two steady states.

Table 5 confirms the intuition above by showing that all measures of wage inequality increase when the economy transits. The first row displays the change of variance in log wages, while the rest display the log differences between different percentiles of the wage distribution. Note again that all measures of wage inequality are only affected by the composition effect because the price effect drops out when relative measures are used.

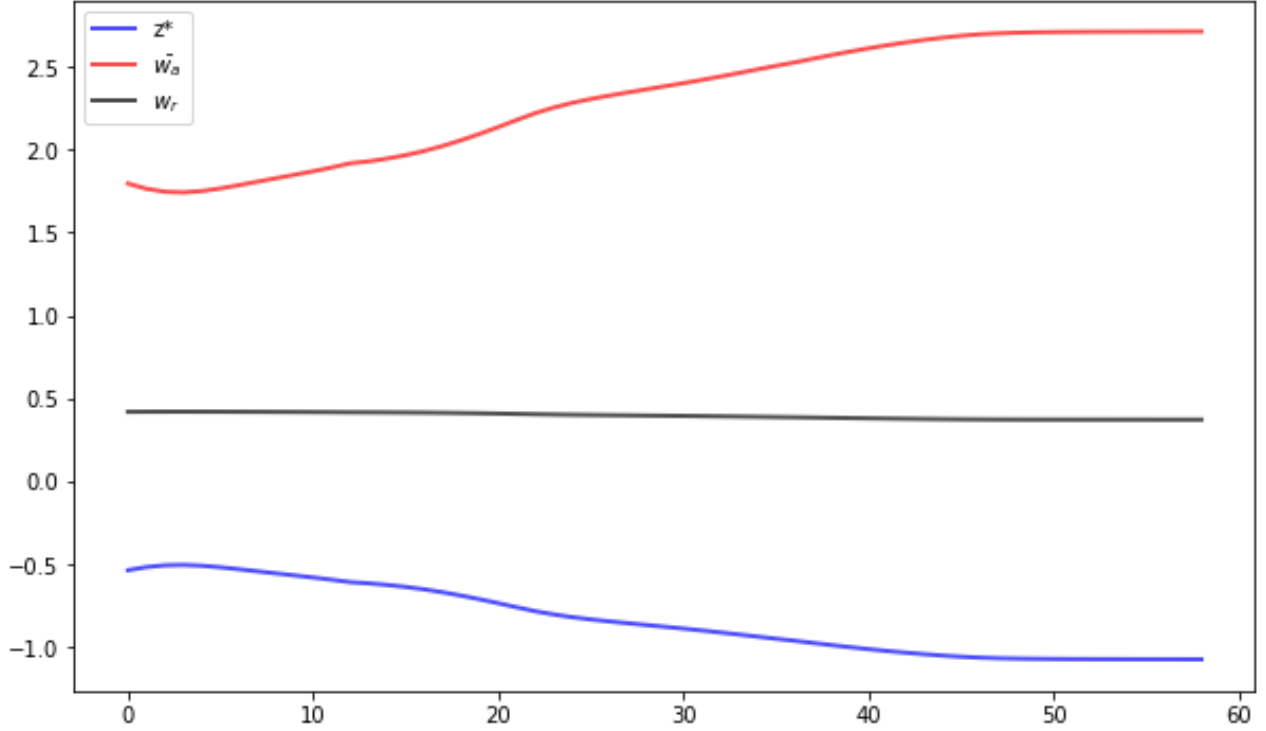
5.2 Transition Dynamics

This section discusses the entire path of the solution of the model and investigates whether the model could generate an acceleration of the growth of wage dispersion within the abstract task throughout the transition path. We know from the steady state analysis above that an investment specific technical change embodied by an increase in q could drive a higher level of investment and capital stock, which replace the input of labor in the routine task. So lower types of workers who are replaced by machines then have to work on the abstract task, which mechanically widens wage inequality within the abstract task due to the composition effect. However, the facts in Section 2 demonstrate that wage inequality was not increasing linearly in the past four decades, but featured an acceleration in its growth starting from 2000. To understand this using our task-based framework requires a complete characterization of the behavior of the model when it transits from one steady state to another.

Figure 5 displays the transition dynamics of the model solution. The main object of interest is the threshold values of worker type z^* , which determine the allocation of workers in the two tasks. Not surprisingly, z^* transits from a high to a low level almost monotonically. But one appealing finding is that the trajectory of z^* is significantly convex, with largely a flat level in the first 15 periods and an accelerated declining rate after that. The kink arises in the second half of the 1990s, coinciding with the timing of the acceleration of wage inequality. Since in this model wage inequality within the abstract task is entirely driven by the composition effect induced by reallocation of labor from the routine to the abstract task, this result suggests that the model might be able to generate a non-linear trend of wage inequality during the transition.

Consistent with the conjecture above, I find that the wage distribution in the abstract task shifts up at an accelerated rate, and this acceleration is more salient for the higher percentiles of wage. Figure 6 shows the distribution of wages measured by varying percentiles throughout the transition path. The 90th percentile of wage exhibits a remarkable kink at the end of the 1990s while the 10th percentile of wage is relatively flat. This suggests that wage inequality within the abstract task should rise at an increasing rate. Indeed, various measures of inequality as shown in Figure 7 reveal that all percentiles of wage in the abstract task shift up relative to the lower percentiles, and the trend of the growth is not linear throughout the transition path

Figure 5: Transition Dynamics of Model Solution



Note: z^* (blue line) is the threshold value of z that determines the assignment of workers; w_a (red line) and w_r (black line) denote the mean wage rate of labor for abstract and routine tasks.

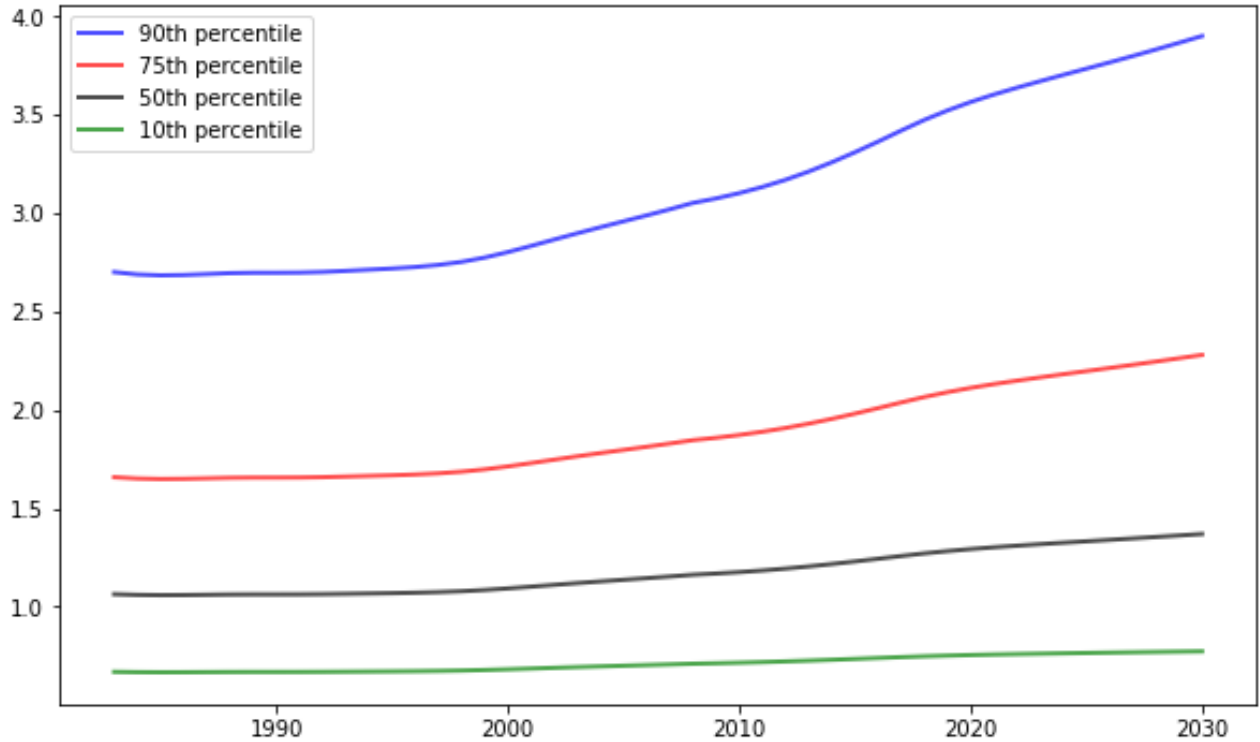
but features an acceleration occurring around 2000, which is in line with the empirical facts we find in Section 2.

To gain a more straightforward view of how the model maps to the data, Figure 8 brings the model back to the data and the result suggests the model could match the trend of wage inequality in the abstract task as measured by the log difference between 90th and 10th percentile of wage qualitatively well.¹⁹ Although the model struggles to capture the bouncing and downward-trending behavior of the data before 2000 but predicts a flat level, it matches well the timing and magnitude of the widening of wage inequality up until now. Given that the wage data in the abstract task is a small sample and the whole sample of skilled workers seems to support a stable and flat trend of wage inequality before 2000, I take the result as a

¹⁹The reason only the 90 – 10 wage inequality is used here is because I fix the distribution of z as standard normal and I calibrate the parameter that affects the spread of wage distribution (α_a) to match the 90 – 10 wage inequality in the initial period. So this measure of wage inequality provides a more reliable comparison between the model and the data.

successful test for the model. In the next section I briefly discuss why this model could work in explaining the non-linear expansion of wage inequality.

Figure 6: Wage Distribution in the Abstract Task



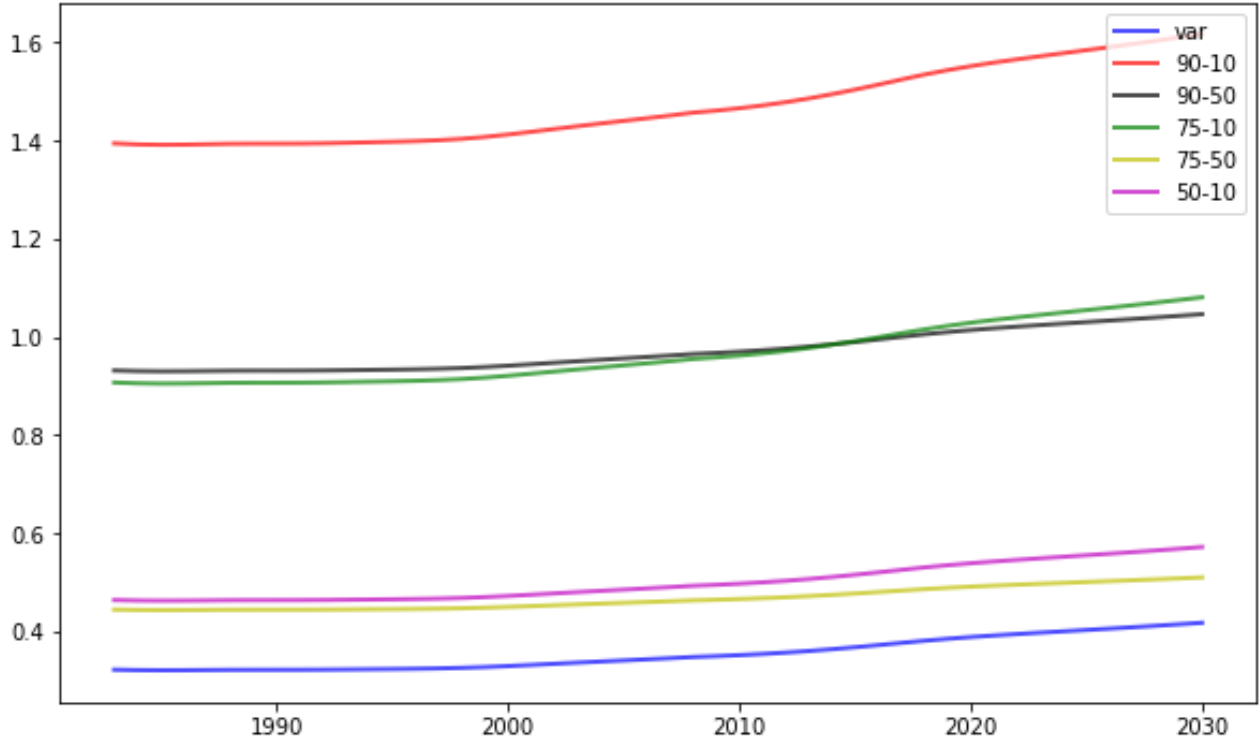
Note: The figure displays the transition of the 90/75/50/10th percentile of wage in the Abstract task solved from the model.

5.3 Discussion

The results of the model imply that a sustained investment specific technical change could induce a non-linear expansion path of inequality throughout the transition dynamics. Interestingly, when revisiting the path of q , which is the sole exogenous driving force of the model, we can find that the growth rate of q is at a peak around 2000. Thus, the widening of wage inequality actually occurs when the increase in q slows down.

This seems counter-intuitive because we know that it is the crowding-out effect induced by investment specific technical change that drives the change of composition of workers, which mechanically enlarges the spread of wage distribution. If this is the case, one might

Figure 7: Wage Inequality in the Abstract Task

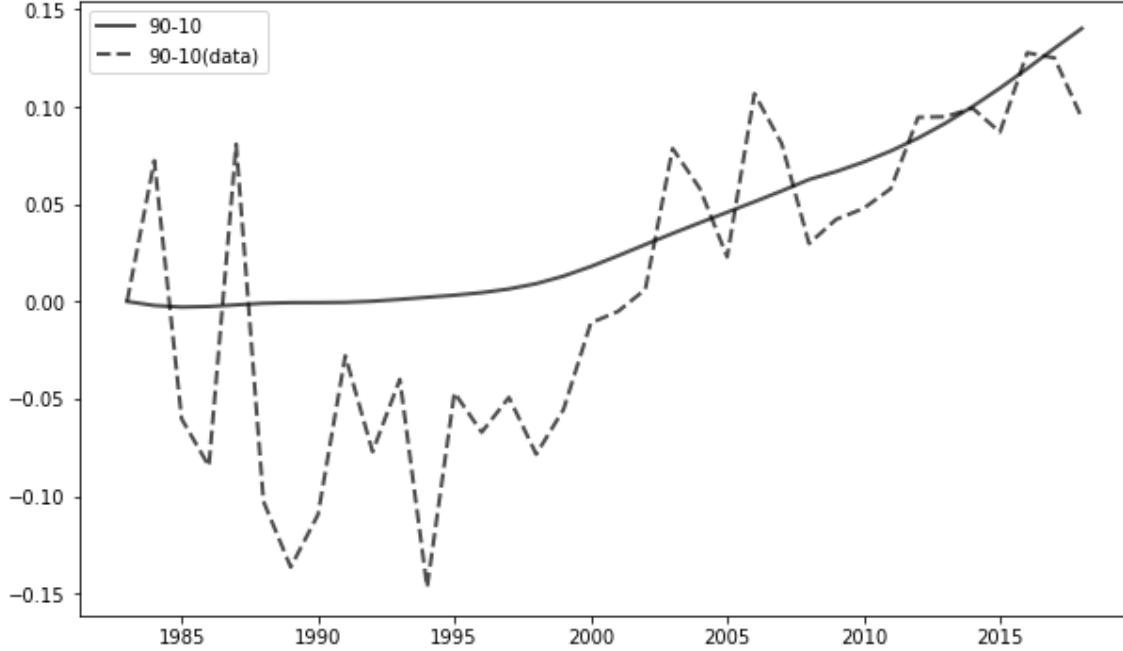


Note: The figure displays the transition of different measures of wage inequality in the abstract task. See the note in Table 5 for interpretation of the measures.

expect the trajectory of wage inequality should largely follow the path of q . In fact, [Aghion \(2002\)](#) demonstrates that there is essentially a one-to-one mapping between the level of wage inequality and the growth rate of a neutral technical change embodied by an increase in TFP. To understand this, it is useful to examine how the capital stock evolves, because ultimately the higher the capital stock, the more workers with low z will be crowded out from the routine task.

Figure 9 shows that the capital stock does not change much before 2000 and rapidly accelerates thereafter. This pattern largely shapes the behavior of other variables, especially the threshold values of z^* . Unlike TFP change as in [Aghion \(2002\)](#), investment specific technical change features a “lagging” response of investment because when agents perfectly anticipate a sustained growth in q , which will make the investment cheaper as the process proceeds, they would not immediately invest to redeem the good news, but choose to postpone the investment to the future when the prices are even lower than when the technical change initiates.

Figure 8: 90-10 Wage Inequality in the Abstract Task (Model and Data)



Note: The solid (dashed) line denotes the log difference between the 90th and 10th percentile of wage in the abstract task throughout the transition path obtained from the model (data). Both measures are normalized to zero in the initial period.

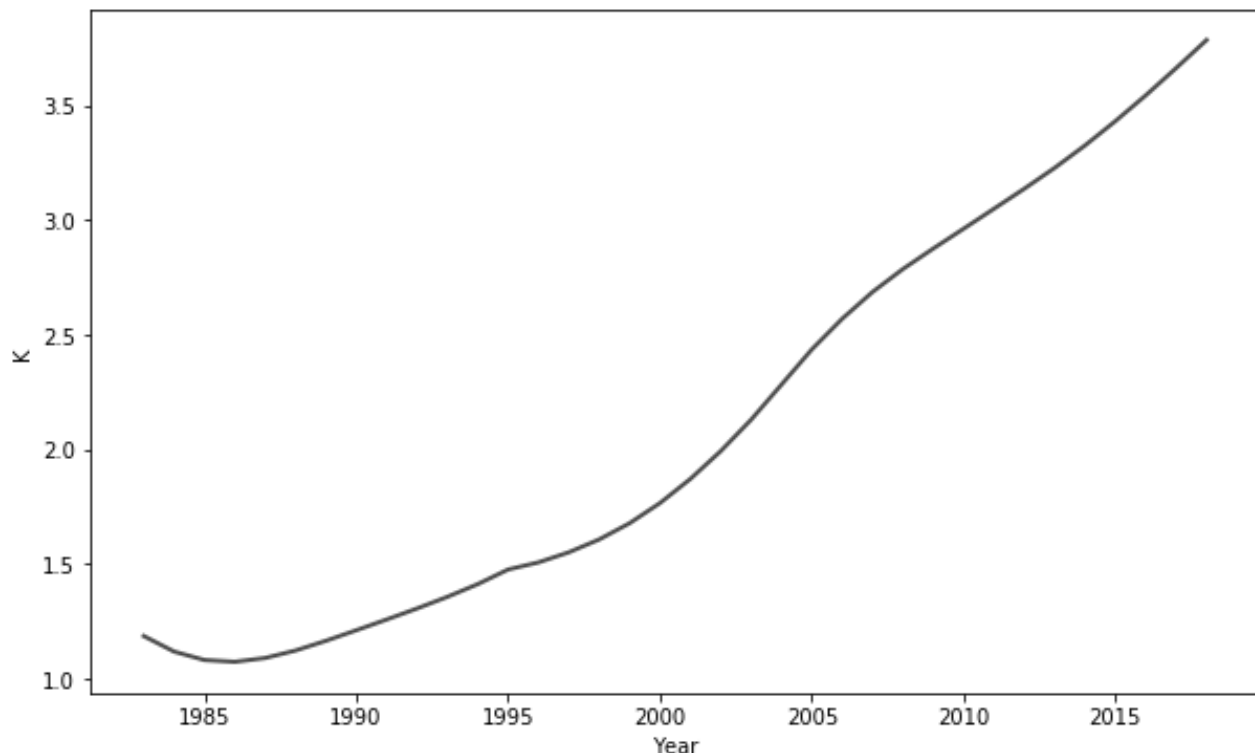
Consequently, the acceleration in investment occurs after the technical change progresses.

We know wage inequality in the abstract task is exclusively driven by the composition effect caused by the change of the threshold values of z^* .²⁰ Then, since the latter is due to the reallocation of labor in the routine task, which is replaced by machines (capital stock), to the abstract task, the fanning-out transition path of capital stock determines that wage inequality in the abstract task also features a non-linear expansion path throughout the transition.

In addition, the log normal distribution of labor efficiencies preserves the convex expansion path of threshold value z^* , so that the path of wage inequality mimics that of z^* . How the results would change when other right-tailed wage distributions and model parameters are used is left to future work, but the quantitative results suggest that the model is able to provide a well-matched timing and magnitude of the change of the trend in wage inequality.

²⁰ Appendix C provides a more rigorous analytical derivation of why measures of wage inequality in the abstract task are smooth functions of z^* and do not depend on wage rates.

Figure 9: Transition of Capital Stock



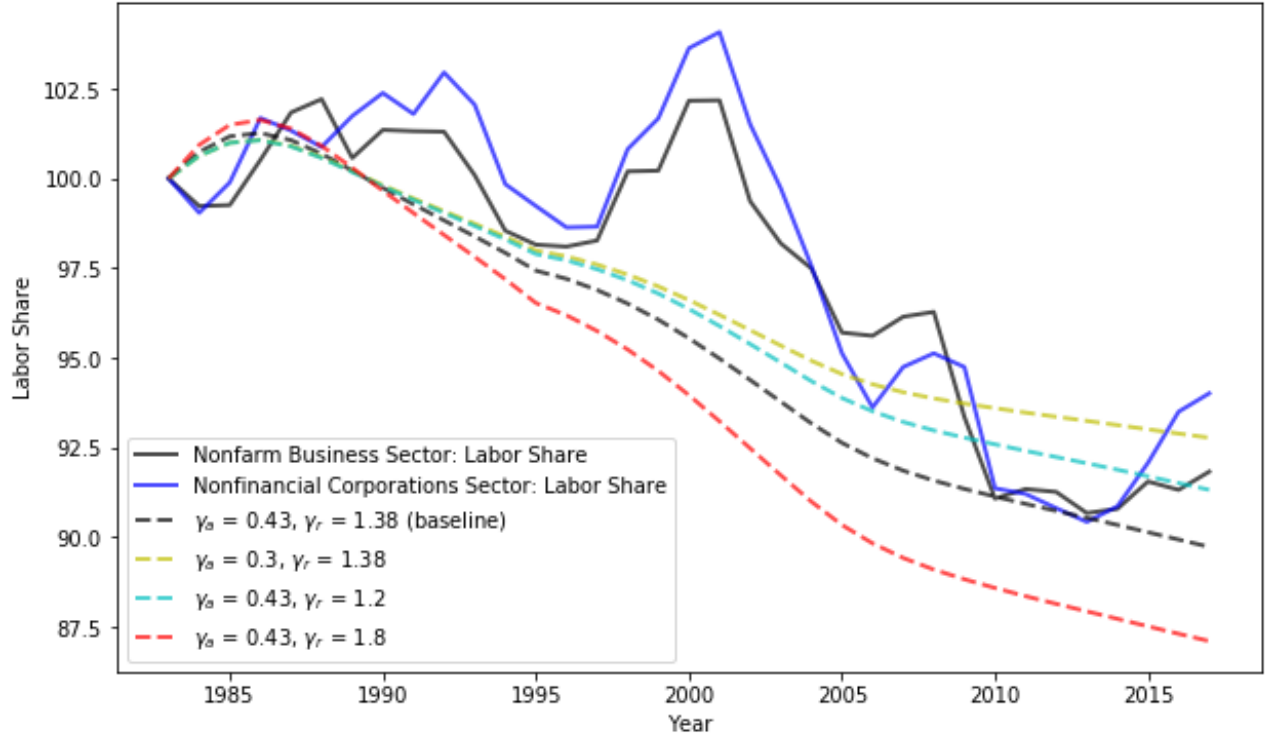
5.4 Broader Implications

While I mainly focus on wage inequality (within the abstract task) in this paper, the model is also able to draw implications that are of broader interest in macroeconomic literature and results qualitatively consistent with the data. I discuss in this section the labor share, investment share and overall inequality and compare them with the data.

Of all the series, labor share is of particular interest because its recent decline triggered numerous works investigating the magnitude and determinants (see e.g. [Elsby et al. \(2013\)](#); [Karabarbounis and Neiman \(2014\)](#)). Since the key mechanism of the model is the replacement of labor by machine, one should expect the model to generate a declining trend of labor share, which is consistent with the observation from data. Indeed, Figure 10 confirms the good fit by comparing two commonly used series of labor share in the data with the series generated by the baseline model (black dotted line).²¹ Despite the jump of the labor share around 2000

²¹The two series are chosen because they are less affected by the problem caused by imputation and therefore is unambiguous and more likely to be reliable (see [Elsby et al. \(2013\)](#); [Karabarbounis and Neiman \(2014\)](#))

Figure 10: Labor Share (1983-2018)



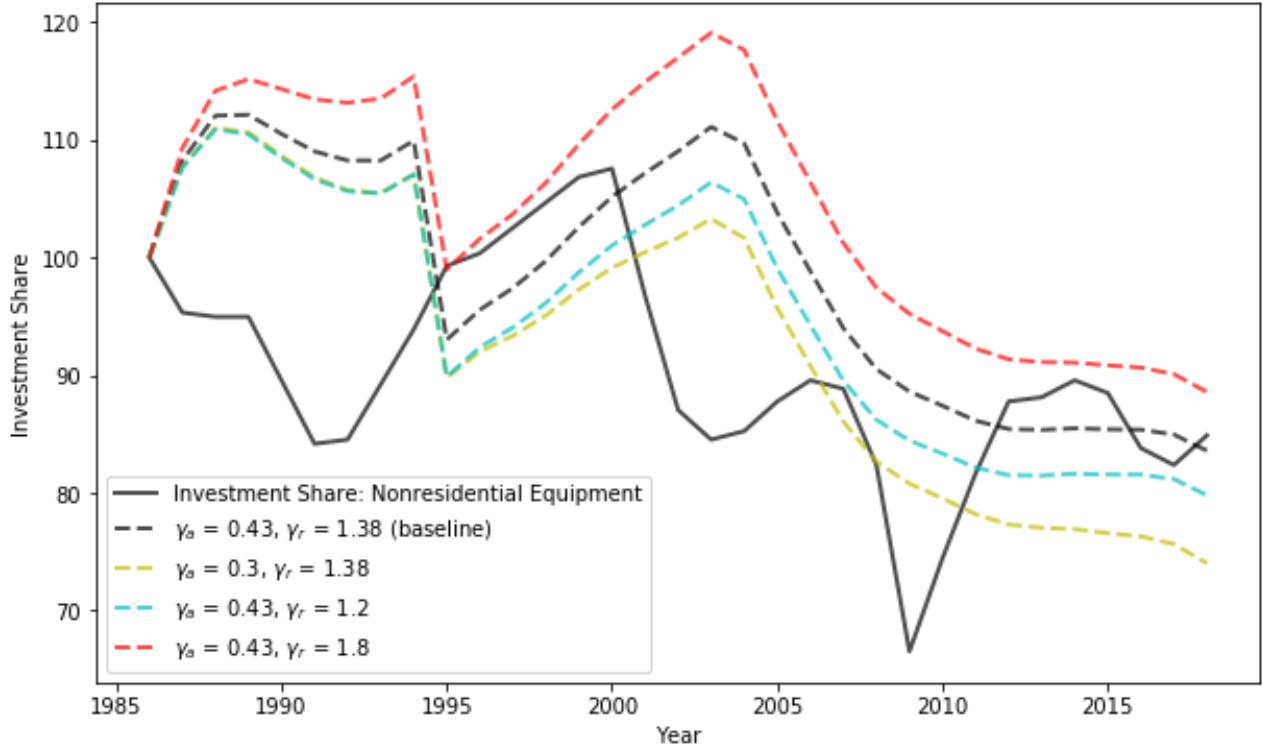
Note: The solid lines denote the labor share of non-farm business sector and non-financial corporation sector reported by BLS. The dotted lines denote the labor share obtained by solving the model under different scenario of elasticities of substitution. All series are normalized to 100 in the initial year (1983).

which was probably caused by cyclical factors like IT bubbles, the model successfully captures the declining trend of labor share in the past four decades with quantitatively comparable magnitude relative to the data.

In addition, by decreasing or increasing the elasticities of substitution (γ_a, γ_r), the model is able to give rise to various magnitudes of the decline of labor share, which reflects the flexibility of the theoretical framework to accommodate different measures of labor share in the data. Therefore, the results suggest a successful and straightforward mechanism both qualitatively and quantitatively in explaining the declining trend of labor share in the last four decades, which is the replacement of labor by machines induced by investment specific technological change. This paper is not the first one to propose this mechanism; rather, it

for more details). The declining trend of labor share is supported by more measures other than those listed, such as share of compensation of employees.

Figure 11: Investment (Equipment) Share (1985-2018)



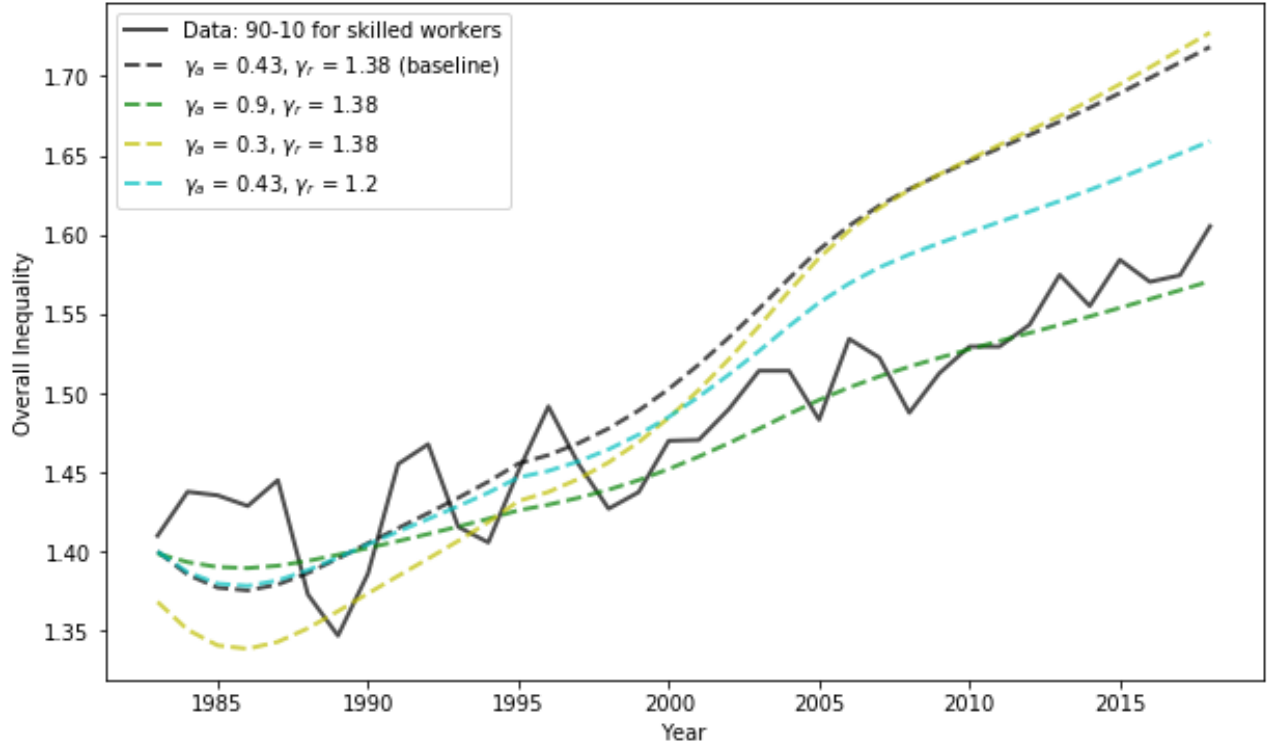
Note: The solid line denotes the investment share of nonresidential equipment reported by BEA. The dotted lines denote the investment share obtained by solving the model under different scenario of elasticities of substitution. All series are normalized to 100 in the initial year (1985).

was extensively examined in the literature and there is a considerable consensus that the acceleration of investment specific technological change contributes to the recent decline of labor share.²² The implication of the model in this paper, nevertheless, serves as a fairly rigorously conducted robustness check of the theory.

The other relevant series is the investment share, which reflects the flow of marginal increment of capital accumulation rather than the stock. This series is of interest because the investment (in equipment) is directly related to the price of equipment production and stimulated by its decline. Given that we have the full path of relative price of investment goods (q_t), we should expect to replicate fairly well the relative movement of the investment share if the long run behavior of investment (in equipment) is mostly dependent on its rental rate r . Figure 11 establishes this conjecture by showing the series of investment derived by the

²²See [Karabarbounis and Neiman \(2014\)](#) for a more rigorous analysis.

Figure 12: Overall Inequality



Note: The solid line denotes the overall inequality measured by the log difference between 90th percentile and 10th percentile of real wage for skilled workers from CPS ASEC data. The dotted lines denote the the log difference between 90th percentile and 10th percentile of real wage for all workers in the model economy.

model co-move with the empirical counterpart except for a lag. This error is most likely to come from the series of q_t , which might be measure with lag, but the successful replication of the peak and trough of the investment share demonstrates the strong dependence of the investment behavior on the relative price (efficiency), which is consistent with the mainstream in the literature of (investment specific) technological change.

Figure 12 displays the last series derived from the model, which is the level of overall inequality measured by the log difference between 90th percentile and 10th percentile of real wage for all workers in the model economy. I compare that with its empirical counterpart for skilled workers because it is the relevant group for the analysis of wage inequality. It should be noted that the overall inequality increases for completely distinct reason from within-group inequality. The latter is driven by composition effect, while the former is all price effect because the rank of individual total wage is unchanged by any reallocation in this economy.

The result should therefore be viewed as a complementary qualitative implication of the model which demonstrates how the other effect should play a role in determining the entirety of wage inequality, but the quantitative significance should not be taken very seriously.

6 Conclusions

The analysis in this paper was motivated by empirical evidence that the growth rate of wage inequality has significantly accelerated since 2000. The main contribution of the paper is to link the task-based framework to wage inequality, which was not fully explored in previous literature. Using a task-based framework as in ALM, I find the change in the trend of wage differential for the whole group of skilled workers was exclusively driven by that within the non-routine analytic task.

A parsimonious theoretic framework of labor reallocation across tasks based on [vom Lehn \(2020\)](#) was then developed. In this model, the labor reallocation is induced by an ongoing investment specific technical change which features a drop in the prices of investment goods. Workers in the routine task are replaced by cheaper machines and then have to relocate to the abstract task. Since workers are heterogeneous in the endowment of efficiency units of labor in the abstract task and the assignment of workers to tasks is based on the comparative advantage, the marginal entrants to the abstract task are lower types of z compared to the incumbents. Wage inequality in the abstract task is then widened due to the composition effect. In addition, since economic agents tend to postpone the investment in machines after the ongoing investment specific technical change takes place for a while, the expansion path of wage inequality is not linear but features an acceleration of wage dispersion in the middle of the technical change. Despite its simple set-up, the quantitative results suggest that the model is able to provide a well-matched timing and magnitude of the change in the trend of wage inequality that is observed in the data.

Appendix A. Solution Method

A.1 Solving the Steady States

Suppose the threshold value of z is z^* in the steady state, it must satisfy the following indifference condition:

$$w_a e^{\alpha_a z^*} = w_r \quad (13)$$

This reduces to

$$z^* = \frac{\ln w_r - \ln w_a}{\alpha_a} \quad (14)$$

Then, solving the Euler equation (9) evaluated at steady state gives rise to steady state value of r . Since N_a and N_r can be expressed as a function of z by utilizing expectation of truncated log-normal distribution and CDF of standard normal distribution, equation (14), combined with equations (10) - (12), consists of a system of four equations and four unknowns, namely z^* , w_a , w_r , K .

A.2 Solving the Transition

The solution to the full model can be found using the standard shooting algorithm. To be specific, I proceed by the following steps:

1. Let the first period be in the initial steady state, so we know all the variables in the first period. Then consider the Euler equation (9) and substitute C by K and Y using the resource constraint (8).
2. Guess a K_2 and solve the system of equations (10) - (12) for period 2, so we know r_2 . Then (9) gives K_3 and following the same procedure I obtain the entire path of the model solution up to any period given the guess of K_2 .
3. Allow roughly two decades of burn-in time. I adjust K_2 such that the model hits the terminal steady state at period 67. If the model does not converge up to an infinitesimal tolerance value, adjust the burn-in time by 1 period.

Appendix B. Definition of DOT Tasks and Classification of COC Occupations

In this appendix, I describe how the classification of tasks is constructed and mapped to occupations using the consistent three-digit Census Occupation Code (COC) in CPS data. The following table displays the definitions of the five categories of tasks and variables used to measure the intensity of tasks put forth in Section 2. Using the values of measures of task intensity by the 1977 Dictionary of Occupational Titles (DOT), I rank the occupations based on consistent three-digit Census Occupation Code (COC) in CPS data extracted from IPUMS for each category of task. For example, the economist might rank the first in the non-routine analytic task because it has the highest score on MATH, but rank the last in the routine manual task because it has the lowest score on FINGDEX. I then categorize each occupation to the task that ranks the highest for this occupation. Economist, for example, is attributed to the non-routine analytic task since its MATH score ranks higher than any of the other scores. Examples of occupations/tasks from CPS data (consistent three-digit Census Occupation Code (COC) on the 1990 basis (occ1990)) are presented in the last column of the table.

Definitions and Examples of Tasks from ALM (2003)

Task	Variable	DOT definition	Typical Example Tasks
Non-routine Analytic	GED Math (MATH)	General educational development, mathematics	Teachers and Instructors, Researchers, Engineers Financial Services
Non-routine Interactive	Direction, Control, Planning (DCP)	Adaptability to accepting responsibility for the direction, control, or planning of an activity	Directors and Supervisors, Administrators and Managers, Private Services
Routine Cognitive	Set Limits, Tolerances, or Standards (STS)	Adaptability to situations requiring the precise attainment of set limits, tolerances, or standards	Clerks and Operators, Proofreaders and Keyers, Precision Production
Routine Manual	Finger Dexterity (FINGDEX)	Ability to move fingers, and manipulate small objects with fingers, rapidly or accurately	Repairers and Mechanics, Fabricators, Assemblers and Hand Working
Non-routine Manual	Eye Hand Foot Coordination (EYEHAND)	Ability to move the hand and foot coordinately with each other in accordance with visual stimuli	Specialists, Guards, Attendants and Inspectors, Health Services

Source: ALM (2003) and IPUMS.

Note: The variables used to measure the intensity of tasks follow ALM (2003) based on definition of tasks from 1977 Dictionary of Occupational Titles. The examples of occupations/tasks are from IPUMS consistent three-digit Census Occupation Code (COC) on the 1990 basis (occ1990).

Appendix C. Technical Details

In this appendix, I provide a rigorous analytic derivation of why measures of wage inequality in the abstract task depend on z^* but not wage rates. I use 90 – 10 wage differential defined in Table 5 as an example but the derivation follows for all measures of wage differential between percentiles.

The logarithm of ratio of the wage for the 90th percentile and 10th percentile of worker in the abstract task the following:

$$\log\left(\frac{w_{at}e^{\alpha_a z_t^{90}}}{w_{at}e^{\alpha_a z_t^{10}}}\right) = \alpha_a(z_t^{90} - z_t^{10}) \quad (15)$$

where z_t^{90} (z_t^{10}) is the 90th (10th) percentile of z in the abstract task, which satisfy:

$$\frac{1}{1 - \Phi(z_t^*)} \int_{z_t^*}^{z_t^{90}} d\Phi(z) = 90 \quad (16)$$

$$\frac{1}{1 - \Phi(z_t^*)} \int_{z_t^*}^{z_t^{10}} d\Phi(z) = 10 \quad (17)$$

where z_t^* is the threshold value at time t . Totally differentiating equation (16) with respect to t gives

$$\frac{\Phi(z_t^*)}{(1 - \Phi(z_t^*))^2} \frac{dz_t^*}{dt} \int_{z_t^*}^{z_t^{90}} d\Phi(z) + \frac{1}{1 - \Phi(z_t^*)} [\phi(z_t^{90}) \frac{dz_t^{90}}{dt} - \phi(z_t^*) \frac{dz_t^*}{dt}] = 0 \quad (18)$$

Rearrange and we get

$$\frac{dz_t^{90}}{dt} = -89 \frac{\phi(z_t^*)}{\phi(z_t^{90})} \frac{dz_t^*}{dt} \quad (19)$$

Similar procedure for the 10th percentile gives

$$\frac{dz_t^{10}}{dt} = -9 \frac{\phi(z_t^*)}{\phi(z_t^{10})} \frac{dz_t^*}{dt} \quad (20)$$

Therefore, using equation (15) and applying (18) and (19), the time derivative of the logarithm of ratio is

$$\frac{d}{dt}(\alpha_a(z_t^{90} - z_t^{10})) = \alpha_a \frac{dz_t^*}{dt} \phi(z_t^*) \left(\frac{9}{\phi(z_t^{10})} - \frac{89}{\phi(z_t^{90})} \right) \quad (21)$$

References

- ACEMOGLU, D. (2002): “Directed technical change,” *The Review of Economic Studies*, 69, 781–809.
- ACEMOGLU, D. AND D. AUTOR (2011): “Skills, tasks and technologies: Implications for employment and earnings,” in *Handbook of labor economics*, Elsevier, vol. 4, 1043–1171.
- ACEMOGLU, D. AND F. ZILIBOTTI (2001): “Productivity differences,” *The Quarterly Journal of Economics*, 116, 563–606.
- AGHION, P. (2002): “Schumpeterian growth theory and the dynamics of income inequality,” *Econometrica*, 70, 855–882.
- AGHION, P., P. HOWITT, AND G. L. VIOLANTE (2002): “General purpose technology and wage inequality,” *Journal of Economic Growth*, 7, 315–345.
- AUTOR, D. H., L. F. KATZ, AND M. S. KEARNEY (2008): “Trends in US wage inequality: Revising the revisionists,” *The Review of economics and statistics*, 90, 300–323.
- AUTOR, D. H., F. LEVY, AND R. J. MURNANE (2003): “The skill content of recent technological change: An empirical exploration,” *The Quarterly journal of economics*, 118, 1279–1333.
- CORTES, G. M. (2016): “Where have the middle-wage workers gone? A study of polarization using panel data,” *Journal of Labor Economics*, 34, 63–105.
- CUMMINS, J. G. AND G. L. VIOLANTE (2002): “Investment-specific technical change in the United States (1947–2000): Measurement and macroeconomic consequences,” *Review of Economic dynamics*, 5, 243–284.
- DAVID, H. AND D. DORN (2013): “The growth of low-skill service jobs and the polarization of the US labor market,” *American Economic Review*, 103, 1553–97.
- ELSBY, M. W., B. HOBIJN, AND A. ŞAHİN (2013): “The decline of the US labor share,” *Brookings Papers on Economic Activity*, 2013, 1–63.

- GOLDIN, C. AND L. F. KATZ (1998): “The origins of technology-skill complementarity,” *The Quarterly journal of economics*, 113, 693–732.
- GORDON, R. J. (1990): *The measurement of durable goods prices*, University of Chicago Press.
- GREENWOOD, J., Z. HERCOWITZ, AND P. KRUSELL (1997): “Long-run implications of investment-specific technological change,” *The American economic review*, 342–362.
- JUHN, C., K. M. MURPHY, AND B. PIERCE (1993): “Wage inequality and the rise in returns to skill,” *Journal of political Economy*, 101, 410–442.
- KARABARBOUNIS, L. AND B. NEIMAN (2014): “The global decline of the labor share,” *The Quarterly journal of economics*, 129, 61–103.
- KATZ, L. F. AND K. M. MURPHY (1992): “Changes in relative wages, 1963–1987: supply and demand factors,” *The quarterly journal of economics*, 107, 35–78.
- KATZ, L. F. ET AL. (1999): “Changes in the wage structure and earnings inequality,” in *Handbook of labor economics*, Elsevier, vol. 3, 1463–1555.
- KRUSELL, P., L. E. OHANIAN, J.-V. RÍOS-RULL, AND G. L. VIOLANTE (2000): “Capital-skill complementarity and inequality: A macroeconomic analysis,” *Econometrica*, 68, 1029–1053.
- LEMIEUX, T. (2006): “Increasing residual wage inequality: Composition effects, noisy data, or rising demand for skill?” *American Economic Review*, 96, 461–498.
- ROY, A. D. (1951): “Some thoughts on the distribution of earnings,” *Oxford economic papers*, 3, 135–146.
- SATTINGER, M. (1975): “Comparative advantage and the distributions of earnings and abilities,” *Econometrica: Journal of the Econometric Society*, 455–468.
- (1993): “Assignment models of the distribution of earnings,” *Journal of economic literature*, 31, 831–880.
- TINBERGEN, J. (1974): “Substitution of graduate by other labour,” *Kyklos: international review for social sciences*.

——— (1975): *Income differences: recent research*.

VIOLANTE, G. L. (2002): “Technological acceleration, skill transferability, and the rise in residual inequality,” *The Quarterly Journal of Economics*, 117, 297–338.

VOM LEHN, C. (2020): “Labor market polarization, the decline of routine work, and technological change: A quantitative analysis,” *Journal of Monetary Economics*, 110, 62–80.

YOUNG, A. (2014): “Structural transformation, the mismeasurement of productivity growth, and the cost disease of services,” *American Economic Review*, 104, 3635–67.