Math 115AH

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1 Materials

Sets: A set is a collection of elements (no formal definition)

Let's say S is a set, we write $x \in S$

$N = \{1, 2, 3, 4, ...\}$ called natural numbers, $Z = \{..., -2, -1, 0, 1, 2, ...\}$ called integers

$Q = \{\frac{a}{b} \,|\, a, b \in Z \, and \, b \neq 0\}$

ϕ = Empty Set

$\text{Mat}_{m \times n}$(R) = $m \times n$ matrices with entries in R

R[x] = Polynomials with coefficients in R

Subsets: say A, B are two sets, we say A is a subset of B and write A $\subseteq$ B to mean:

$$\text{If x} \in \text{A then x} \in \text{B}$$

Equality: say A, B are sets, we say A = B to mean:

$$\text{x} \in \text{A if and only if x} \in \text{B}$$

Another way to say this: A $\subseteq$ B and B $\subseteq$ A

Union, Intersection and Complement:

Here A, B are both sets

A $\cup$ B = "A union B" = set of elements that are in A or B

A $\cap$ B = "A intersect B" = set of elements in both A and B

A $\setminus$ B = "A complement B" = set of elements in A and not in B

Quantifiers:

$\forall$ means "for all"

eg. $\forall x \in N$, $x > 0$

$\exists$ means "there exists"

eg. $\exists I \in Mat_{2\times 2}(R)$ such that $I \cdot A = A$ for every $A \in Mat_{2\times 2}(R)$

2 Practice

(1) Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Sol:

First we'll show $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

Say $x \in A \cup (B \cap C)$, this means $x \in A$ or $x \in B \cap C$

Case 1: If $x \in A$, then $x \in A \cup B$ and $x \in A \cup C$

then $x \in (A \cup B) \cap (A \cup C)$

Case 2: If $x \in B \cap C$, then $x \in B$ and $x \in C$

So $x \in A \cup B$ and $x \in A \cup C$ So $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

Now we need to show $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$

Pick $y \in (A \cup B) \cap (A \cup C)$, then $y \in A \cup B$ and $y \in A \cup C$

Since $y \in A \cup B$, $y \in A$ or $y \in B$, if $y \in A$ we win

So say $y \in B$, similarly $y \in A \cup C$, if $y \in A$ we win

So say $y \in C$, then $y \in B \cup C$

So we win

(2) Prove that there are sets A, B, C so that $(A \cup B) \cap C \neq A \cup (B \cap C)$

For example, A = $\{1\}$, B = ϕ and C = ϕ

Then $(A \cup B) \cap C = \phi$ and $A \cup (B \cap C) = \{1\}$