Oligopoly and Strategic Interaction

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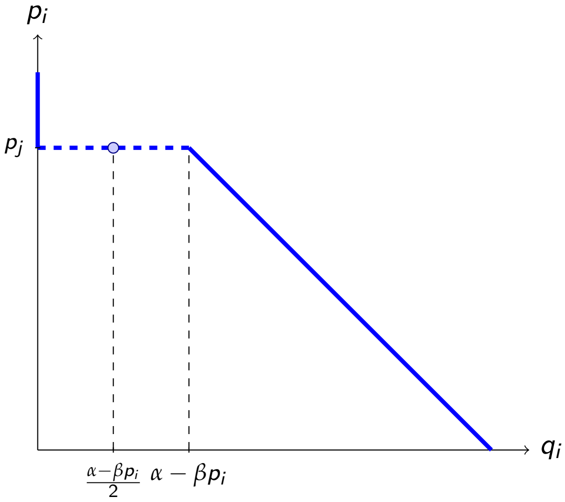
Betrand's model of Oligopoly: Price Competition

- Firms i and j sell identical product in a market.
- Compete with each other by choosing their prices p_i and p_j respectively.
- Firm i's demand function is given by:

$$q_i(p_i, p_j) = \begin{cases} \alpha - \beta p_i & \text{if} \quad p_i < p_j \\ \frac{\alpha - \beta p_i}{2} & \text{if} \quad p_i = p_j \\ 0 & \text{if} \quad p_i > p_j \end{cases}$$

- For simplicity, assume that marginal costs of production are constant c.
- Each firm cares about its own profits only.

Firm i's Demand Function under Bertrand Competition



- Demand is discontinuous.
- The discontinuity in demand carries over to profits.

Firm i's Profits

- Firm i's profit function is given by:

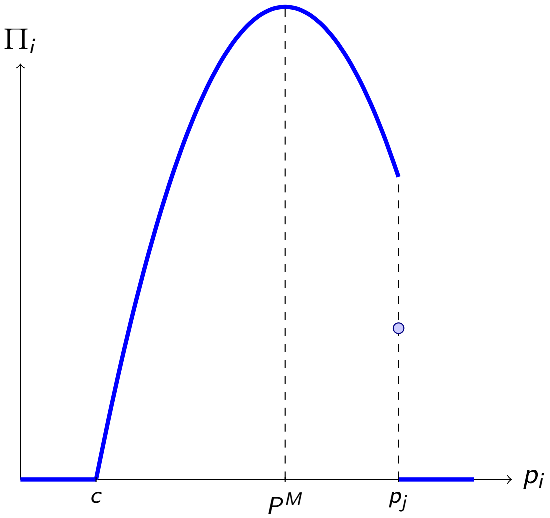
$$\Pi_i(p_i, p_j) = \begin{cases} 0 & \text{if} \quad p_i < c \\ (p_i - c)(\alpha - \beta p_i) & \text{if} \quad c < p_i < p_j \\ (p_i - c)\frac{\alpha - \beta p_i}{2} & \text{if} \quad p_i = p_j \\ 0 & \text{if} \quad p_i > p_j \end{cases}$$

- Firm's profits are dependent on its own prices as well as its competitor's prices.

- Will firm i ever choose $p_i > p_j$?
  - $p_i > p_j$ is strictly dominated by $p_i \leq p_j$.

- Will firm i ever choose $p_i \leq c$?
  - $p_i \leq c$ is strictly dominated by $c < p_i$.

Firm i's Profits



Firm i's Best Response Function

Best response function

Function identifying strategy (or strategies) which produce the "most favorable outcome" for a player, taking other players' strategies as given.

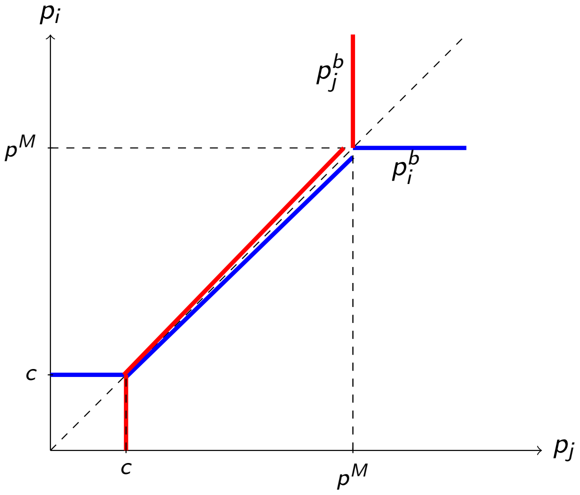
- Firm i's optimization problem becomes

$$\max_{p_i \leq p_j} (p_i - c)(\alpha - \beta p_i)$$

- Firm i will never charge a price higher than that charged by a monopolist. Why?
- Firm i's best response function is given by:

$$p_i(p_j) = \begin{cases} p^M & \text{if} \quad p_j > p^M \\ p_j - \epsilon & \text{if} \quad c < p_j \leq p^M \\ c & \text{if} \quad p_j \leq c \end{cases}$$

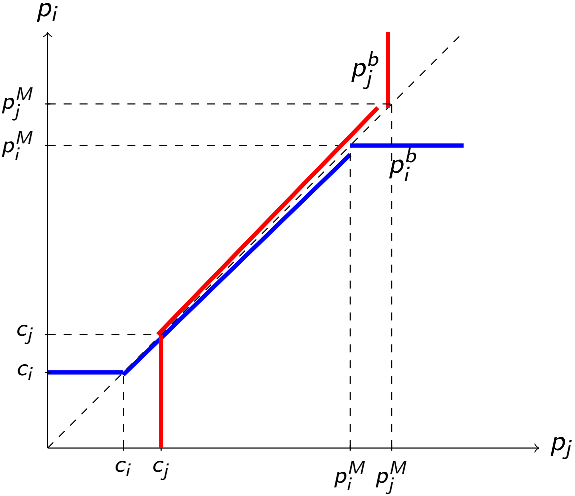
Firms' Best Response Functions



Competition among firms leads to marginal cost pricing and zero profits in equilibrium.

Bertrand Competition with Different Marginal Costs

What if $c_i < c_j$?



Competition among firms leads to $p_i^* = p_j^* = c_j$ and firm i makes non-zero profits in equilibrium.

Cournot Model of Oligopoly: Quantity Competition

- Firms i and j sell identical product in a market.
- Compete with each other by independently choosing their production q_i and q_j.
- Industry prices P are determined by both q_i and q_j, that is,

$$P(Q) = P(q_i + q_j)$$

- For simplicity, assume that marginal costs of production are constant c.
- Firm i chooses q_i to maximizes its own profits:

$$\max_{q_i} [P(Q) - c] q_i$$

Cournot Model of Oligopoly: Quantity Competition

- Partially differentiating firm's objective function with respect to q_i yields:

$$
\begin{aligned}
P(Q) + q_i \frac{\partial P(Q)}{\partial q_i} &= c \\
\iff P(Q) + q_i \frac{dP(Q)}{dQ} &= c \text{ Why?}
\end{aligned}
$$

- In determining q_i, firm considers the impact of its production on industry prices.

- Optimal q_i is also dependent on q_j and vice versa.

Response to an increase in competitor's output

- If $q_j = 0$, then $q_i = Q$ and firm's optimization condition becomes identical to monopolist's condition:

$$P(Q) + Q\frac{dP(Q)}{dQ} = c$$

$\implies$ If $q_j = 0$, then $q_i^* = Q^M$.

- What happens as q_j increases?
  The change in firm's marginal revenue is

$$\frac{\partial P(Q)}{\partial q_j} + q_i \frac{\partial^2 P(Q)}{\partial q_j \partial q_i}$$

- As long as $P(Q)$ is concave or not "too" convex, the q_i^* is decreasing in q_j.

- If $q_j = Q^c$ where Q^c is the competitive output, then $q_i^* = 0$

Firm output under cournot competition is less than the monopoly output.

$$
\begin{aligned}
\frac{P(Q) - c_i}{P(Q)} &= -\frac{q_i}{P(Q)} \frac{dP(Q)}{dQ} \\
&= -\frac{q_i}{Q} \frac{Q}{P(Q)} \frac{dP(Q)}{dQ} \\
&= \frac{s_i}{E_D}
\end{aligned}
$$

where $s_i = \frac{q_i}{Q} < 1$ is firm i's market share and E_D is (industry) price elasticity of demand.

- Presence of another firm reduces the market power of firm i.
- Firms' market power is limited by price elasticity of demand, as well as market shares.

Under Cournot competition, firms exercise less market power than a monopoly.

Cournot industry output versus monopoly output

- Summing up the first order conditions for Cournot firms, we get:

$$\begin{aligned} \sum_i (P(Q) + q_i \frac{dP(Q)}{dq_i}) &= \sum_i c \\ P(Q) + \frac{Q}{2}\frac{dP(Q)}{dQ} &= c \end{aligned}$$

- Recall that the first order condition for monopoly is:

$$P(Q) + Q\frac{dP(Q)}{dQ} = c$$

- As $Q\frac{dP(Q)}{dQ} < \frac{Q}{2}\frac{dP(Q)}{dQ} < 0 \implies$

Industry output under cournot competition exceeds monopoly output and is lower than perfectly competitive output.

Cournot output as number of firms increases

- Suppose there are N identical firms in the industry.
- Summing up the first order conditions for N Cournot firms, we get:

$$P(Q) + \frac{Q}{N}\frac{dP(Q)}{dQ} = c$$

- As $\frac{Q}{N} > \frac{Q}{N+1} \implies$ industry output increases with N.
- As $N \to \infty$, $\frac{Q}{N}\frac{dP(Q)}{dQ} \to 0 \implies P(Q) \to c$.

Under cournot competition firms' market power reduces as N increases and industry output converges to perfectly competitive output.

Cournot competition: linear demand and identical costs

- Let the industry demand curve be:

$$P = \alpha - \beta Q = \alpha - \beta(q_i + q_j)$$

- Firm's have identical constant marginal costs $c < \alpha$.
- Firm i's first order optimization condition becomes:

$$\frac{\partial \pi_i}{\partial q_i} = \alpha - c - \beta q_j - 2\beta q_i = 0$$

- Re-arranging terms, firm i's *best response function* is given by:

$$q_i^b(q_j) = \frac{\alpha - c}{2\beta} - \frac{q_j}{2}$$

- Similarly firm j's *best response function* is:

$$q_j^b(q_i) = \frac{\alpha - c}{2\beta} - \frac{q_i}{2}$$

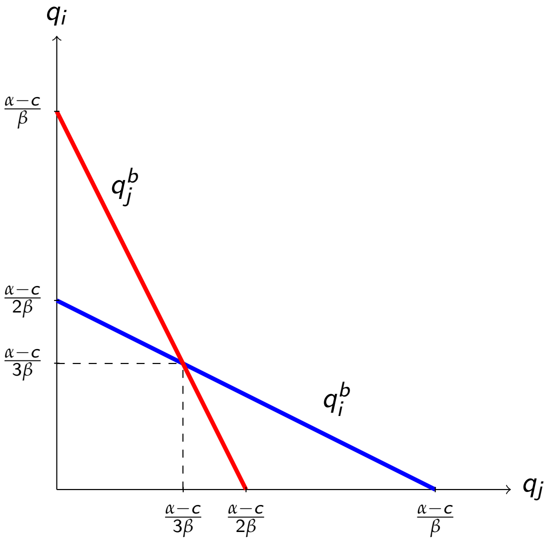
Cournot competition: linear demand and identical costs

- Equilibrium requires $q_i^b(q_j)$ and $q_j^b(q_i)$ to hold simultaneously.
- Simultaneously solving best response functions yields:

$$q_i^* = q_j^* = \frac{\alpha - c}{3\beta}$$

- Industry output is: $Q^* = q_i^* + q_j^* = \frac{2}{3}\left(\frac{\alpha - c}{\beta}\right)$
- Industry prices are: $P^* = \frac{\alpha + 2c}{3}$
- Lerner Index is: $\frac{P^* - c}{P^*} = \frac{\alpha - c}{\alpha + 2c}$
- Demand elasticity at equilibrium price is: $E_D = \frac{\alpha + 2c}{2(\alpha - c)}$

Best response functions



Cournot oligopoly vs other market structures

- If industry is perfectly competitive, then
  - Industry price is: $P^C = c < P^*$.
  - Industry output is: $Q^C = \frac{\alpha - c}{\beta} > Q^*$
- If there is only one firm in the industry, then its first order condition is:

$$\alpha - 2\beta Q = c$$

  - Industry output is: $Q^M = \frac{\alpha - c}{2\beta} < Q^*$.
  - Industry price is: $P^M = \frac{\alpha + c}{2} > P^*$

2 observations

1. If $q_j = 0$, then $q_i^b(q_j) = Q^M$.
2. If $q_j \geq Q^c$, then $q_i^b(q_j) = 0$.

Cournot Competition: many firms

- Suppose there are N firms in the industry.
- Industry prices are $P(Q) = P(q_i + Q_{-i}) = \alpha - \beta(q_i + Q_{-i})$ where Q_{-i} is the output of other not i firms.
- Firm i's first order condition becomes:

$$\frac{\partial \pi_i}{\partial q_i} = \alpha - c - \beta Q_{-i} - 2\beta q_i = 0$$

- Firm i's best response function is:

$$q_i^b(Q_{-i}) = \frac{\alpha - c}{2\beta} - \frac{Q_{-i}}{2}$$

Cournot Competition: Many firms

- Firm i's output is:

$$q_i = \frac{1}{N+1}\frac{\alpha - c}{\beta}$$

- Industry output is:

$$Q = \frac{N}{N+1}\frac{\alpha - c}{\beta}$$

- Prices are:

$$P = \frac{\alpha + Nc}{N+1}$$

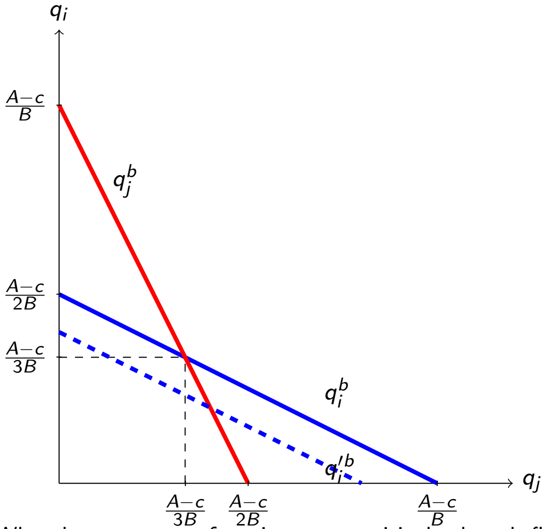
- What happens to q_i, Q and P as $N \to \infty$? (Hint: Apply L'Hopital's rule for taking limits)

Properties of cournot equilibrium: summary

- $Q^* \in (Q^M, Q^C)$ where Q^C and Q^M stand for the competitive and monopoly output level.
- $Q^*(N)$ is increasing in N where N stand for number of firms.
- $\lim_{N\to\infty} Q^*(N) = Q^C$.
- More realistic prediction than Bertrand competition.
- Unfortunately less realistic situation.
    - Most situations involve price competition.

Best response functions and strategic substitution

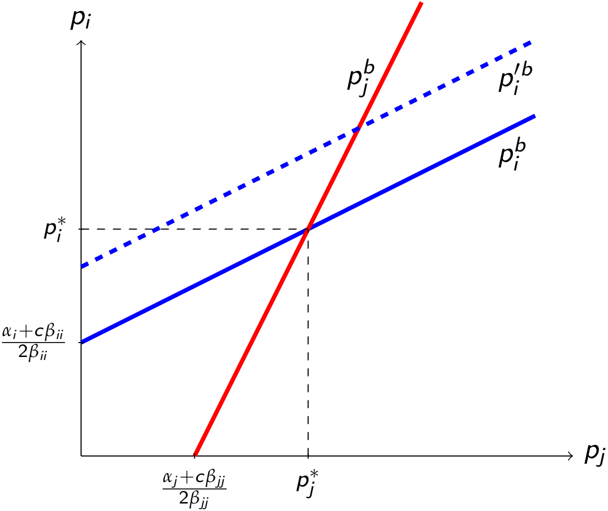
What happens if only firm i's marginal cost of production increase?



When best response functions are positively sloped, firms' strategies are *strategic substitutes* $\rightarrow$ Firms respond very aggressively to strategies of their competitors.

Best response functions and strategic complementarity

What happens if only firm i's marginal cost of production increase?



When best response functions are negatively sloped, strategies are *strategic complements*
$\rightarrow$ Firms do not respond very aggressively to strategies of their competitors.

Models of Product Differentiation

- Two classes of models:
  - Reduced form: interdependence between the two goods is captured by the two demand functions $q_i(p_i, p_j)$ and $q_j(p_i, p_j)$.

  - Vertical and Horizontal Differentiation Models:
    - Horizontal Differentiation: firms produce products that cater to different tastes.
    - Vertical Differentiation: firms produce products that cater to consumers with different income/willingness to pay for quality.

Bertrand Competition with Differentiated Product

- What if consumers view the products sold by two firm's differently?
- Firms' demand functions are:

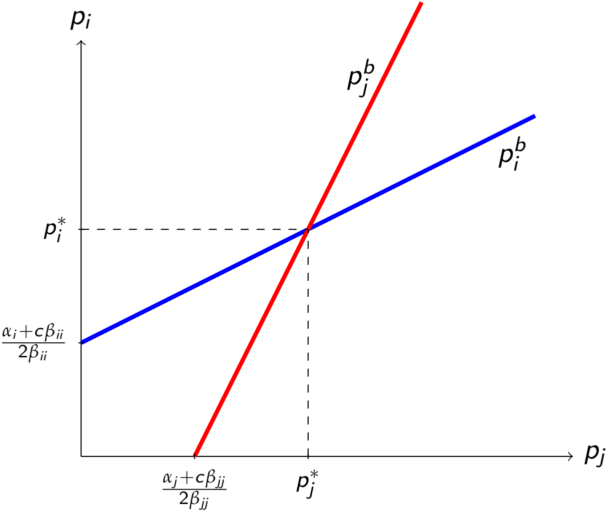
$$\begin{aligned} q_i &= \alpha_i - \beta_{ii} p_i + \beta_{ij} p_j \\ q_j &= \alpha_j - \beta_{jj} p_j + \beta_{ji} p_i \end{aligned}$$

where $\beta_{ii}, \beta_{jj} > 0$ (Law of Demand) and $\beta_{ij}, \beta_{ji} > 0$ (substitutes in consumption) and $c < \min\{\frac{\alpha_i}{\beta_{ii} - \beta_{ij}}, \frac{\alpha_j}{\beta_{jj} - \beta_{ji}}\}$ (marginal cost pricing leads to positive demand)

- Firms' best response functions are:

$$\begin{aligned} p_i^b(p_j) &= \frac{c}{2} + \frac{\alpha_i + \beta_{ij} p_j}{2\beta_{ii}} > c \\ p_j^b(p_i) &= \frac{c}{2} + \frac{\alpha_j + \beta_{ji} p_i}{2\beta_{jj}} > c \end{aligned}$$

Bertrand Competition with Differentiated Product



Bertrand Competition

- If firms are viewed symmetrically, that is, $\alpha_i = \alpha_j = \alpha$ $\beta_{ii} = \beta_{jj} = 1$ and $\beta_{ij} = \beta_{jj} = \beta$.
- Firms' best response functions are:

$$\begin{aligned} p_i^b(p_j) &= \frac{c}{2} + \frac{\alpha + \beta p_j}{2} \\ p_j^b(p_i) &= \frac{c}{2} + \frac{\alpha + \beta p_i}{2} \end{aligned}$$

- Symmetric pricing strategies:

$$p_i^* = \frac{\alpha - c}{2 + \beta} > c$$

- Symmetric firms adopt symmetric pricing strategies.
- If products are differentiated, Bertrand competition does not erode all profits.