Monopoly
(Ch 2.1, 4, 5.2, 5.3 (PRN))

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March 24, 2021

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Firm's output decision

- A firm's goal is to maximize profit.
- Profits $\equiv$ Revenue $-$ Costs.
- We already know how a firm's costs change with its production.
- Revenue changes with price and quantity sold.
- Many firms face a trade-off between price and quantity sold.
- Law of demand:

$$Q_D = f(P) \quad \textit{with} \quad f' < 0 \quad \text{(Demand Curve)}$$
$$P = f^{-1}(Q) \equiv g(Q) \quad \textit{with} \quad g' < 0 \quad \text{(Inverse Demand Curve)}$$

- To sell more, firms must charge less.
- It decides to produce a level of output that maximizes its profits.

Profit Maximization

- First order condition of optimization:

$$\begin{aligned} \pi(q) &\equiv R(q) - C(q) \\ \frac{d\pi(q)}{dq} &= \frac{dR(q)}{dq} - \frac{dC(q)}{dq} \end{aligned}$$

- Marginal revenue: $MR = \frac{dR(q)}{dq}$: incremental revenue from one more unit sold
- Marginal cost: $MC = \frac{dC(q)}{dq}$: incremental cost of one more unit produced

Profit maximization implies that marginal profits $\frac{d\pi(q)}{dq} = 0$, which is equivalent to

$$MR = MC$$

- This is only a necessary (and not sufficient condition) for optimization.

More on Marginal Revenue

Q: aggregate quantity
q: individual quantity
Let $Q = \sum_0^n q_i$

$$\begin{aligned} R(q) &= P(Q) \times q \\ MR = \frac{dR(q)}{dq} &= P(Q) + \frac{\partial P(Q)}{\partial q} q \end{aligned}$$

- $\frac{\partial P(Q)}{\partial q} \leq 0$ due to Law of Demand.
- Positive direct effect: Additional $P(Q)$ from selling marginal-unit.
- Negative indirect effect: Lower prices for all the units sold.
- $MR \leq P(Q)$, that is, Marginal Revenue Curve never lies above inverse demand curve.

MR when n is large

$$\frac{\partial P(Q)}{\partial q} = \frac{\partial P(Q)}{\partial Q}\frac{\partial Q}{\partial q}$$

Consider a market consisting of large number of firms.

- $\frac{\partial Q}{\partial q} \approx 0$
- $\frac{\partial P(Q)}{\partial q} \approx 0 \Rightarrow$ Each firm is "price-taker".
- Marginal Revenue curve is independent of q_i and coincides with inverse demand curve $P(Q)$

Output decision when n is large

- Profit maximizing rule implies each firm chooses q_i^s such that the following condition holds:

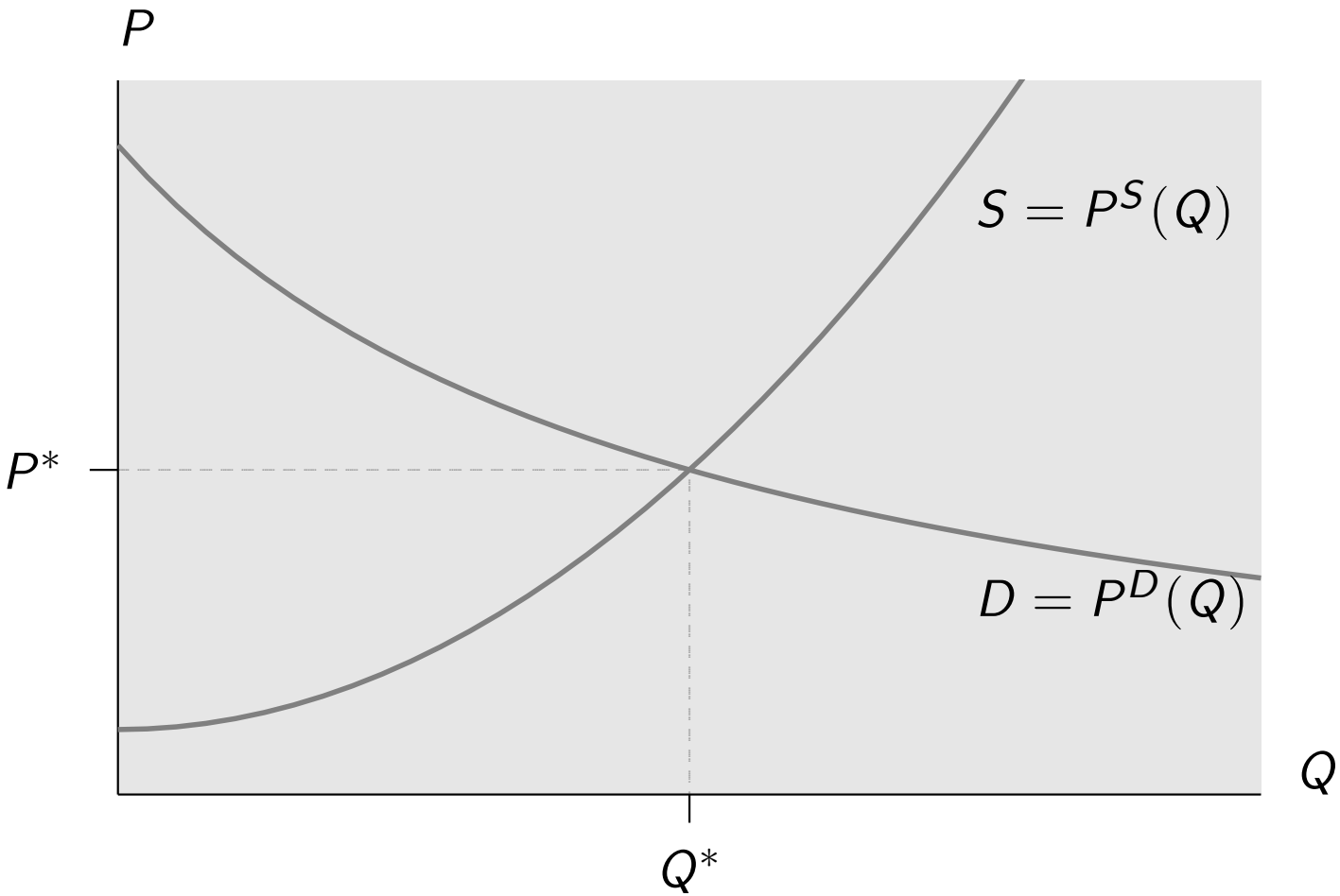
$$MR(q_i^s) = P(Q) = MC(q_i^s)$$

- Industry supply at any given price is then given by:

$$Q_S = \sum_{0}^{n} q_i \qquad \text{at} \qquad P$$

- How are Q_S and P related to each other?
  - Intensive margin: Quantity supplied by each existing seller?
  - Extensive margin: Number of sellers in the industry?
- Industry demand is identified by aggregating individual demands at a price.
- Industry prices are set by the interaction of demand and supply sides.

Determination of industry output when n is large



Alternative interpretation of demand curve

- Suppose consumers have unit demand for the product.
- Consumers are heterogeneous in terms of the value or benefit they derive from this product.
- Consumers are ranked by their valuations with $v(i) > v(i+1)$.
- Under this interpretation, demand cure coincides with the consumer's valuation curve.
- Total value generated by Q units is given by:

$$\text{Total Value} = \int_0^Q P^D(Q)dQ$$

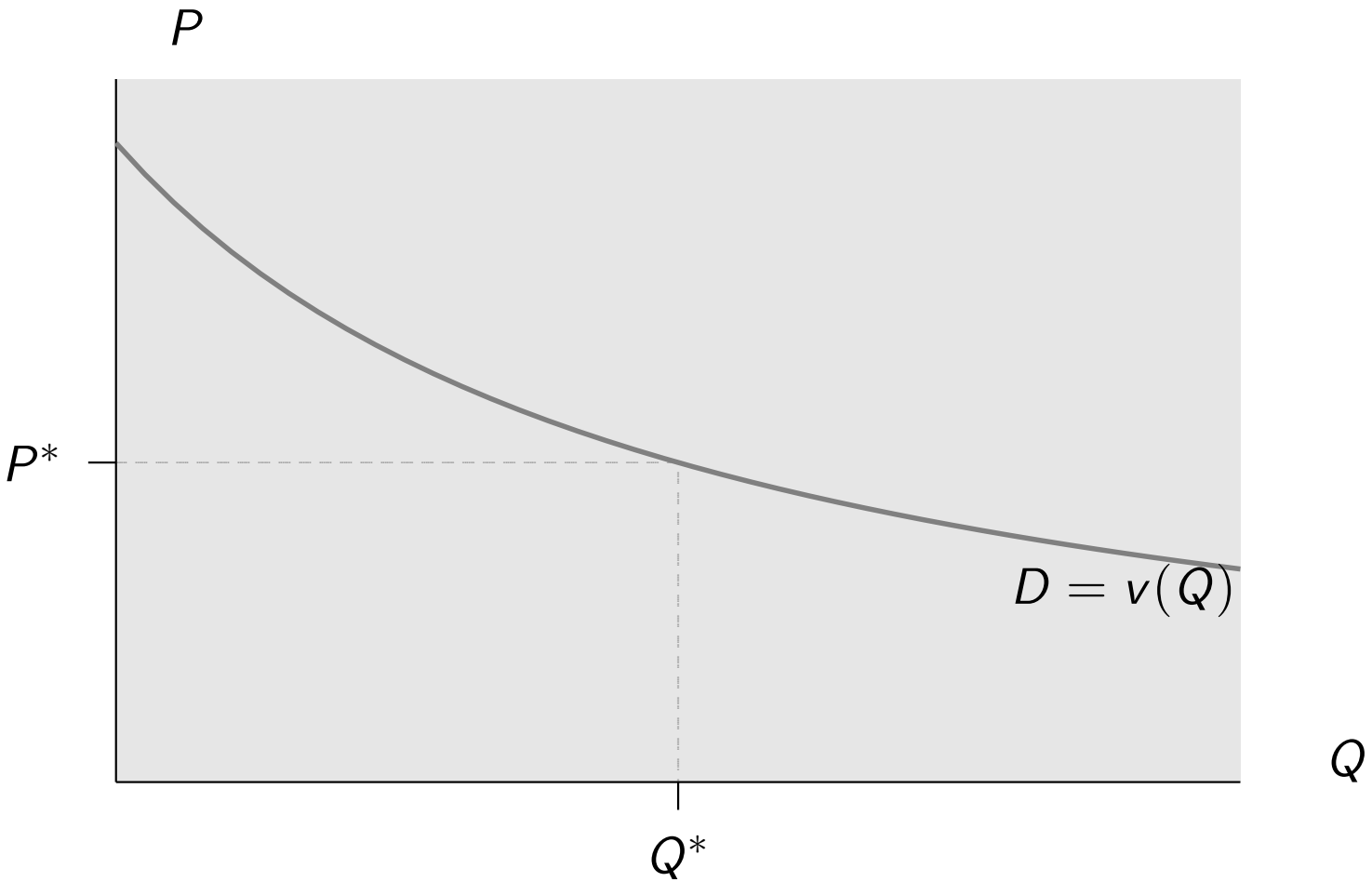
- In order to receive this value from Q units, consumers spend:

$$\text{Total Expenditure} = P(Q)Q$$

- Consumer Surplus:

$$\int_0^Q P^D(Q)dQ - P(Q)Q$$

Consumers' gain from market participation



Alternative interpretation of supply curve

- Suppose producers have unit supply for the product.
- Producers are heterogeneous in terms of the cost of production.
- Producers are ranked by their valuations with $c(i) < c(i+1)$.
- Under this interpretation, supply cure coincides with marginal cost of production curve.
- Total cost of producing Q units is given by:

$$\text{Total Cost} = \int_0^Q P^S(Q)dQ$$

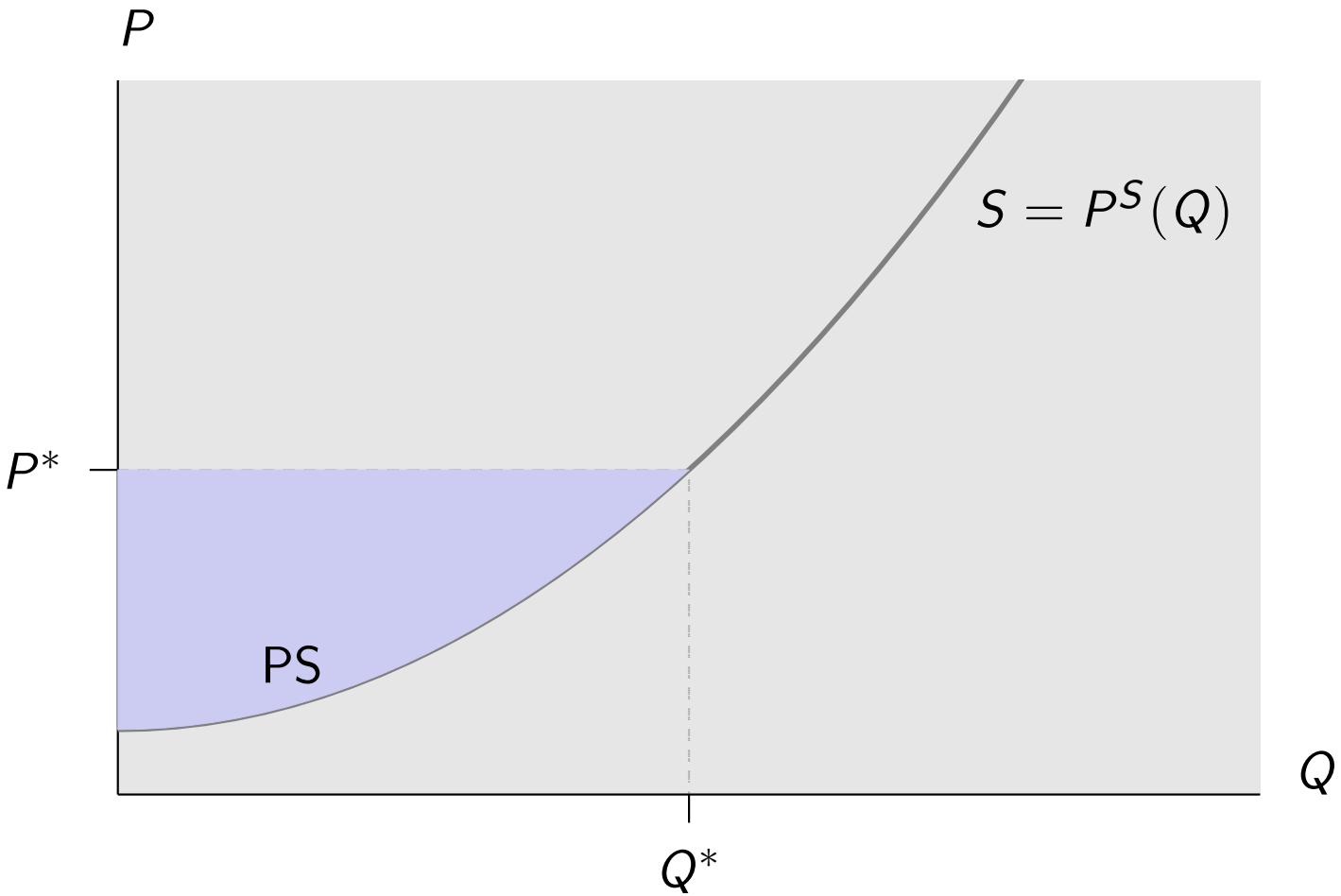
- Producers' benefit from selling Q units is given by:

$$\text{Total Revenue} = P(Q)Q$$

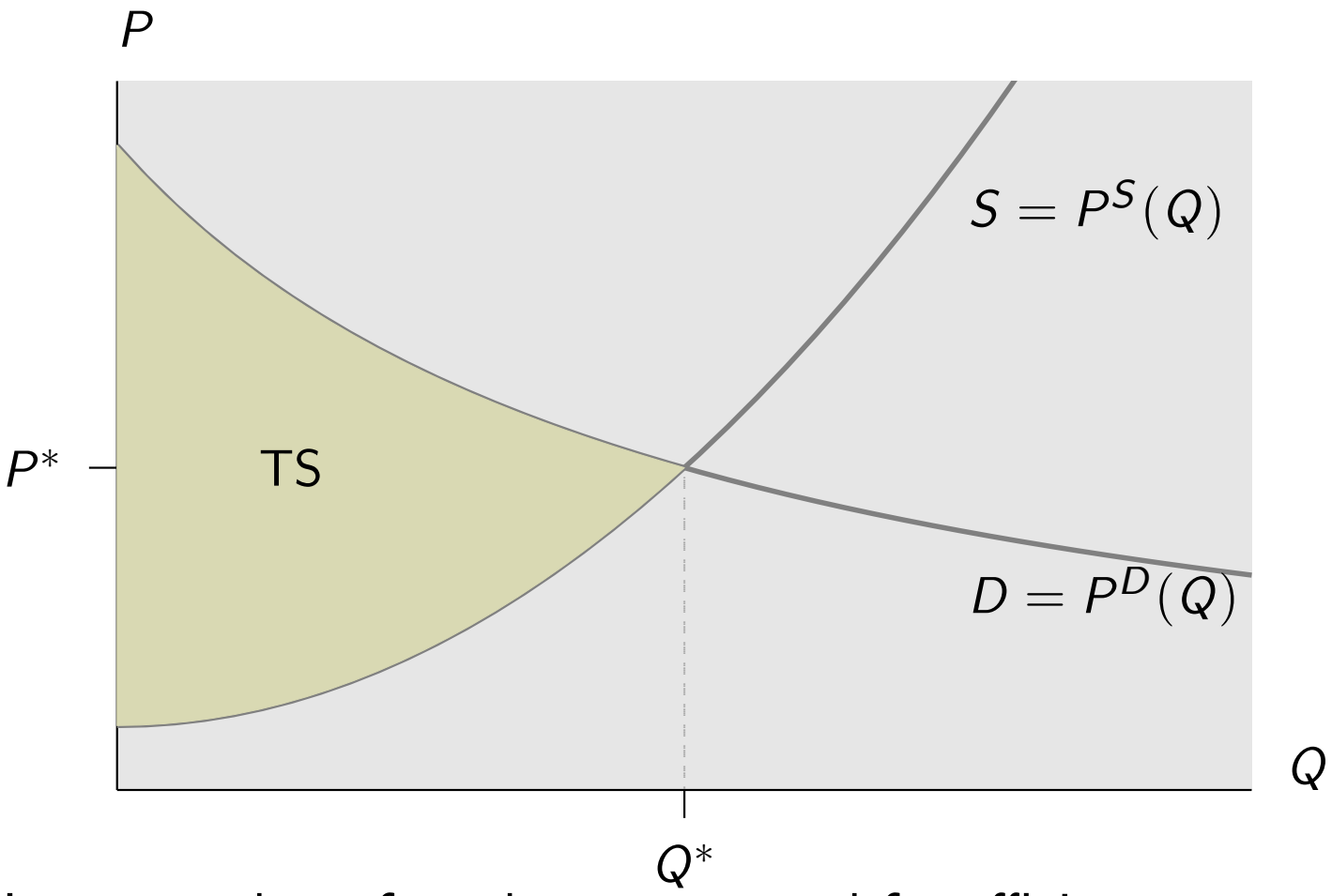
- Producer Surplus:

$$P(Q)Q - \int_0^Q P^S(Q)dQ$$

Producers' gain from market participation

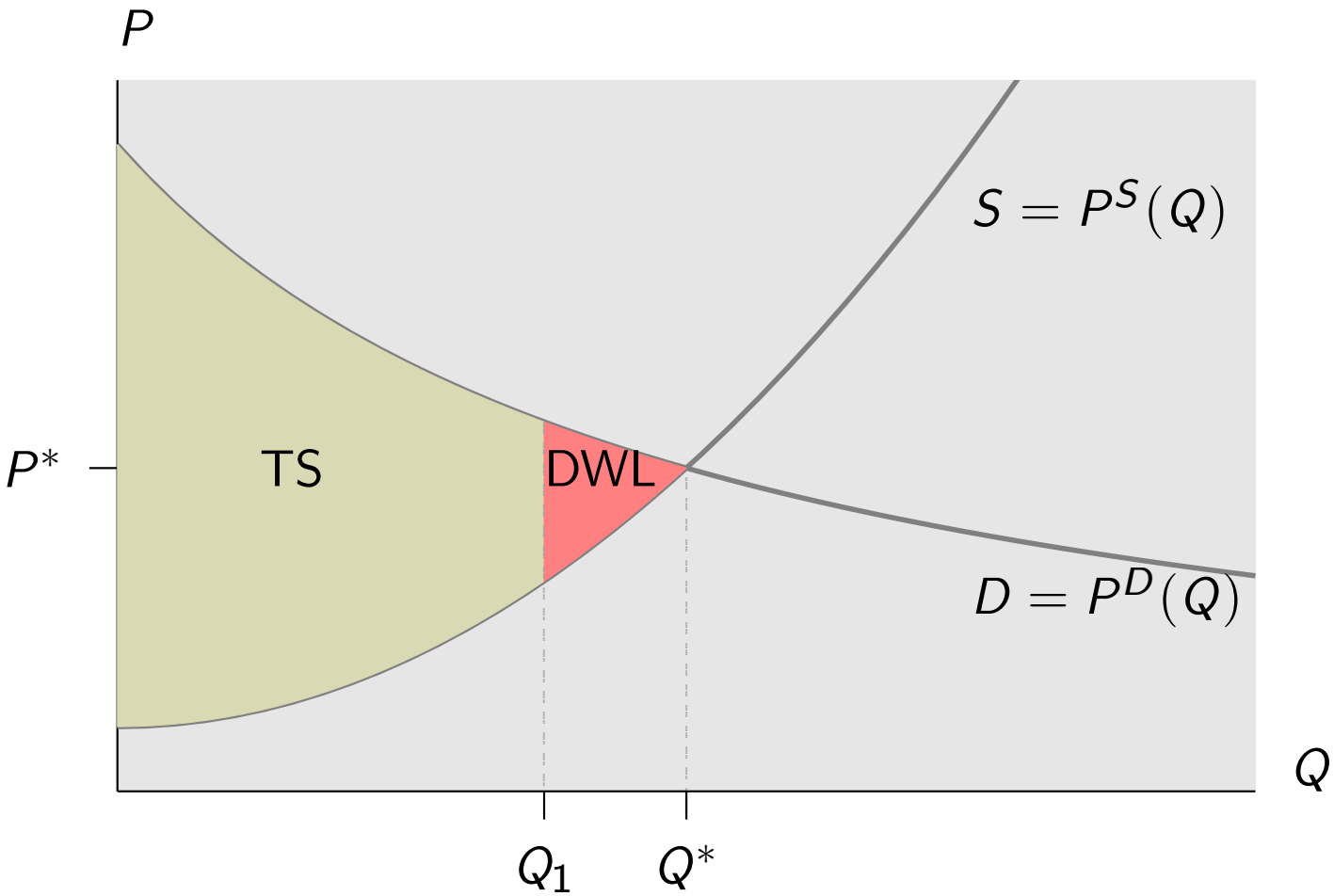


Total Social Surplus



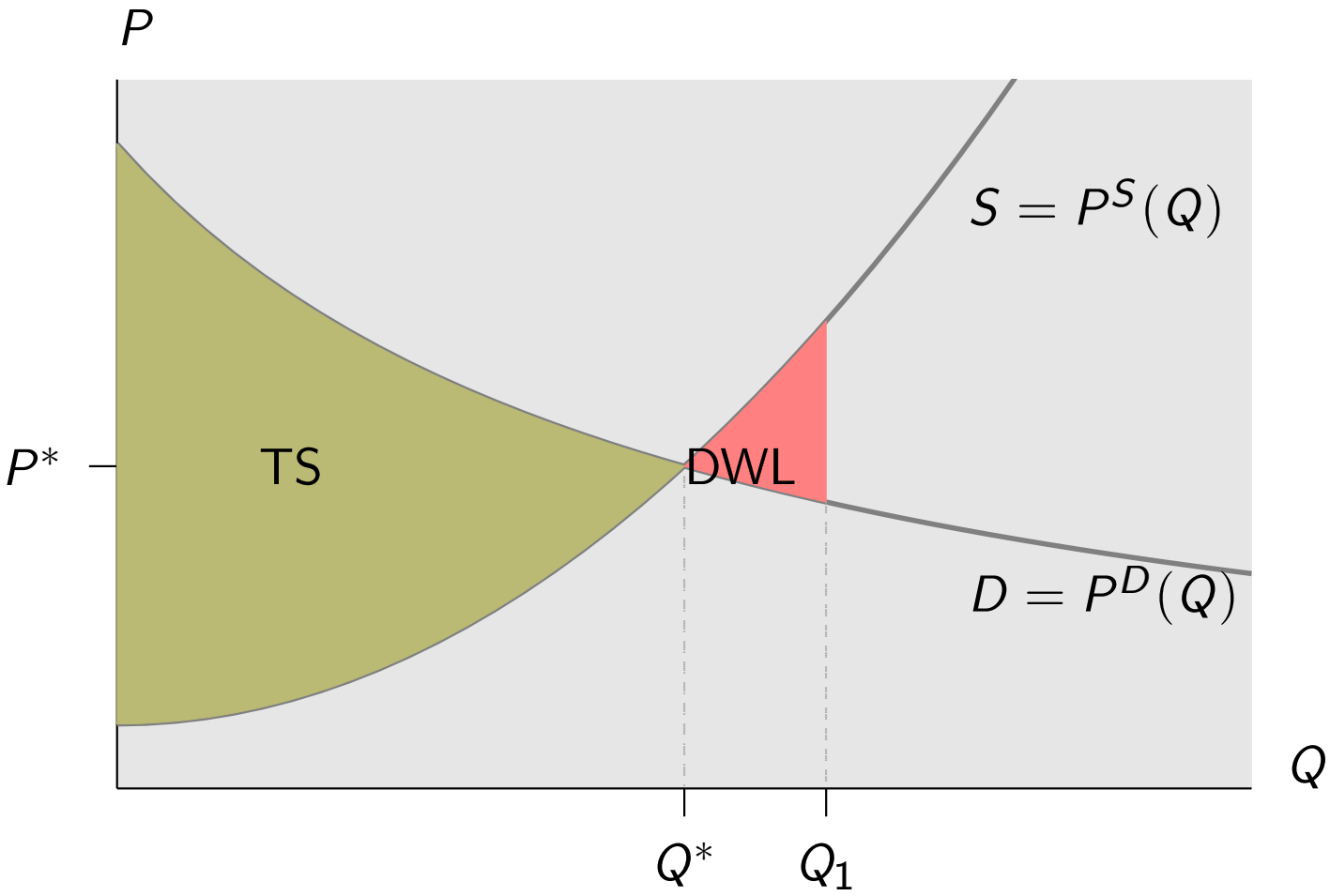
Large number of producers are good for efficiency.

Underproduction



Deadweightloss: loss in total surplus due to inefficient production

Overproduction



Practice Problem

Consider an industry with demand curve:

$$Q_D = 24 - P$$

and total costs:

$$TC = 12 + Q^2$$

If all firms are price taker and have identical costs, find the industry level of output,prices, consumer surplus, producer surplus and total social surplus.

MR when n is small

$$\frac{\partial P(Q)}{\partial q} = \frac{\partial P(Q)}{\partial Q}\frac{\partial Q}{\partial q}$$

Consider a market consisting of only few firms.

- $\frac{\partial Q}{\partial q} > 0$
- $\frac{\partial P(Q)}{\partial q} < 0 \Rightarrow$ Each firm is "price-maker".
- Marginal Revenue curve lies below the inverse demand curve.
- Is marginal revenue always positive?

More on Marginal Revenue

$$\begin{aligned} MR &= P(Q) + \frac{\partial P(Q)}{\partial q} q \\ &= P(Q)\left(1 + \frac{\partial P(Q)}{\partial q}\frac{q}{P(Q)}\right) \\ &= P(Q)\left(1 - \frac{1}{|E_D|}\right) \end{aligned}$$

where E_D denotes firm's price elasticity of demand.
It is immediate from above expression that MR is increasing in $|E_D|$.

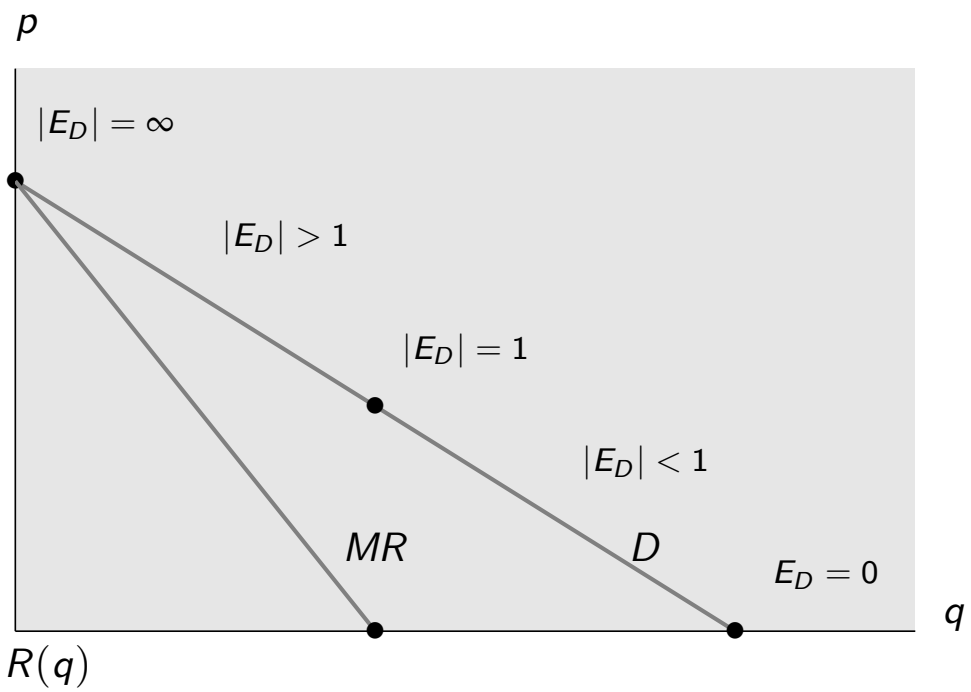
Marginal Revenue and Total Revenue

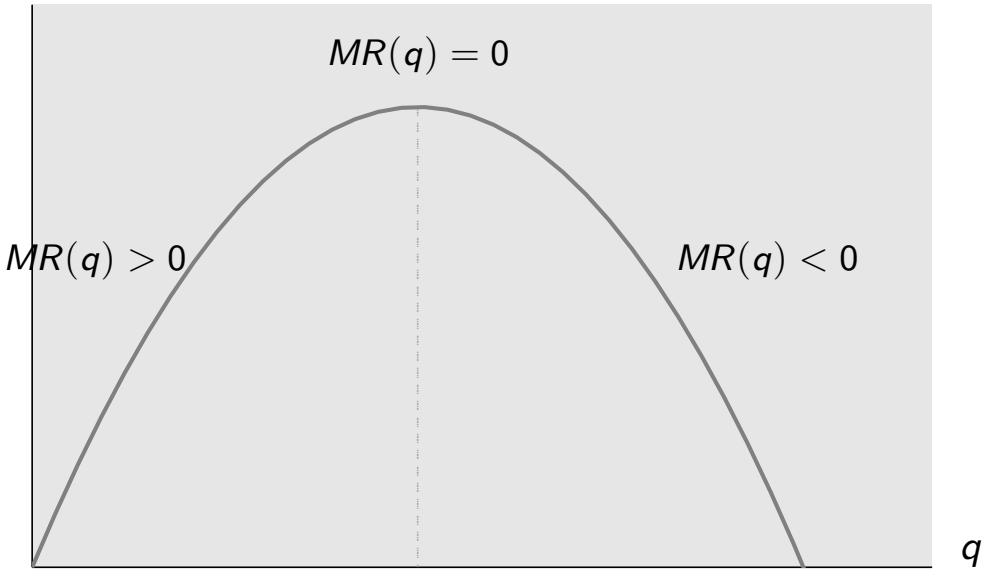
- Relation between MR and E_D.
  - If demand is elastic ($|E_D| > 1$), then $MR > 0$.
  - If demand is unit-elastic ($|E_D| = 1$), then $MR = 0$.
  - If demand is inelastic ($|E_D| < 1$), then $MR < 0$.

  Will a profit maximizing firm ever operate along the inelastic part of demand curve?
- Total Revenue: Revenue generated by all units sold.

- If $MR > 0$, $R(q)$ is increasing in q.
- If $MR < 0$, $R(q)$ is decreasing in q.

Marginal revenue with linear demand





Output decision when n is small

Profit maximizing firms choose q such that

$$\begin{aligned} MR(q) &= MC(q) \\ P(Q)\left(1-\frac{1}{|E_D|}\right) &= = MC(q) \end{aligned}$$

- Rearranging the above expression, it can be seen that

$$P(Q) - MC(q) = \frac{P(Q)}{|E_D|} > 0$$

- Firms are able to charge a mark-up above their marginal costs.
- This mark-up is decreasing in $|E_D|$.

Market Power and Lerner Index

Market Power

Ability to affect the market price.

Lerner Index

The ratio of the difference between price and marginal cost to the price:

$$\frac{P(Q) - MC(q)}{P(Q)} = \frac{1}{|E_D|}$$

The Lerner index is decreasing in $|E_D|$.

- If $E_D = -1.01$, only slightly elastic, the markup is 0.99 (99%)
- If $E_D = -3$, more elastic, the markup is 0.33 (33%)
- If $E_D = -\infty$, perfectly elastic, the markup is zero

The Lerner Index ranges between 0 to 1.

Estimating industry-wide Lerner Index

Firms within an "industry" are often different from each other.

- If the product in question is homogeneous and all firms sell it at exactly same price, market-wide Lerner Index can be calculated as

$$L_1 = \frac{P - \sum_{i=1}^{N} s_i MC_i}{P}$$

where s_i is the market share of $i-$th firm and N is the total number of firms.

- If the product in question is not homogeneous then market-wide Lerner Index can be calculated as

$$L_2 = \sum_{i=1}^{N} s_i \left(\frac{P_i - MC_i}{P_i} \right)$$

where s_i is the market share of $i-$th firm and N is the total number of firms.

- Estimating Marginal Cost can be challenging. Hall (1988, JPE) suggest a method for estimating these costs.

Market Power and Market Structure

1. Perfect competition
   - Many firms
   - Identical product
   - No barriers to entry
   - No control over price (and both firms and buyers are well-informed about prices and products of all firms)
   - Examples: agricultural products, financial instruments
2. Monopolistic competition
   - Large number of small firms acting independently
   - Differentiated products
   - No barriers to entry
   - Some control over price
   - Examples: Boutiques, restaurants, repair shops

Market Structure

3. Oligopoly
   - small number of large mutually interdependent firms
   - Identical or differentiated products
   - Moderate barriers to entry
   - Considerable control over price
   - Non-price competition is important
   - Examples: Airlines, cell phone providers
4. Monopoly
   - One firm
   - One product with no close substitutes
   - High barriers to entry
   - Firms set price subject to demand, regulation
   - Examples: Pharmaceuticals with patents

Market Concentration

- Market Concentration $\Rightarrow$ degree to which a small number of firms provide a major portion of the industry's total production.
- If concentration is low, then the industry is considered to be more competitive.
- If the concentration is high, then the industry will be viewed as less competitive.
- Government agencies such as the U.S. Department of Justice examine concentration within an industry when deciding to approve potential mergers between its firms.
- Measures of concentration:
  - Four-firm concentration ratio
  - Herfindahl-Hirschman Index

Four-firm concentration ratio

Let N be the number of firms in an industry.Let $sales_i$ be the individual output/revenues of firm i for $i = 1, 2...N$.The **market share** of firm i is given by

$$s_i \equiv \frac{sales_i}{Total\ Industry\ Sales} \times 100$$

Four-firm concentration ratio

Percentage of total industry sales accounted for by the four largest firms in the industry.

$$C_4 \equiv \sum_{i=1}^{4} s_i$$

- Ranges from 0 (for perfect competition) to 100 (for monopoly) with higher values indicating less competition.
- Ratio of less than 40 percent is indicative of monopolistic competition.
- Ratio of more than 40 percent is indicative of oligopolistic competition.

Herfindahl-Hirschman Index

Let N be the number of firms in an industry. Let $sales_i$ be the individual output/revenues of firm i for $i = 1, 2...N$.The **market share** of firm i is given by

$$s_i \equiv \frac{sales_i}{Total\ Industry\ Sales} \times 100$$

Herfindahl-Hirschman Index

Sqaure of the percentage market share of each firm summed over the 50 largest firms in the market.

$$HHI \equiv \sum_{i=1}^{50} s_i^2$$

- Ranges from 0 to 10000
- HHI less than 2500 $\Rightarrow$ high degree of competition
- HHI more than 2500 $\Rightarrow$ low degree of competition

Example

Company	Market share (% of sales)
A	32
B	17
C	15
D	10
E	7
F	7
G	5
H	4
I	2
J	1

- What is the four-firm concentration ratio in this market?
- What is the Herfindhal-Hirschman Index for this market?

What is a market?

- No clear consensus
  - Market for automobiles: Should market for passenger cars include light trucks, pick-ups SUVs?
  - Market for soft drinks: Do carbonated drinks like Pepsi and Coca Cola compete only with each other or do they also compete against other beverages such as fruit juices, iced teas, flavored milk, etc?
- Should "close substitutes" be included in a market?

Empirical estimation of market

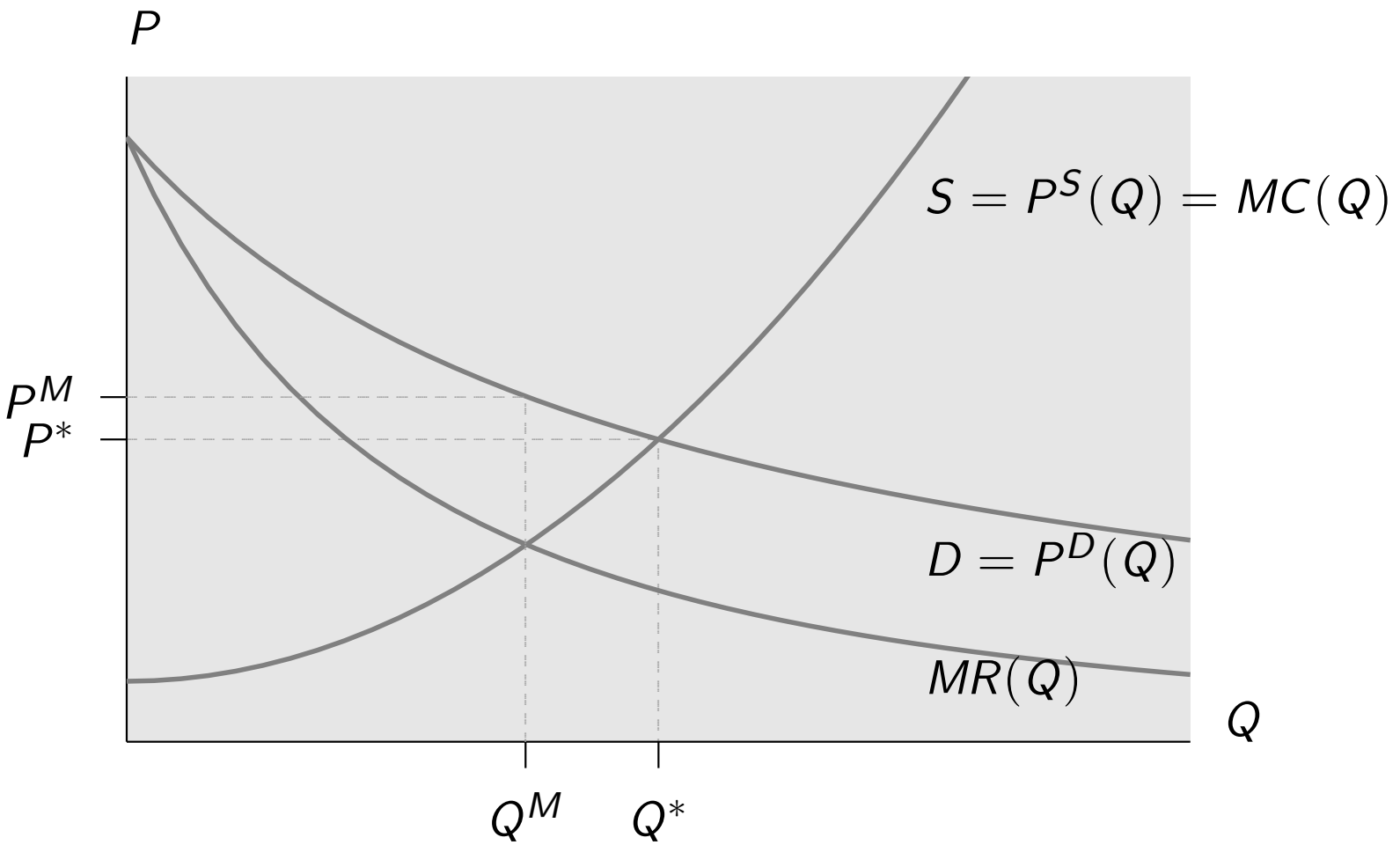
- In the US, Census Bureau is the custodian of market definitions.
- Employs North American Industrial Classification System (NAICS)
  - First classify the output of business unit into broad sectors, such as manufacturing, metals, agriculture, etc.
  - Then classify into further sub-sector, then into an industry and then sub-industry.
- groups establishments on the basis of similarity in production techniques than on the basis of substitutability in consumption.
  - Wood, Ceramic tile and linoleum have different NAICS codes.

Limitations of a Concentration Measure

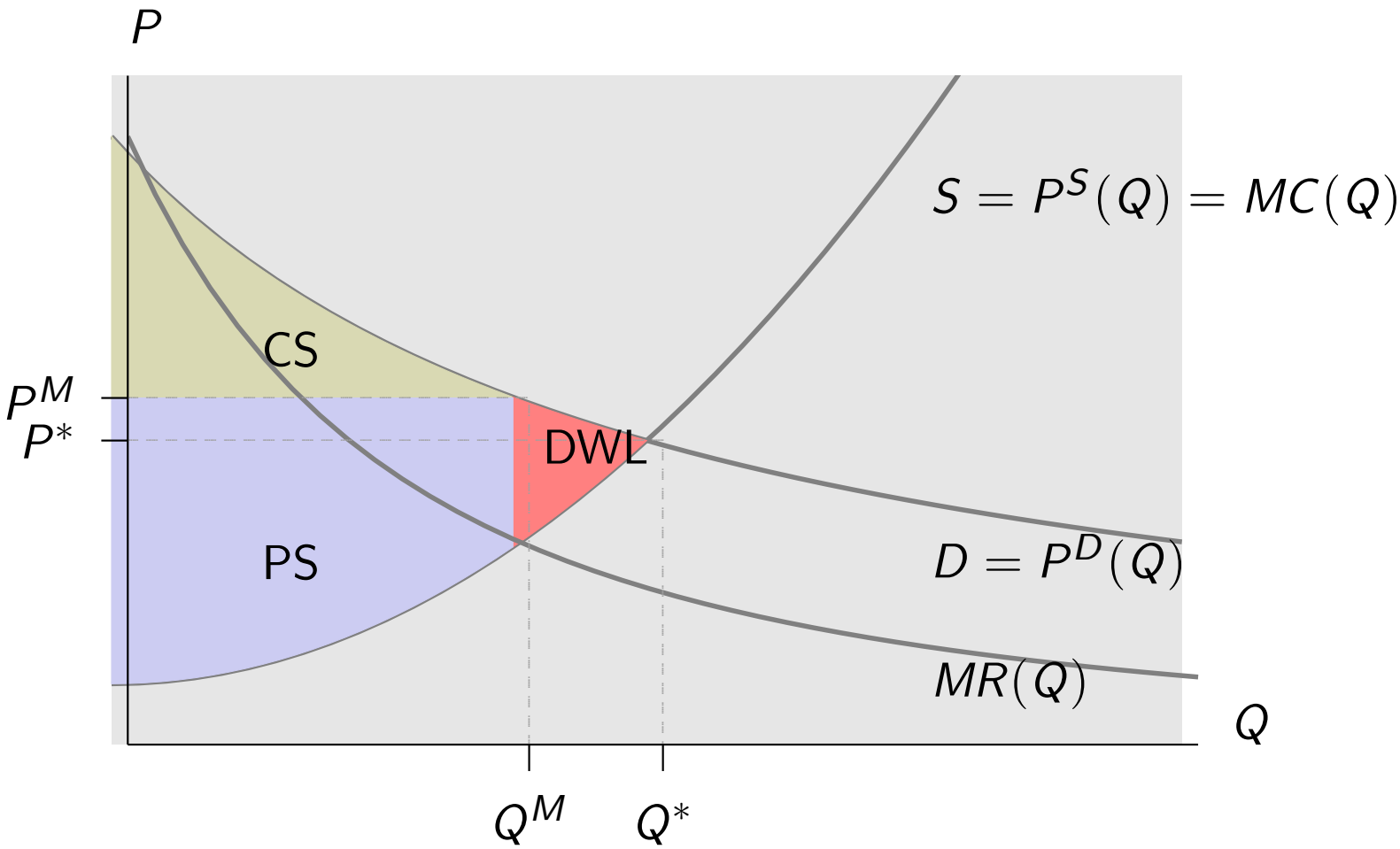
- Hinges on market definition (denominator in HHI calculation) $\Rightarrow$ Narrowly defined product would appear to be more concentrated.
  - geographical scope
  - product market (market v. industry)
  - ignores foreign production in measuring competitiveness of domestic industry
- Presents a static view - doesn't account for barriers to entry, firm turnover thereby overstating degree of competition
- Ignores competition from other products which could be considered substitutes in consumption
- Ignores that market structure is endogenously determined by production technology and costs.

Output decision of a monopoly

- In case of monopoly, firm supply and industry supply are identical, that is, $q = Q$.
- Monopolist chooses Q such that $MR(Q) = MC(Q)$.



Welfare Loss due to Monopoly



Practice Problem

Consider an industry with demand curve:

$$Q_D = 24 - P$$

and total costs:

$$TC = 12 + Q^2$$

If there is only one firm in the industry, find the industry level of output,prices, consumer surplus, producer surplus and total social surplus. Calculate the Lerner Index of monopoly for each level of output and verify that it equals the inverse of $|E_D|$.

How bad is monopoly power?

- The welfare loss associated with a monopoly is:

$$\begin{aligned} \text{DWL} &= \int_{Q^*}^{Q^M} P^D(Q) - P^S(Q) \\ &\approx \frac{1}{2}(P^M - MC(Q^M))(Q^* - Q^M) \end{aligned}$$

- An industry's total revenue is $P^M Q^M$. Then welfare loss as a proportion of industry sales is:

$$\frac{\text{DWL}}{P^M Q^M} \approx \frac{1}{2}\frac{(P^M - MC(Q^M))}{P^M}\frac{(Q^* - Q^M)}{Q^M}$$

- Multiplying and dividing the right hand side by

$$\frac{P^M - P^*}{P^M} = \frac{P^M - MC(Q^*)}{P^M}$$

we get,

How bad is monopoly power?

$$
\begin{aligned}
\frac{\text{DWL}}{P^M Q^M} &\approx \frac{1}{2}\frac{(P^M - MC(Q^M))}{P^M}\frac{(Q^* - Q^M)}{Q^M} \\
&= \frac{1}{2}\frac{(P^M - MC(Q^M))}{P^M}\frac{P^M - MC(Q^*)}{P^M}\frac{\frac{(Q^* - Q^M)}{Q^M}}{\frac{P^M - P^*}{P^M}} \\
&= \frac{1}{2}(\text{Lerner Index})^2|E_D| \\
&\approx \frac{1}{2}\frac{1}{|E_D|}
\end{aligned}
$$

- DWL is inversely related to $E_D|$.
- Keep in mind that under imperfect competition, MC may not be minimized, so $P - MC$ may be artificially low.

Non-uniform pricing practices

- So far, we focused on the output decision of a firm which employs uniform or single-pricing strategy.

Single Price

All the units are sold at the same price to all the customers.

- However, a monopolist may be able to earn higher profits by employing nonuniform pricing strategies whereby it charges different prices from different consumers or for different units sold to the same consumer.

- Examples of non-uniform/discriminatory pricing include:
  - Personalized Pricing
  - Two-part pricing
  - Block Pricing
  - Peak-load pricing
  - ...

Price Discrimination

Price Discrimination

Practice of selling different units of a good or service for different prices.

- Products with identical costs are sold in different markets at different prices, or
- The ratio of price to marginal cost differs for similar products.
- To be able to price discriminate, a monopoly must:
  - Identify and separate different buyer types.
  - Sell a product that cannot be resold.
- Price differences that arise from cost differences are not price discrimination.
- There are three types of price discrimination:
  1. First-degree price discrimination or perfect price discrimination
  2. Second-degree price discrimination or nonlinear price discrimination
  3. Third-degree price discrimination or group price discrimination

Perfect Price Discrimination

Also known as, First degree discrimination

Perfect Price Discrimination

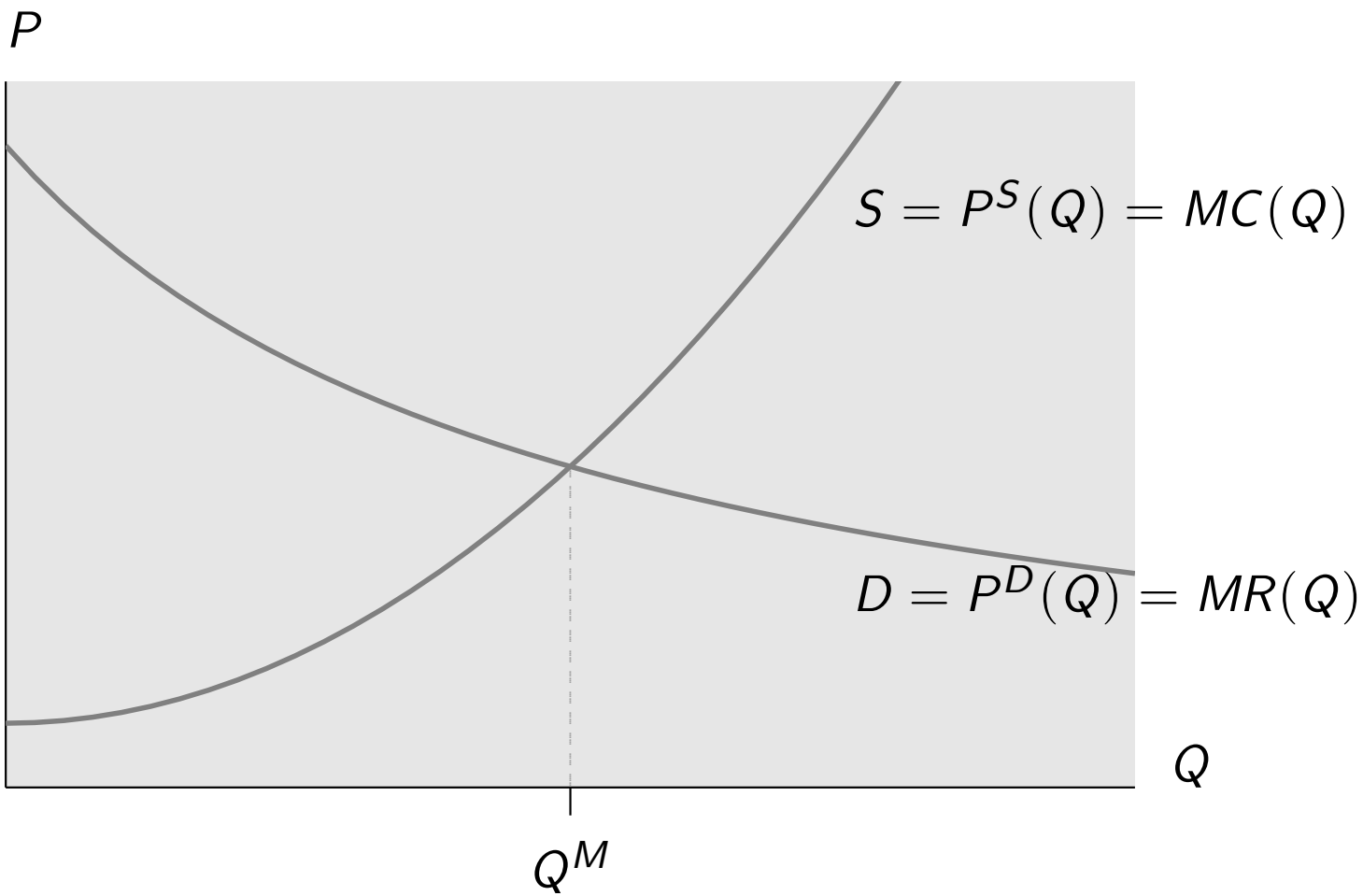
Selling each unit of output for the highest price anyone is willing to pay.

- Seller can identify where each consumer lies on the demand curve and charges each consumer the highest price the consumer is willing to pay.

$$P(Q) = v(Q) = MR(Q)$$

  - MR=P for every unit sold
  - As a result, MR curve coincides with the demand curve
- Allows the seller to extract the greatest amount of profits.
- Requires a considerable amount of information about the consumer.

First-degree price discrimination



- All consumer surplus is captured by the producer.
- Monopoly no longer creates productive allocative inefficiency.

Two-part pricing

Two-part pricing

The firm charges each consumer a lump-sum access fee for the right to buy as many units of the good as the consumer wants at a per-unit price.

- A consumer's overall expenditure (and firm's revenue) for amount Q consists of an access fee, A, and a per-unit price, P. So, expenditure is $E = A + PQ$
- Two-part pricing schemes transfer the consumer surplus from the buyer to the seller.
- In order to effectively employ two-part pricing, a firm must have market power, know how individual demand curves vary across its customers, and prevent resale.
- Can you think of any firms which employ two-part pricing?
- Examples include: health clubs, cable companies
- With identical customers, a firm can set a two-part price that is efficient (p = MC) and all total surplus goes to the firm (CS = 0).

Two-part pricing

- Firm's profit function is given by:

$$\pi = A + P(Q)Q - C(Q)$$

- Incentive compatibility for consumers requires:

$$A \leq \int_0^Q v(Q)dQ - P(Q)Q = CS(Q)$$

- If a firm is profit maximizing, then incentive compatibility constraint will be binding for consumers.
- Firm's profit function becomes:

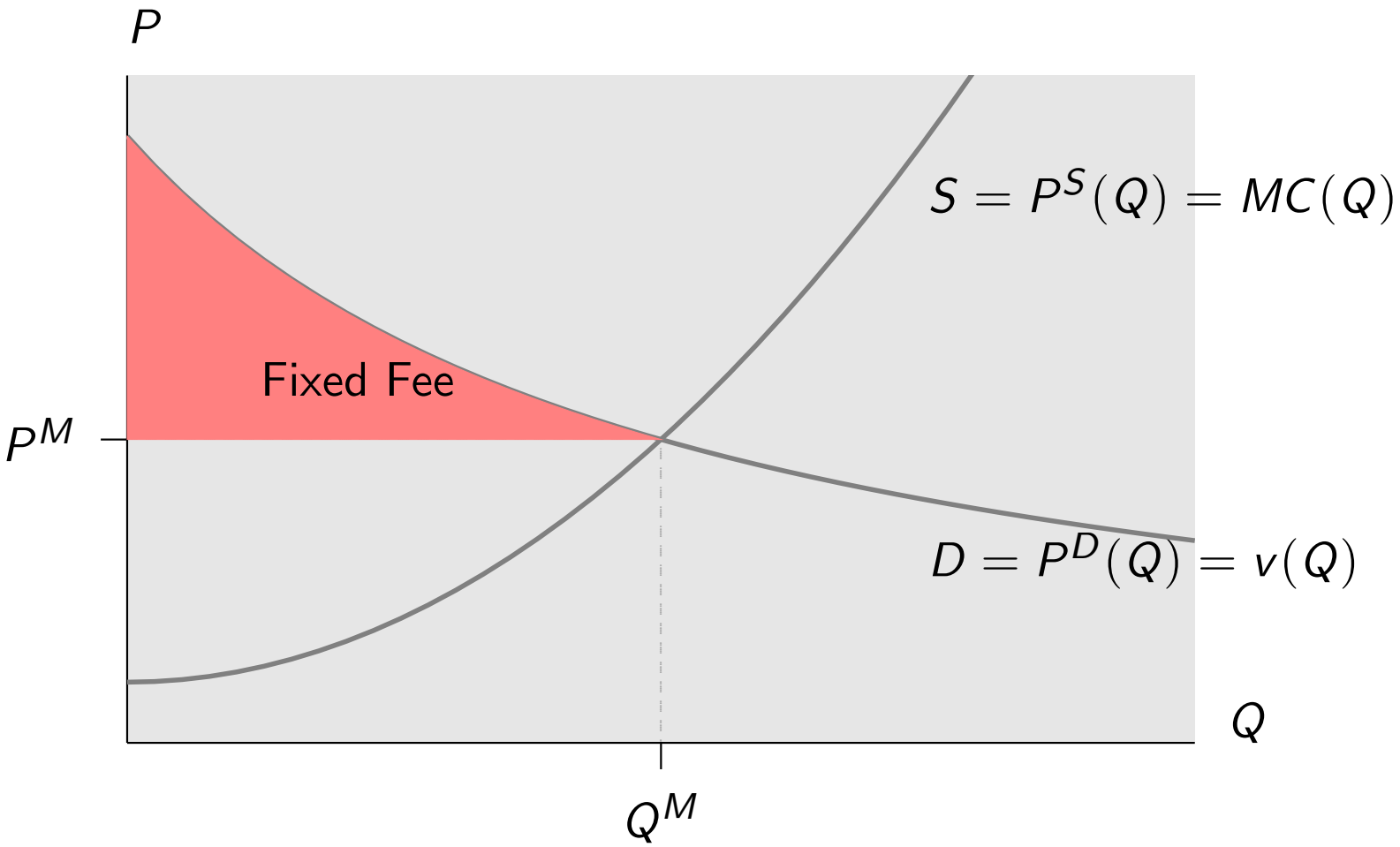
$$\pi = \int_0^Q v(Q)dQ - P(Q)Q + P(Q)Q - C(Q) = \int_0^Q v(Q)dQ - C(Q)$$

- First order optimization condition yields:

$$v(Q^M) = MC(Q^M) = P^M$$

and fixed fee equal to $\int_0^Q v(Q)dQ - P^M Q^M$

First-degree price discrimination

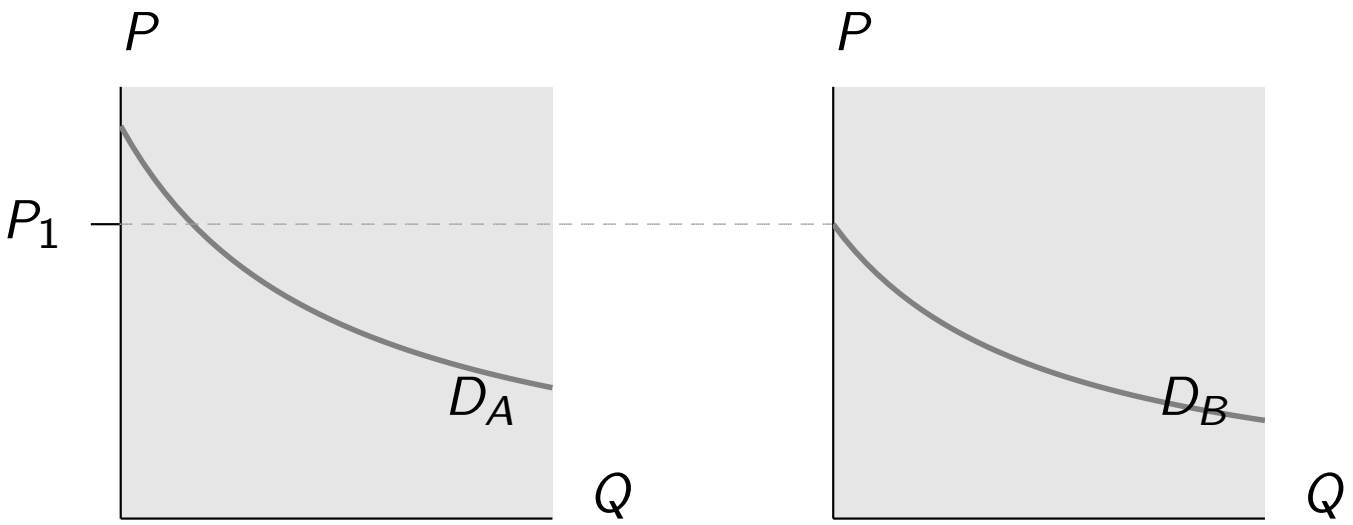


Third Degree Price Discrimination

Discrimination based on observable characteristics.

- Customers are segregated into different markets and charged different prices in each.
- The market segmentation can be based on any characteristic such as age, location, gender, income, etc.
- Assume the firm operates in two markets, A and B.
- The demand in market A is less elastic than the demand in market B.

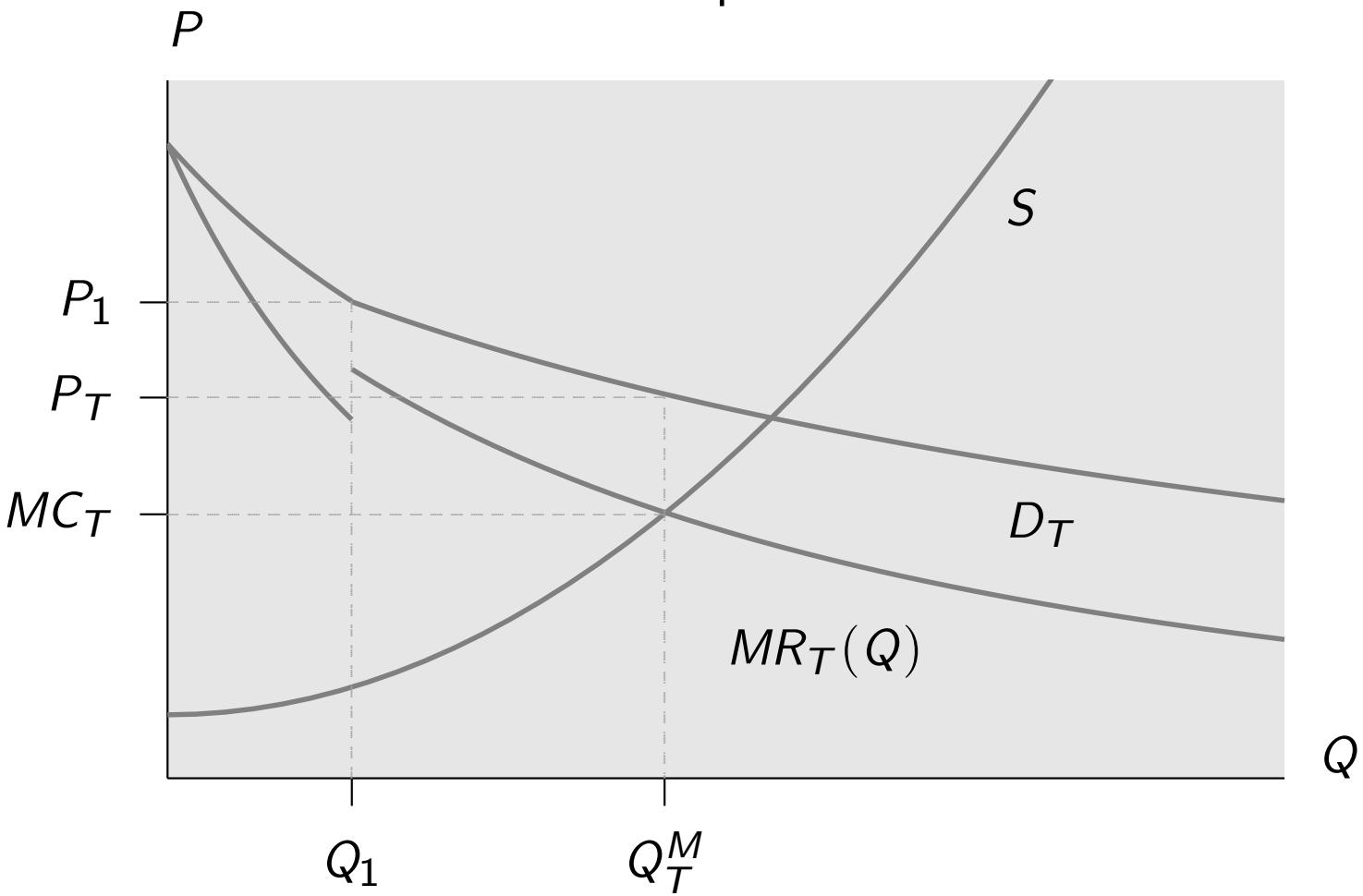
Third-degree price discrimination



The entire market faced by the firm is described by the horizontal sum of the demand and marginal revenue curves.

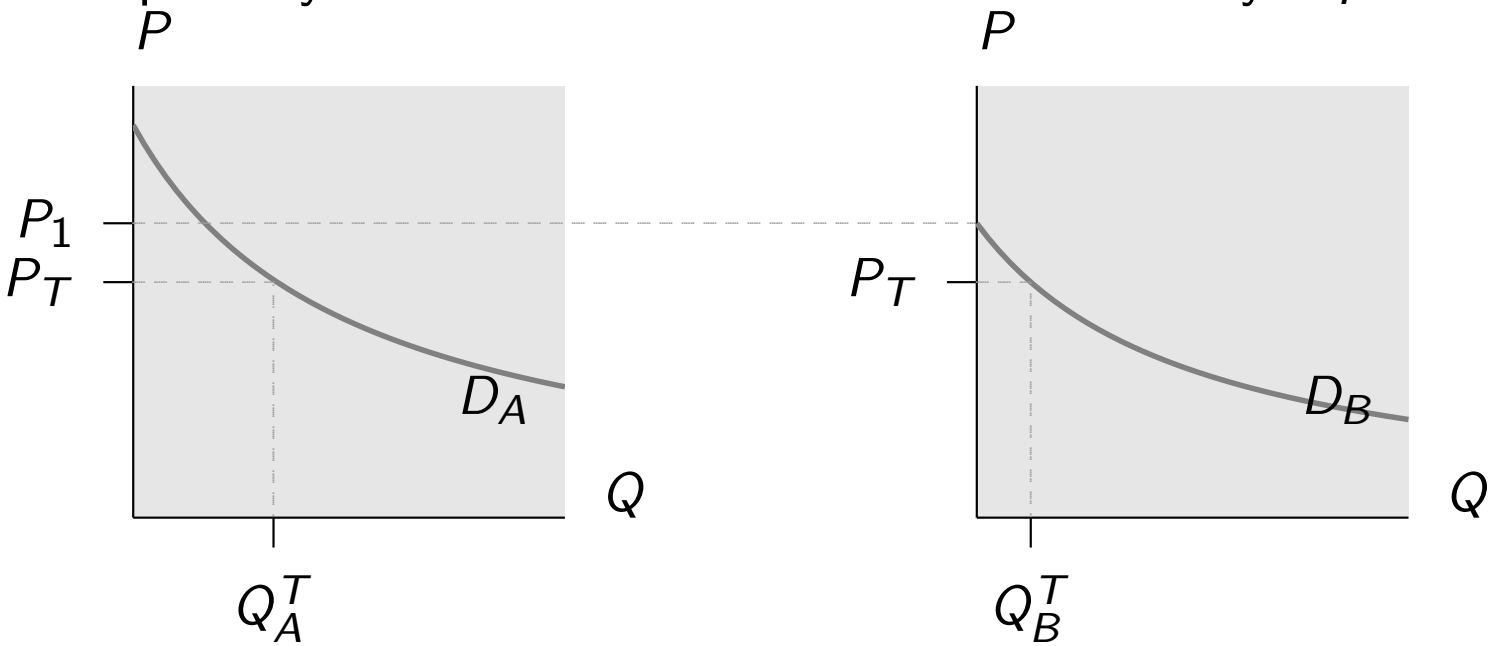
Third-degree price discrimination

What is firm is not allowed to price discriminate?



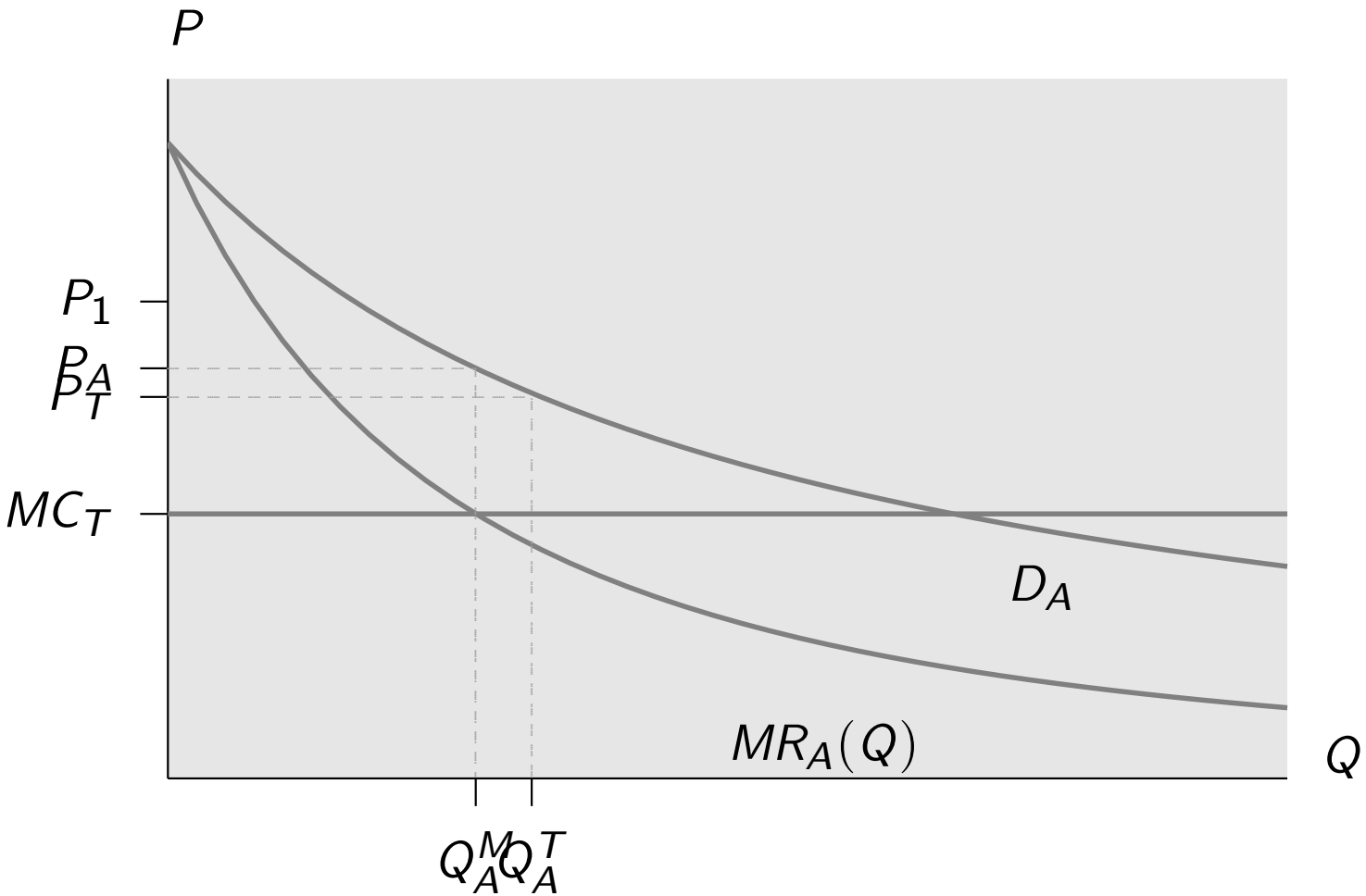
Third-degree price discrimination

The quantity sold in each market is determined by P_T



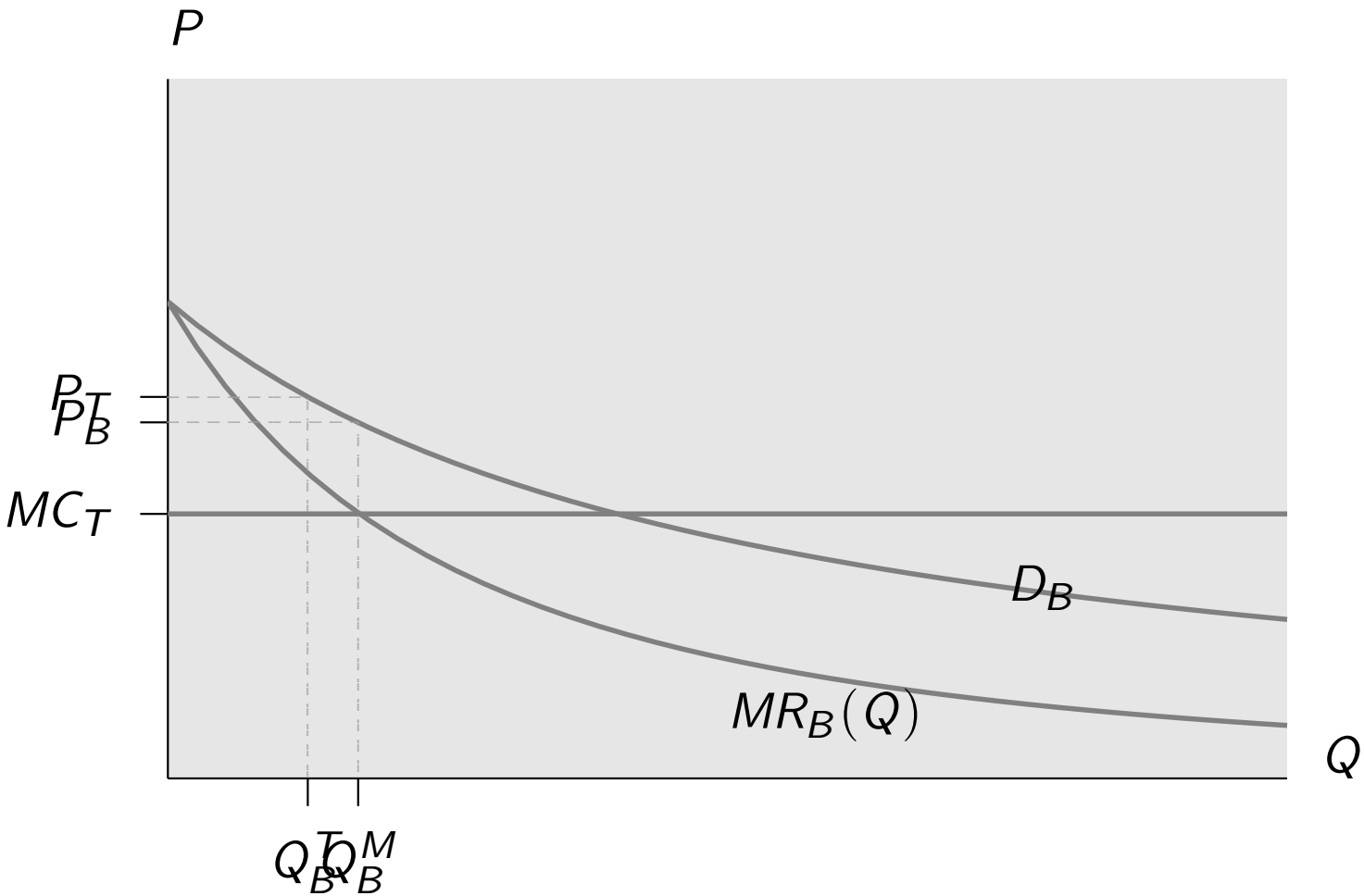
What if a firm is allowed to price discriminate?

Market A under third-degree price discrimination



Notice that at Q_A^T, $MR_A(Q_A^T) < MC_T(Q_T)$. Profits can be increased by lowering sales.

Market B under third-degree price discrimination



Notice that at Q_B^T, $MR_A(Q_A^T) > MC_T(Q_T)$. Profits can be increased by selling more in this market.

Price Discrimination

- The firm finds the total amount to produce by equating the marginal revenue and marginal cost in the market as a whole: Q_T.
- If the firm were forced to charge a uniform price P_T, it would find the price by examining the aggregate demand D_T at the output level Q_T.
- Q_T is then used to determine the price P_T using the demand curves D_T.
- The firm can increase its profits by charging a different price in each market.
- In order to find the optimum price to charge in each market, draw a horizontal line back from the $\frac{MR_T}{MC_T}$ intersection
- Where this line intersects each sub-market's MR curve determines the amount that should be sold in each market: Q_A^M and Q_B^M
- These quantities are then used to determine the price in each market using the demand curves D_A and D_B

Price discrimination and elasticity

If

$$MR_A < MC_T < MR_B$$

seller can increase its profits by lowering prices in B and increasing prices in A.

At the optimum,

$$MR_A = MR_B = MC_T$$

$$p_A \left(1 - \frac{1}{|E_D^A|}\right) = p_B \left(1 - \frac{1}{|E_D^B|}\right) = MC$$

where $E_D = \frac{\Delta Q_D}{\Delta P} \frac{P}{Q}$ If $|E_D^A| < |E_D^B|$, then $p_A > p_B$

Under third degree price discrimination, a seller should charge a lower price in those market segments with greater price elasticity.

Practice Problem

Consider a monopolist with a constant marginal cost of $MC = 1$ who faces a linear inverse demand function of

$$P_A = 6 - 0.5Q_A$$

in country A and linear inverse demand function of

$$P_B = 9 - Q_B$$

in country B.

1. Find the profit maximizing level of output, prices and corresponding profits when monopolist engages in third degree price discrimination.
2. Find the profit maximizing level of output, prices and corresponding profits when monopolist cannot price discriminate.

Relevant Readings

- Slides # 1–19 Ch 2.1 (PRN)
- Slides # 20–35 Ch 4 (PRN)
- Slides # 36– 42 Ch 5.2 (PRN)
- Slides # 43–51 Ch 5.3 (PRN)