Producer Theory
Production and Costs
Ch.3 (PRN)

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Decision Time Frames

Short Run

Time frame in which the quantity of one or more resources used in production is fixed.

For most firms, capital, land, and entrepreneurship are fixed factors of production and labor is the variable factor of production.

Long Run

Time frame in which the quantities of all resources can be changed.

- A firm plans in the Long Run, but operates in the Short Run.

Production Function

- Production function: summarizes how a firm converts inputs into outputs using one available technology
  - defines the relationship between inputs and the maximum amount that can be produced within a given period of time with a given level of technology
  - shows only efficient production processes because it gives the maximum output.

$$Q = f(X_1, X_2, ...X_n)$$

where Q = output and $X_1, X_2...X_n$ are inputs/factors of production

Production in the Short Run

- We will assume that a firm uses only two inputs: capital (K) and labor (L).
- Production function: $Q = f(L, K)$.
- Production function summarizes the characteristics of existing technology at a given time.
- Shows the technological constraint firm/manager faces.
- How can the firm increase its output in the short run? $\Rightarrow$ By employing more labor
- Depending upon the context, inputs may also be referenced as factors of production or resources.
- Depending upon the context, output may also be referenced as Quantity (Q), Total Product (TP) and Product

Total product (TP)

output produced in a given period

Product Schedule

An example of a short-run production function can be:

$$Q = f(L) = 30L + 20L^2 - L^3$$

Production Schedule

A tabular representation of various levels of output which a firm can produce by employing various amount of labor, given capital/technology.

Labor (L)	TP or Q
0	0
1	4
2	10
3	13
4	15
5	16
..	...

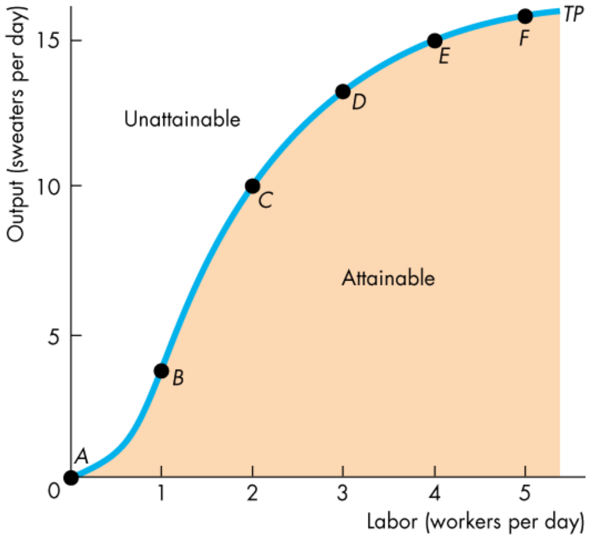
Graphing Total Product Curve

Graph the production function given by: $Q = 30L + 20L^2 - L^3$.

Sidenote: Steps involved in sketching a function

1. Identify if the function is linear or non-linear.
2. Look for points of discontinuity
3. Look for points where the function may not be differentiable.
4. If function is linear, then identify any two points and connect them.
5. If the function is non-linear then you have to do extra work.
6. If differentiable, Calculate first derivative and identify critical points.
7. Classify critical points using either first or second derivative.
8. Identify the co-ordinates where function crosses the x-axis and y-axis.
9. Identify the limiting behavior of the function as x goes to plus/minus infinity.
10. Bear in mind that a non-linear function cannot have a linear graph.

Total Product Curve



Note: This is not the graph of $Q = 30L + 20L^2 - L^3$.

Total Product Curve Contd.

- The total product curve separates efficient outcomes from inefficient ones in the short run.
- The total product curve separates attainable output levels from unattainable output levels in the short run.
- Why does total product curve take this shape?
- The answer to this question lies in **marginal product**.

Marginal Product of factor X

Marginal product of labor (MP_L)

The change in total product that results from a one-unit increase in the quantity of labor employed, with all other inputs remaining the same.

$$MP_L = \frac{\Delta TP}{\Delta \# L} \stackrel{as \Delta \to 0}{=} \frac{\partial Q}{\partial L} \equiv f_L$$

MP_L is a function of L and K.

Marginal product of capital (MP_K)

The change in total product that results from a one-unit increase in the quantity of capital employed, with all other inputs remaining the same.

$$MP_K = \frac{\Delta TP}{\Delta \# K} \stackrel{as \Delta \to 0}{=} \frac{\partial Q}{\partial K} \equiv f_K$$

MP_K is a function of L and K.

Practice Problems

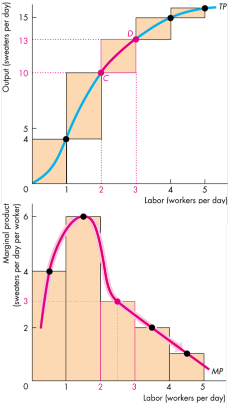
Suppose the production function for flyswatters during a particular period is given by:

$$Q = 6K^2L^2 - K^3L^3$$

1. Find MP_L.
2. Find MP_K.
3. Sketch MP_L when $K = 1$.

Marginal Product Curve Again

Marginal product: slope of the total product curve



(b) Marginal product

L	TP or Q	MP_L
0	0	
1	4	
2	10	
3	13	
4	15	
5	16	

Marginal Product Curve Contd.

- Almost all production processes are like the one shown here and have:
  - Increasing marginal returns initially
  - Diminishing marginal returns eventually
- Check out this **link**.

Law of Diminishing Marginal Returns

As a firm uses more of a variable input with a given quantity of fixed inputs, the marginal product of the variable input eventually diminishes.

Diminishing Marginal Productivity

1. $$\frac{\partial MP_L}{\partial L} \equiv f_{LL} < 0 \text{ for high enough L}$$

2. $$\frac{\partial MP_K}{\partial K} \equiv f_{KK} < 0 \text{ for high enough K}$$

3. $$\frac{\partial MP_L}{\partial K} \equiv f_{LK} > 0$$

Marginal Product and Total Product

How do you think are the shapes of total product curve and marginal product curves are related?

1. MP $> 0 \Rightarrow$ Slope of the tangent to the points on TP is increasing.
   - MP is increasing $\Rightarrow$ TP is convex.
   - Highest point on MP curve $\Rightarrow$ inflection point, steepest point on TP curve.
   - MP is decreasing $\Rightarrow$ TP is concave.
2. MP $= 0 \Rightarrow$ TP curve is flat, at its highest point.
3. MP $< 0 \Rightarrow$ TP curve is falling.

Average Product of factor X

Another concept which is often used to describe the relationship between input and output is Average Product.

Average product of labor (AP_L)

Output per unit of labor employed.

$$AP_L = \frac{TP}{\#L}$$

Labor(workers per day)	TP or Q	AP_L
0	0	
1	4	
2	10	
3	18	
4	25	
5	30	

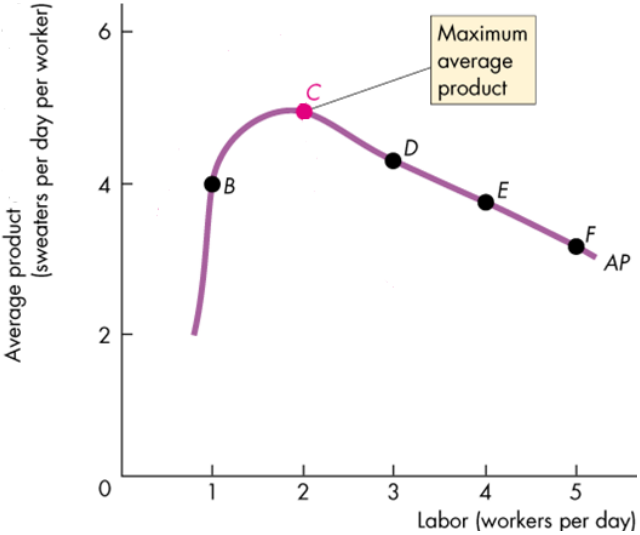
Practice Problems

Suppose the production function for flyswatters during a particular period is given by:

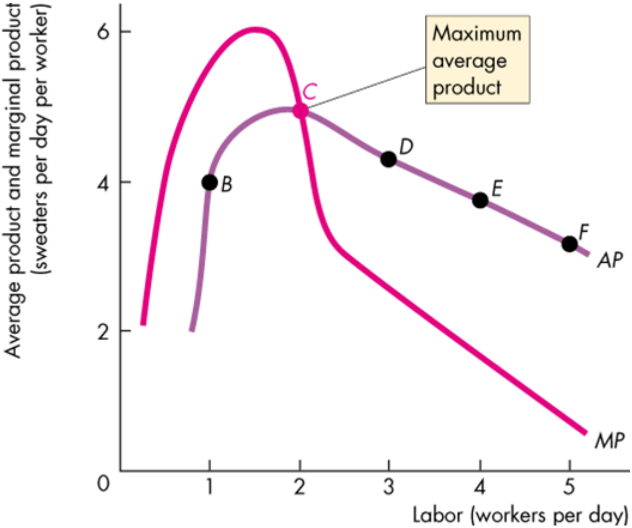
$$Q = 6K^2L^2 - K^3L^3$$

1. Find AP_L.
2. Find AP_K.
3. Sketch AP_L when $K = 1$.

Average Product Curve



Average product: slope of line from origin to TP curve



Relation between Marginal Product and Average Product

$$\begin{aligned}
\frac{dAP_L}{dL} &= \frac{d}{dL}\left(\frac{Q}{L}\right) \\
&= \frac{L\frac{dQ}{dL} - Q}{L^2} \\
&= \frac{dQ}{dL} - \frac{Q}{L} \\
&\overset{sign}{=} MP_L - AP_L
\end{aligned}$$

- Where $MP > AP$, average product increases.(From point B to C)
- Where $MP < AP$, average product decreases. (From point C on)
- Where $MP = AP$, average product is at its max. (At point C)

Elasticity of output

Elasticity of output

The elasticity of output with respect to an input measures the proportionate change in output relative to a proportionate change in input and is given by:

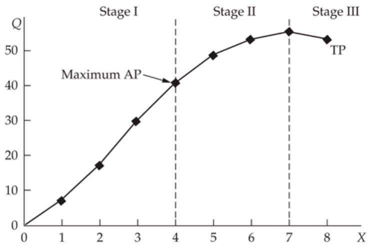
$$E_{Q,L} = \frac{\partial Q}{\partial L}\frac{L}{Q}$$

and

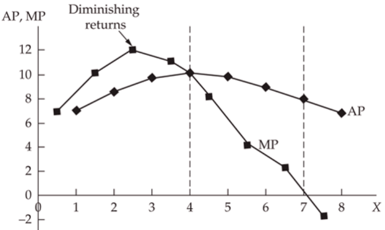
$$E_{Q,K} = \frac{\partial Q}{\partial K}\frac{K}{Q}$$

Consider $Q = 30L + 20L^2 - L^3$ and calculate $E_{Q,L}$.

The Three Stages of Production



(a) Total product



(b) Average and marginal products

The Three Stages of Production

Based on the behavior of MP and TP, the short-run production function can be divided into three distinct stages of production:

- **Stage 1** $MP > AP \Rightarrow$ Fixed input is underutilized
- **Stage 2** $AP > MP \geq 0$ Fixed input is properly utilized
- **Stage 3** $MP < 0$ Fixed input capacity is reached

A competitive firm strives to operate in Stage 2 of production function. More on this after discussion on costs!!

Costs in the Short Run

In short run,

- **Total cost** (TC) = costs of producing a given level of output can be broken into two components:
  1. **Total fixed cost** (TFC) - cost of the firm's fixed inputs. Does not change with output. Examples:
     - Employees that are under long-term contracts.
     - Rents of buildings that are under lease.
     - Electricity that must be paid for buildings.
  2. **Total variable cost** (TVC) - cost of the firm's variable inputs. Do change with output.

Total Cost Schedule

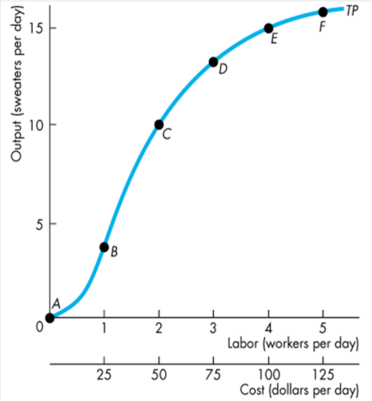
- Total Costs $=$
  $w \times$ quantity of labor employed $+\, r \times$ quantity of capital employed.
- $\#L$ and $\#K$ depend on the level of output to be produced and time frame in question.
- In short run, capital is fixed and production function is given by $Q = f(L, K_0)$
- This can be used to derive labor demand function $L_D(Q, K_0) = f^{-1}(Q, K_0)$...
- .. which in turn can be used to derive a firm's cost function.

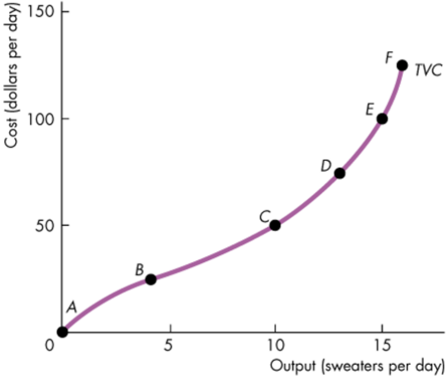
Total Cost function

Relationship between a total costs of production and level of output to be produced:

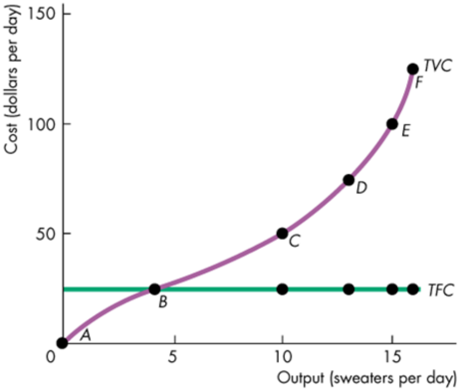
$$TC(Q) \text{ or } C(Q) = TFC + TVC(Q) = rK_0 + wL_D(Q, K_0)$$

Relationship between TP and TVC





(Total) Cost Curves



Marginal Costs

Why do the Total Variable Cost curve and Total Cost Curve take this shape? The answer to this question lies in **marginal costs**.

- Change in costs from producing one more unit of output.
- Calculated as

$$MC = \frac{dTC}{dQ} = \frac{dTVC}{dQ}$$

- Are (mostly!) unaffected by fixed costs.
- What shape should these curves take? Why?

- Almost all production processes are like the one shown here and have:
  - Decreasing marginal costs initially
  - Increasing marginal costs eventually

How are MP and MC related?

$$
\begin{aligned}
\text{MC} &= \frac{\text{change in TVC}}{\text{change in output}} \\
&= \frac{\text{wage} \times \text{change in labor}}{\text{change in output}} \\
&= \frac{w}{MP_L}
\end{aligned}
$$

MC and MP_L are inversely related to each other.

- MC is at its minimum at the same output level at which MP is at its maximum.
- Over the output range with increasing marginal returns, marginal cost falls as output increases.
- Over the output range with diminishing marginal returns, marginal cost rises as output increases.

Average Costs

Average cost is calculated from each of the total cost measures.

- **Average fixed cost** (AFC) is total fixed cost per unit of output.
- **Average variable cost** (AVC) is total variable cost per unit of output.
- **Average total cost** (ATC) is total cost per unit of output.

$$ATC = AFC + AVC$$

Average Costs

- Cost per-unit of output produced.
- Calculated by dividing total costs by number of units produced.

$$ATC = \frac{TC}{Q} = \frac{TFC + TVC}{Q} = AFC + AVC$$

- Are affected by fixed costs.
- What shape should these curves take?
  - AFC curve: average fixed cost falls as output increases.
  - AVC curve: U-shaped
    - As output increases, average variable cost falls to a minimum and then increases.
  - ATC curve: U-shaped, and getting closer to AVC

How are AP and AVC related to each other?

$$
\begin{aligned}
\text{AVC} &= \frac{\text{TVC}}{\text{output}} \\
&= \frac{\text{wage} \times \text{L}}{\text{Q}} \\
&= \frac{w}{AP_L}
\end{aligned}
$$

AVC and AP_L are inversely related to each other.

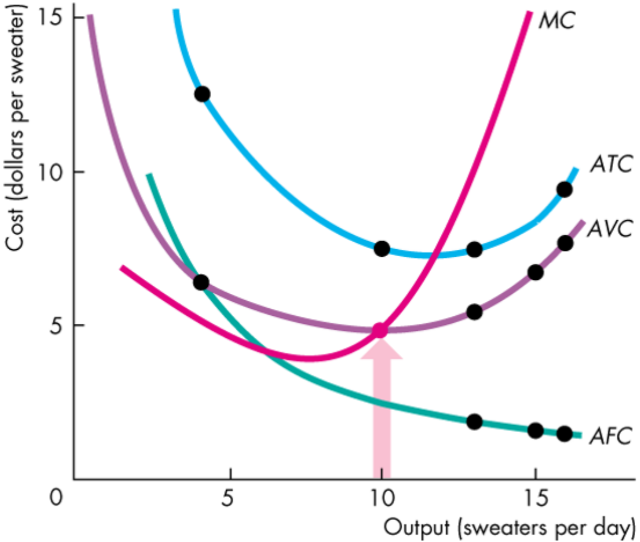
- Initially, $MP > AP$, which brings rising AP and falling AVC.
- Eventually, $MP < AP$, which brings falling AP and rising AVC.
- AVC is at its minimum at the same output level at which average product is at its maximum.

MC and AC curves

How are MC and AC Curves related?

- When AVC is falling, MC is below AVC.
- AVC is rising, MC is above AVC.
- At the minimum AVC, MC equals AVC.
  Similarly,
- when ATC is falling, MC is below ATC.
- when ATC is rising, MC is above ATC.
- At the minimum ATC, MC equals ATC.

MC and AC curves



Practice Problem

Consider the Cobb-Douglas Production function $q = L^{0.5}K^{0.5}$. Suppose labor is available at a wage rate of "w" and capital is available at a rental rate of "v". Further, in the short run, capital is fixed at K_1

1. Given K_1, what is the cost minimizing amount of labor L^* employed by the firm for producing Q units?
2. Given L^* above, express total costs as a function of v, w, K_1 and Q.
3. Consider $v = 3$, $w = 12$ and $K_1 = 80$. Express total costs as a function of Q.
4. How much of these costs are fixed? What are variable costs?
5. Calculate and graph marginal cost, average fixed cost, average variable cost and average total cost functions.
6. Consider $v = 3$, $w = 12$ and $K_1 = 80$. What is the cost of producing 100 units of output?

Shifts in the Cost Curves

The position of a firm's cost curves depend on two factors:

- Technology
  - What is a technological change?
  - An increase in productivity shifts the average and marginal product curves upward and the average and marginal cost curves downward.
  - If technological advance brings more capital and less labor into use, fixed costs increase and variable costs decrease.
    - What happens to ATC in this case?
- Prices of factors of production
  - An increase in a fixed cost shifts the total cost (TC) and average total cost (ATC) curves upward but does not shift the marginal cost (MC) curve.
  - An increase in a variable cost shifts the total cost (TC), average total cost (ATC), and marginal cost (MC) curves upward.

Deriving firm's supply curve from Cost Curves

- A firm's goal is to maximize profit.
- It decides to produce a level of output that maximizes its profits.
- Given price P, an optimizing firm chooses a level output such that

$$P = MC$$

Profits or Losses

- If P$>$ATC $\Rightarrow$Economic Profits
- If P $=$ min. ATC $\Rightarrow$Break Even Point (Firm makes zero profits/losses)
- If P$<$ATC $\Rightarrow$Economic Losses

Temporary Shutdown Decision

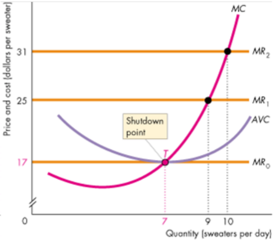
- What if maximum profit is a loss?
- If losses are permanent, then go out of business.
- If losses are temporary, then compare losses from operating against losses from losses from shutting down temporarily.

$$
\begin{aligned}
\text{Economic loss} &= \text{TC - TR} \\
&= \text{TFC} + \text{TVC - TR} \\
&= \text{TFC} + (\text{AVC - P}).\text{Q}
\end{aligned}
$$

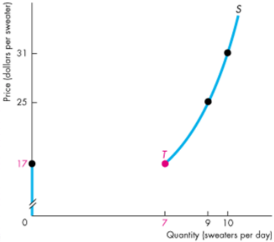
Shutdown or operate

- If P$>$AVC $\Rightarrow$Continue operating
- If P $=$ min. AVC $\Rightarrow$Shut down point (Firm is indifferent between producing and shutting down)
- If P$<$AVC $\Rightarrow$Shut down

Firm's Supply Curve



(a) Marginal cost and average variable cost



(b) Campus Sweaters short-run supply curve

Long Run Production Function

- In the long run, all inputs are variable.
- Diminishing returns set on for **all** factors of production.

		Output		
Labor	K = 1	K =2	K=3	K=4
1	4	10	13	15
2	10	15	18	20
3	13	18	22	24
4	15	20	24	26
5	16	21	25	27

- How would TP, MP_L and AP_L curves change when capital is changed?
  - All the curves shift.

Long Run Production Function

- The long run production process is described by the concept of returns to scale
- Returns to scale = the resulting increase in total output as all inputs increase by same proportion
- If all inputs into the production process are doubled, three things can happen:
    1. output can more than double $\Leftrightarrow$ "increasing returns to scale" (IRTS)
    2. output can exactly double $\Leftrightarrow$ "constant returns to scale" (CRTS)
    3. output can less than double $\Leftrightarrow$ "decreasing returns to scale" (DRTS)

Returns to scale can also be described using the following equation

$$\beta Q = f(\alpha L, \alpha K)$$

- if $\beta > \alpha$ then the firm experiences "increasing returns to scale" (IRTS)
- if $\beta = \alpha$ then the firm experiences "constant returns to scale" (CRTS)
- if $\beta < \alpha$ then the firm experiences "decreasing returns to scale" (DRTS)

Question

Following are the different algebraic expressions of the production function. Identify whether each one has increasing, constant or decreasing returns to scale.

1. $Q = 5K + 7L \Rightarrow$ Constant Returns to Scale
2. $Q = 5K^2 + 7L^2 \Rightarrow$ Increasing Returns to Scale
3. $Q = K^{0.3}L^{0.2} \Rightarrow$ Decreasing Returns to Scale

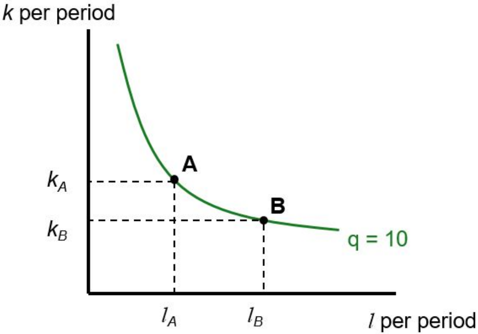
Isoquants

- Often it is possible to substitute one input for another.

Isoquants

An isoquant shows those combinations of K and L that can produce a given level of output (Q_0)

$$f(K, L) = Q_0$$



Practice Problems

Sketch the isoquants for the following production functions:

1. $$f(L,K) = \alpha K + \beta L$$

2. $$f(L,K) = \min\{\alpha K, \beta L\} \quad \alpha, \beta > 0$$

3. $$f(L,K) = L^{\alpha} K^{\beta} \quad \alpha + \beta = 1$$

Marginal Rate of Technical Substitution

- The slope of the isoquant shows how one input can be substituted for another while holding the output constant.

Marginal Rate of Technical Substitution(MRTS)

MRTS of labor for capital shows the rate at which labor can be substituted for capital while holding the output constant.

$$MRTS_{LK} = -\left.\frac{dK}{dL}\right|_{Q=Q_0}$$

- $MRTS_{LK} > 0$ always.
- $MRTS_{LK}$ is also given by the ratio $\frac{MP_L}{MP_K}$. Can you prove this?
- $MRTS_{LK}$ is decreasing in L. Is law of diminishing marginal productivity sufficient to ensure this?

Practice Problems

Calculate the $MRTS_{LK}$ for the following production functions:

1. $$f(L,K) = \alpha K + \beta L$$

2. $$f(L,K) = \min\{\alpha K, \beta L\} \quad \alpha, \beta > 0$$

3. $$f(L,K) = L^{\alpha} K^{\beta} \quad \alpha + \beta = 1$$

Costs in the Long Run

Long Run

Time frame in which the quantity of ALL resources used in production is variable.

- By definition, all the costs becomes variable and there are zero fixed costs.
- In the long run, the firm adjusts all its inputs so that its cost of production is as low as possible.

Isocosts

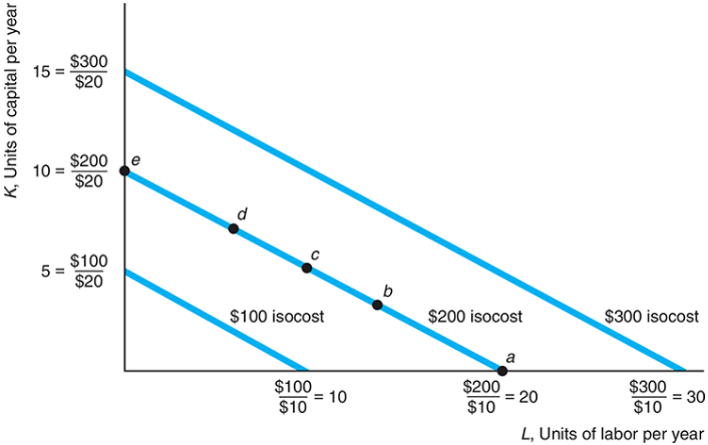
Isocosts

An isocost represents all the combinations of inputs that have the same total cost $\overline{C}$.

Properties of Isocosts

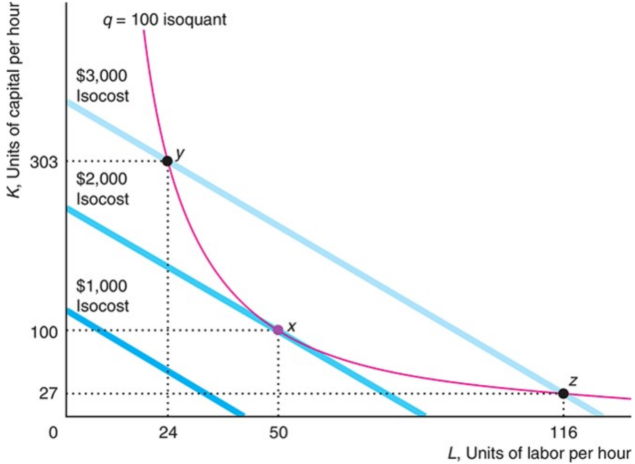
- The points at which the isocost lines hit the capital and labor axes depends on the firm's cost, and on the input prices.
- Isocost lines that are farther from the origin have higher costs than those closer to the origin.
- The slope of each isocost line is the same: $\frac{\Delta K}{\Delta L} = -\frac{w}{r}$, the rate at which the firm can trade capital for labor in input markets.

A Family of Isocost Lines



Combining production and costs

The firm minimizes its cost by using the combination of inputs on the isoquant that is on the lowest isocost line that touches the isoquant.



Cost Minimization

There are three equivalent rules to minimize costs in the long run:

1. The Lowest Isocost Rule The firm minimizes its cost by using the combination of inputs on the isoquant that is on the lowest isocost line that touches the isoquant.

2. The Tangency Rule
$$MRTS_{LK} = -\frac{w}{r}$$
At the minimum-cost bundle, x, the isoquant is tangent to the isocost line. The slope of the isoquant ($MRTS_{LK}$) and the slope of the isocost are equal.

3. The Last-Dollar Rule
$$\frac{MP_L}{w} = \frac{MP_K}{r}$$
Cost is minimized if inputs are chosen so that the last dollar spent on labor adds as much extra output as the last dollar spent on capital.

Practice Problem

Consider

$$Q = L^{0.5}K^{0.5}$$

What is the lowest cost of producing 100 units when

1. the price of capital is \$ 10 per unit and price of labor is \$10 per unit.
2. the price of capital is \$ 10 per unit and price of labor is \$5 per unit.
3. the price of capital is \$ 3 per unit and price of labor is \$12 per unit.

Long Run Costs

- Just like before, 3 concepts are used to explain the relationship between costs and output:
  1. **Long Run Total cost** (LRTC) = costs of producing a given level of output (depends on the amount of inputs used).
  2. **Long Run Marginal cost** (LRMC) = Change in costs from producing one more unit of output.
  3. **Long Run Average cost** (LRAC) = Costs per unit of output produced.
- Just like in short run, in the total costs are increasing, MC and AC are U-shaped.

Economies and Diseconomies of Scale

- **economies of scale** - falling long-run average cost as output increases.
  - Arises with Increasing Returns to Scale
  - Arises due to specialization of labor and capital
- **diseconomies of scale** - rising long-run average cost as output increases.
  - Arises with Decreasing Returns to Scale
  - Arises due to coordination problems and rising input costs
- **Constant economies of scale** - constant average cost as output increases.
  - Arises with Constant Returns to Scale

Short-Run Cost and Long-Run Cost

For each plant size, **diminishing marginal product of labor** creates a set of short run, U-shaped costs curves for MC, AVC, and ATC

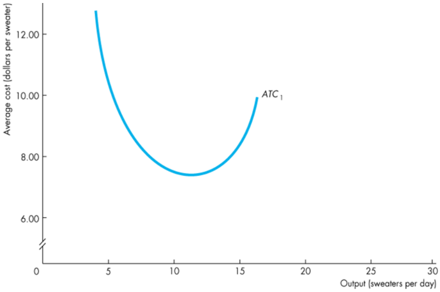


Figure: ATC curve for a plant with 1 knitting machine

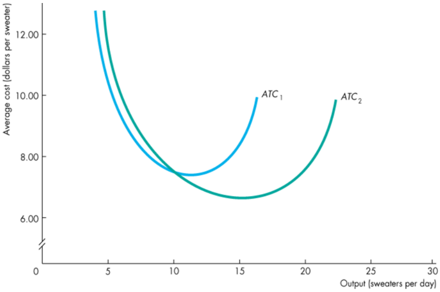


Figure: ATC curve for a plant with 2 knitting machines

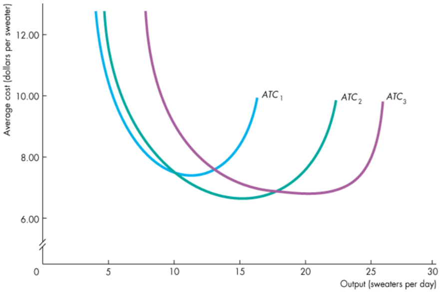


Figure: ATC curve for a plant with 3 knitting machines

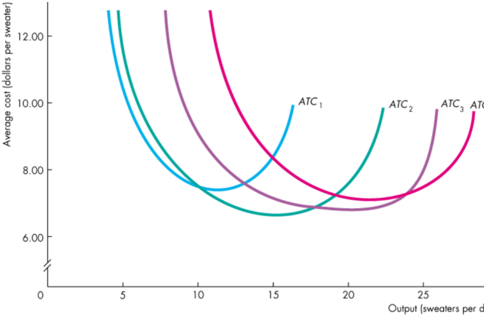
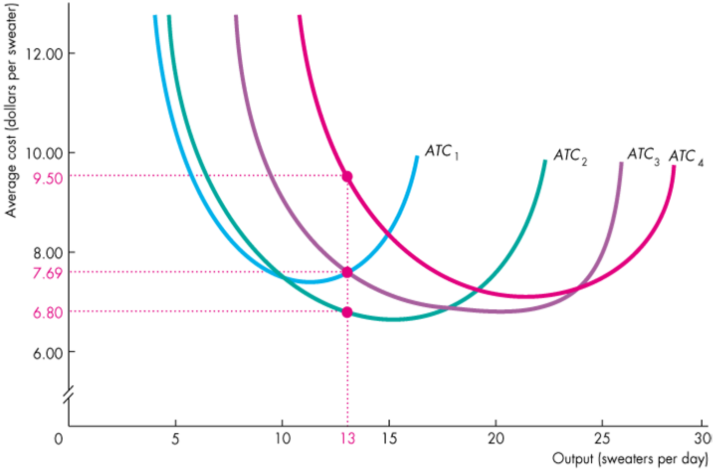


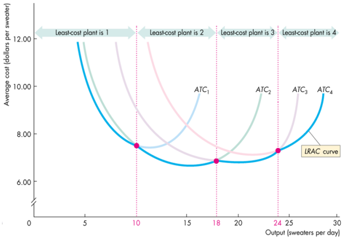
Figure: ATC curve for a plant with 4 knitting machines

Economically Efficient Plant

In the long run, a firm uses an economically efficient plant for producing given output ⇒ plant has lowest ATC



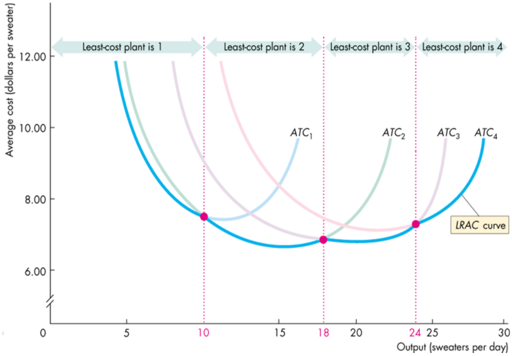
Long-run average cost curve



Long-run average cost curve

Relationship between the lowest attainable average total cost and output when both the plant size and labor are varied.

- It is the Planning curve- tells the cost minimizing combination of labor and capital for each level of output.
- For a plant to be economically efficient for a given level of output, it is not necessary that it operates at the lowest average cost in the short run.



Minimum Efficient Scale

The smallest quantity that corresponds to the lowest LRAC.

- Means that economies of scale are fully exploited.
- Useful in determining how many firms we expect to see in an industry.

Example

As part of its promotional efforts, PetSmart produces 100 small-scale promotional signs per month at each of its 75 retail stores. On average, monthly production costs are estimated to be $5,000 per machine at each location - $1,000 for installation, $3,000 for printing, and $1,000 for maintenance. Production costs company-wide total approximately $375,000 per month.

- Can the retailer benefit by consolidating this operation?

Economies and Diseconomies of Scope

Economies of Scope

If the joint cost of producing two products jointly is less than the cost of producing those two products separately.

Complements in production often display economies of scope.

Diseconomies of Scope

If the joint cost of producing two products jointly is more than the cost of producing those two products separately.

Example: Mergers

A multi-concept restaurant incorporates two or more restaurants, typically chains, under one roof. Sharing facilities reduces costs of both real estate and labor. The multi-concept restaurants typically offer a limited menu, compared with full-sized, stand-alone restaurants. For example, KMAC operates a combination Kentucky Fried Chicken (KFC)/Taco Bell restaurant. The food preparation areas are separate, but orders are taken at shared point-of-sale (POS) stations. If Taco Bell and KFC share facilities, *ceteris paribus* they reduce fixed costs by 30%.

- What do this imply for the decision of whether to open a shared facility versus two separate facilities?
- Further consider that sales in joint facilities are 20% lower than sales in two separate facilities. How would this alter your last decision?

Example: Acquisition

Founded in 1906, three entrepreneurs started a battery production company, later known as 'Rayovac' that grew to rival Energizer and Duracell. In 1996, a private equity firm - Thomas H. Lee Company acquired 'Rayovac'. Sequentially, taking advantage of easy credit availability, the company then bought many other battery production companies as well. The company justified these acquisitions by saying that it would allow them to take advantage of efficiencies and economies of scale. The company also bought many unrelated companies at the same time as the battery binge ? the reasoning being that because of synergies, if they centralized the production of many different goods the costs of production would be lower. By February 2009 the new conglomerate was bankrupt

- What things could have gone wrong here?

Horizontal Integration

The acquisition of additional business activities that are at the same level of the supply chain in similar or different industries.

- Cost considerations play an important role in this decision.
- For example: In May 2010, Astellas Pharma Inc. said Monday it will buy OSI Pharmaceuticals Inc. for $4 billion in an all-cash deal, the latest Japanese drug maker to pay a hefty premium for products to boost flagging profits.
- For example: In 2012, Volkswagon acquired Porsche.

Check out this video.

Vertical Integration

The acquisition of additional business activities that are at either higher or lower level of the value chain in the industry.

In March 2013, Starbucks Corp. has bought its first farm, with plans to use the 600-acre property in Costa Rica to develop new coffee varieties and test methods to eradicate a fungal disease known as coffee rust that is vexing the industry.