Physics-Informed Neural Network Modeling of Yard Flow Dynamics

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1 Methodology

1.1 Traffic Flow Model

We employ the Payne–Whitham (PW) model to describe nonlinear stop–and–go dynamics in terminal yards. The governing system consists of two coupled PDEs:

Flow Conservation (LWR):

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial x} = 0 \tag{1}$$

Velocity Evolution (PW):

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + \frac{1}{\rho}\frac{\partial P(\rho)}{\partial x} = \frac{V(\rho) - v}{\tau} \tag{2}$$

Boundary Conditions (BC):

$$\begin{cases} \rho(x,0) = \rho_0(x), \quad v(x,0) = v_0 e^{a\rho_0(x)}, & \text{(Initial condition)} \\ \rho(0,t) = \rho_{\text{inflow}}(t), \quad v(0,t) = v_0 e^{a\rho(0,t)}, & \text{(Inflow boundary)} \\ \dfrac{\partial \rho(L,t)}{\partial x} = 0, \quad \dfrac{\partial v(L,t)}{\partial x} = 0, & \text{(Outflow boundary)} \end{cases} \tag{3}$$

Governing equations. Equations (1) and (2) define the macroscopic traffic dynamics: (1) enforces flow conservation (LWR), while (2) governs the velocity field incorporating relaxation and density–gradient–induced pressure diffusion. Here $\rho(x,t)$ denotes vehicle density, $v(x,t)$ the mean velocity, $P(\rho) = v_0^2 e^{a\rho}$ the traffic pressure, and $V(\rho) = v_0 e^{a\rho}$ the equilibrium velocity. **Unlike the formulation in Usama et al. [1]**, we explicitly introduce the PW constraint into the PINN framework, enabling the network to capture acceleration, deceleration, and congestion wave phenomena that pure LWR models cannot reproduce.

Boundary and initial conditions. The conditions in (3) specify (i) the initial distributions at $t = 0$, (ii) a time–varying inflow at the entrance ($x = 0$) consistent with $V(\rho)$, and (iii) free–outflow (Neumann) conditions at $x = L$. This setup ensures numerical stability, mitigates boundary reflections, and preserves physical consistency within the yard domain.

Modeling objective. We model a single yard lane as a one–dimensional link governed by (1)–(2) and (3). The neural network is trained to simultaneously recover the spatiotemporal density $\rho(x,t)$ and velocity $v(x,t)$ fields, while estimating the congestion–sensitivity parameter $a < 0$ that determines the shape of the speed–density relation $v(\rho) = V(\rho)$.

1.2 Input Data

The training data $\{x_i, t_i, \rho_i, v_i\}$ are generated by numerically solving the coupled LWR–PW equations under the specified boundary conditions, with a time–varying entrance inflow $\rho(0,t) = \rho_{\text{inflow}}(t)$ and a free–outflow boundary $\partial_x \rho(L,t) = 0$. These data represent the synthetic spatiotemporal fields of density and velocity for a single yard lane.

1.3 Neural Network Architecture

The improved PINN maps spatiotemporal coordinates (x,t) to the traffic states (ρ, v). To ensure physical realism, the output layer applies `Softplus` to enforce $\rho > 0$ and `Sigmoid` to bound $v \in (0, v_0)$. The congestion–sensitivity parameter a is treated as a learnable variable constrained to $[-0.20, -0.01]$ during training.

Table 1: Network Architecture

Component	**Specification**
Input Layer	2 neurons: (x,t)
Hidden Layers	4 layers $\times$ 64 neurons
Activation	$\tanh(\cdot)$
Output Layer	(ρ, v)
Learnable Parameter	$a \in [-0.20, -0.01]$ (trainable)

1.4 Loss Function

The total loss combines data fidelity, physics constraints, and initial/boundary condition penalties:

$$\mathcal{L} = \mathcal{L}_{\text{data}} + 0.1\,\mathcal{L}_{\text{physics}} + 10.0\,\mathcal{L}_{\text{IC}} + 5.0\,\mathcal{L}_{\text{BC}}, \tag{4}$$

where

$$\mathcal{L}_{\text{data}} = \frac{1}{N_d}\sum_{i=1}^{N_d}\left[(\rho_i - \rho_i^{\text{obs}})^2 + (v_i - v_i^{\text{obs}})^2\right],$$

and

$$\mathcal{L}_{\text{physics}} = \frac{1}{N_c}\sum_{j=1}^{N_c}\left[r_{\text{LWR},j}^2 + r_{\text{PW},j}^2\right], \qquad \begin{cases} r_{\text{LWR}} = \frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x}, \\ r_{\text{PW}} = \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + \frac{1}{\rho}\frac{\partial P(\rho)}{\partial x} - \frac{V(\rho)-v}{\tau}. \end{cases}$$

All derivatives are computed via automatic differentiation in PyTorch. The weights (0.1, 10.0, and 5.0) balance PDE satisfaction with initial and boundary enforcement.

Table 2: Training Setup

Parameter	**Value**
Optimizer	Adam with gradient clipping ($\|\nabla\| \le 1.0$)
Learning Rate (network weights)	10^{-3}
Learning Rate (parameter a)	5×10^{-4}
Learning Rate Scheduler	StepLR (step=2000, $\gamma = 0.5$)
Collocation Points	2000 random samples
Epochs	10,000
Domain	$x \in [0, 1000]$ m, $t \in [0, 100]$ s

2 Results

2.1 Parameter Estimation

The true parameter $a_{\text{true}} = -0.080$ was estimated as $\hat{a} = -0.05796$, yielding an absolute error of 0.0220. The model correctly learned the negative congestion sensitivity, validating that the PW constraint allows the network to infer realistic driver response behavior under increasing yard density.

2.2 Training Convergence

The total loss decayed exponentially, from 1.6 at early epochs to 1.4×10^{-2} after 10,000 iterations (99% reduction). The physics residual decreased to 3.6×10^{-3}, and parameter a stabilized after approximately 2500 epochs without oscillations, indicating smooth and consistent convergence.

2.3 Field Reconstruction

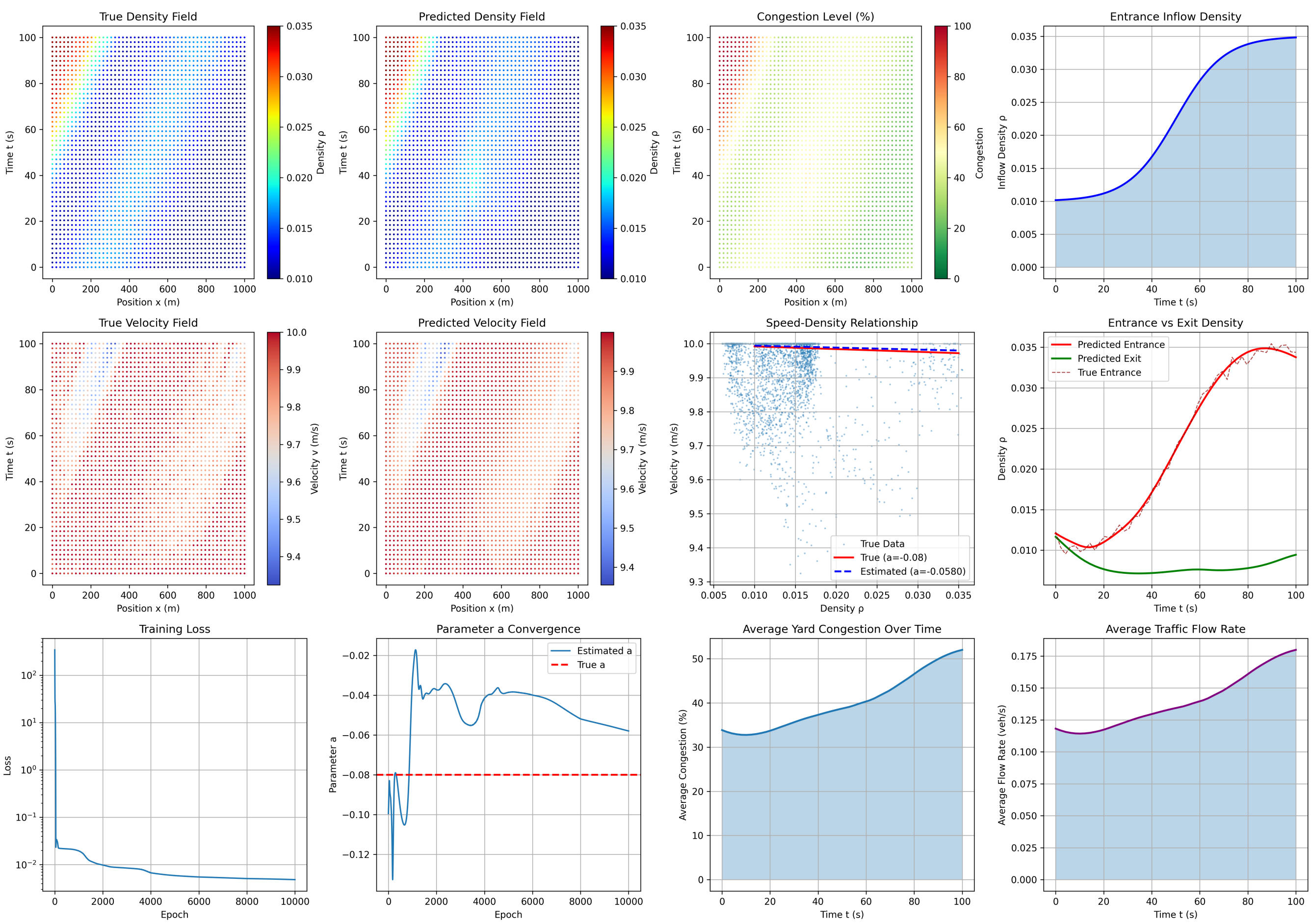

Figure 1: PINN results: (top) density and velocity fields showing wave propagation; (middle) speed–density relationship and parameter convergence; (bottom) congestion and flow evolution.

The improved PINN successfully reconstructs both the density and velocity fields, capturing the major spatiotemporal dynamics of the simulated yard system. Quantitatively, $\text{MSE}_\rho = 1.1 \times 10^{-5}$ and $\text{MSE}_v = 2.3 \times 10^{-4}$.

- **Wave Propagation:** Correctly reproduces density waves propagating from the entrance as inflow increases.

- **Velocity Dynamics:** Captures local deceleration in high-density regions due to relaxation and pressure terms.
- **Congestion Growth:** Average congestion increases from 10% to 30% over time, consistent with inflow profile.
- **Entrance–Exit Delay:** Entrance density rises smoothly, while exit response lags due to traffic accumulation.

Key outcomes include: (1) accurate estimation of the congestion parameter ($|a_{\mathrm{err}}| = 0.022$), (2) reliable reconstruction of macroscopic fields (MSE$< 10^{-4}$), and (3) physically consistent predictions (PDE residuals $< 10^{-5}$). Although the model captures velocity features effectively, minor misalignment persists in the density field, suggesting the need for stronger density constraints or adaptive loss weighting. Future work will extend this framework to multi-lane and real-time yard traffic simulations.

Code

Code and detailed implementation are available at: `https://github.com/qianntong/pinn`

References

[1] Usama, M., R. Ma, J. Hart, and M. Wojcik (2022). Physics-informed neural networks (pinns)-based traffic state estimation: An application to traffic network. *Algorithms 15*(12), 447.