SOFR So Good? New Benchmark Interest Rate and Crowding-Out Effect

Qian Wu*

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Abstract

I study the scarce collateral channel through which the government debt can result in an additional crowding-out effect on both asset prices and macroeconomic variables under the SOFR regime. The collateral channel suggests that a higher supply of Treasuries reduces their scarcity value, which drives up Treasury holder's borrowing cost on the Repo market. That is, SOFR increases. I first provide empirical evidence of an additional rise in SOFR in response to an increase in government debt. To proceed, I borrow the structure of agents' problems and markets set up from Jermann (2019) and then incorporate a no-arbitrage condition in the spirit of Nyborg (2019). In LIBOR economy, business coupon rate is determined by the interbank borrowing cost, while in SOFR economy, it is determined by outside no-arbitrage condition on liquidity markets. The scarce collateral channel enables public debt to affect the real economy even without distortionary taxes, although quantitative analysis shows that in general equilibrium, this effect is very small. The findings from this paper abate the worries about extra crowding-out effect from the benchmark rate transition.

Keywords: scarcity value, bank risk, SOFR, LIBOR, public debt
JEL Codes: G21, G28, H63, E32

*Indiana University

1 Introduction

The London Interbank Offered Rate (LIBOR), a widely used benchmark interest rate for a large variety of financial assets, is being retired due to manipulation scandals during the financial crisis and a shrinking interbank lending market. [1] This pushed a transition from the current benchmark interest rate, LIBOR, to an alternative benchmark interest rate. The transition can induce large effect in economies that used to adopt LIBOR, as the market size of assets indexed by LIBOR is very large. In the US, among all cash products, business loans have the highest exposure to USD LIBOR. As of the end of 2016, there were $3.4 trilliion outstanding business loans indexed by USD LIBOR. [2] The critical link between financial institutions and real sector firms causes an impending demand to understand the possible real economic effect from this transition in benchmark interest rate.

The Secured Overnight Financing Rate (SOFR) was announced by the ARRC as the recommended USD LIBOR replacement. Unlike LIBOR, which is a rate based on leading banks' estimated borrowing cost on unsecured interbnk market, SOFR is an overall measure of borrowing cost on the Treasury Repo market. Transactions on Treasury Repo market utilize Treasury securities as a collateral for borrowing and lending. As closely connected with the Treasury security market, SOFR can be highly sensitive to government's financing activities, especially under the case of scarce collateral. LIBOR does not have this property. SOFR has been selected as the alternative to LIBOR due to its robust determination, it remains a question to what extent the government borrowing activity could volatilize SOFR through the collateral channel and what the macroeconomic effect might be.

In this paper, I am considering the collateral channel through which the government debt may result in additional crowding-out effect on the real economy. Intuitively, the collateral channel works as follows. Suppose that Treasury securities as collateral are relatively scarce, then Treasury holders have an advantage of collateralized borrowing on the Repo market. The scarcity value allows Treasury holders to borrow at lower rate. A higher supply of Treasury reduces the scarcity value of government bond on the Repo market, which drives up Treasury holder's borrowing cost on the Repo market, that is, SOFR increases.

The scarcity value of Treasury as collateral has been studied by empirical literature. D'Amico et al. (2018) estimate a positive and significant scarcity premium for specific collateral Treasury Repo. They show that the scarcity premium is persistent and larger for

[1] The publication for one-week and two-month USD LIBOR ceased on December 31, 2021; The publication for the USD LIBOR with other terms ceased on June 30, 2023.

[2] See the second report of the alternative reference rate committee (ARRC (2018)).

shorter-term securities. Infante et al. (2020) study the re-use of Treasury securities in the financial system. They use supervisory data to estimate a negative association between Treasury re-use and supply of securities. Arrata et al. (2020) focuse on the euro area. In particular, they investigate the scarcity effect of the Public Sector Purchase Programme (PSPP) - the European version of quantitative easing - on Repo rates. Due to the specialness of government bonds, a quantitative easing decreases the supply on the market and makes bonds scarcer. It follows that Repo rate drops below the ECB deposit facility rate. This thread of literature empirically examines the effect of scarce collateral on Repo rate, but does not touch the macroeconomic side.

Another relevant thread of literature studies the discrepancy between LIBOR and SOFR in terms of bank funding shock insuring feature. LIBOR closely track leading banks' unsecured borrowing cost, so it provides a partial insurance against banks' funding cost shock. SOFR does not reflect banks' borrowing cost, therefore it has no insuring feature. Jermann (2019) develops a stylized model to compare the economic scenarios under the two benchmark interest rates. A negative funding cost shock (higher funding cost) results in higher bank default rate in the SOFR economy than that in the LIBOR economy. Therefore, the investment cut is larger in the SOFR economy than that in the LIBOR economy.

To verify the existence of a scarce collateral channel and analyze the macroeconomic effect of this channel, I first provide empirical evidence of an additional rise in SOFR in response to an increase of government debt. The OLS result reveals a significant and positive correlation between SOFR and government debt outstanding. To complement, Local Projection is used to estimate empirically the impulse responses of SOFR and LIBOR, respectively, to a random variation in borrowing. Consistently, the Local Projection result indicates a persistent additional response from SOFR relative to LIBOR. I proceed to construct a theoretical model, with the main structure of agents problems and markets set up borrowed from Jermann (2019). The model then incorporates a no-arbitrage condition in the spirit of Nyborg (2019) to derive the equilibrium SOFR. The model focuses on the floating rate long-term business loans, with which firms finance investment. Banks supply (demand) interbank lending (borrowing), with an endogenous probability to default on their interbank debt position. In LIBOR economy, business coupon rate is determined by the interbank borrowing cost, while in SOFR economy, it is determined by a no-arbitrage condition on liquidity markets. The scarce collateral channel enables public debt to affect real economy even without distortionary taxes, although quantitative analysis shows that this effect is small.

The paper proceeds as follows. Section 2 describes institutional background of Treasuries Repo market and the associated scarcity value of Treasuries as collateral. In Section 3 , I present empirical evidence that verifies the existence of scarce collateral channel for SOFR. In Section 4 , I discuss the theoretical model. I begin with the model set up, then move to calibration and quantitative implications in Section 5 . Finally, concluding remarks are offered in Section 6 .

2 Scarcity Value of Treasury Collateral

The Repo market allows asset holders to borrow cash against collateral. As the underlying marekt of SOFR, Treasury overnight Repo market provides secured overnight funding to Treasury holders. An overnight repo transaction involves two counterparties: cash lender lends out cash against the collateral at day t, at an interest rate r to the cash borrower who holds the collateral. At day $t+1$, the trade is reversed. the cash lender receives principal and interest payment, and cash borrower recovers the position of the collateral.

As a highly risk-free and liquid asset, Treasuries have been accounting for a huge share of of the collateral posted in the US Repo market.[3] This is even more so since the financial crisis. [4] Despite the risk-free feature of repo contracts, the repo rate is documented to subject to demand and supply factors. This paper specifically investigates the scarce collateral effect of repo rates. Treasuries as scarce collateral will result in a "scarcity premium" which markets the Treasury holders to borrow at a lower rate on the Repo market.[5] This "scarcity premium" have potentially important implications for the government financing policies. A higher supply of Treasuries on the market decreases the scarcity value, which makes Treasury holders borrow at a higher rate on the Repo market. The higher government debt level can affect not only the asset prices, but also real sectors, given that SOFR is supposed to serve as benchmark interest rate for many loans including business loans and floating-rate mortgages.

The transactions underlying SOFR comprises two segments: bilateral Treasury Repo and tri-party Treasury Repo. In a bilateral Repo, the settlement is handled directly by the trading parties rather than by a third-party clearing bank as in a tri-party repo. Tri-party transactions are secured by General Collateral (GC) pools of accepted Treasury securities, any of which can be delivered as collateral by the cash borrower. Unlike tri-pary transaction,

[3]See Copeland et al. (2014).

[4]See Gorton et al. (2012).

[5]Scarce Treasuries as collateral as been empirically studies by D'Amico, Fan, and Kitsul (2018) and Infante and Saravay (2020).

bilateral transactions feature Specific Collateral (SC) as lenders and borrowers can designate specific securities as collateral. It follows that the incentive for lenders entering the bilateral repo market can be to seek a specific security. Therefore, the collateral scarcity premium is more likely to happen in the bilateral Repo market, as most transactions in the bilateral market feature SC. Figure 1 plots shares of the GC and SC Repo transactions by time. In

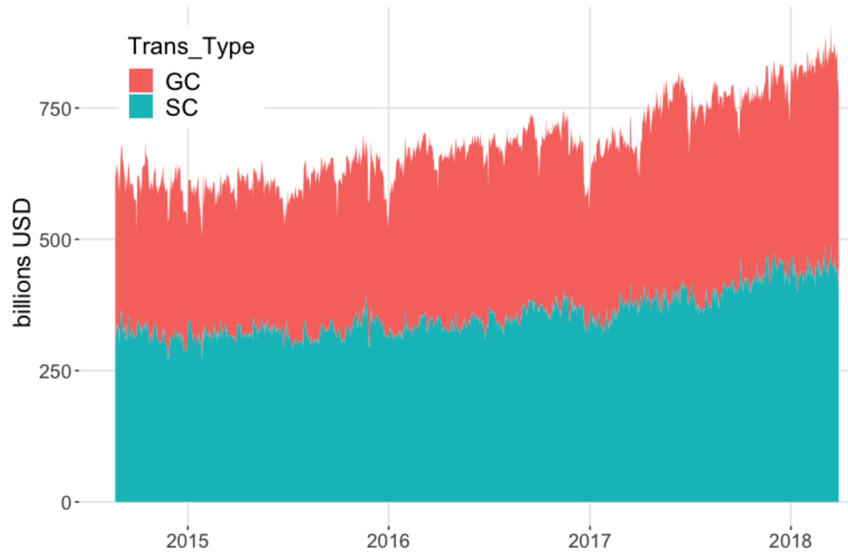

Figure 1. Volume of General Collateral (GC) and Specific Collateral (SC) transactions on Treasury overnight Repo market. *Note:* Data are from Federal Reserve Bank of New York (New York Fed).

almost all periods, SC transactions make up over a half of the Repo market. Due to the significant share of SC transactions underlying SOFR, the scarcity value of Treasuries implies that government's financing policy changing federal debt level may induce non-negligible consequence on the real economy through the collateral channel of the Treasury Repo market.

When the volume of outstanding Treasuries is larger, the Treasuries as collateral become less scarce, so the scarcity value reduces. Defining the scarcity premium as the spread between bilateral and tri-party Repo rates,[6] intuitively when government debt supply increases, scarcity premium should increase. This is due to the fact that a larger supply alleviates the scarcity of Treasuries as collateral, Repo cash borrowers then are forced to borrow at a higher rate, especially in SC transactions. To verify the scarcity value of Treasuries, Table 1 presents the results when regressing the scarcity premium with respect to the change in the Treasuries outstanding.Two measures of available government securities are employed:

[6]The spread between bilateral and tri-party rates is rigorously minus of scarcity premium, as a higher bilateral rate implies less scarcity value.

	Dependent variable:			
	SC repo rate - GC repo rate			
	(1)	(2)	(3)	(4)
Δlog debt	$1,259.139^{***}$	$1,233.700^{***}$		
	(158.963)	(160.411)		
Δlog Treasuries			738.579^{***}	731.915^{***}
			(145.558)	(145.273)
Δlog GC volume		-0.471		1.857
		(4.049)		(4.120)
Δlog SC volume		5.228		7.492^{**}
		(3.270)		(3.322)
Constant	-0.121	-0.098	-0.020	0.011
	(0.143)	(0.144)	(0.145)	(0.145)
Observations	768	768	768	768
R^2	0.076	0.079	0.033	0.039
Adjusted R^2	0.075	0.075	0.031	0.036

Table 1: Scarcity value of Treasuries on repo market

Δlog debt is the change rate of federal government debt held by public. Data are from FiscalData database. Δlog Treasuries is the change rate of tradable Treasuries outstanding. This variable is computed based on Treasury auction data using Kingler and Syrstad (2021) method. Δlog GC and Δlog SC are change rates of GC and SC transaction volume, respectively. Data are from New York Fed. The dependent variable is the scarcity premium defined as the spread between GC and SC rates. Data are from New York Fed. Estimation frequency is daily. Data covers the period from 14-August to 18-March. *Note:* *p<0.1; **p<0.05; ***p<0.01. Robust standard errors in parenthesis.

the government debt outstanding held by public and the marketable Treasuries outstanding. Results indicate that GC Repo rate rises relative to SC Repo rate in response to a higher supply of government securities, which is consistent with the intuition.

The scarcity value of Treasuries on Repo market implies an additional collateral channel for government financing policy to affect the real economy under SOFR regime. In an economy with LIBOR as the benchmark interest rate, such a collateral channel does not exist, because LIBOR is not determined by transactions on the Treasury Repo market. In fact, LIBOR, as an estimated cost of interbank borrowing, is not based on any real market transactions. That is one major reason why LIBOR is vulnerable to manipulation. The transition in benchmark interest rate thus may give rise to additional crowding-out effect from government debt through this collateral channel.

3 Empirical Evidence

In this section, I present empirical evidence indicating additional response from SOFR when there is a change in the supply of public borrowing. The conceptual framework and the empirical evidence in section 2 lead to the following prediction: Compared with LIBOR, an increase in the supply of US Treasuries causes additional rise in SOFR as a result of scarcity value of Treasuries. LIBOR has no such a collateral channel because it is not based on transactions on Treasury Repo market. To test this prediction, a series of empirical experiments have been conducted, with a purpose to verify the extra crowding-out effect of government debt under the SOFR regime through the scarce collateral channel.

3.1 OLS

The empirical analysis begins with simple OLS regressions in order to get a sense of the correlation between benchmark rates and government debt. Note that this paper centers on the additional collateral channel of government debt crowding out effect, without touching other standard channels through which government debt affects real economy. Figure 2 plots sequences of daily SOFR and USD LIBOR. SOFR data are taken from the New York Fed, while USD LIBOR data are taken from St. Louis Fed. Spikes happen to SOFR and LIBOR in the end and beginning of each month, corresponding to reporting dates. To remove fluctuations in SOFR and LIBOR due to policy changes, I subtract Effective Federal Funds Rate (FFR) from the two benchmark rates. Besides, to eliminate the ”window dressing” effect from reporting regulations and larger outliers, the last, second-last, and first trading day of each month as well as five days after September 15, are removed from the sample.[7] Since the sample is a time series, I take change rate for volume variables to avoid the possible instationarity. The baseline specification is to regress the SOFR or LIBOR spread from FFR with respect to the change in supply of Treasuries. To get rid of the effect from auto-correlation, I add lagged rates as control variables.

Two measures are used to quantify the supply of Treasuries, with the first being the government debt held by public outstanding. The government debt data is taken from the FiscalData database [8] created by the Department of the Treasury and the Bureau of the Fiscal Service. Alternatively, I also borrow from Kingler and Syrstad (2021) to compute a measure of outstanding marketable Treasury securities based on Treasury auction data,

[7] This strategy follows Kingler and Syrstad (2021).

[8] `https://fiscaldata.treasury.gov/`

with the auction data taken from the TreasuryDirect database [9]. The data frequency is daily. For government debt measure, the sample covers the period between 14-August and 21-October, while for Treasuries outstanding measure, the sample covers a shorter period between 14-August and 19-December.

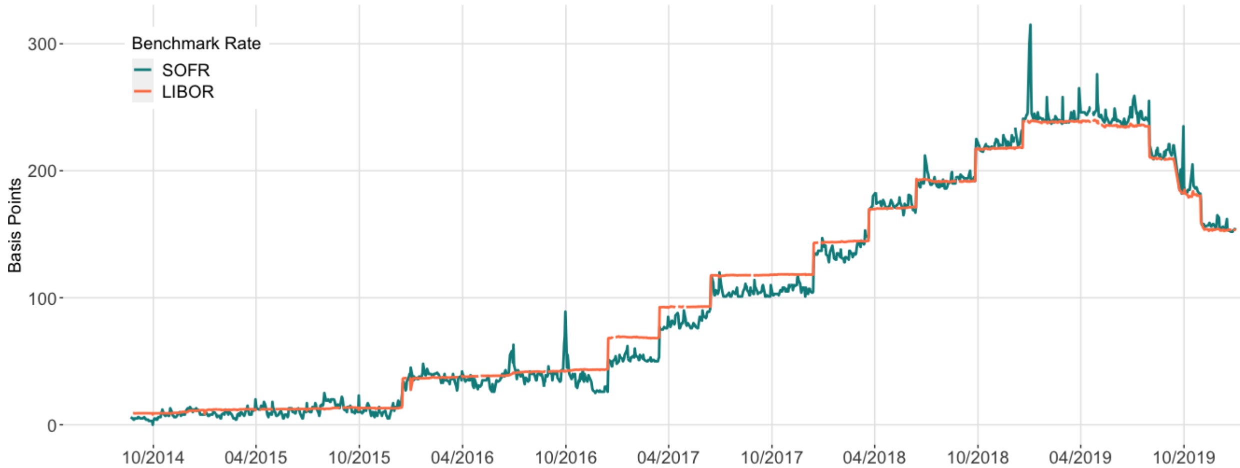

Figure 2. SOFR and LIBOR over time. *Note:* SOFR data are from New York Fed, USD LIBOR data are from FRED database.

Table 2 summarizes the regression results. The two panels correspond to specifications with government debt and Treasuries outstanding as the measure of public borrowing, respectively. In each panel, four regression equations are evaluated. Column (1) and (2) investigate SOFR's change while column (3) and (4) investigate LIBOR's change, in response to a variation in public borrowing. Confirming the prediction, in both panels SOFR spread obtains positive and significant coefficient of government borrowing, relative to LIBOR spread. A one-percentage increase in public borrowing is estimated to result in roughly 10 bps increase in SOFR net of policy effect, with Treasuries outstanding as the measure of public borrowing. This effect shrinks to roughly 4 bps if with government debt as the measure for public borrowing. Interestingly, LIBOR exhibits a negative response to a higher level of public borrowing, although the magnitude is very small. When using government debt as explaining variable, the additional response from SOFR is not as large as when using Treasuries outstanding, simply because government debt is a more noisy measure. It includes not only marketable Treasury securities (Treasury bills, notes, bonds and Treasury Inflation-Protected Securities (TIPS)), but also the non-marketable portion (Floating Rate

[9] `https://www.treasurydirect.gov/`

Table 2. Additional rise in SOFR in response to a higher level of public borrowing. *Note:* *p<0.1; **p<0.05; ***p<0.01. Robust standard errors in parenthesis.

Panel A: Gov't debt outstanding as the measure of borrowing

	Dependent variable:			
	SOFR		LIBOR	
	(1)	(2)	(3)	(4)
Δlog debt	608.277***	604.886***	−36.413*	−33.851*
	(87.712)	(86.476)	(18.708)	(18.599)
SOFR(-1)		0.174***		
		(0.029)		
LIBOR(-1)				−0.159***
				(0.030)
Constant	−0.277***	−0.241***	0.008	0.008
	(0.087)	(0.086)	(0.019)	(0.019)
Observations	1,134	1,131	1,102	1,083
R^2	0.041	0.071	0.003	0.029
Adjusted R^2	0.040	0.070	0.003	0.027
Residual Std. Error	2.911 (df = 1132)	2.869 (df = 1128)	0.616 (df = 1100)	0.612 (df = 1080)

Panel B: Treasuries outstanding as the measure of borrowing

	Dependent variable:			
	SOFR		LIBOR	
	(1)	(2)	(3)	(4)
Δlog Treasuries	995.000***	994.614***	−72.438***	−66.848***
	(90.566)	(88.833)	(19.771)	(19.647)
SOFR(-1)		0.177***		
		(0.028)		
LIBOR(-1)				−0.154***
				(0.030)
Constant	−0.358***	−0.314***	0.014	0.013
	(0.085)	(0.084)	(0.019)	(0.019)
Observations	1,134	1,129	1,100	1,079
R^2	0.096	0.129	0.012	0.036
Adjusted R^2	0.096	0.127	0.011	0.034
Residual Std. Error	2.826 (df = 1132)	2.770 (df = 1126)	0.613 (df = 1098)	0.608 (df = 1076)

Notes (FRNs), Domestic Series, Foreign Series, State and Local Government Series (SLGS), United States Savings Securities, and a portion of Government Account Series (GAS) securities).

3.2 Local Projection

OLS is a good start as it provides a simple framework to analyze the correlation between benchmark rates and government borrowing. However, OLS method is not capable of deriving implications about the persistence of the additional effect from the collateral channel. To show that the extra response in SOFR with respect to government borrowing is not only significantly large but also persistent, I conduct time series analysis using Local Projection with monthly data covering the period between 2014/08 and 2019/12. The identification strategy follows Bayer, Born, and Luetticke (2020). In the first stage, I identify the government borrowing shock using the following equation:

$$b_t = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + A(L)X_{t-1} + \epsilon_t^b,$$

where b_t denotes log(govt debt); X_t denotes controls including six lags of log(govt debt), log(industrial production index), and log(stock price) [10]; and $\hat{\epsilon}_t^b$ is adopted as the identified borrowing shock. The Industrial Production Index data are taken from FRED, and I use S&P 500 as the overall stock price measure. In the second stage, I estimate the following equation to generate the impulse response functions of SOFR and LIBOR to a standard deviation of government borrowing shock.

$$r_{t+h} = \beta_0 + \psi_h \hat{\epsilon}_t^b + \Gamma(L)Z_{t-1} + u_{t+h},$$

where r_{t+h} denotes SOFR or LIBOR spread at horizon h; Z_{t-1} denotes controls including three lags of SOFR or LIBOR spread, log(govt debt), and log(industrial production). The IRFs are given in graphs below. Note that SOFR spread exhibits a positive and persistent response to government borrowing shock for as long as 12 horizons (months) after the shock happens. A simple numerical analysis reveals that a 1-percentage increase in government debt outstanding, which is equivalent to a 0.75-percentage change in GDP [11], results in a 0.8-bps rise in SOFR right after the borrowing shock. This effect remains positive within the following 12 months and peaks at 3.3-bps in 8 months after the shock. On the other

[10] Identifying the government borrowing shock using twelve lags obtains similar result.

[11] In dollar value, this is roughly 146 billion.

hand, LIBOR spread responds ambiguously. This finding is consistent with my conceptual framework, with the key mechanism being that SOFR features an additional collateral channel while LIBOR does not. Two robustness checks are conducted. In the first alternative specification, I replace industrial production index with unemployment rate as a measure of output, the resulting IRFs are very similar. Also, in order to exclude the possible effect from price level (I already partially control the price effect by including stock price), CPI is employed as a control in the second alternative specification, and the results remain unchanged.

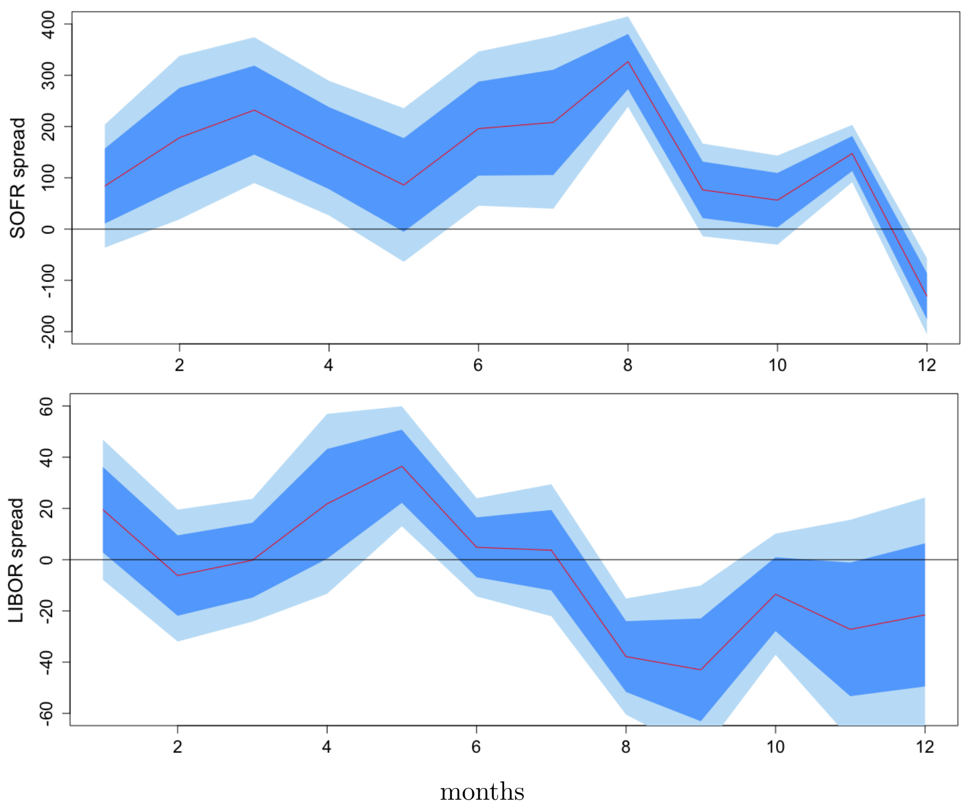

Figure 3. Impulse responses to a government borrowing shock. Solid red line represents mean of IRF. Dark blue band represents 90% confidence bounds and light blue band represents 68% confidence bounds.

Overall, this section provides empirical evidence showing that in economy with SOFR as the benchmark interest rate, public debt can induce additional crowding-out on real economy through the collateral channel from Treasury Repo market. It would be helpful to estimate IRFs of macroeconomic variables to the government borrowing shock and compare how they differ in SOFR and LIBOR economy. However, such a comparison is not possible at least at this point, since the transition is still ongoing and SOFR has not fully replaced LIBOR's role. In next section, I use a model to study SOFR and LIBOR regimes and draw implications about how the government debt crowding-out effect differs under the two regimes.

4 Model

The theoretical model is built based on Jermann (2019). The model is used to compare macroeconomic effects of the government borrowing shock under SOFR and LIBOR regimes. In those two regimes, SOFR and LIBOR serve as the index of the coupon payment of the floating rate business loan, respectively. Floating rate business loan is the only approach for firms to finance investment, and it is with coupons adjusted periodically to the current market conditions. In the LIBOR economy, since LIBOR is not connected with government bond, a government borrowing shock results in no real economic effects, assuming Recardian Equivalence holds by lump-sum tax. In the SOFR economy, no-arbitrage condition implies a close connection between SOFR and government bond. It follows that a higher government debt induces a higher borrower cost faced by firms, productive investment thus declines.

4.1 Banks

There are a continuum of banks. Each bank j issues multi-period floating rate business loan with amortization schedule fixed and coupons adjusted each period. A period in the model is one quarter. Denote the price of the business loan with q, coupon rate i, amortization rate λ, and business loan outstanding b_j, the value of bank j's business loan is given by

$$(i + \lambda + (1 - \lambda)q)b_j.$$

They finance themselves with one-period unsecured and defaultable interbank debt as well as equity. Conditional on not defaulting, banks issue new interbank debt and business loans. Banks suffer from aggregate and idiosyncratic funding cost shocks, which are multiplicative to the size of business loan. Denote bank j's interbank debt outstanding with d_j, price of interbank debt p, and the aggregate and idiosyncratic funding shocks Ψ and z_j, respectively, the dividend payout to shareholders is

$$[i + \lambda + (1 - \lambda)q]b_j - qb'_j - d_j + pd'_j(1 + \tau) - \Psi z_j b_j,$$

where the idiosyncratic funding shock $z_j \in [\underline{z}, \bar{z}]$ is iid across time and individuals and can be fully characterized by the CDF $\Phi(z)$ with $E(z) = 0$. For quantitative analysis, I follow Jermann (2019) to give a quadratic density function for z in the form of $\phi(z) = \omega z^2 + \eta$. I specify the aggregate funding shock to be $\Psi = \bar{\Psi}\psi$, where Ψ is a constant and $log\psi$ follows an AR(1) process. The aggregate funding shock affects volatility of funding cost, so it can be seen as a measure of bank risk. I follow Jermann (2019) to assume that defaulting banks will reconstruct within the period and next period they will face the same optimization problem. Bank j's problem is as follows:

$$\begin{aligned} V(b_j, d_j) = \max_{d'_j, b'_j} \Big\{ & [i + \lambda + (1 - \lambda)q]b_j - qb'_j - d_j + pd'_j(1 + \tau) - \Psi z_j b_j \\ & + EM' \int_{\underline{z}}^{z^{*'}(.)} V(b'_j, d'_j) d\Phi(z'_j) \Big\}. \end{aligned}$$

Banks default when their value is lower than the cost of funding, i.e., $V(b_j, d_j) < \Psi z_j b_j$. This implies a cutoff idiosyncratic funding shock z^* such that all banks receiving a smaller shock survive while all banks receiving a larger shock default. Focusing on symmetric equilibrium, I can drop the subscript j from now on. Banks' optimization conditions are:

$$p(1 + \tau) = EM' \int_{\underline{z}}^{z^{*'}(.)} d\Phi(z'), \tag{1}$$

$$q = EM' \int_{\underline{z}}^{z^{*'}(.)} [i' + \lambda + (1 - \lambda)q' - \Psi' z'] d\Phi(z'). \tag{2}$$

In symmetric equilibrium, since market clearing condition requires that all banks' interbank positions sum up to zero, the equilibrium implication is that

$$d' = 0 \tag{3}$$

4.2 Firms

Firms rely on floating rate business loans to finance their production. There is a representative firm whose productivity is affected by an exogenous TFP shock. Each period, for the business loan stock outstanding, firm pays back both the coupon and the amortization to banks. Therefore, the dividend payout to shareholders is

$$Af(k) - b(i + \lambda),$$

where A is TFP shock with $log(A)$ following an AR(1) process. Newly borrowed business loan is used as productive investment. Thus, capital accumulation follows the equation

$$k' = q(b' - (1 - \lambda)b) + (1 - \delta)k. \tag{4}$$

The representative firm's problem is given by

$$\begin{aligned}
&W(b, k) = \max_{b',k'} \left\{ Af(k) - b(i + \lambda) + EM'W(b', k') \right\} \\
&s.t. \\
&k' = q(b' - (1 - \lambda)b) + (1 - \delta)k.
\end{aligned}$$

The optimization conditions are

$$EM'[(i' + \lambda) + \mu' q'(1 - \lambda)] = \mu q, \tag{5}$$

$$EM'[A' f'(k') + \mu'(1 - \delta)] = \mu. \tag{6}$$

4.3 Government

Government issues lump-sum tax T and one-period risk free bond G' at the risk free rate r to finance the repayment of government bonds issued last period. In this economy, I assume that there is no government consumption. The government's budget constraint is

$$G' + T = (1 + r^G)G, \tag{7}$$

$$r^G = 1/EM' - 1. \tag{8}$$

The government bond is modeled as $G = \bar{G}g$ with $log(g)$ following an AR(1) process.

4.4 Equilibrium Consumption

The representative household receives dividend payout from firm and banks, purchase newly issued government bonds, get return from government bonds purchased last period, pay tax, and suffer default losses. The net income is given by

$$
\begin{aligned}
&Af(k) - b(i+\lambda) + \int_{\underline{z}}^{\bar{z}} \Big\{[i+\lambda+(1-\lambda)q]b - qb' - d + pd'(1+\tau) - \Psi zb\Big\} d\Phi(z) \\
&\qquad\qquad - (1-\Phi(z^*))(1-\kappa)b[i+\lambda+(1-\lambda)q] + (1+r)G - G' - T \\
&= Af(k) - (1-\Phi(z^*))(1-\kappa)b[i+\lambda+(1-\lambda)q] - q(b'-(1-\lambda)b),
\end{aligned}
$$

with the parameter κ controlling the recovery rate in the case of default. Note that the net output in the economy after accounting for loss from default is given by

$$
y = Af(k) - (1-\Phi(z^*))(1-\kappa)b[i+\lambda+(1-\lambda)q], \tag{9}
$$

and the investment is

$$
q(b'-(1-\lambda)b),
$$

so the recourse constraint is established as

$$
c = y - inv. \tag{10}
$$

The stochastic discount factor is

$$
M' = \beta \frac{u'(c')}{u'(c)}, \tag{11}
$$

where the utility function is CRRA with γ as the parameter of risk aversion.

4.5 LIBOR Regime

In the LIBOR economy, the coupon rate of business loan is indexed by LIBOR. LIBOR, as a measure of interbank borrowing cost, is assumed to represent the unsecured funding rate of interbank debt. Hence, under the LIBOR regime the business loan coupon rate is given by

$$
i' = i^L = 1/p - 1, \tag{12}
$$

where p, the price of interbank debt, is endogenously determined in the model.

4.6 SOFR Regime

An alternative regime is the SOFR regime. In the SOFR economy, the coupon rate of the business loan is indexed by SOFR, with SOFR measuring overall borrowing cost of transactions collateralized by government securities. In this model, SOFR is determined by the no-arbitrage condition from three financing markets, borrowed from Nyborg (2019).

Suppose that a borrower holding G units of government bond needs L units of liquidation. I assume that $G < L$ so that the collateralized borrowing constraint is binding. For a bond holder, there are three markets to finance: a government bond Repo market, a secondary government bond market, and an unsecured borrowing market.

Repo Market Let the price of government bond on the Repo market be normalized as 1. The collateralized borrowing rate on the Repo market is given by the SOFR rate i^S. Assuming no haircut of government bond as a collateral, then the borrower holding G units of government bonds can borrow up to G units of liquidation on the Repo market.

Secondary Market A bond holder can also finance liquidity through the government bond secondary market. Let $1 - \epsilon$ denote the price of government bond on the secondary market, where the term ϵ allows a discrepancy of valuation. I assume that the secondary market borrowing rate is the same as that of the primary market, that is, the government bond rate r^G.

Unsecured Market There exists an unsecured financing market as well, with the borrowing rate equal to the LIBOR i^L.

To satisfy the L units of liquidity requirement, consider the following two financing strategies:

Strategy 1 Borrow G units of liquidity on Repo market, then borrow the remaining $L - G$ units on unsecured market.

Strategy 2 Sell ω government bonds on secondary market to raise $\omega(1-\epsilon)$ units of liquidity, then borrow the remaining $L - \omega(1 - \epsilon)$ units on unsecured market. To achieve an interior solution of optimal fraction to refinance on secondary market, ω^*, I assume that the borrower only receives a noisy signal about the secondary market borrowing rate, $\tilde{r}^G \sim N(r^G, \sigma^2)$.

The cash flows created by these two strategies are summarized by Table.

Table 3. Cash flows of two strategies to finance liquidity.

		t	$t+1$
Strategy 1	Repo	G	$-G(1+i^S)$
	Unsecured	$L-G$	$-(L-G)(1+i^L)$
	Sum	L	$-[G(1+i^S)+(L-G)(1+i^L)]$
Strategy 2	Sell	$w(1-\epsilon)$	—
	Buy	—	$-w(1-\epsilon)(1+r^G)$
	Unsecured	$L-w(1-\epsilon)$	$-[L-w(1-\epsilon)](1+i^L)$
	Sum	L	$-[(1+i^L)L+w(1-\epsilon)(r^G-i^L)]$

To derive the expression for SOFR i^S, I first solve for the optimal amount to liquidize on secondary market ω^*. For a borrower with a CRRA utility with risk aversion parameter γ, the certainty equivalent cash outflow of $w(1-\epsilon)\tilde{r}^G$ is $w(1-\epsilon)r^G + \frac{\gamma}{2}\sigma^2[\omega(1-\epsilon)]^2$. For borrower who would like to minimize the funding cost of strategy 2, her objective function is

$$\begin{aligned}&\min_{\omega}\{w(1-\epsilon)(\tilde{r}^G - i^L)\}\\ &= \min_{\omega}\{w(1-\epsilon)r^G + \frac{\gamma}{2}\sigma^2[\omega(1-\epsilon)]^2 - w(1-\epsilon)i^L\}\end{aligned}$$

The optimal amount to liquidize on the secondary market is given by

$$\omega^* = \frac{i^L - r^G}{\gamma\sigma^2(1-\epsilon)}$$

Since the two strategies result in the same cash inflow at time t, the no-arbitrage condition implies that they must come with the same cash outflow at time $t+1$. Solving this equivalence obtains a close-form expression for the SOFR rate:

$$i^S = i^L - \frac{(i^L - r^G)^2}{G\gamma\sigma^2}. \tag{13}$$

Equation (13) indicates that SOFR is always lower than LIBOR since SOFR is the collateralized borrowing cost while LIBOR is the unsecured borrowing cost. When bond holder is more risk averse, she is willing to bear a higher certainty equivalent cash outflow. The cost of financing on the secondary bond market increases. It follows that optimal amount to liquidize on the secondary market decreases. In this case, the financing cost of strategy 2 rises since the bond holder finance a larger share of liquidity through the expensive unsecured market, which drives up SOFR due to the no-arbitrage condition of the two strategies. Likewise, the effect of a more noisy signal of secondary bond market rate is similar as that of a higher risk aversion. A larger supply of government bond G increases SOFR. This is because a higher supply of collateral reduces scarcity value of government bond, which forces bond holders to borrow at a higher rate. Due to this additional collateral channel, government debt can affect the real economy in the SOFR regime even without distortionary taxes.

5 Quantitative Analysis

To obtain quantitative implications from the model, in this section I first assign values to parameters by calibrating the model, then I used the calibrated model to compare the SOFR and LIBOR regimes. Parameters are determined either by literature, according to Jermann (2019), or matching significant moments. The focus of the quantitative analysis is to compare the effect of the additional collateral channel on the real economy. Since such an additional channel is present in SOFR but not LIBOR regime, the effect is measured by the difference of macroeconomic variables in two regimes.

5.1 Calibration

I compute a linearized version of model near the neighborhood of the non-stochastic steady state. The model is calibrated to match the US data. A period in the model is a quarter. The parameter values are given by Table 4. The discount factor is set as 0.9901 so that the steady state risk free rate, i.e., the government bond rate in this model, is 1%. The density for the iid idiosyncratic funding cost shock is slected to be a quadratic function with the form $\phi(z) = \omega z^2 + \eta$. For an eligible density function, the corresponding cumulative density function must take the value between 0 and 1, and this leaves us with only one free parameter, which is set as the maximal value of the shock in my calibration. I follow Jermann (2019) to calibrate the interbank debt advantage (cost) and the maximal value of the idiosyncratic funding cost shock jointly to match the steady state TED spread and default rate. The TED spread is set to be 5 basis points and the default rate 0.5%. For the steady state aggregate funding shock, I follow Jermann (2019) to set it as the steady state price of business loan under LIBOR and SOFR, respectively. The steady state government borrowing is set as 12, which results in a 0.8 ratio of government debt to output. The persistence and the volatility of the TFP shock and the bank risk shock are calibrated jointly. I use Simulated Method of Moment (SMM) by matching the model implied autocorrelation and standard deviation for output and TED spread with those from real data. The data of real GDP and TED spread cover the periods between 2003 and 2019, and are taken from the FRED database. In specific, for each parameter set, I simulated sequences of endogenous variables from the theoretical model. It follows that with each parameter set, I can compute one group of model implied moments: the autocorrelation and the standard deviation of the overall output and the TED spread. Then the parameter values are chosen to be the ones obtaining the least distance of the model moments from the data moments. One remaining parameter that is determined by moment matching is the noise of the bond rate on the secondary market. I calibrate this parameter for a 4 basis point of steady state spread between LIBOR and SOFR.

Table 4. Parameter values.

Symbol	Parameter	Value	Implied moment	Value
		Literature		
γ	Risk aversion	2	—	—
δ	Depreciation	0.025	—	—
ρ_g	Govt borrowing shock persistence	0.9	—	
σ_g	Govt borrowing shock SD	0.12	—	
		Jermann (2019)		
α	Capital share	0.5	—	—
$1/\lambda$	Loan maturity	12	—	—
		Moment matching		
β	Discount factor	0.9901	$r^G = 1/\beta - 1$	0.01
τ	Interbank debt advantage	-0.0045	TED spread	0.0005
z_{max}	Idios. funding shock max	1.096	$\Phi(z^*)$	0.995
σ	Secondary bond price signal noise	0.0114	$i^L - i^S$	0.0004
q^L_{ss}	SS bank risk with LIBOR	1.0651	q^L	1.0651
q^S_{ss}	SS bank risk with SOFR	1.0646	q^S	1.0646
$\bar{G}$	SS govt debt	12	G/Y	0.8
σ_ψ	Bank risk SD	0.001	std(TED)	0.002
ρ_ψ	Bank risk persistence	0.91	autocorr(TED)	0.97
σ_a	TFP shock SD	0.04	std(Y)	0.006
ρ_a	TFP shock persistence	0.89	autocorr(Y)	0.85

A second set of parameters are borrowed from the current literature. To characterize household's utility, I assume that the utility function is CRRA with risk aversion controlled by γ. The household's risk aversion is set to be 2, which is a commonly used number in standard RBC and DSGE models. To proceed, the quarterly capital depreciation rate is set to be 2.5%, implying a 10% annual depreciation rate for capital stock. Finally, to pin down the persistence of the government borrowing shock, I refer to related literature such as Ravn et al. (2006), Gali et al. (2007) and Zubairy (2014). In particular, Ravn et al. (2006) select a 0.9 government spending persistence for their deep habit formation model to study how government spending affects private consumption and wages. Gali et al. (2007) calibrate the government spending persistence to be 0.9 to match the empirical IRFs generated from a VAR model. Zubairy (2014) estimates the persistence parameter in a DSGE model with Bayesian method, the resulting posterior mean is 0.92. Following the literature, in this paper, I set the persistence of the government borrowing shock to be 0.9. Then the volatility is set as one percentage of the steady state public debt.

The remaining two parameters are directly taken from Jermann (2019). To facilitate the calibration, I assume that the representative firm's production function is given by $f(k) = Ak^{\alpha}$. The value of capital share in output is set to be 0.5. Finally, for the multi-period floating rate business loan, the repayment schedule λ is equal to 1/12, suggesting that loans mature in 3 years.

5.2 Results

Figure 4 displays the impulse responses to a one standard deviation (calibrated as also one percentage) shock of public debt. Responses in LIBOR and SOFR economy are indicated with green and blue curve, respectively. A higher government bond supply means a lower scarcity value of Treasuries on Repo market, which drives the SOFR to rise. Thus, in the SOFR economy, the business loan coupon rate, indexed by SOFR, responds with an increase. The higher coupon rate translates into a higher price of business loan, since loan issuers now enjoy higher capital gains. Banks gain value from the higher coupon payment of outstanding loans and thus face a lower probability of default. Bank's lower probability of default, i.e., higher probability of survival, causes a rise in the market value of the interbank debt. On the firm side, a higher coupon rate implies higher borrowing costs, which reduces net return of capital. It follows that firms demand less business loans from banks. Since firms in the model rely solely on business loans to finance investment, the aggregate investment decreases as well (decrease in business loan volume outweighs increase in price), which further results in a decrease in capital accumulation. Interestingly, net output responds with an immediate rise, then tend to drop about 6 quarters after the shock. The change in net output is a

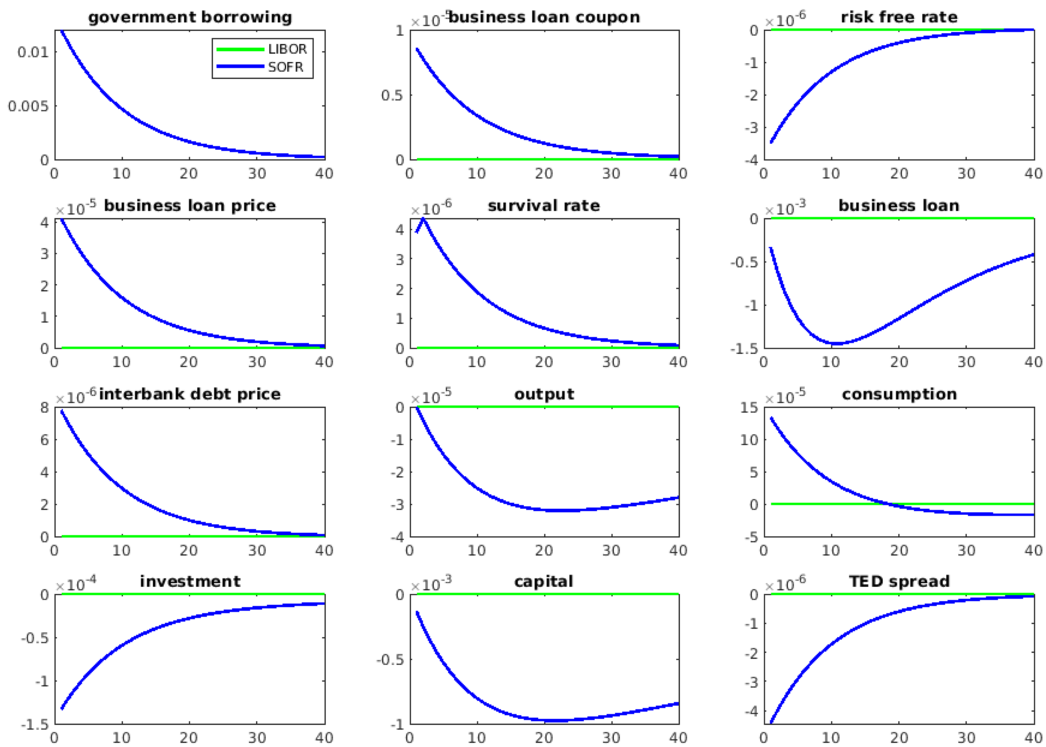

Figure 4. Impulse responses to a one standard deviation shock in government borrowing shocks.

composite of two forces with opposite directions. Recall that the net output, implied by equation (9), is defined as the difference between gross output and bank default loss. As the capital accumulation is harmed, the gross output is also reduced. On the other hand, a lower default rate along with a lower market value of business loans lead to a smaller default loss. The two forces with opposite direction together shape the dynamics in the net output. The response from gross output is with smaller magnitude but higher persistence, compared with the change in the value of business loans, and that is why the net output increases in the beginning but decreases periods later. Representative household receives lower equity payout from firms (due to the lower gross output and the higher business loan repayment) but higher payout from banks (due to the higher price of outstanding business loans and lower newly issued business loans). In the mean time, she pays less default loss.

The net effect on consumption is positive, which eventually results in an increasing stochastic discount factor and a decreasing risk free rate (also the government bond rate).

The above dynamics are purely effects from the scarce collateral channel which is only possible in a SOFR economy. Previous analysis indicates that such effects are not possible in LIBOR economy without this additional channel. Consistent with the conceptual framework, a government borrowing shock does not induce any significant responses of macroeconomic variables in the LIBOR regime. Quantitatively, the effect of a government borrowing shock on the real economy is quite small. The spread between SOFR and risk free rate moves by approximately 0.1 bps instantaneously. This effect reduces by a half about 6 quarters later and almost fully dissipates after 8 years. It takes the aggregate capital 5 years to reach the lowest point, 10 bps from the steady state level, and this effect exhibits high persistence. The aggregate consumption has an instantaneous rise by 1.5 bps then reverts back to the steady state level after 4 years. The disruption caused by more costly business borrowing from the higher coupon rate is diluted by the response in bank default loss. Even though a higher borrowing cost harms the gross output, this cost is well compensated by a higher net bank value. This explains the finding that in general equilibrium, the economic aggregate effect of scarce collateral channel on government debt crowding-out is quite small.

Figure 5 plots responses to a one standard deviation increase in aggregate bank risk, which implies that the realization of z is more spread out. The funding cost risk directly affects default probability and then transitions to prices. A more volatile funding cost increases bank's probability of default thus decreases interbank debt price. A key difference is the response of benchmark interest rate in the SOFR and LIBOR regimes. In the LIBOR economy, the benchmark interest rate rises in response to higher inter-bank default probability, which drives up the business loan price. Indicated by equation (13), SOFR is equal to LIBOR minus a default risk premium. The variation in SOFR comes from two sources. First, when LIBOR increases, the first term in equation (13) suggests that SOFR also increases by the same amount. In the mean time, a more volatile funding cost increases LIBOR by a larger amount than the risk-free rate. Thus, the default risk premium for SOFR also rises. It turns out that the rise in risk premium outweighs the rise in LIBOR, which results in a decrease in SOFR. Business loan takes changes of different signs in the two regimes. When business loan is indexed by LIBOR, due to a higher coupon rate, the loan demand shrinks. When the loan is indexed by SOFR, its demand increases in response to a lower coupon rate. Since in SOFR economy, the business loan coupon rate is lower, the business loan price is also lower. In the LIBOR economy, the capital gain from higher coupon payment offsets part of the negative effect on bank default probability, the disruption in financial intermediaries is less severe, which can be viewed from the less reduced bank survival rate. This finding is the same as Jermann (2019), suggesting that when funding risk is high as in a financial crisis, LIBOR might be a better choice than SOFR. The effect on macroeconomic variables are all with the same signs in the two regimes, and the quantitative discrepancy is pretty mild.

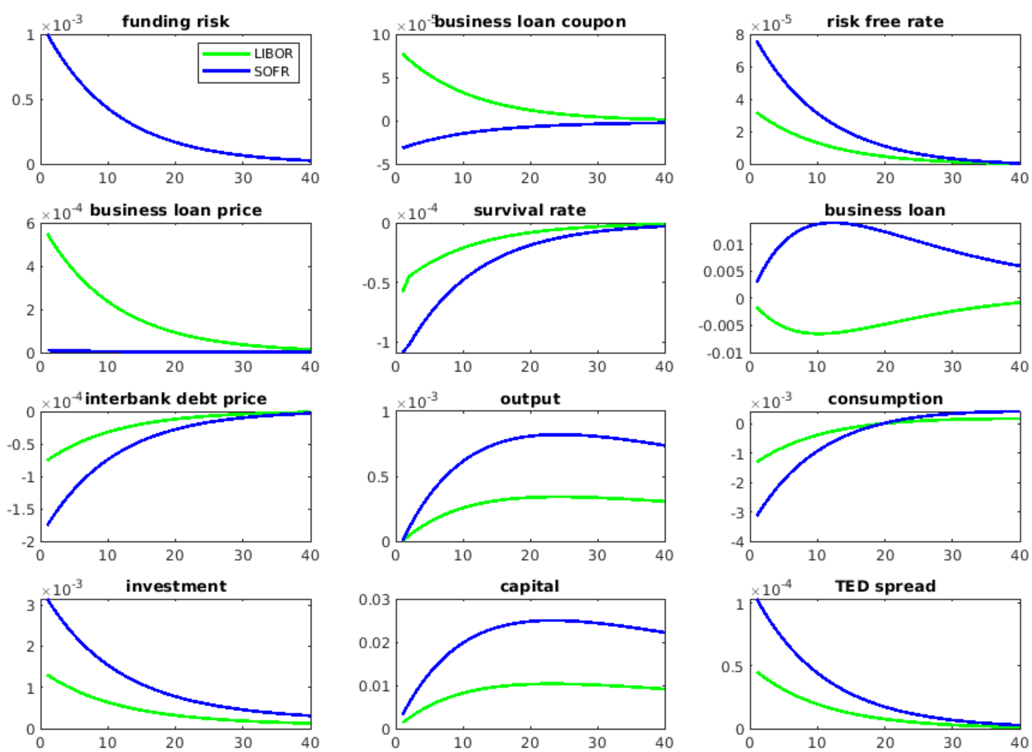

Figure 5. Impulse responses to a one standard deviation innovation in bank funding risk.

Figure 6 presents the impulse responses to a one standard deviation innovation in the capital productivity. For the main macroeconomic variables, their responses are consistent with the expectation from a real business cycle model. Interest rates rise as there is a higher demand for funds. Also, macroeconomic variables such as output, investment, and consumption all increase. The is no essential difference between the SOFR and LIBOR economy.

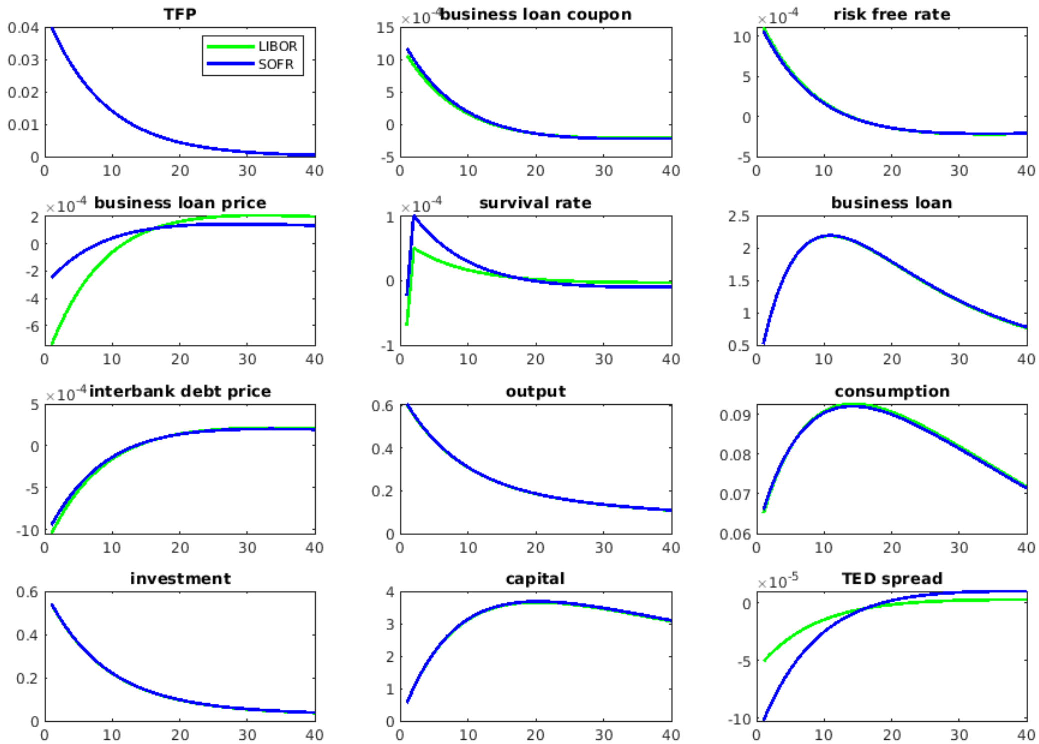

Figure 6. Impulse responses to a one standard deviation innovation in TFP shocks.

6 Conclusion

In this paper, I have illustrated the asset pricing and macroeconomic effect of the collateral channel by comparing SOFR and LIBOR regimes. SOFR with underlying transactions collateralized by Treasuries is exposed to shocks in government bond supply. A higher supply of Treasuries reduces the scarcity value of government bonds on Repo market, which forces bond holders to borrow with higher rates. LIBOR does not have this property. To begin with, I presented empirical evidence of the existence of the scarce collateral channel. I showed that compared with LIBOR, SOFR has additional response to the random variation in public debt. This additional response comes mainly from the specific collateral transactions. Then I proceed to adopt a modified version of Jermann (2019) model, with SOFR pinned down by no-arbitrage condition on the financing markets in a spirit of Nyborg (2019), to quantitatively analyze the effect of additional collateral channel on the real economy. I showed that the collateral channel allows public debt to crowd out investment even without distortionary taxes, though the effect magnitude is very small. In general equilibrium, the negative effect of higher SOFR on private investment is offset by bank capital gain from the increase in both the market value and the coupon payment for business loans.

Findings in this paper have important policy implications. The national debt level has been a significant subject for US domestic policy. On top of that, the federal government still relies heavily on debts to finance future expenditures. My findings point out the fact that with SOFR as the new benchmark interest rate, future debt financing may induce additional crowding out to the real economy by harming private investment. The good news is, the additional crowding-out effect is with an negligibly small size. Although this paper does not find a large extra crowding-out effect moving from LIBOR to SOFR, it sheds some light on a better understanding of the underlying mechanism and possible economic consequences when making fiscal policies.

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Appendix A. Robustness Checks

In this section, I provide results of robustness checks for empirical evidence. First, I examine whether the additional rise in SOFR indicated by the OLS results holds if I control for more lagged periods. Proceeding to the Local Projection, I conduct two checks with the first one including overall inflation as an exogenous variable and the second one replacing industrial production index with unemployment rate. The results of all three checks are robust, suggesting the validity of the scarce collateral channel for SOFR.

A.1. OLS

In the baseline specification (Table 2), one period lagged SOFR or LIBOR is included as a control variable. One concern is that the SOFR or LIBOR is autocorrelated for more than one period, and thus the additional positive correlation between SOFR and government borrowing is attributed to higher-order autocorrelations. To show that this concern is not valid, I include higher-order lagged dependent variables. The results are given in Table A.1.

Table A.1. Additional rise in SOFR in response to a higher level of public borrowing. *Note:* *p<0.1; **p<0.05; ***p<0.01. Robust standard errors in parenthesis. This table provides results of alternative specificatons with additional lagged variables.

Panel A: Gov't debt outstanding as the measure of borrowing				
	Dependent variable:			
	SOFR		LIBOR	
	(1)	(2)	(3)	(4)
Δlog debt	599.523***	566.669***	−36.952**	−49.357**
	(85.753)	(86.329)	(18.423)	(20.942)
Constant	−0.249***	−0.283***	0.008	0.007
	(0.086)	(0.087)	(0.019)	(0.020)
SOFR(-1:-2)	*Yes*	*Yes*	*No*	*No*
SOFR(-3:-7)	*No*	*Yes*	*No*	*No*
LIBOR(-1:-2)	*No*	*No*	*Yes*	*Yes*
LIBOR(-3:-7)	*No*	*No*	*No*	*Yes*
Observations	1,128	1,113	1,065	978
R^2	0.083	0.096	0.065	0.105
Adjusted R^2	0.080	0.090	0.062	0.098
Residual Std. Error	2.844 (df = 1124)	2.830 (df = 1104)	0.605 (df = 1061)	0.605 (df = 969)

Table A.1 Continued. Additional rise in SOFR in response to a higher level of public borrowing. *Note:**p<0.1; **p<0.05; ***p<0.01. Robust standard errors in parenthesis. This table provides results of alternative specificatons with additional lagged variables.

Panel B: Treasuries outstanding as the measure of borrowing

	Dependent variable:			
	SOFR		LIBOR	
	(1)	(2)	(3)	(4)
Δlog Treasuries	994.114***	−66.111***	981.394***	−65.053***
	(88.035)	(19.648)	(90.593)	(20.557)
Constant	−0.326***	0.012	−0.368***	0.010
	(0.083)	(0.019)	(0.085)	(0.020)
SOFR(-1:-2)	*Yes*	*Yes*	*No*	*No*
SOFR(-3:-7)	*No*	*Yes*	*No*	*No*
LIBOR(-1:-2)	*No*	*No*	*Yes*	*Yes*
LIBOR(-3:-7)	*No*	*No*	*No*	*Yes*
Observations	1,124	1,059	1,099	962
R^2	0.142	0.069	0.154	0.108
Adjusted R^2	0.139	0.066	0.148	0.101
Residual Std. Error	2.745 (df = 1120)	0.603 (df = 1055)	2.743 (df = 1090)	0.606 (df = 953)

Regardless of the measure of the government borrowing, additional positive change happens to SOFR when there is an increase in government borrowing. The results show that the additional rise in SOFR cannot be explained by higher-order autocorrelations, which supports the existence and uniqueness of the scarce collateral channel for SOFR. Note that the regression frequency is daily, which means the control set includes one-day lagged to one-week lagged dependent variables. For even higher-order autocorrelations, I checked the specifications with up to two-week lagged SOFR or LIBOR, and the results are very similar so ignored here.

A.2. Local Projection

The specification for local projection may be considered restrictive. To verify that the additional positive response from SOFR holds in alternative specifications, in this section, I present the results of two robustness checks. First, to control the possible effect on asset prices from the inflation, I include CPI as an additional exogenous variable. Results corresponding to this version are given in Figure A.1. The directions and magnitudes of the effects are very similar to the baseline specification. Controlling for the price effect does not corrode the persistent and positive response in SOFR to a government borrowing shock.

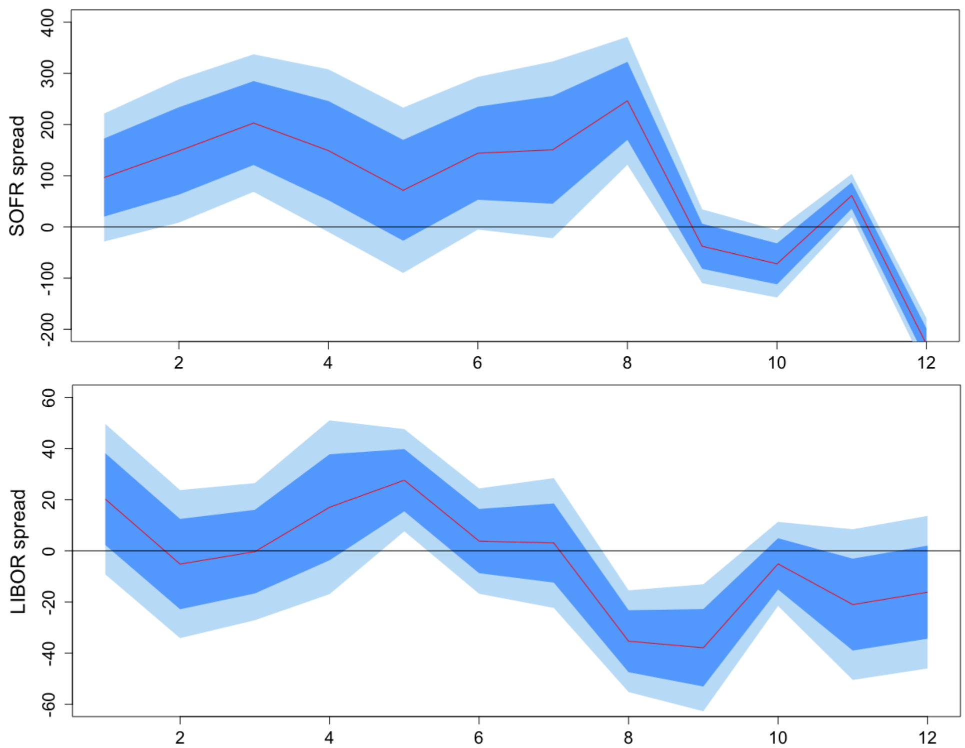

Figure A.1. Impulse responses to a government borrowing shock. Solid red line represents mean of IRF. Dark blue band represents 90% confidence bounds and light blue band represents 68% confidence bounds. This figure provides results of alternative specification with CPI as an additional control variable.

Moreover, the baseline version uses industrial production index to describe the output level. To measure the overall output level more broadly, ideally, the gross domestic output (GDP) should be employed, since in recent periods a larger share of growth is driven by wholesale and retail industries rather than goods-producing industries. [12] However, GDP is only available at a quarterly frequency, which causes inconsistency in regression frequency. To reconcile the conflict, in the second robustness check, I replace the industrial production index with the unemployment rate, which is a broader measure of economic outcome available at a monthly frequency. The results are presented using Figure A.2. The results continue to be robust in this robustness check.

[12] U.S. Bureau of Economic Analysis, FAQ: "How do the industrial production index (IPI) and the gross domestic product (GDP) compare conceptually?"

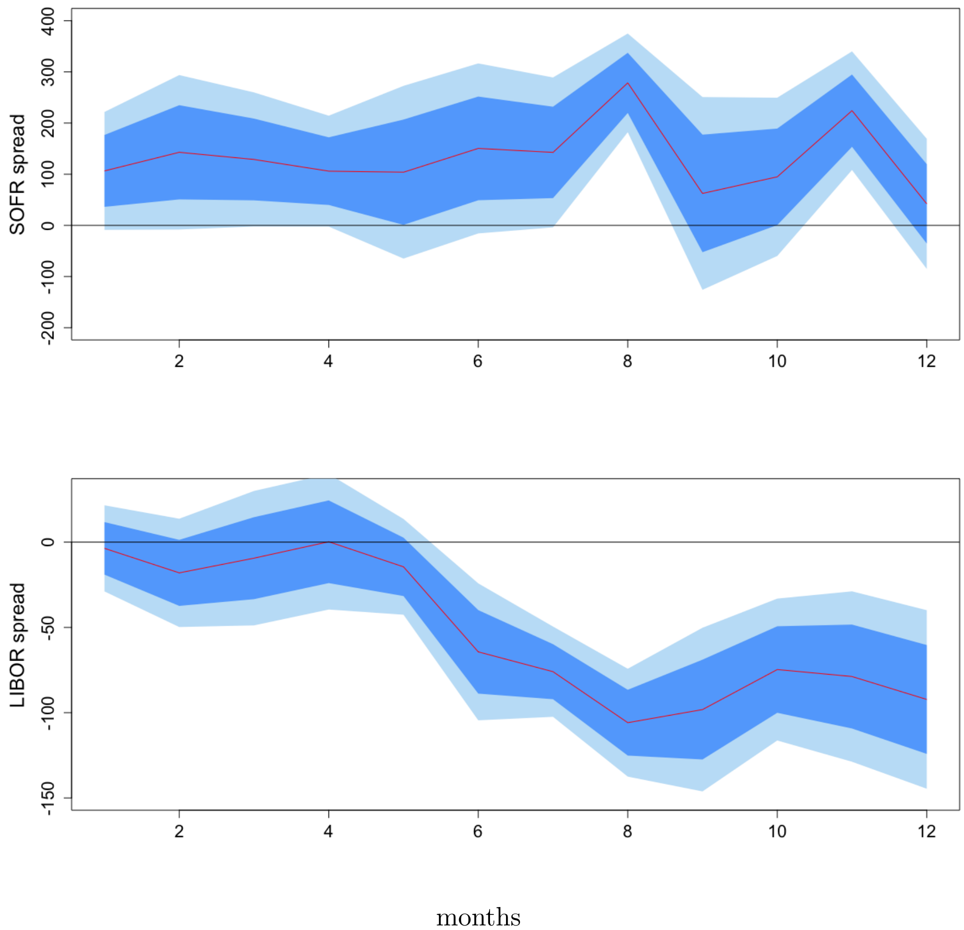

Figure A.2. Impulse responses to a government borrowing shock. Solid red line represents mean of IRF. Dark blue band represents 90% confidence bounds and light blue band represents 68% confidence bounds. This figure provides results of alternative specification replacing industrial production index with unemployment rate.

Appendix B. Equilibrium Conditions

In this appendix, I present conditions to characterize the equilibrium economy and the implied steady state structures. The recursive equilibrium is given by

1. Bank value function $V(b, d)$;
2. Bank policy functions of interbank borrowing d' and business loan supply b';
3. Firm value function $W(b, k)$;
4. Firm policy functions of capital demand k' and business loan demand b';
5. Lump-sum tax schedule T;
6. Private consumption c;
7. Interbank debt price p, business loan price q and government bond rate r^G;

such that given interbank debt price p and business loan price q, bank policy functions b' and d' maximize bank dividend payouts, and the associated value function is $V(b, d)$; given interbank debt price p and business loan price q, firm policy functions k' and b' maximize firm dividend payouts, and the associated value function is $W(b, k)$; the consumption plan c satisfies resource constraint; the lump-sum tax T adjusts to satisfy government budget constraint; interbank lending market clears; business loan market clears.The equilibrium economy is fully characterized by the following equations:

$$[i + \lambda + (1 - \lambda)q]b - d = \Psi z^* b \tag{B.1}$$

$$\Phi(z^*) = \omega/3 * (z^{*3} + z_{max}^3) + \eta(z^* + z_{max}) \tag{B.2}$$

$$E(z|z < z^*) = \omega/4(z^{*4} - z_{max}^4) + \eta/2 * (z^{*2} - z_{max}^2) \tag{B.3}$$

$$p(1 + \tau) = EM'\Phi(z^{*'}) \tag{B.4}$$

$$q = EM'[i' + \lambda + (1 - \lambda)q' - \Psi' E(z'|z' < z^{*'}] \tag{B.5}$$

$$d = 0 \tag{B.6}$$

$$EM'[(i' + \lambda) + \mu' q'(1 - \lambda)] = \mu q \tag{B.7}$$

$$EM'[A' f'(k') + \mu'(1 - \delta)] = \mu \tag{B.8}$$

$$k' = q(b' - (1 - \lambda)b) + (1 - \delta)k \tag{B.9}$$

$$inv = q(b' - (1 - \lambda)b) \tag{B.10}$$

$$c = y - inv \tag{B.11}$$

$$G' + T = (1 + r^G)G \tag{B.12}$$

$$r^G = 1/EM' - 1 \tag{B.13}$$

$$M' = \beta(c'/c)^{-\gamma} \tag{B.14}$$

$$i^L = 1/p - 1 \tag{B.15}$$

$$i^S = i^L - \frac{(i^L - r^G)^2}{G\gamma\sigma^2} \tag{B.16}$$

$$log(\psi') = \rho_\psi log(\psi) + e_\psi, e_\psi \sim N(0, \sigma_\psi^2) \tag{B.17}$$

$$log(A') = \rho_a log(A) + e_a, e_a \sim N(0, \sigma_a^2) \tag{B.18}$$

$$log(G') = \rho_g log(G) + e_g, e_g \sim N(0, \sigma_g^2) \tag{B.19}$$