



# 图像处理-形态学

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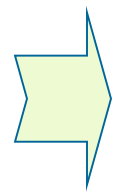




什么是数学形态学 (Mathematical Morphology)?

Mathematical  
Morphology

集合理论  
形态、形状



数学形态学是用集合论研究图像形状、形态的学科



集合的对象是什么？集合操作有哪些？

- 集合的每个对象叫元素；集合操作有：交、并、差



数学形态学在图像处理中有何用途？

形态学滤波(如去噪)、提取边缘、骨架提取、空洞填充、形态学重构等。

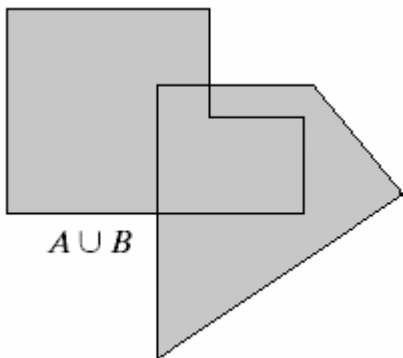
## 9.1 预备知识

### § 9.1.1 集合论的基本概念

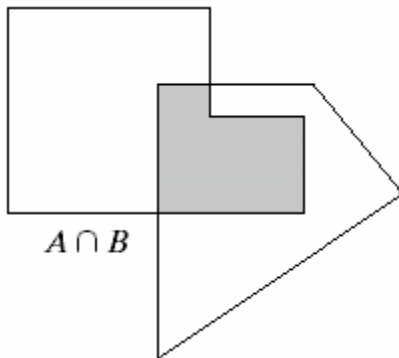
- ① 研究的集合是图像中各个物体的集合
- ② 对于二值图像，集合是由二维整数空间 $\mathbf{Z}^2$ 的元素组成的  
即  $\mathbf{a}=(x,y)^T, \mathbf{a}\in\mathbf{A}\subset\mathbf{Z}^2$ 的，其中 $(x,y)$ 是图像坐标
- ③ 对于灰度图像，集合是由三维整数空间 $\mathbf{Z}^3$ 的元素组成的  
即  $\mathbf{a}=(x,y,f(x,y))^T, \mathbf{a}\in\mathbf{A}\subset\mathbf{Z}^3$ 的，其中 $(x,y)$ 是图像坐标， $f(x,y)$ 是像素在 $(x,y)$ 处的灰度值

🧐 如何表示一个集合S？  $S = \{\text{元素} \mid \text{该集合的元素所满足的条件}\}$

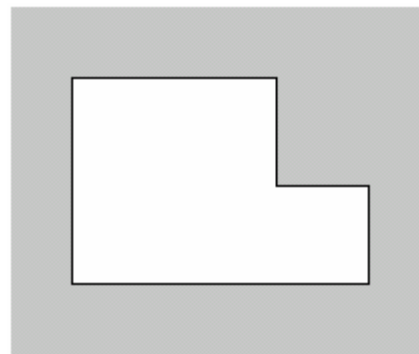
◆ 并集  $C = A \cup B$



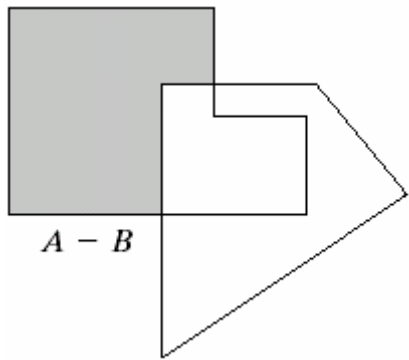
◆ 交集  $C = A \cap B$



◆ 补集  $(A)^c = \{w \mid w \notin A\}$



◆ 差集  $A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$



◆ 集合互斥  
 $A \cap B = \emptyset$

➤ 数学形态学中常用的两个集合定义：反射集合和位移集合

that are used extensively in morphology but generally are not found in  
basic texts on set theory

① 集合的反射

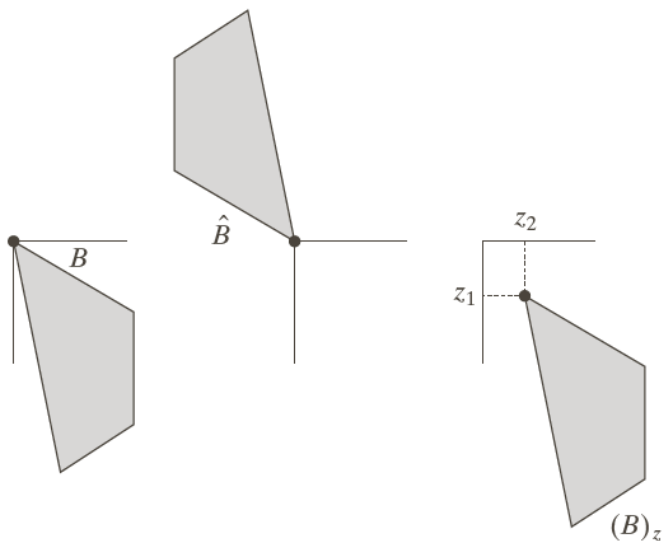
集合B的反射

$$\hat{B} = \{w \mid w = -b, \text{ for } b \in B\}$$

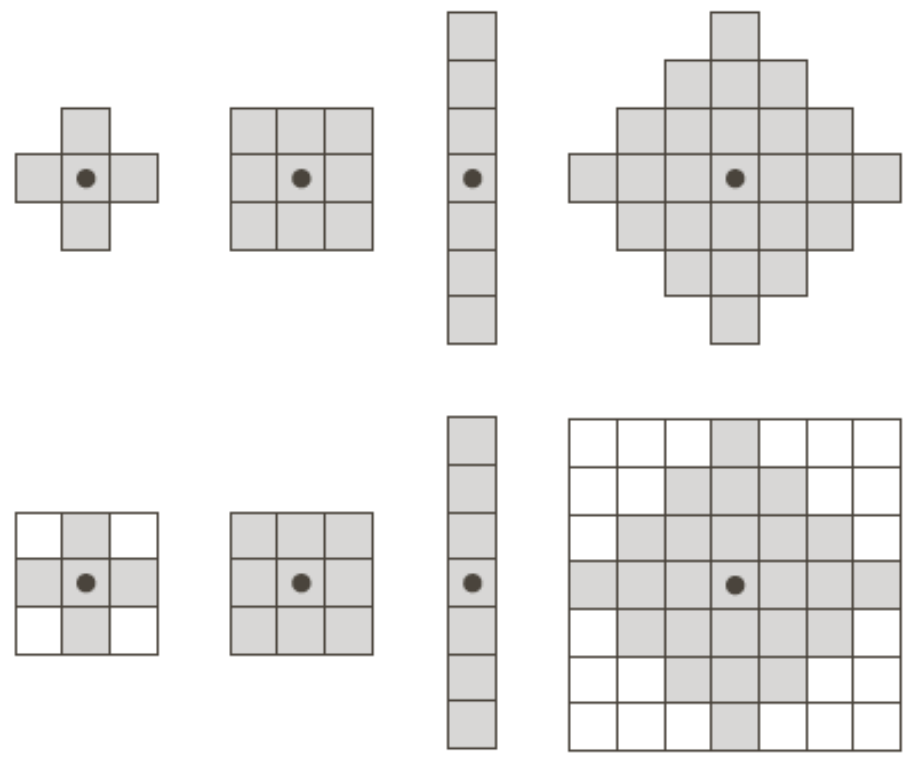
② 集合的位移

集合A的位移

$$(A)_z = \{c \mid c = a + z, \text{ for } a \in A\}$$

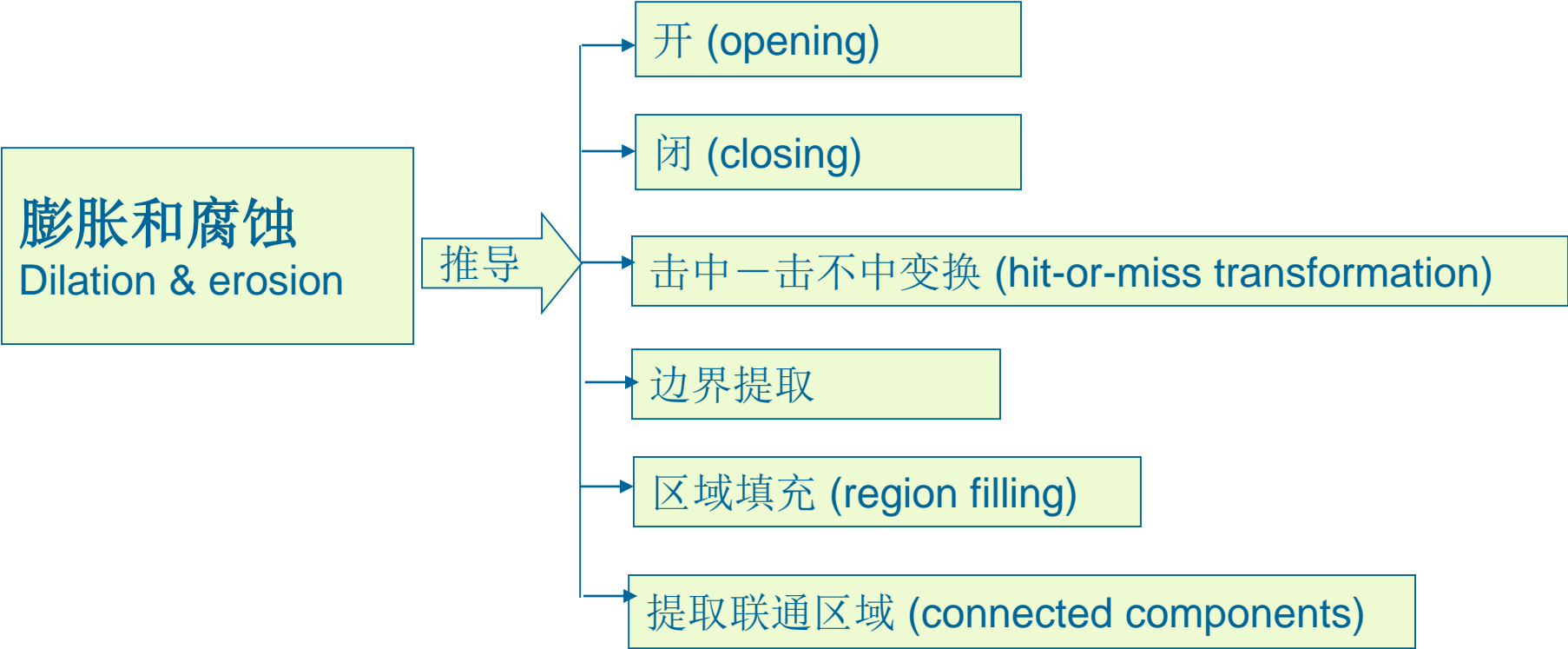


➤ 结构元素—SE



# 9.2 膨胀和腐蚀

✓ 膨胀和腐蚀是数学形态学各方法的基础





## 9.2.1 膨脹



膨脹是什么意思呢？

比喻：蒸馒头







## 9.2.1 膨胀

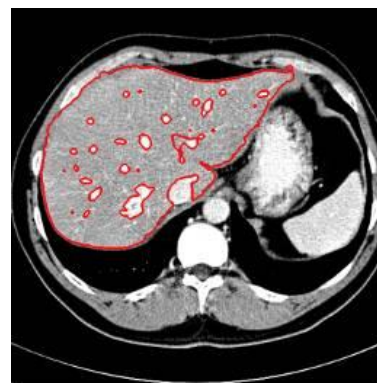


膨胀是什么意思呢？有什么效果？

比喻：脸部化浓妆



● Dilation is used to fill holes and narrow gulfs in objects.



● It increases the object size

数学形态学中也有一把刷子



## 9.2.1 膨胀

### ➤ 膨胀的定义 $A \oplus B$

#### ✓ 公式描述

集合 **B** 对集合 **A** 造成的膨胀

$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \emptyset\} = \{z \mid [(\hat{B})_z \cap A] \subseteq A\}$$

其中 **B** 为结构元素

化妆用的刷子和粉

$\hat{B}$  是集合 **B** 的反射集合, 即  $\hat{B} = \{w \mid w = -b, \text{ for } b \in B\}$

$(\hat{B})_z$  是集合  $\hat{B}$  的位移集合, 即  $(\hat{B})_z = \{c \mid c = b + z, \text{ for } b \in \hat{B}\}$

#### ✓ 文字描述

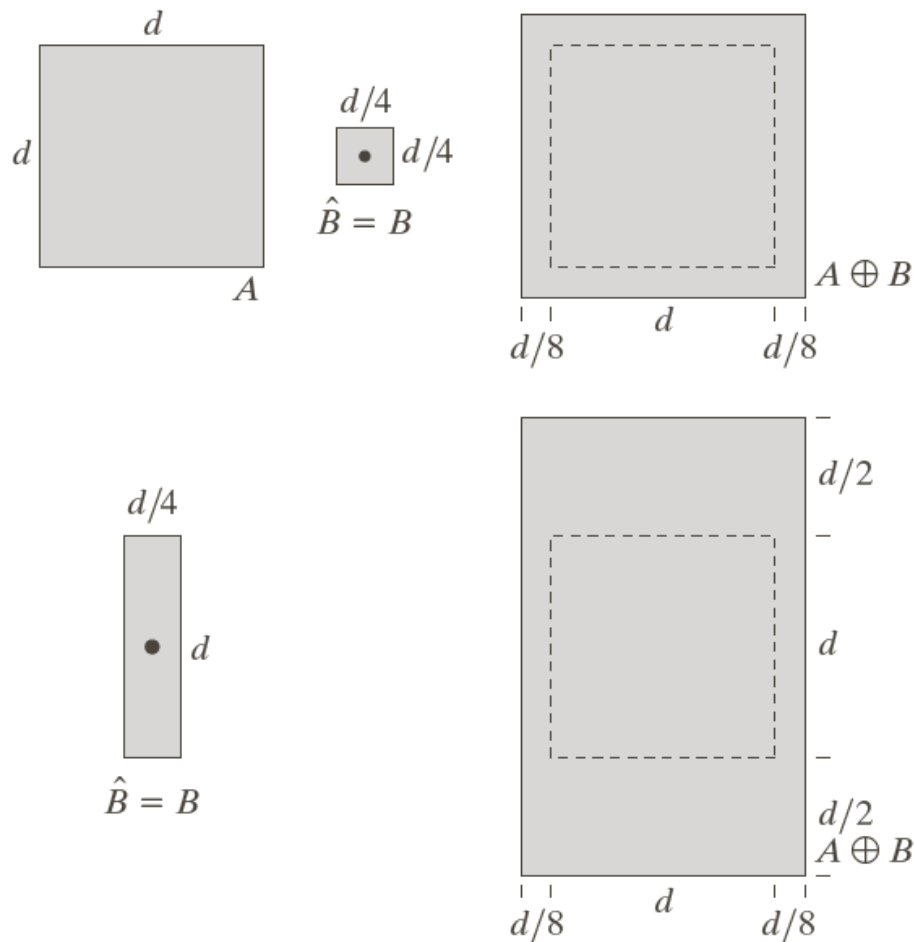
The dilation of **A** by **B** is the set of all displacements, **z**, such that  $(\hat{B})_z$  and **A** overlap by at least one element.

可见膨胀是 用反射集合、位移集合的交集形式表示的  
这个形式非常像 信号的卷积 🤖



## 9.2.1 膨脹

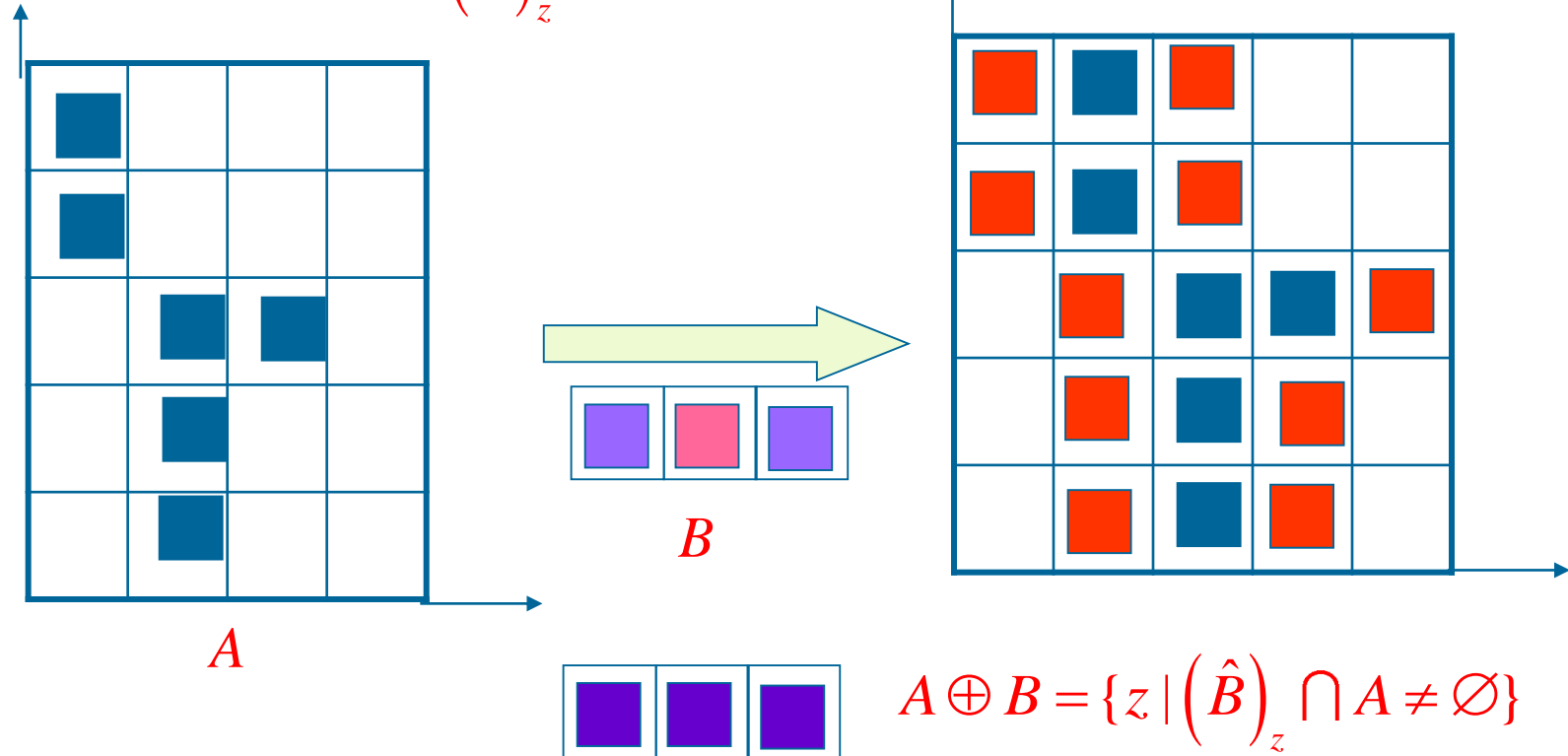
➤ **示例1**  $A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \emptyset\}$





## 9.2.1 膨脹

➤ 示例2  $A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \emptyset\}$



膨脹运算满足交换律和结合律

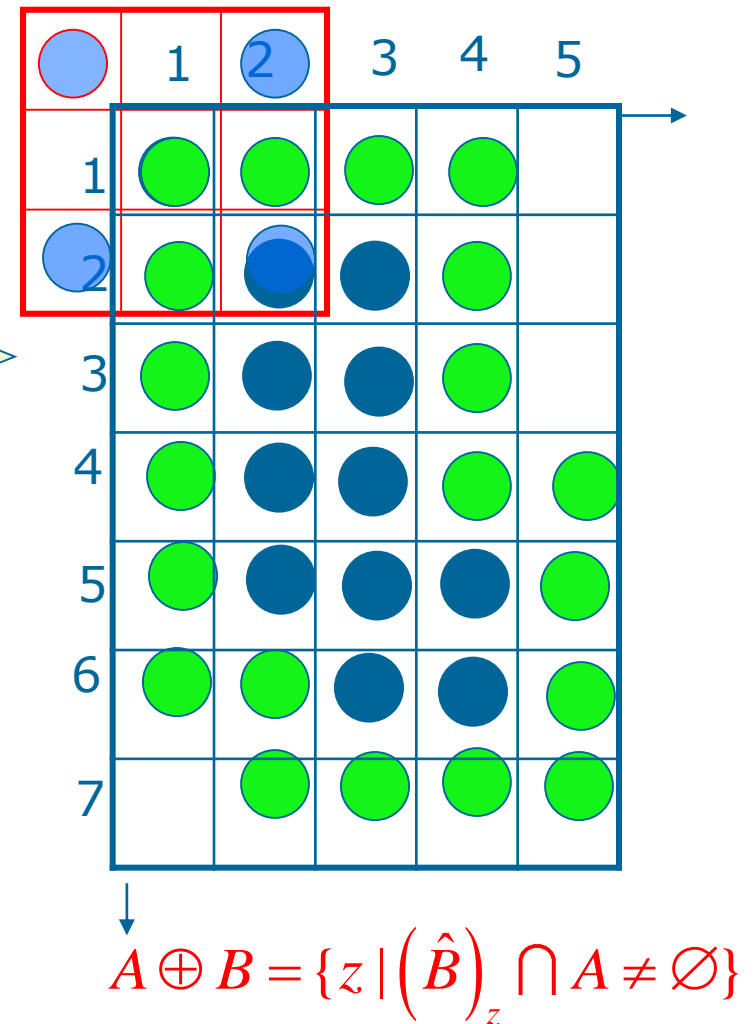
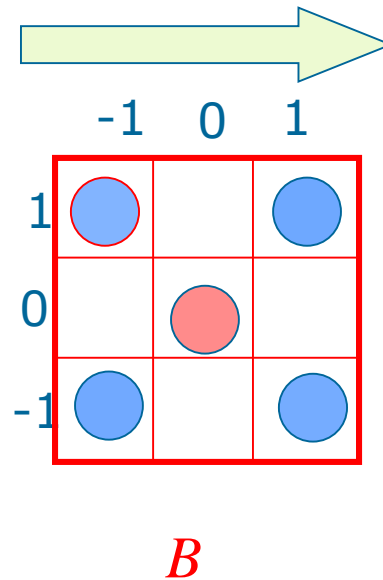
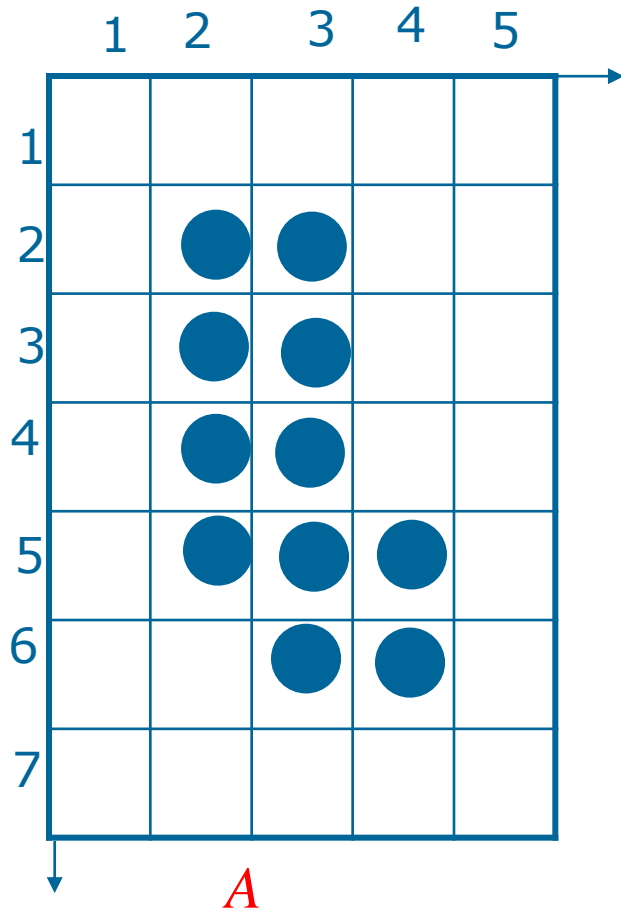
$$A \oplus B = B \oplus A$$

$$A \oplus (B \oplus C) = (A \oplus B) \oplus C$$



## 9.2.1 膨脹

➤ 示例3  $A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \emptyset\}$



## 9.2.1 膨脹

### ➤ 实际应用 $A \oplus B = \{z | (\hat{B})_z \cap A \neq \emptyset\}$

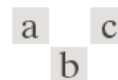
Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



0	1	0
1	1	1
0	1	0



**FIGURE 9.7**

(a) Sample text of poor resolution with broken characters (see magnified view).  
(b) Structuring element.  
(c) Dilation of (a) by (b). Broken segments were joined.



## 9.2.1 膨脹

### ➤ 实际应用 $A \oplus B = \{z | (\hat{B})_z \cap A \neq \emptyset\}$

膨脹操作与高斯低通滤波器平滑操作的比较

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



$$H(u,v) = e^{\frac{-D^2(u,v)}{2\sigma^2}}$$



腐蚀是什么意思呢？有什么效果？

◆ Dilation causes objects to dilate or grow in size

◆ Erosion causes objects to shrink



➤ 腐蚀的定义  $A \ominus B$

集合  $B$  对集合  $A$  造成的腐蚀  $A \ominus B = \{z \mid (B)_z \subseteq A\}$

其中  $B$  为结构元素 腐蚀物

其中  $(B)_z$  是集合  $B$  的位移集合，即  $(B)_z = \{c \mid c = b + z, \text{ for } b \in B\}$

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The erosion of  $A$  by  $B$  is the set of all points  $z$  such that  $B$ , translated by  $z$ , is contained in  $A$ .

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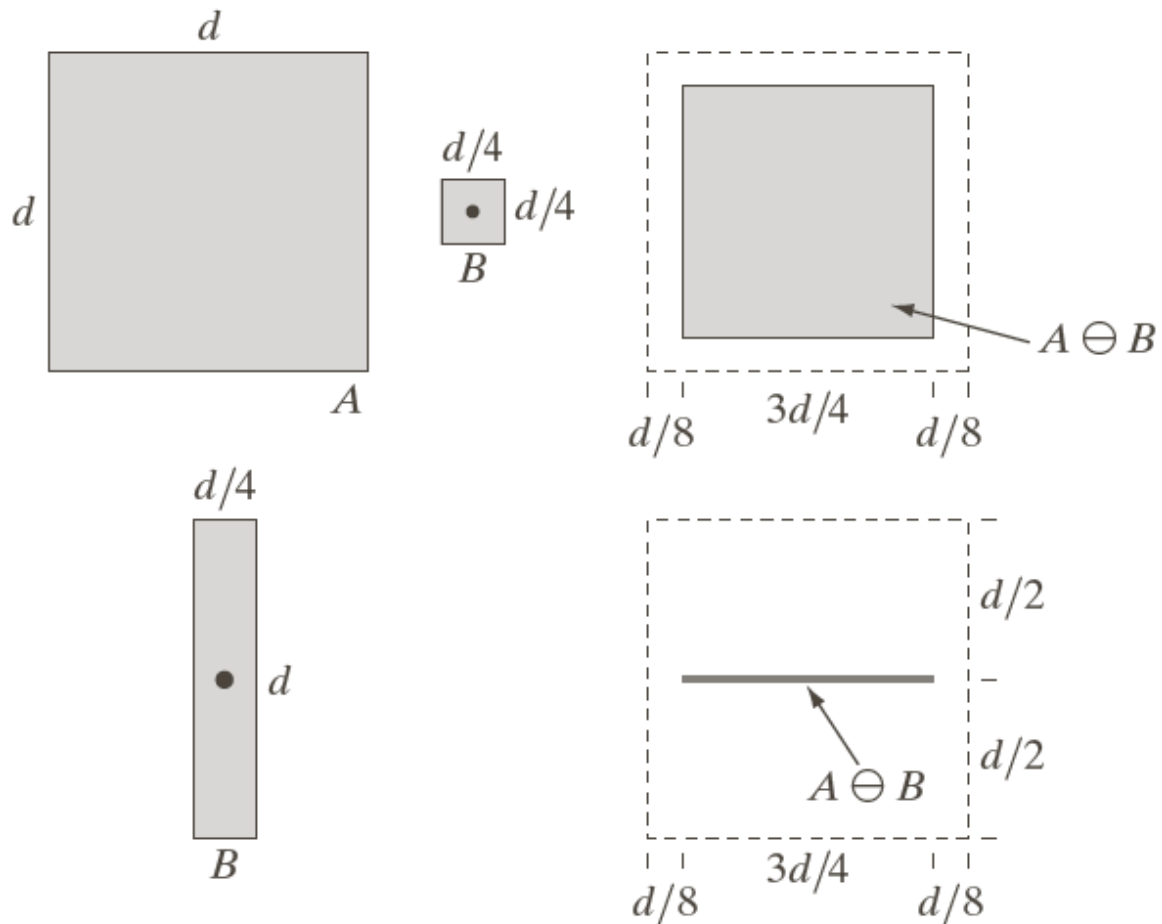
可见腐蚀运算也非常像卷积运算



## 9.2.2 腐蚀

### ➤ 腐蚀举例1 $A \ominus B$

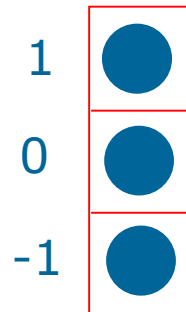
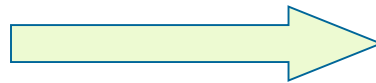
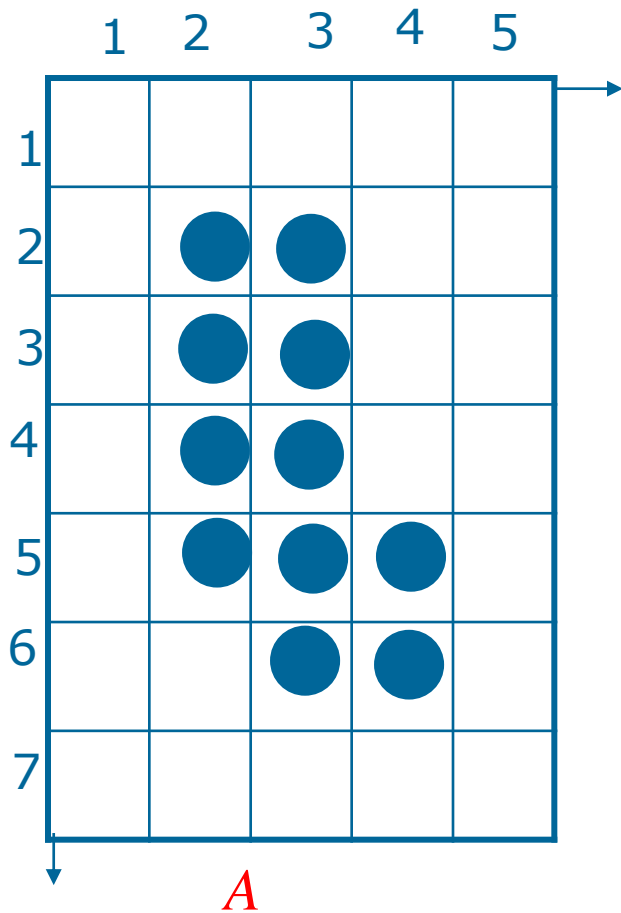
$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$



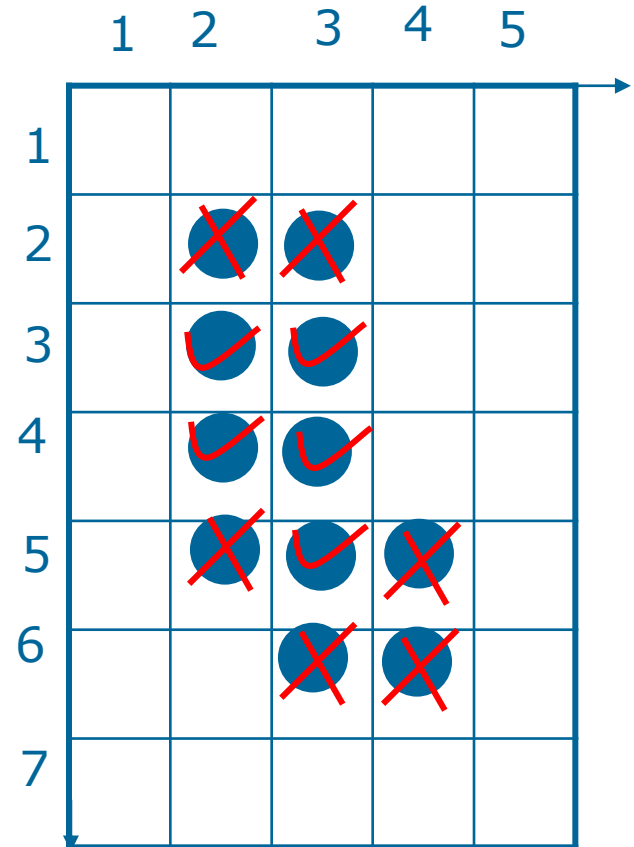


## 9.2.2 腐蚀

➤ 示例2  $A \ominus B = \{z \mid (B)_z \subseteq A\}$

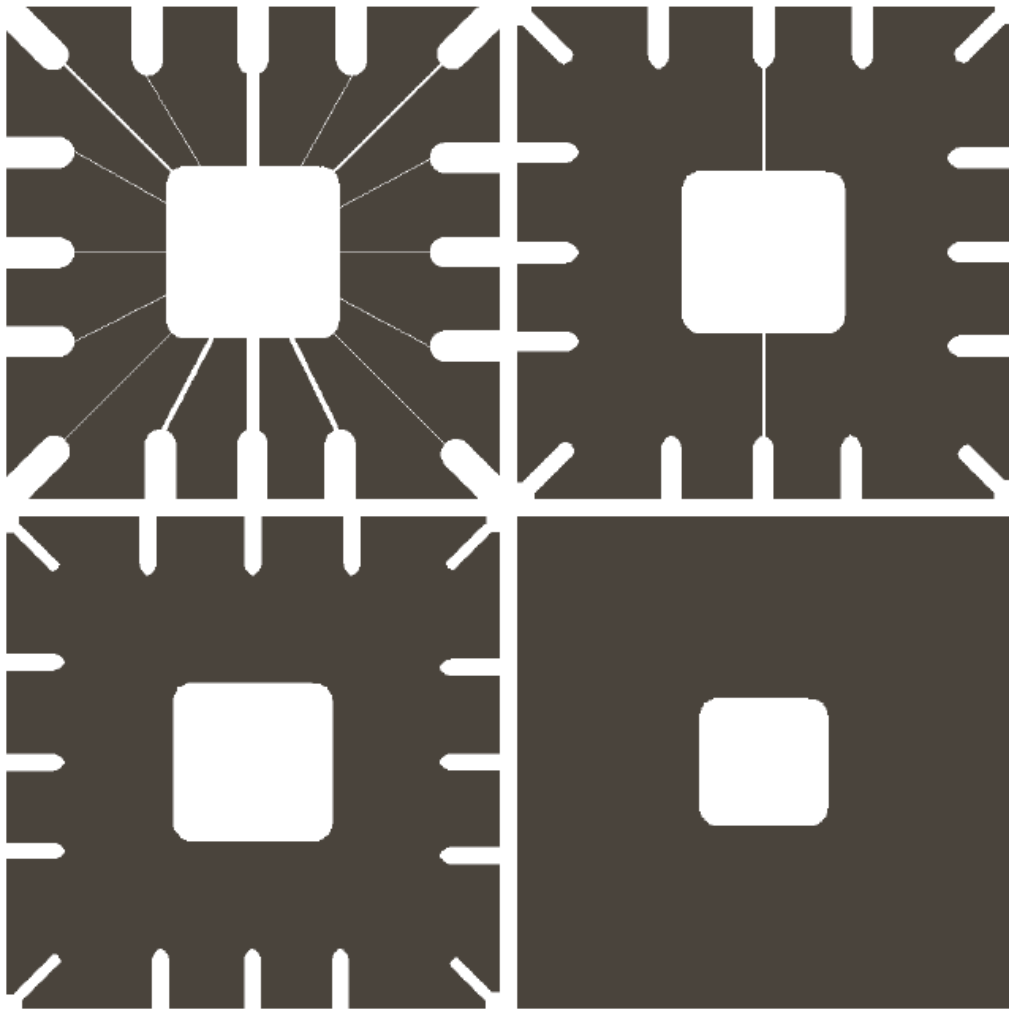


$B$



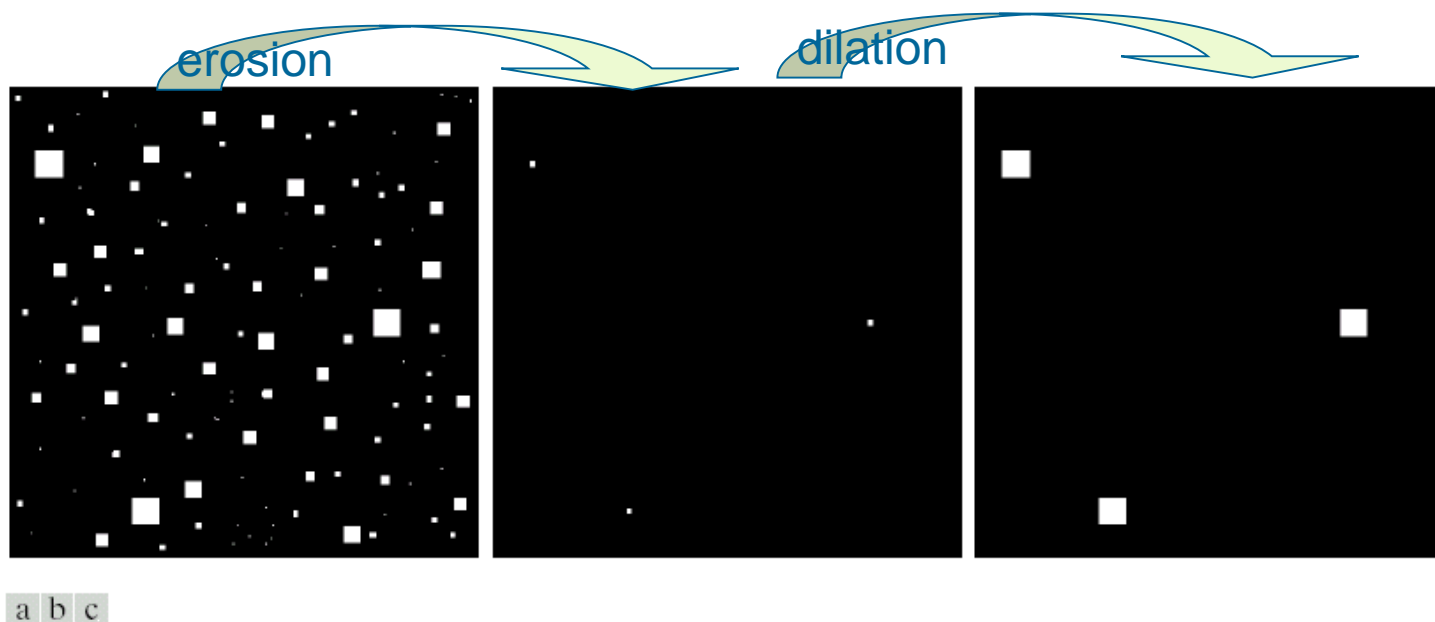
$A \ominus B = \{z \mid (B)_z \subseteq A\}$

➤ 示例3  $A \ominus B = \{z \mid (B)_z \subseteq A\}$



486×486的图像  
分别被  
11×11  
15×15  
45×45  
的结构元素腐蚀

➤ 示例4  $A \ominus B = \{z \mid (B)_z \subseteq A\}$



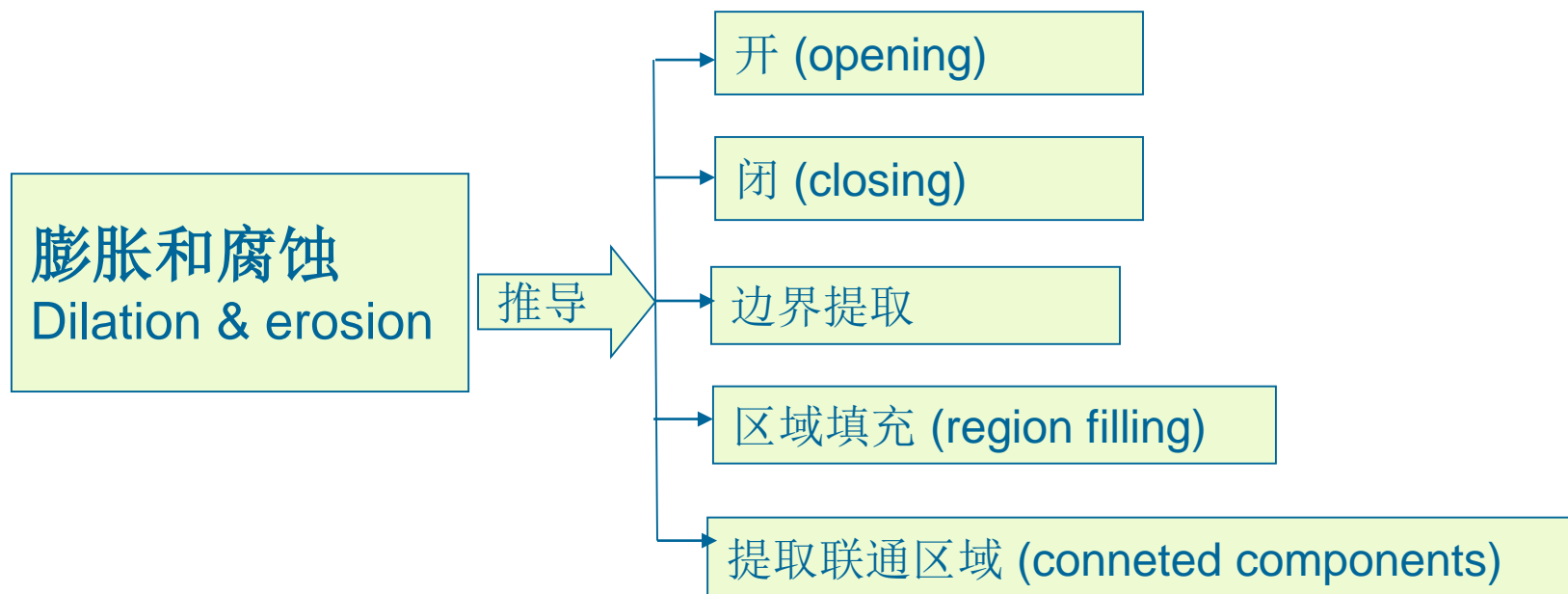
**FIGURE 9.7** (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.



图像经过腐蚀后，再经过膨胀能保持图像不变化吗？ 答:不能

## 9.3 开和闭

✓ 膨胀和腐蚀是数学形态学各方法的基础





## 9.3 开和闭

### ✓ 开Opening的定义 $A \circ B$

结构元素B对集合A的开运算为  $A \circ B = (A \ominus B) \oplus B$

即：The opening A by B is the erosion of A by B, followed by a dilation of the result by B

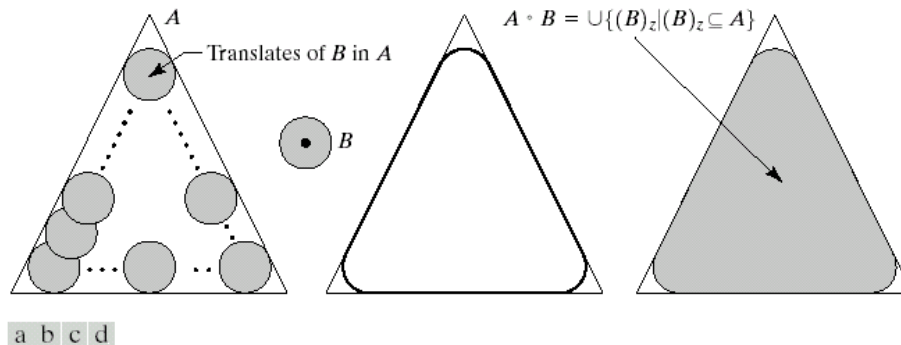


为什么叫“开”？



因为先执行腐蚀操作，而腐蚀操作有开的作用

### ✓ 开运算的几何拟合特性 $A \circ B = (A \ominus B) \oplus B = \bigcup \{(B)_z \mid (B)_z \subseteq A\}$



**FIGURE 9.8** (a) Structuring element  $B$  “rolling” along the inner boundary of  $A$  (the dot indicates the origin of  $B$ ). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).



## 9.3 开和闭

### ✓ 闭Closing的定义 $A \bullet B$

结构元素B对集合A的闭运算为  $A \bullet B = (A \oplus B) \ominus B$

即：The closing A by B is the dilation of A by B, followed by an erosion of the result by B

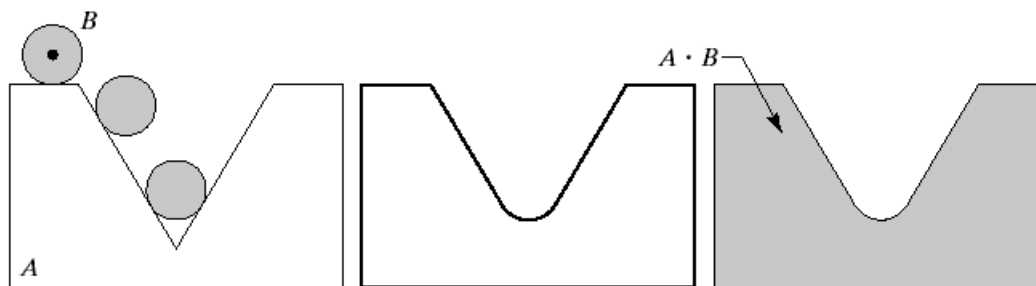


为什么叫“闭”？



因为先执行膨胀操作，而膨胀操作有闭的作用

### ✓ 闭运算的几何拟合特性 $A \bullet B = (A \oplus B) \ominus B = \{w | (B)_z \cap A \neq \emptyset, w \in (B)_z\}$



a b c

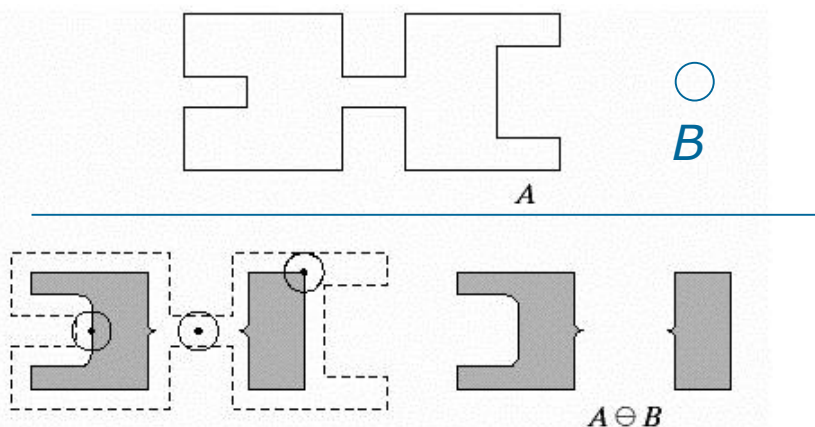
FIGURE 9.9 (a) Structuring element B “rolling” on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).



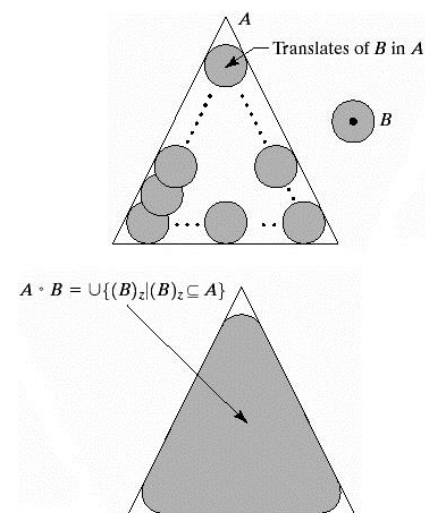
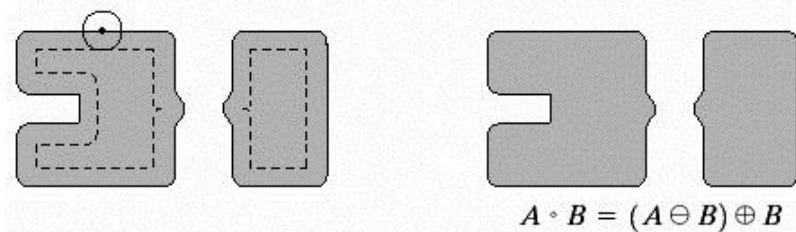
## 9.3 开和闭

✓ **Opening** 示例  $A \circ B = (A \ominus B) \oplus B$

① 先腐蚀



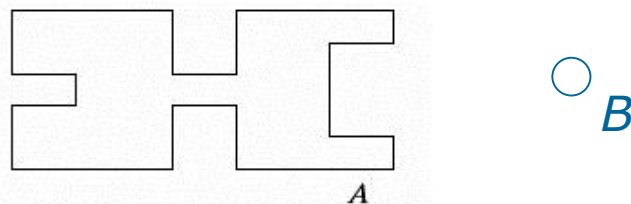
② 再膨胀



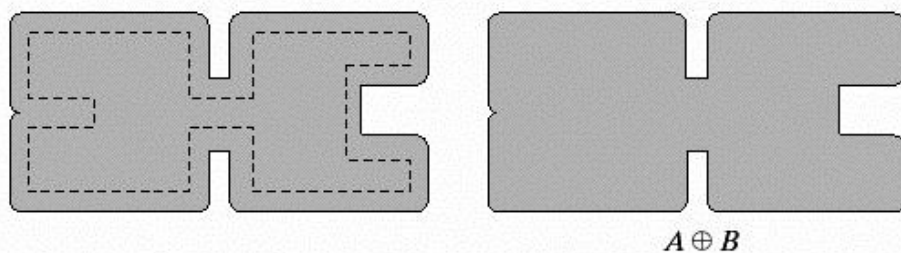


## 9.3 开和闭

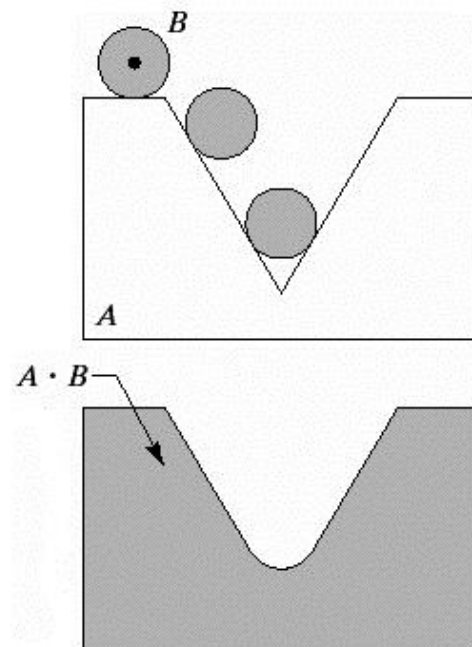
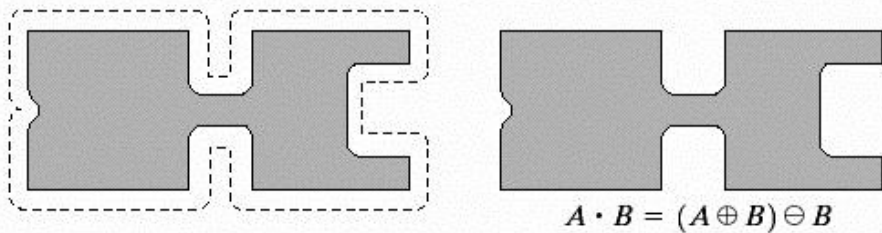
✓ **Closing** 示例  $A \bullet B = (A \oplus B) \ominus B$



① 先膨胀



② 再腐蚀

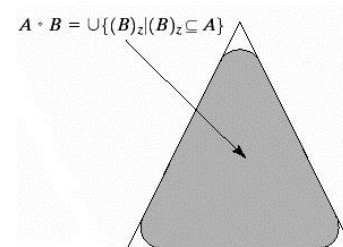
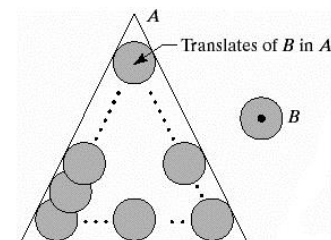




## 9.3 开和闭

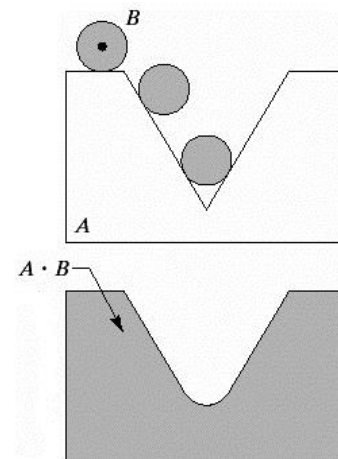
### ✓ Opening的几个特性

- 1)  $A \circ B$  是  $A$  的子集 (子图像)
- 2) 如果  $C$  是  $D$  的子集, 则  $C \circ B$  是  $D \circ B$  的子集
- 3)  $(A \circ B) \circ B = A \circ B$



### ✓ Closing的几个特性

- 1)  $A$  是  $A \bullet B$  的子集 (子图像)
- 2) 如果  $C$  是  $D$  的子集, 则  $C \bullet B$  是  $D \bullet B$  的子集
- 3)  $(A \bullet B) \bullet B = A \bullet B$





## 9.3 开和闭

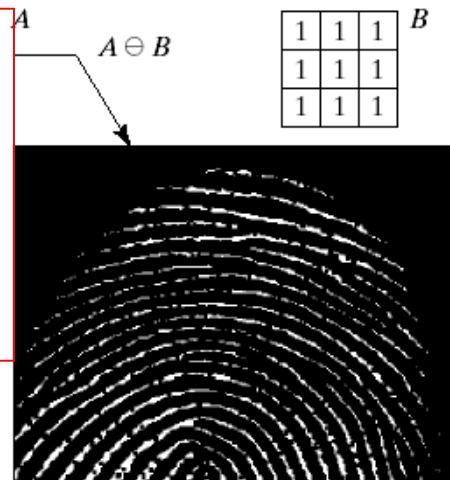
### ✓ Opening和Closing的实际应用举例

原始图像A;

暗背景上亮噪声;

指纹脊线上暗噪声

开了



$$B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$(A \ominus B) \oplus B = A \circ B$$

$$[(A \circ B) \oplus B] \ominus B = (A \circ B) \cdot B$$



$$\begin{bmatrix} a & b \\ d & c \\ e & f \end{bmatrix}$$

FIGURE 9.11

(a) Noisy image.  
(c) Eroded image.  
(d) Opening of  $A$ .  
(d) Dilation of the opening.  
(e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)



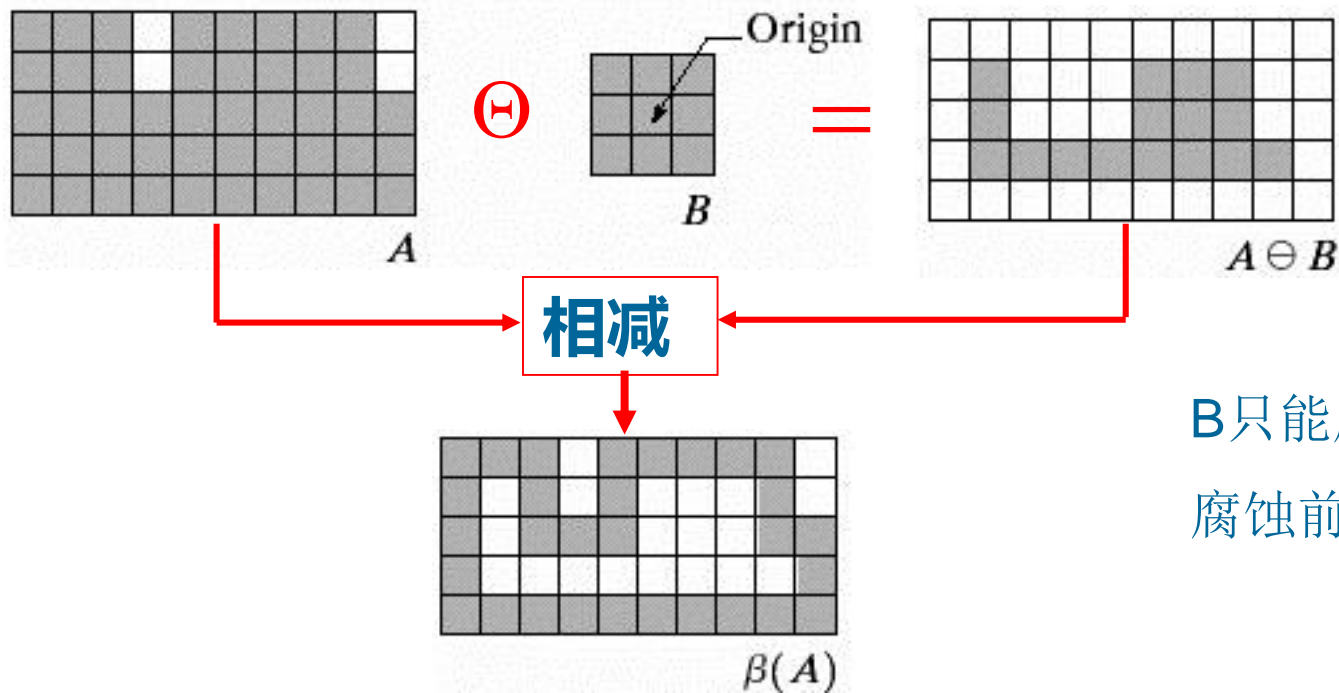
## 9.4 形态学基本算法

### § 9.4.1 边界提取 (boundary extraction)

集合 **A** 的边界  $\beta(A)$  可以由集合 **A** 减去它的腐蚀得到，即

$$\beta(A) = A - (A \ominus B)$$

✓ 示意图



$B$  只能腐蚀  $A$  的边界；  
腐蚀前后的变化就是边界。

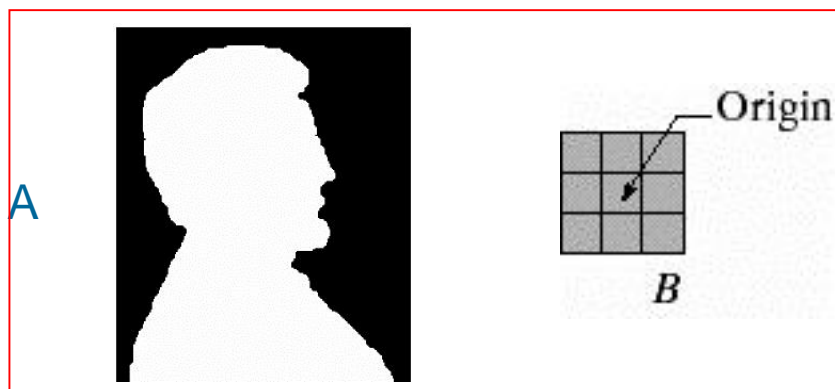


## 9.4.1 边界提取

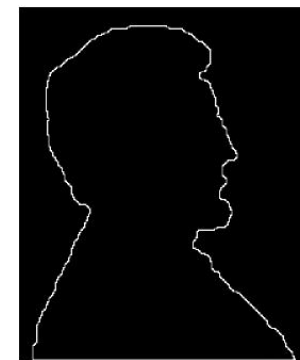
集合A的边界 $\beta(A)$ 可以由集合A减去它的腐蚀得到，即

$$\beta(A) = A - (A \ominus B)$$

### ✓ 实际应用



$$\beta(A) = A - (A \ominus B)$$



如果B是 $5 \times 5$ ，提取的边界有什么变化？



既然是由差别找边界，是否可用膨胀代替腐蚀？

$$\beta(A) = (A \oplus B) - A \quad ?$$



## 9.4.2 空洞填充

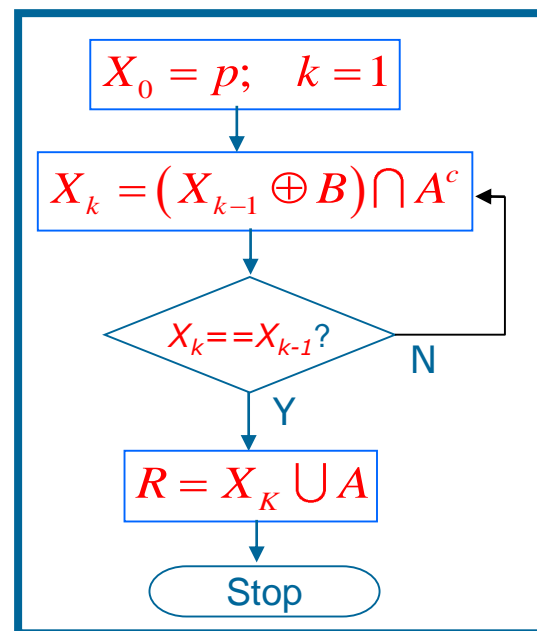
### ■ 边界提取vs.空洞填充

边界提取：给定区域**R**找边界。  $\beta(A) = A - (A \oplus B)$

空洞填充：给定边界找区域**R**。

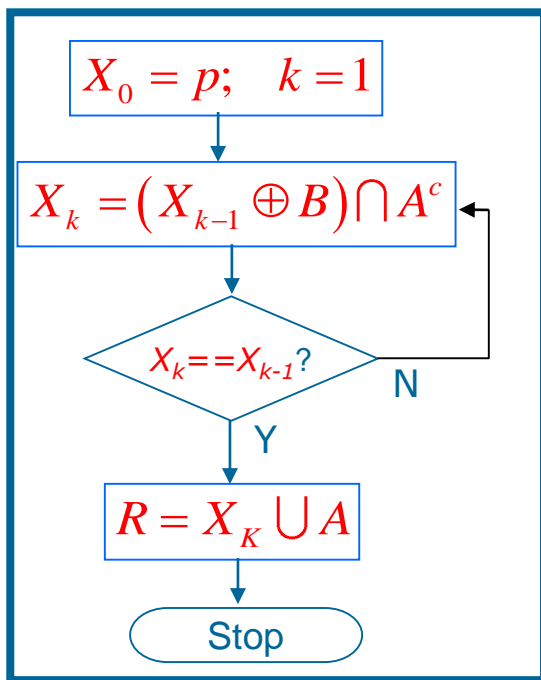
$X_k = (X_{k-1} \oplus B) \cap A^c$  迭代, 直到收敛; 最后  $R = X_K \cup A$

➤ 空洞填充：设某区域的边界是**A**, 设**p**是区域内的一点(不在边界上)并设 **$X_0 = p$** , 选定一个结构元素**B**, 则边界**A**所构成的区域(点的集合)可以通过如下迭代得到：

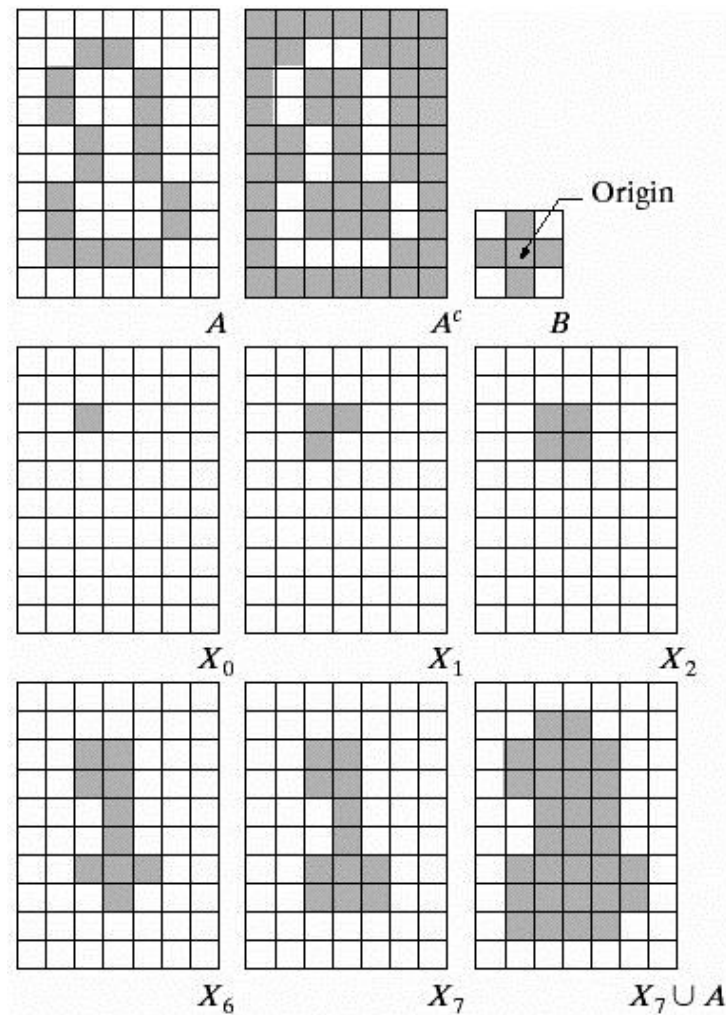


## 9.4.2 空洞填充

### ● 单个空洞填充示例



为什么最后要并A  
呢?  $R = X_7 \cup A$



$$X_1 = (X_0 \oplus B) \cap A^c$$

$$X_2 = (X_1 \oplus B) \cap A^c$$

$$X_3 = (X_2 \oplus B) \cap A^c$$

$$X_4 = (X_3 \oplus B) \cap A^c$$

$$X_5 = (X_4 \oplus B) \cap A^c$$

$$X_6 = (X_5 \oplus B) \cap A^c$$

$$X_7 = (X_6 \oplus B) \cap A^c$$

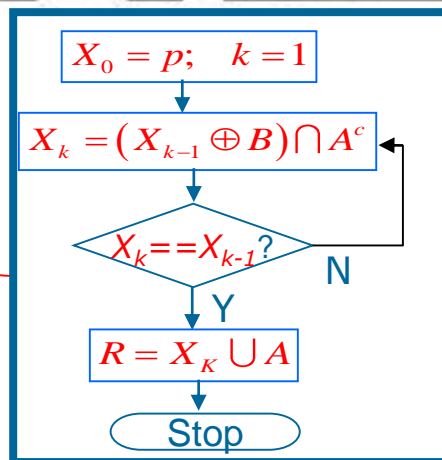
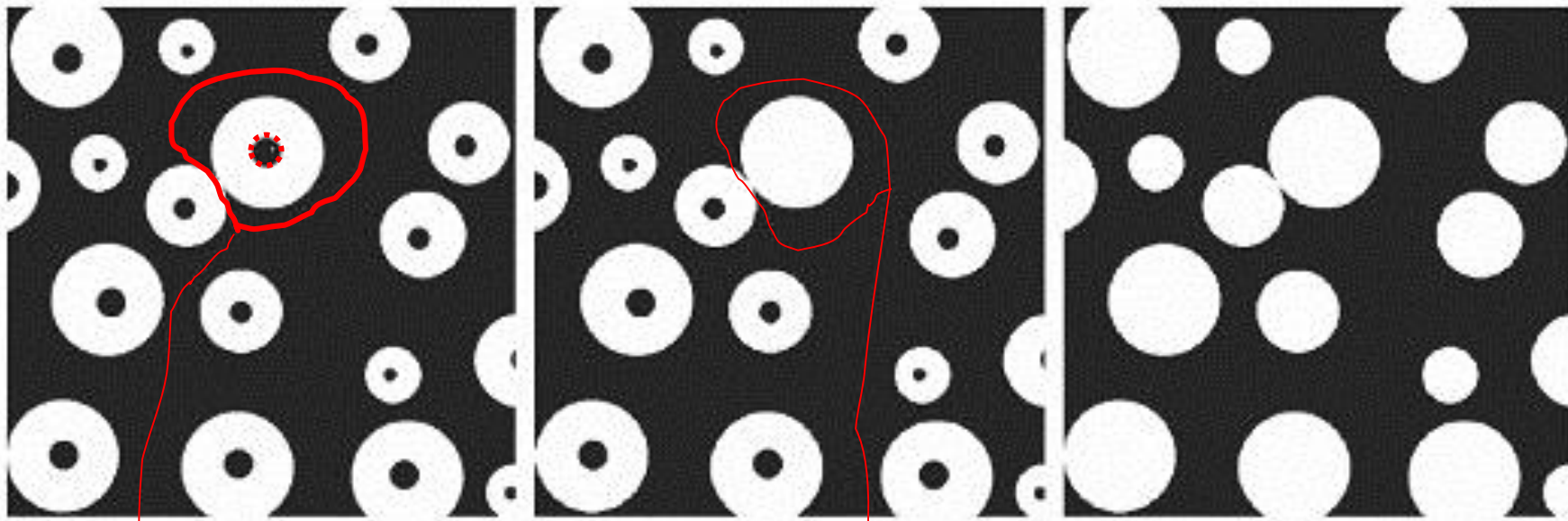
$$X_8 = (X_7 \oplus B) \cap A^c = X_7$$

$$R = X_7 \cup A$$



## 9.4.2 空洞填充

### ● 多个空洞填充示例



**Assumption: A point inside each boundary is given**

## 9.4.3 提取连通区域

**Dilation**

$$X \oplus B$$

**Conditional dilation**

$$(X \oplus B) \cap (\text{condition})$$

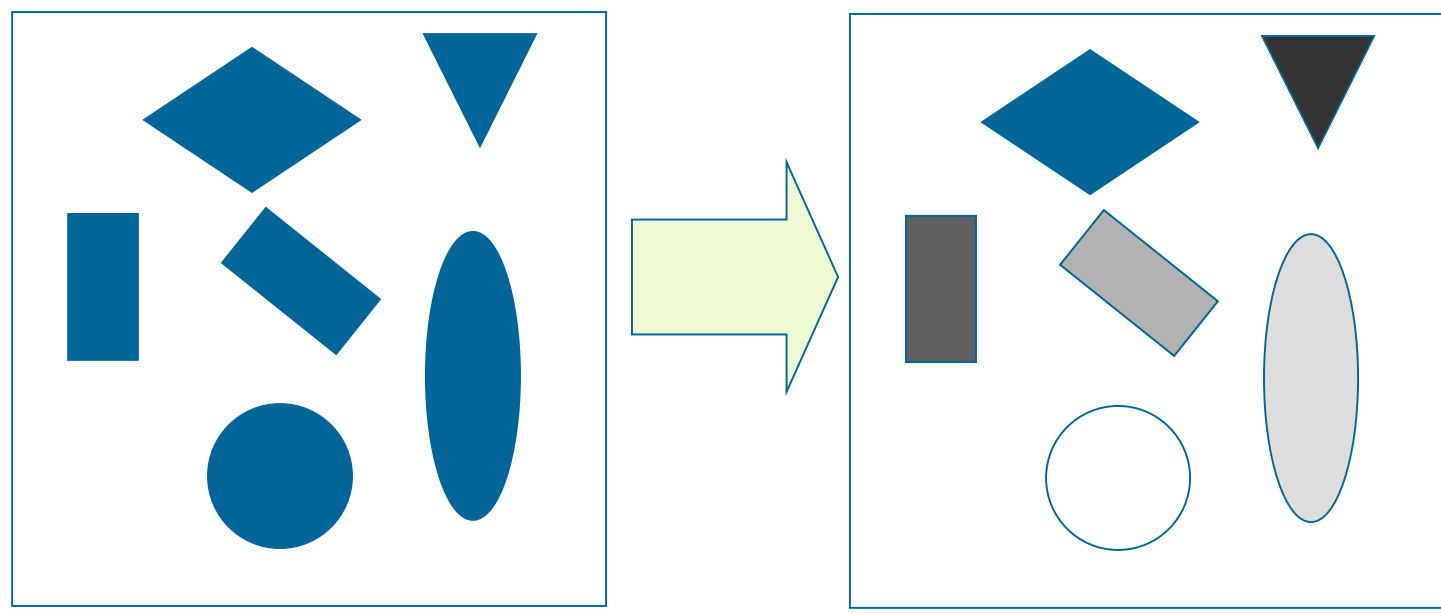
**Region filling**

$$(X \oplus B) \cap A^c$$

**Extraction of connected components**

$$(X \oplus B) \cap (?)$$

 什么是连通区域呢？什么是连通区域的提取？提取连通区域有什么用途？



原始图像：6个不同的连通区域(物体)

Connected component labeling

 如何提取？



## 9.4.3 提取连通区域

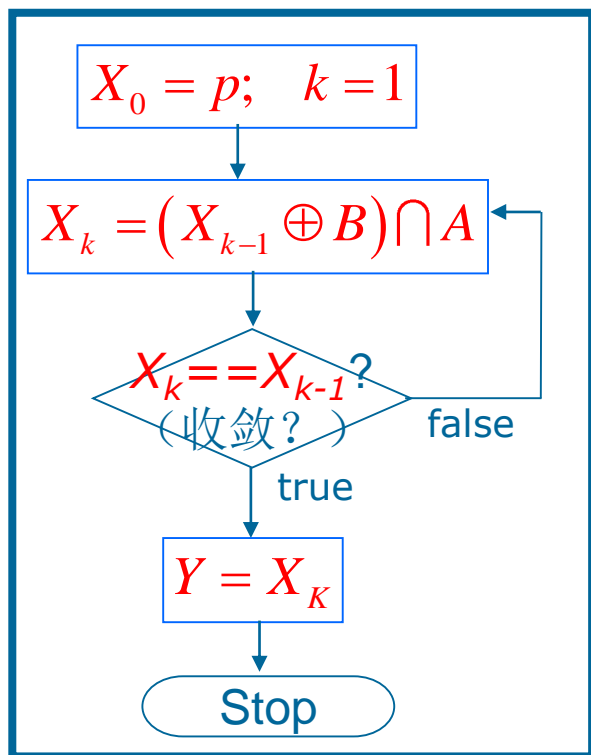


设由集合 **A** 表示的连通区域为 **Y**. 如果已知 **Y** 上的一点 **p**, 则连通区域 **Y** 中的所有元素可由如下的迭代公式求得：

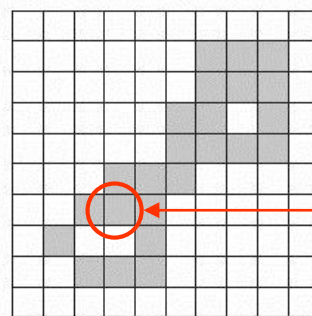
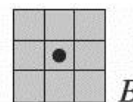
$$X_k = (X_{k-1} \oplus B) \cap A$$

where  $X_0 = p, k = 1, 2, 3, \dots$

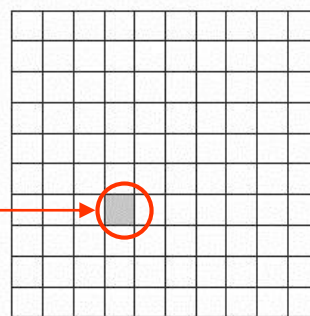
➤ 流程图



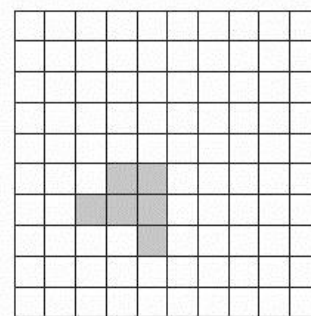
➤ 示例



A

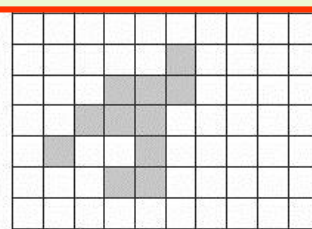


$X_0$

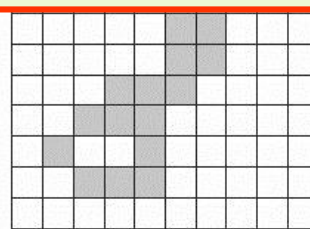


$X_1$

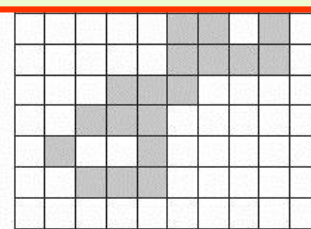
子曰：星星之火，可以燎原！



$X_2$



$X_3$



$X_6$



## 9.4.3 提取连通区域

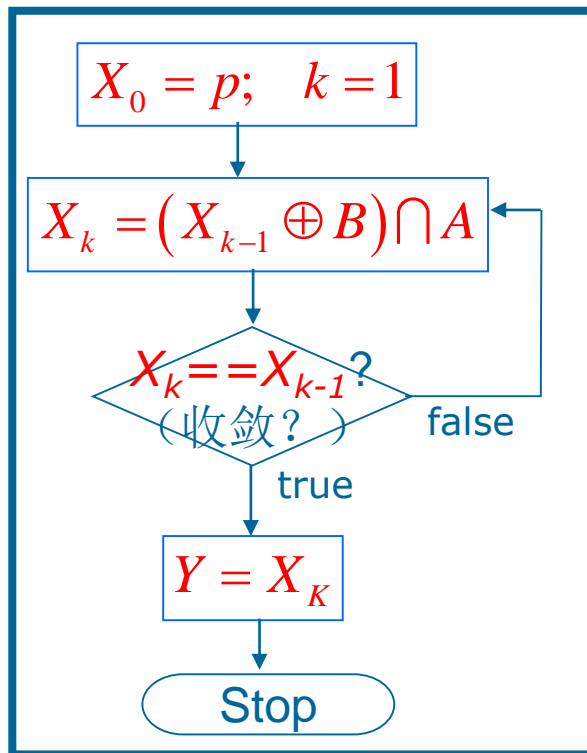


设由集合 **A** 表示的连通区域为 **Y**. 如果已知 **Y** 上的一点 **p**, 则连通区域 **Y** 的所有元素可以如下的迭代公式求得:

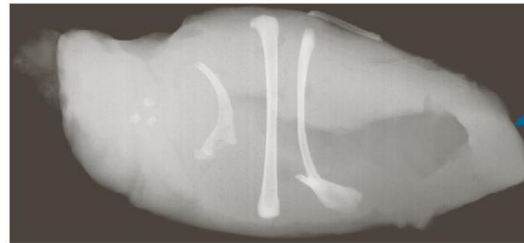
$$X_k = (X_{k-1} \oplus B) \cap A$$

where  $X_0 = p, k = 1, 2, 3, \dots$

### 流程图



### 实际应用



X-ray image of chicken file with broken bone fragments



Threshold image (binary)



Image eroded with a 5\*5 structuring element

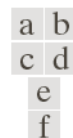
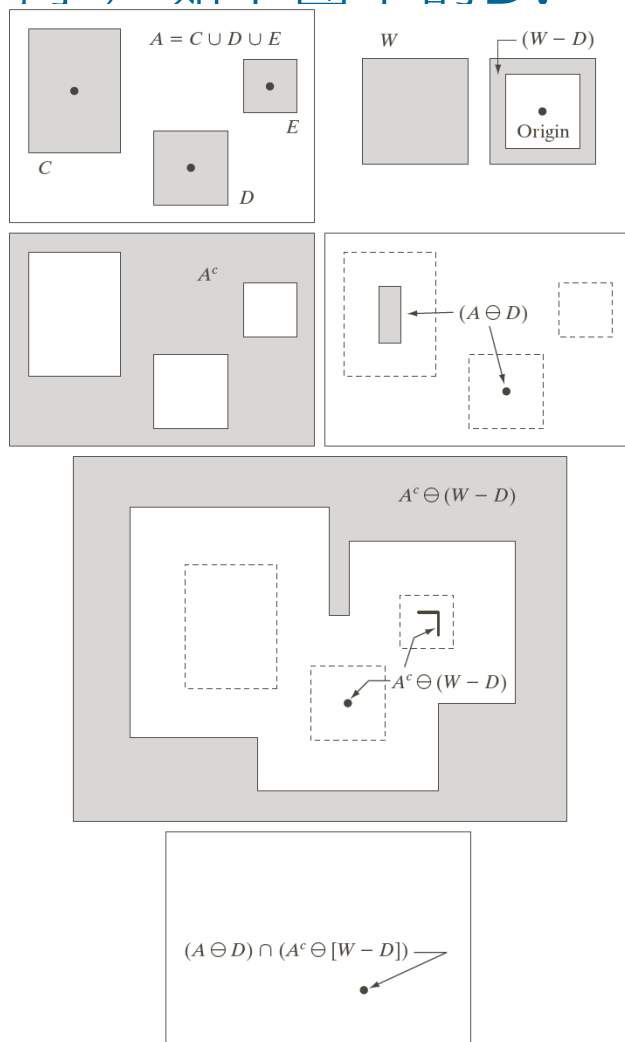
Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

$$X_k = (X_{k-1} \oplus B) \cap A$$



# 击中击不中变换

形态学击中与否变换是形状检测的基本工具。先看一个形状定位的例子，如下图中的**D**：



**FIGURE 9.12**

(a) Set  $A$ . (b) A window,  $W$ , and the local background of  $D$  with respect to  $W$ ,  $(W - D)$ .

(c) Complement of  $A$ . (d) Erosion of  $A$  by  $D$ .

(e) Erosion of  $A^c$  by  $(W - D)$ .

(f) Intersection of (d) and (e), showing the location of the origin of  $D$ , as desired. The dots indicate the origins of  $C$ ,  $D$ , and  $E$ .

在各个操作步骤中，图 (d)中A被D腐蚀的结果可以看作D的所有原点位置的集合，在这些点上，D从A中发现了一次匹配，或者说D击了一次A。

同样，图 (e)可以看作D的背景击中A所得到的集合。





什么是（一次）细化？  $A \otimes B$

When thinning, a part of the boundary of the object is subtracted from it by the set difference operation.



给定集合A,设计一个使A只在边界上和结构元素B击中的结构元素B,则结构元素B对A的细化为：

$$A \otimes B = A - (A(*)B)$$

即细化是通过去掉物体A的边界来实现的。



通常的细化指多次细化：

序列结构元素  $\{B\} = \{B^1, B^2, B^3, \dots, B^n\}$ , 集合A的细化结果为：

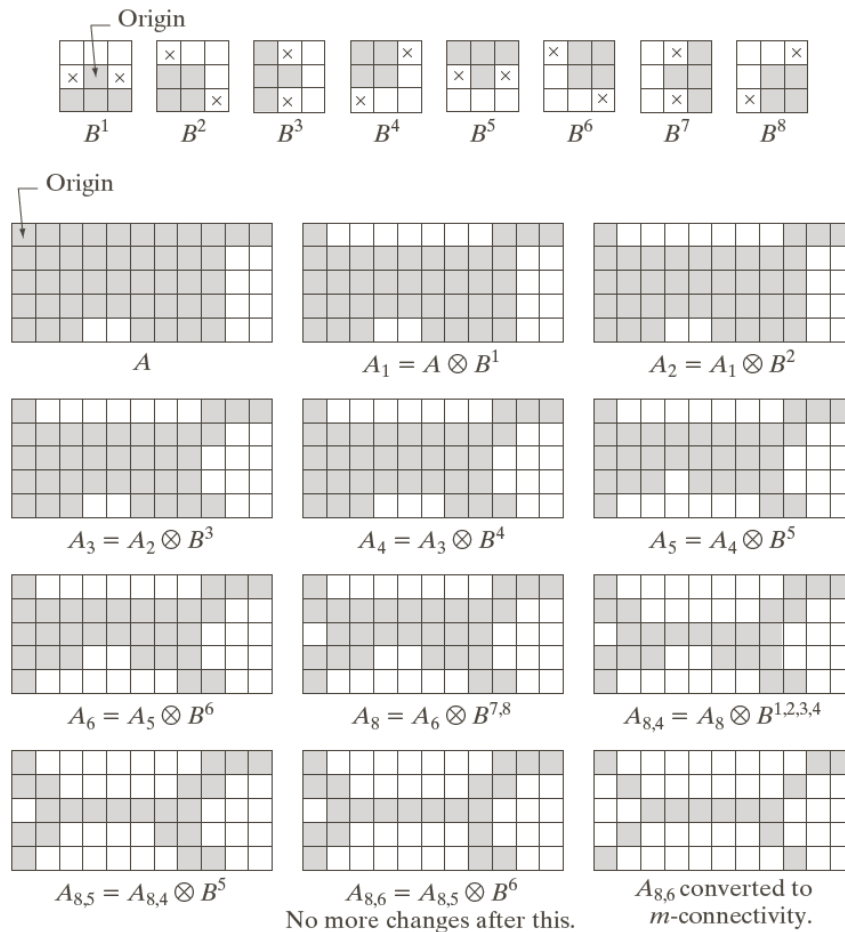
$$A \otimes \{B\} = \left( \left( \left( A \otimes B^1 \right) \otimes B^2 \right) \cdots \right) \otimes B^n$$



## 9.4.4 细化



## 细化过程示例



$$A \otimes B = A - (A(*)B)$$

$$A \otimes \{B\} = (((A \otimes B^1) \otimes B^2) \cdots) \otimes B^n$$

即细化是通过去掉物体A的边界来实现的。

**FIGURE 9.21** (a) Sequence of rotated structuring elements used for thinning. (b) Set  $A$ . (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first four elements again. (l) Result after convergence. (m) Conversion to  $m$ -connectivity.

a
b c d
e f g
h i j
k l m



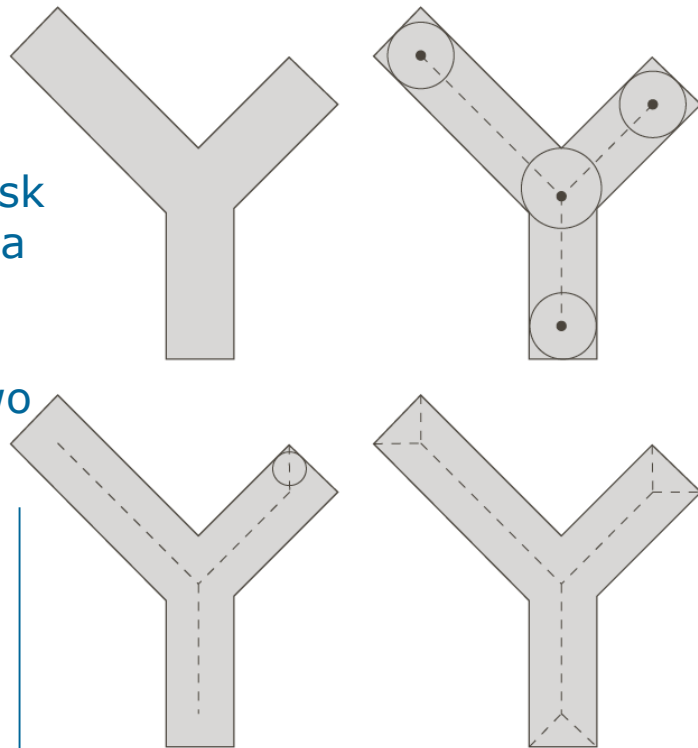
## 9.4.5 骨架



### 什么是骨架？

(a) If  $z$  is a point of  $S(A)$  and  $(D)_z$  is the largest disk centered at  $z$  and contained in  $A$ , one cannot find a larger disk (not necessarily centred at  $z$ )

(b) The disk  $(D)_z$  touches the boundary of  $A$  at two or more different places



集合  $A$  的骨架  $S(A)$  等于 
$$S(A) = \bigcup_{k=0}^K S_k(A)$$

其中

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

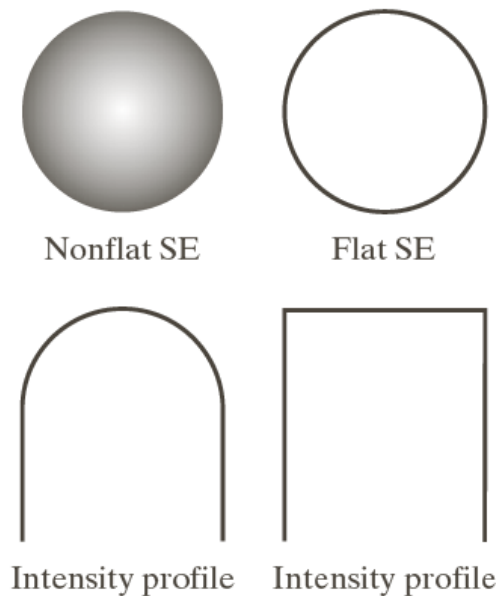
$$A \ominus kB = (\dots(A \ominus B) \ominus B) \ominus \dots) \ominus B$$

$$K = \max \{k \mid (A \ominus kB) \neq \emptyset\}$$

## 9.5 灰度级形态学

### 结构元素

#### 非平坦和平坦结构元素



a	b
c	d

**FIGURE 9.34**

Nonflat and flat structuring elements, and corresponding horizontal intensity profiles through their center. All examples in this section are based on flat SEs.

# 9.5.1 膨胀和腐蚀

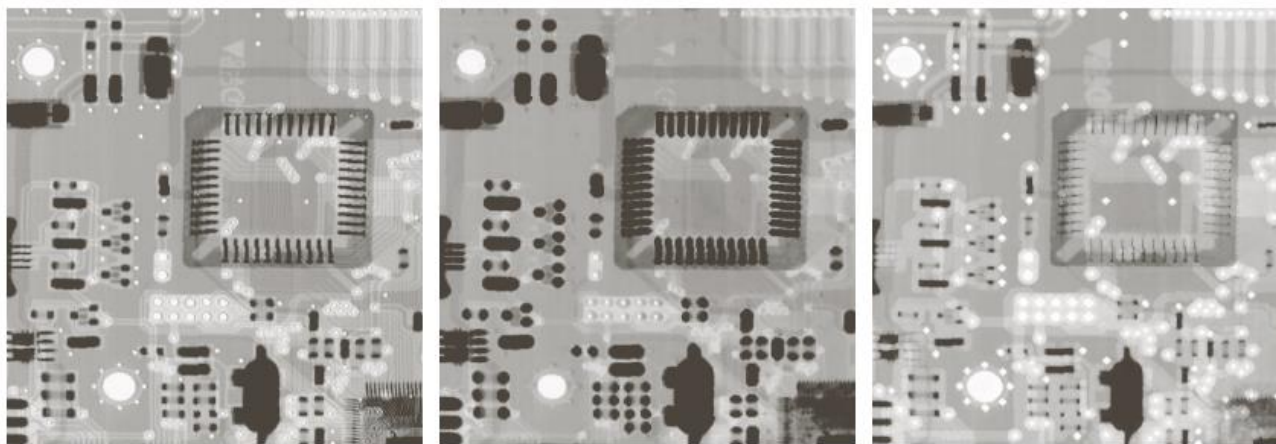
$$[f \oplus b](x, y) = \max_{(s,t) \in b} \{f(x - s, y - t)\}$$

当 $\hat{b}$ 的原点位于 $(x,y)$ 时，用一个平坦的结构元素 $b$ 在 $(x,y)$ 处对图像 $f$ 的膨胀定义为图像 $f$ 中与 $\hat{b}$ 重合区域的最大值。

$$[f \ominus b](x, y) = \min_{(s,t) \in b} \{f(x + s, y + t)\}$$

当 $b$ 的原点位于 $(x,y)$ 时，用一个平坦的结构元素 $b$ 在 $(x,y)$ 处对图像 $f$ 的腐蚀定义为图像 $f$ 中与 $b$ 重合区域的最小值。

## 9.5.1 膨胀和腐蚀



a b c

**FIGURE 9.35** (a) A gray-scale X-ray image of size  $448 \times 425$  pixels. (b) Erosion using a flat disk SE with a radius of two pixels. (c) Dilation using the same SE. (Original image courtesy of Lixi, Inc.)



## 9.5 灰度级形态学

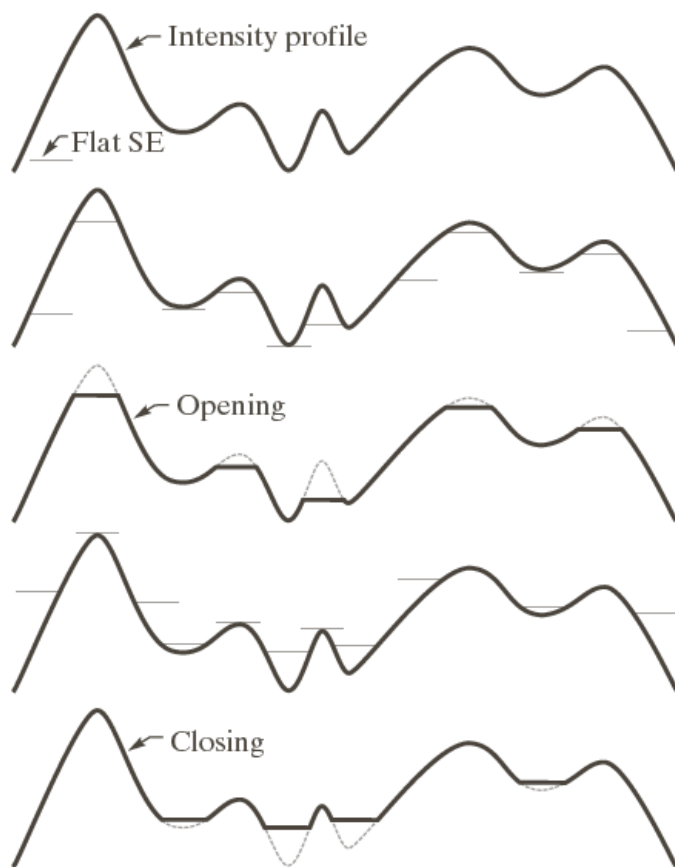
### 9.5.2 开操作和闭操作

$$f \circ b = (f \ominus b) \oplus b$$

$$f \bullet b = (f \oplus b) \ominus b$$

开操作：当从f的下表面向上推动结构元素时，b的任何部分所到达的最高值。常用于去除较小的明亮细节，而保持整体灰度级和较大的明亮特征相对不变。

闭操作：当结构元素从曲线的上侧滑动时，通过寻找结构元素的任何部分所到达的最低值。常用于去除较小的暗细节，而保持整体灰度级和明亮特征相对不变。



a  
b  
c  
d  
e

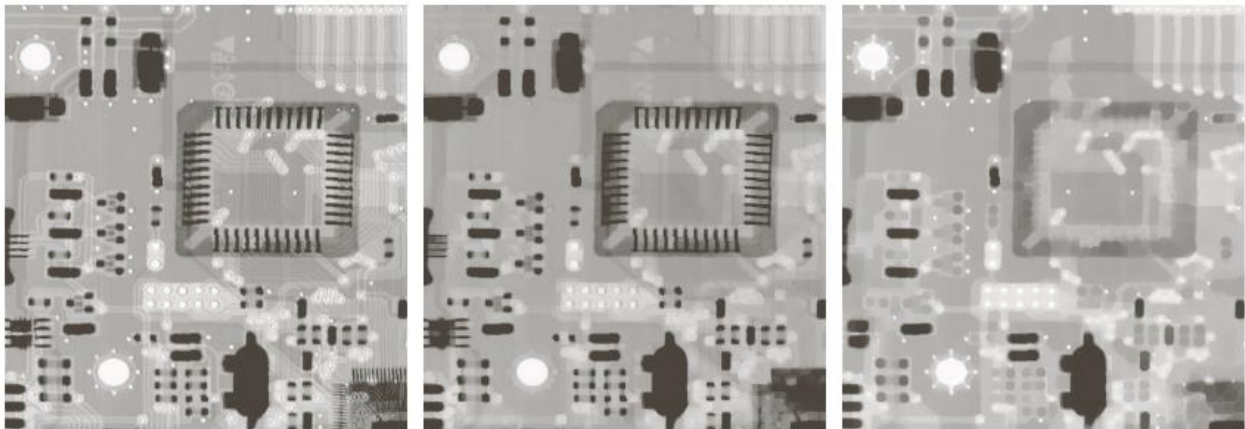
**FIGURE 9.36**

Opening and closing in one dimension. (a) Original 1-D signal. (b) Flat structuring element pushed up underneath the signal. (c) Opening. (d) Flat structuring element pushed down along the top of the signal. (e) Closing.

## 9.5.2 开操作和闭操作

$$f \circ b = (f \ominus b) \oplus b$$

$$f \bullet b = (f \oplus b) \ominus b$$



a

b

c

**FIGURE 9.37** (a) A gray-scale X-ray image of size  $448 \times 425$  pixels. (b) Opening using a disk SE with a radius of 3 pixels. (c) Closing using an SE of radius 5.

性质：
$$(f \circ b) \circ b = f \circ b$$

$$(f \bullet b) \bullet b = f \bullet b$$



## 9.5 灰度级形态学

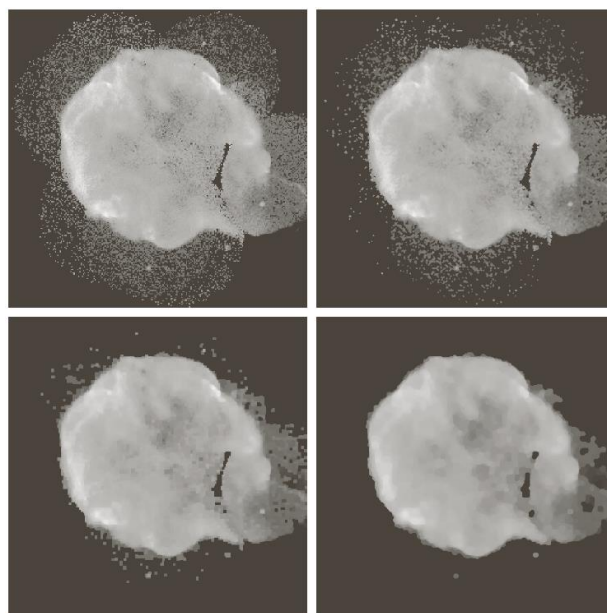
### 9.5.3 一些基本的灰度级形态学算法

$$f \circ b = (f \ominus b) \oplus b$$

$$f \bullet b = (f \oplus b) \ominus b$$

#### 1、形态学平滑

开操作抑制比结构元素小的亮细节，闭操作抑制暗细节，所以常常相互结合用于平滑图像或滤除噪声



a b  
c d

**FIGURE 9.38**

(a)  $566 \times 566$  image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope. (b)–(d) Results of performing opening and closing sequences on the original image with disk structuring elements of radii, 1, 3, and 5, respectively. (Original image courtesy of NASA.)





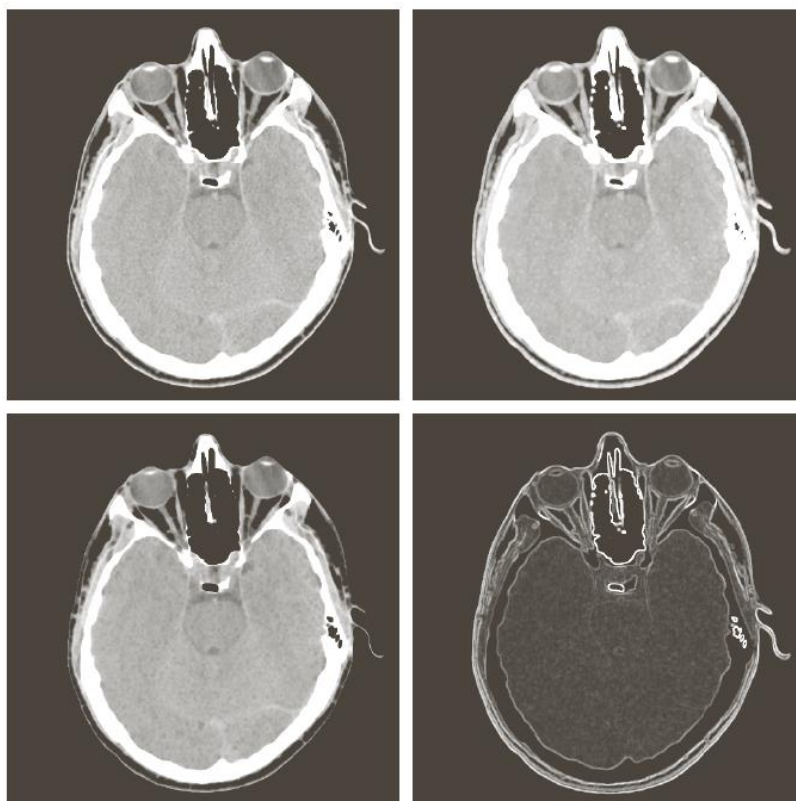
## 9.5 灰度级形态学

### 9.5.3 一些基本的灰度级形态学算法

#### 2、形态学梯度

$$g = (f \oplus b) - (f \ominus b)$$

增强边缘，抑制同质区域



a	b
c	d

**FIGURE 9.39**

(a)  $512 \times 512$  image of a head CT scan.  
(b) Dilation.  
(c) Erosion.  
(d) Morphological gradient, computed as the difference between (b) and (c). (Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)



## 9.5 灰度级形态学

### 9.5.3 一些基本的灰度级形态学算法

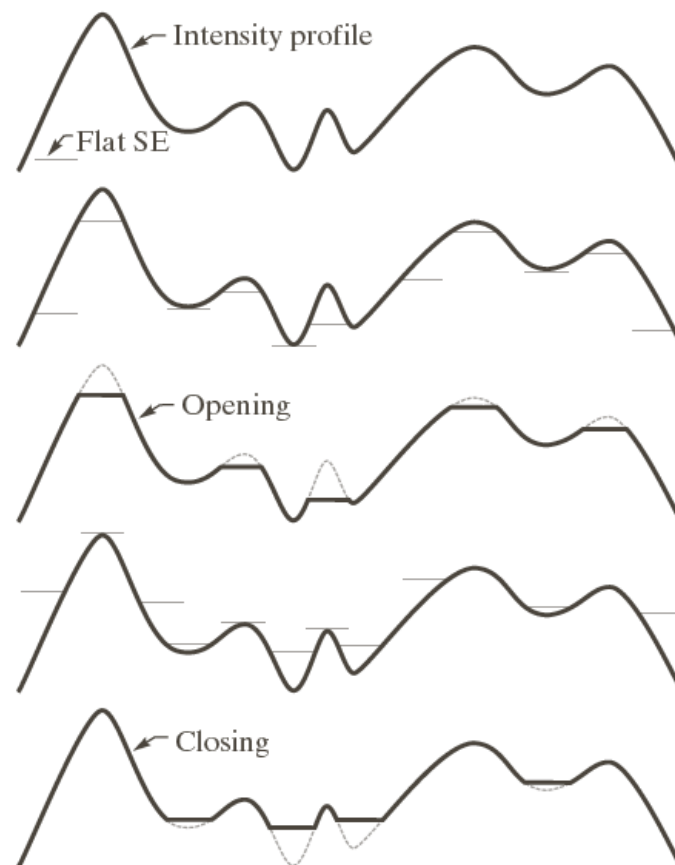
#### 3、顶帽变换和底帽变换

$$T_{hat} = f - (f \circ b) \quad B_{hat} = (f \bullet b) - f$$

**应用目的：**开闭操作用于从一幅图像中删除物体，差操作则得到一副仅保留已删除分量的图像。

顶帽变换用于暗背景上的亮物体——**白顶帽变换**

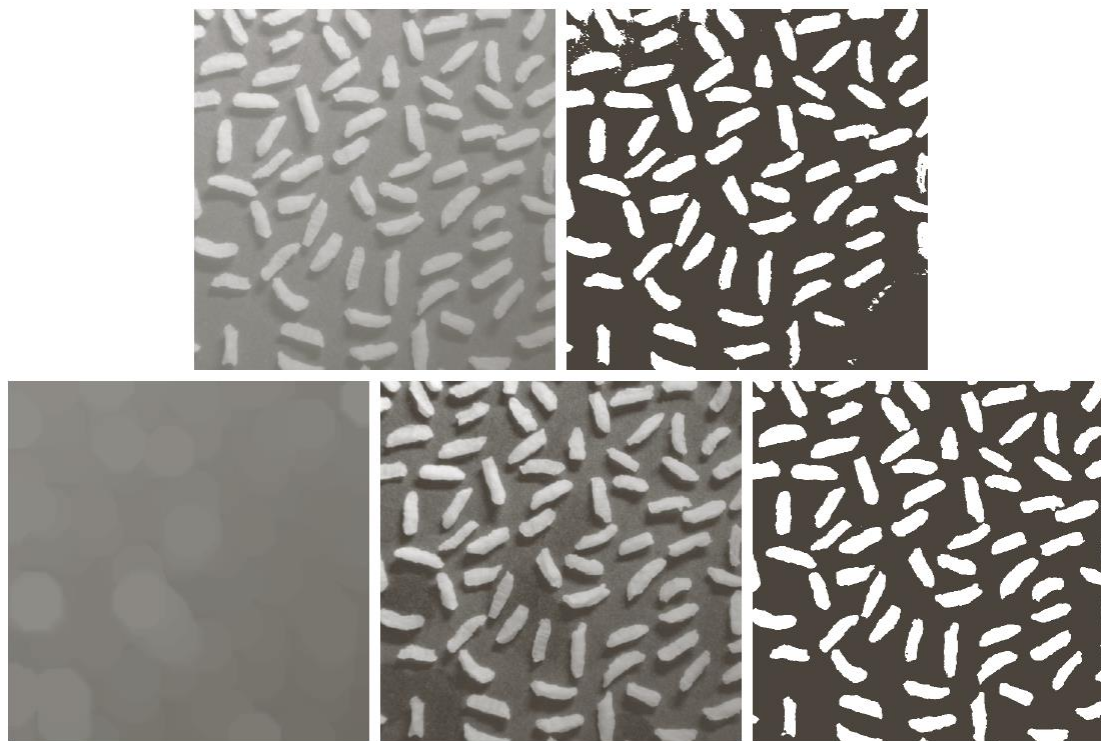
底帽变换用于亮背景上的暗物体——**黑底帽变换**



## 9.5 灰度级形态学

### 9.5.3 一些基本的灰度级形态学算法

#### 3、顶帽变换的应用(校正不均匀光照的影像)



a b  
c d e

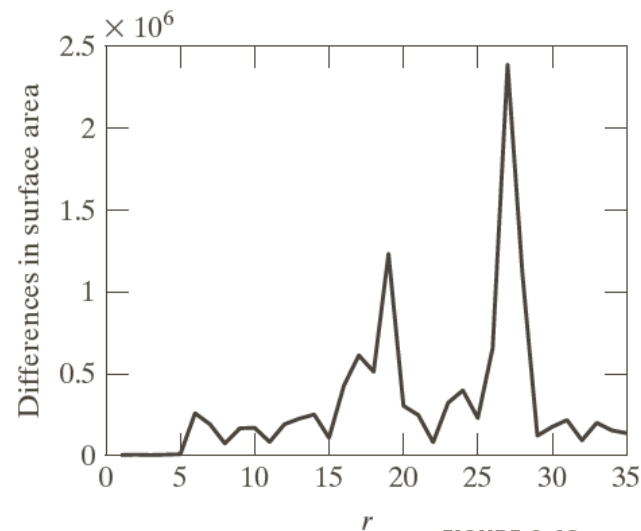
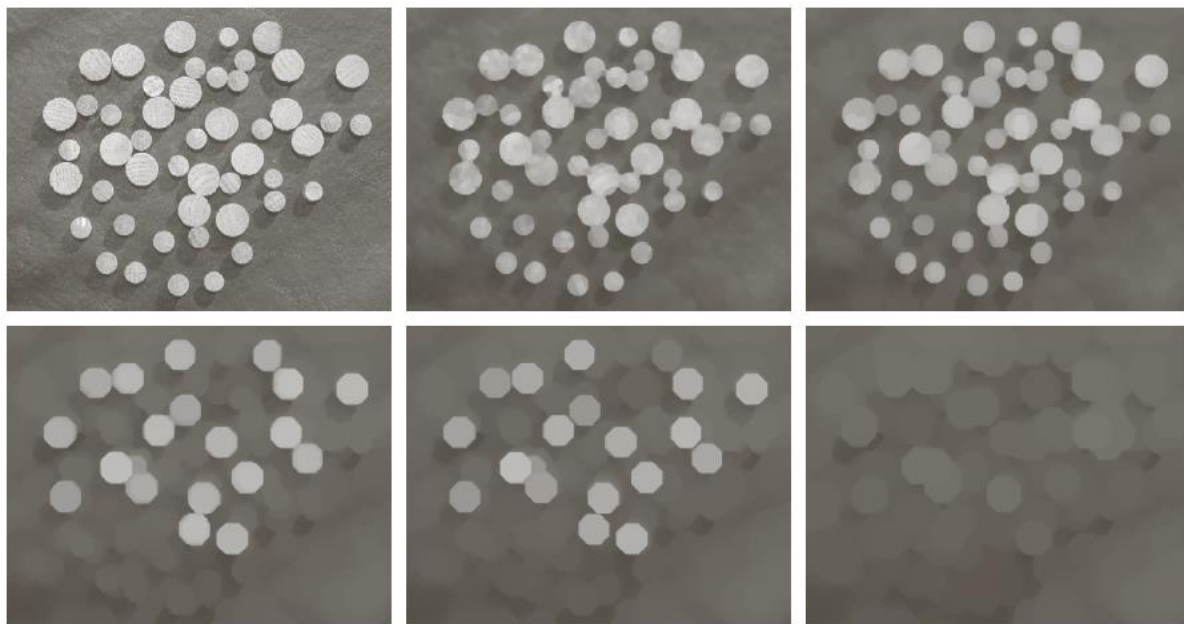
**FIGURE 9.40** Using the top-hat transformation for *shading correction*. (a) Original image of size  $600 \times 600$  pixels. (b) Thresholded image. (c) Image opened using a disk SE of radius 40. (d) Top-hat transformation (the image minus its opening). (e) Thresholded top-hat image.



## 9.5 灰度级形态学

### 9.5.3 一些基本的灰度级形态学算法

#### 4、粒度测定：用于判断图像中颗粒的尺寸分布



**FIGURE 9.42** Differences in surface area as a function of SE disk radius,  $r$ . The two peaks are indicative of two dominant particle sizes in the image.

a	b	c
d	e	f

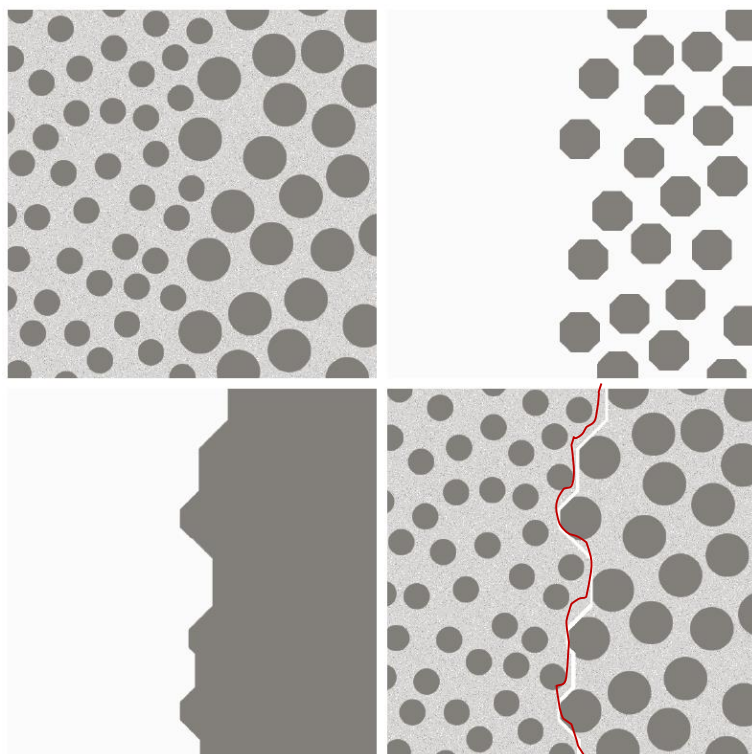
**FIGURE 9.41** (a)  $531 \times 675$  image of wood dowels. (b) Smoothed image. (c)–(f) Openings of (b) with disks of radii equal to 10, 20, 25, and 30 pixels, respectively. (Original image courtesy of Dr. Steve Eddins, The MathWorks, Inc.)



## 9.5 灰度级形态学

### 9.5.3 一些基本的灰度级形态学算法

#### 5、纹理分割：以纹理内容为基础找到两个区域的边界



a b  
c d

**FIGURE 9.43**

Textural segmentation.  
(a) A  $600 \times 600$  image consisting of two types of blobs. (b) Image with small blobs removed by closing (a). (c) Image with light patches between large blobs removed by opening (b). (d) Original image with boundary between the two regions in (c) superimposed. The boundary was obtained using a morphological gradient operation.

# 本章小结



**J. Serra.** *Image Analysis and Mathematical Morphology.* Academic Press, 1982.

膨胀和腐蚀  
Dilation & erosion

推导

开 (opening)

闭 (closing)

边界提取

区域填充 (region filling)

提取联通区域 (connected components)

提取骨架 (Skeleton)