

$$2.1 \quad Q = \sum_{i=1}^n (y_i - \beta x_i)^2$$

$$\text{F.O.C: } \frac{\partial Q}{\partial \beta} = -2 \sum_{i=1}^n x_i (y_i - \hat{\beta} x_i) = 0$$

$$\Rightarrow \sum_{i=1}^n (x_i y_i - \hat{\beta} x_i^2) = 0$$

$$\Rightarrow \sum_{i=1}^n x_i y_i = \hat{\beta} \sum_{i=1}^n x_i^2$$

$$\Rightarrow \hat{\beta} = \frac{\sum x_i y_i}{\sum x_i^2}$$

$$2.3 \quad Q = \sum_{i=1}^n w_i (y_i - \beta x_i)^2$$

$$\text{F.O.C: } \frac{\partial Q}{\partial \beta} = -2 \sum_{i=1}^n w_i x_i (y_i - \hat{\beta} x_i) = 0$$

$$\Rightarrow \sum_{i=1}^n (w_i x_i y_i - \hat{\beta} w_i x_i^2) = 0$$

$$\Rightarrow \sum_{i=1}^n w_i x_i y_i = \sum_{i=1}^n \hat{\beta} w_i x_i^2$$

$$\Rightarrow \hat{\beta} = \frac{\sum w_i x_i y_i}{\sum w_i x_i^2}$$

$$2.8 \quad y - 69 = r(x - 68)$$

$$\text{When } r = 0.25: \quad y_{\text{tall}} - 69 = 0.25 \cdot 4 = 1 \Rightarrow y_{\text{tall}} = 70$$

$$y_{\text{short}} - 69 = 0.25 \cdot (-4) = -1 \Rightarrow y_{\text{short}} = 68$$

$$\text{When } r = 0.75: \quad y_{\text{tall}} - 69 = 0.75 \cdot 4 = 3 \Rightarrow y_{\text{tall}} = 72$$

$$y_{\text{short}} - 69 = 0.75 \cdot (-4) = -3 \Rightarrow y_{\text{short}} = 66$$

Thus, as  $r$  increases,  $y$  for the same level of  $x$  moves further away from  $\bar{y}$