The goal of this assignment was to analyze the quadratic formula (1) and its associated errors.

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad and \quad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \qquad (1)^*$$

One contributor of such error is cancellation error; this occurs when b is sufficiently large compared to *a* and *c*. When evaluated in the first equation, the terms in the numerator essentially *cancel* each other out, leaving a very small number that has lost significant figures. To avoid cancellation error, the formula is refactored to provide these new equations that are resistant to cancellation error (2) (3).

$$x_1 = -\frac{2c}{b + \sqrt{b^2 - 4ac}} \quad and \quad x_2 = -\frac{2c}{b - \sqrt{b^2 - 4ac}} \qquad (2)\ \&\ (3)^*$$

The equations above are still sensitive to cancellation error but are opposite in sign to the equations in (1). The choice of which equation to use depends on the sign of b. If b is positive, (2) will eliminate cancellation error, and if b is negative, (3) will eliminate cancellation error.

To demonstrate the ideas above and to compare absolute and relative errors, I wrote a simple script in MATLAB. The script computes the roots of a given equation using "infinite" significant digits, then again with 4 significant digits. The absolute and relative errors could be calculated and analyzed using the following equations.

$$err_{abs} = |x - fl(x)| \qquad (4)^*$$

$$err_{rel} = \frac{|x - fl(x)|}{|x|} \qquad (5)^*$$

To reduce error as much as possible, the script determines which equations to use based upon the sign of *b,* using the general rule stated above. In creating this script, any roots to a quadratic equation can be quickly and accurately be approximated. In addition to the approximated roots, both the relative and absolute errors are displayed, allowing for more in-depth analysis of the approximation.

Answers to Question 16 from chapter 1.2* (Found using my MATLAB script)

16	Real Root	Approx. Root	Absolute Error	Relative Error
a)	1.9023	1.9030	6.5352e-4	3.4353e-4
	7.4340e-1	7.4300e-1	4.0483e-4	5.4456e-4
b)	-7.8409e-2	-7.8400e-2	8.7938e-6	1.1215e-4
	-4.0596	-4.0600	3.8027e-4	9.3672e-5
c)	1.2229	1.2230	1.2977e-4	1.0612e-4
	-2.2229	-2.2230	1.2977e-4	5.8380e-5
d)	6.2368	6.2350	1.7591e-3	2.8206e-4
	-3.2068e-1	-3.2080e-1	1.2063e-4	3.7617e-4

**All questions, equations and proofs were taken from Chapter 1.2 of Numerical Analysis, 9th Edition, Burden and Faires*