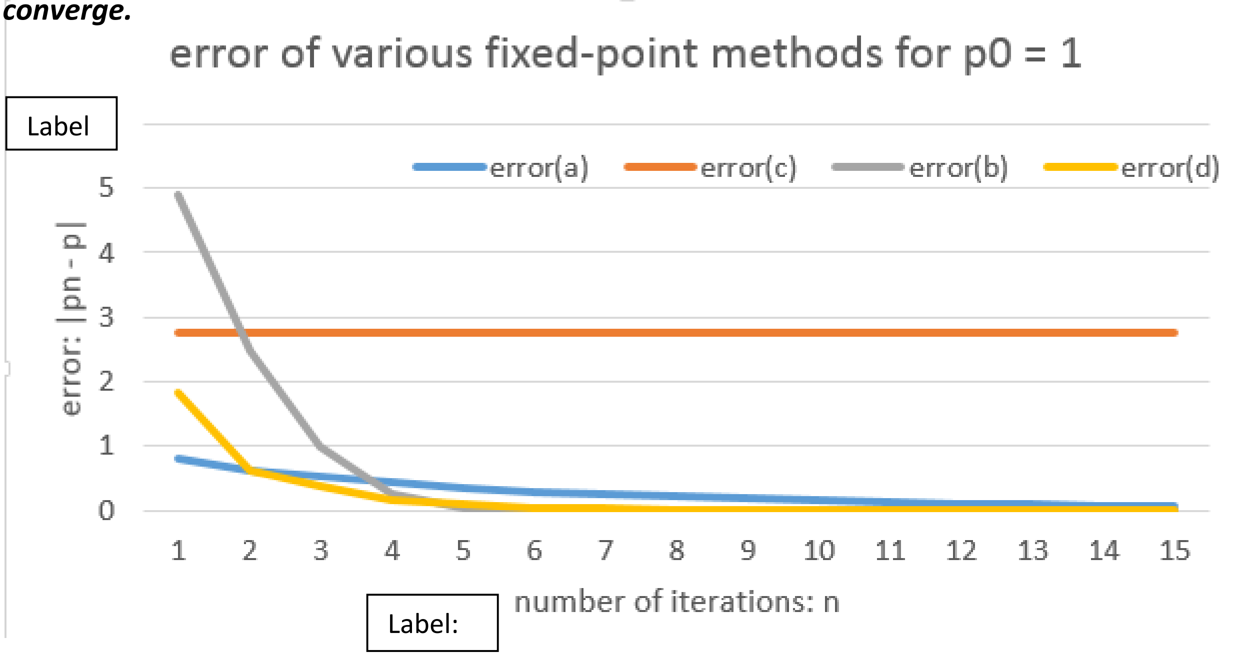
Rank the speed of convergence:

In this assignment, the purpose is to use fixed-point iteration to compute $21^{1/3}$ and rank the certain sequences in order, based on speed of convergence. According to *Numerical analysis tenth edition*, written by RICHARD L. BURDEN, DOUGLAS J. FAIRES, ANNETTE M. BURDEN, (2014), the algorithm2.2 (page59) was used to iterate values and find the approximated results of $21^{1/3}$ by following sequences:

a. $p_n = \dfrac{20p_{n-1} + 21/p_{n-1}^2}{21}$ **b.** $p_n = p_{n-1} - \dfrac{p_{n-1}^3 - 21}{3p_{n-1}^2}$

c. $p_n = p_{n-1} - \dfrac{p_{n-1}^4 - 21p_{n-1}}{p_{n-1}^2 - 21}$ **d.** $p_n = \left(\dfrac{21}{p_{n-1}}\right)^{1/2}$

As been tested before, 15 times of iteration could get almost accurate results of $21^{1/3}$. The results of *<u>(a) = 2.7586, (b) = 2.7589, (d) = 2.7589</u>*, **<u>(c) = invalid because pn will always be 0 and it will never get the solution, so this (c) does NOT converge, and it will NOT be discussed in term of order of convergence</u>**. As the graph shows below, (a), (b), (c) are convergence and ***<u>the order in descending speeds of convergence is (b), (d), (a).</u>*** The sequence in ***(c) does NOT converge.***



Order of convergence:

By the definition 2.7, α is the order and λ is asymptotic error constant. According to the *proof of theorem 2.8* (page 79), "if g' (p) ≠ 0, then $\lim_{n\to\infty} \frac{|p_{n+1} - p|}{|p_n - p|} = |g'(p)|.$ ", and the proof of theorem 2.9 (page 80), "if g' (p) = 0, and g''(p) ≠ 0, then $\lim_{n\to\infty} \frac{|p_{n+1} - p|}{|p_n - p|^2} = \frac{|g''(p)|}{2}$ ". Therefore, the following table represents the orders of convergence α with asymptotic error constant λ.

Q:	(a): p = 2.7586	(b): p = 2.7589	(d): p = 2.7589
g' (p)	***≠0***	***0***	***≠0***
convergence:	***Linearly:***	***quadratically:***	***linearly:***
α:	1	2	1
λ:	0.8571	0.7249	0.5