MACM 316 Computing Assignment 1

Error Approximation for Generic Quadratic Equation

In this assignment the task was to write a Matlab code to solve the general quadratic equation $\mathrm{a}x^2 + \mathrm{b}x + \mathrm{c} = c^2$ using four-digit rounding arithmetic. The two roots can be found using the formulas:

$$x1 = (-b + \sqrt{(b\text{^}2 - 4ac)})/2a \qquad \text{Equation 1.}$$

$$x2 = (-b - \sqrt{(b\text{^}2 - 4ac)})/2a \qquad \text{Equation 2.}$$

Since Matlab can only store a finite number of digits, there is loss of accuracy in our calculation of the quadratic roots due to rounding. We look at two specific cases when we get a poor approximation in one of the two roots. Otherwise we use the above equations.

The first case is when $b^2 \gg= 4 * a * c$, whereby we get a poor approximation of *x1* because the numerator in equation 1 involves the subtraction of nearly equal numbers. To obtain a more accurate four digit approximation for *x1* we change the form of equation 1 by rationalizing the numerator to obtain:

$$x1 = -2c/(-b + \sqrt{(b\text{^}2 - 4ac)})$$

The second case is when $b^2 < 0$, whereby we get a poor approximation of *x2* because the numerator in equation 2 involves the subtraction of nearly equal numbers and also the division by the small result of this subtraction. To obtain a more accurate four digit approximation for *x2* we change the form of equation 2 by rationalizing the numerator to obtain:

$$x2 = -2c/(-b - \sqrt{(b\text{^}2 - 4ac)})$$

I used the Matlab code to solve question 16 in the textbook and found their respective absolute and relative errors as shown in table 1 below.

Table 1:

Questions	*X1*	*Absolute Error(X1)*	*Relative Error(X1)*	*X2*	*Absolute Error(X2)*	*Relative Error(X2)*
A	1.9020	3.46e-4	1.82e-4	0.7432	2.05e-4	2.76e-4
B	-0.07840	9.0e-6	1.12e-4	-4.060	3.80e-4	9.40e-5
C	1.223	1.30e-4	1.06e-4	2.223	1.30e-4	5.8e-5
D	6.237	2.41e-4	3.9e-5	-0.3208	1.21e-4	3.76e4

In the table above question A and D used the second case formulas since they had a negative $b^2 < 0$ coefficient, B used the first case where $b^2 \gg= 4 * a * c$ and C used equation 1 and 2 directly.