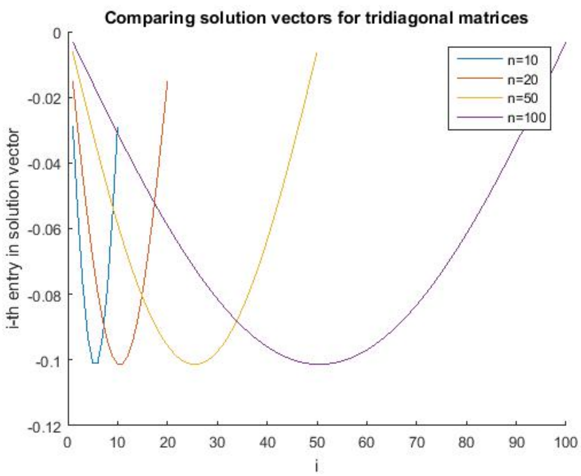
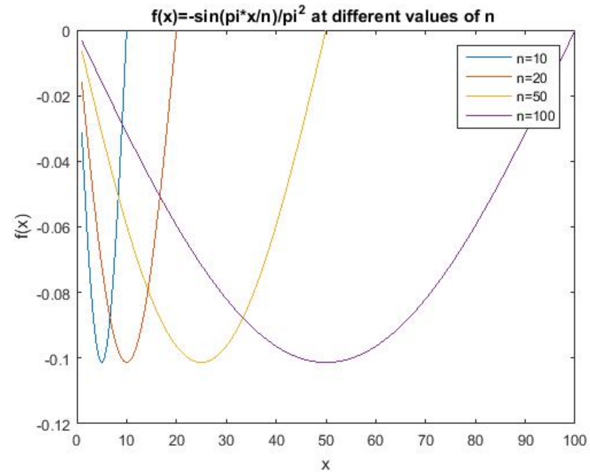
1 Solving tri-diagonal matrices using Crout factorization

In this assignment, I explored A, the $n \times n$ tridiagonal matrix given by $a_{ii} = -2(n+1)^2$, $a_{i,i+1} = a_{i,i-1} = (n+1)^2$ for each $i = 2, ..., n-1$ and $a_{11} = a_{nn} = -2(n+1)^2$, $a_{12} = a_{n,n-1} = (n+1)^2$. And **b**, the n-dimensional column vector given by $b_i = \sin(\pi(\frac{i}{n+1}))$ for each $i = 1, 2, ..., n$. I looked at using Crout factorization to factor them into L*U and finally solving LUx=b.The question to analyze was: what happens to the solution vector, when N gets large?(tends to infinity)



(a) Calculated solution vectors



(b) Plotting $f(x) = -\frac{\sin(\pi x)}{\pi^2}$

x(solution vector)	Min of x	Min of f	Max of x	Max of f
10x1 double	-0.100974364509406	-0.101321183642338	-0.028740300709808	0
20x1 double	-0.101226516202632	-0.101321183642338	-0.015129334058501	0
50x1 double	-0.101305158842937	-0.101321183642338	-0.006239396597221	0
100x1 double	-0.101317098729661	-0.101321183642338	-0.003151328927519	0

The graph on the left shows the solution vectors when solving Ax=b, for different sizes of A and b. The graph on the right shows $f(x) = -\frac{\sin(\pi x/n)}{\pi^2}$ plotted for different values of n. This function is the solution to the boundary value problem $f_{xx} = \sin(\pi x)$, $f(0) = 0$, $f(1) = 0$. I noticed that as n increased, the max and min points of the solution vectors seemed to converge (confirmed by the data in the chart above). Indeed, the min and max points of the solution vector converge to the min and max points given by $f(x) = -\frac{\sin(\pi x)}{\pi^2}$. By $n = 100$ The curves on the left and right seem almost identical (they are plotted on the same axis for comparison).

In conclusion, it seems that the solution vector x, for solving $Ax = b$ converges to $f(u) = -\frac{\sin(\pi u/n)}{\pi^2}$ $u = 1..n$ as n gets large.