Robust Control for Quantum Systems

learning optimal control under noise

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Introduction to quantum system

- simplest quantum system: **qubit**

  classical bit is either 0 or 1, **qubit** can be both.

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examples:

$$\psi_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \psi_2 = \begin{pmatrix} 0.6 \\ 0.8 \end{pmatrix} \quad \psi_3 = \begin{pmatrix} \cos\theta \\ i \sin\theta \end{pmatrix}$$

Introduction to quantum system

evolution of quantum state vector $\psi(t) \in \mathbb{C}^d$:

$$\text{Schrödinger's equation} \quad \dot{\psi}(t) = -iH(t; f(t))\psi(t).$$

- Hamiltonian H: $\mathbb{C}^{d\times d}$, Hermitian, $(t; f(t))$-dependent.
- $f(t)$: reflects our control.

our Hamiltonian model:

$$H(t) = \omega \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} + f(t) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Demonstration of "control":

assume $H(t) := f(t)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, start from $\psi(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

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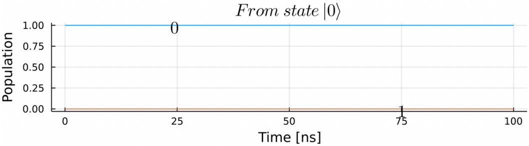
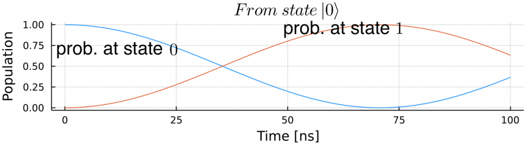
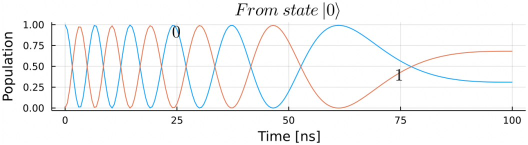
$$f(t) \equiv 0$$



$$f(t) \equiv const$$



$$f(t) = c \cdot (100 - t)^2$$



Control problem

given:

$$\begin{cases} \dot{\psi} = -iH(t; f(t))\psi, \\ \psi(0) = \psi_0 := \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \end{cases}$$

we want:

$$\psi(T) = \psi_{tg} := \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

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loss function:

$$L(f(t)) = 1 - \left|\langle \psi(T), \psi_{tg} \rangle\right|^2$$

Q: when does $L(f) = 0$?

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Noise attack

the optimization problem

$$\min_{f:[0,T]\to\mathbb{R}} L(f)$$

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$$\epsilon \sim \text{Unif}(-\delta, \delta), \qquad H(\epsilon) := H + \epsilon A$$

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noise-aware loss function:

$$L_{NA} = \mathbb{E}^{\epsilon}[e^{L(f;\epsilon)}] = \frac{1}{2\delta}\int_{\epsilon} e^{L(f;\epsilon)} d\epsilon.$$

Why is this challenging?

updated optimization problem

$$\min_{f:[0,T]\to\mathbb{R}} \mathbb{E}^{\epsilon}[e^{\mu L(f;\epsilon)}]$$

$$s.t. \quad \begin{cases} \dot{\psi} = -iH(t;f(t);\epsilon)\psi, \\ \psi(0) = \psi_0. \end{cases}$$

- f in infinite-dim space
- need $\partial_f L$ to update f
- ODE constraint
- integral $\int_\epsilon$ in loss

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We use Juqbox, a Julia package!

- f in infinite-dim space
- need $\partial_f L$ to update f
- ODE constraint
- integral $\int_\epsilon$ in loss

- approximate f by finite basis
- get gradient by adjoint method
- quadrature/Monte-Carlo sampling

Test problem: 1-dim uniform noise

Hamiltonian

$$H(\epsilon) = \omega A + f(t)B + \epsilon C$$

where

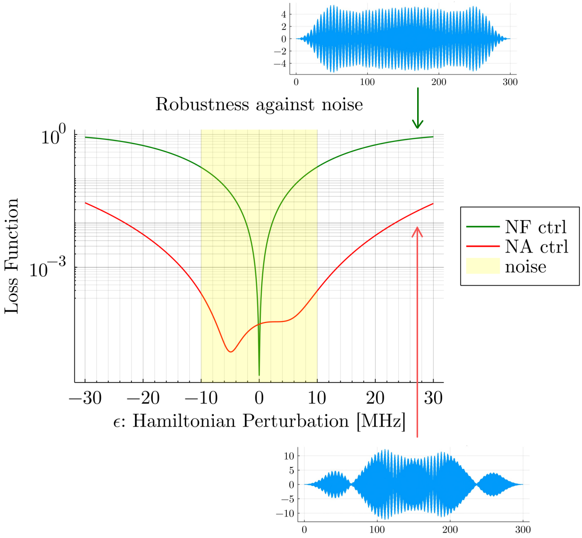
- ωA : natural evolution,
- $f(t)B$: human control,
- ϵC : uniform noise,
- A, B, C are fixed, Hermitian matrices $\in \mathbb{C}^{3\times 3}$.

our goal

start from $\psi(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, arrive at $\psi(T) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.

Comparison between NF and NA methods

- noise-free method is only accurate for $\epsilon \approx 0$;
- noise-aware method behaves robustly under large noise;
- good generalization ability.



Test problem: high-dim noise

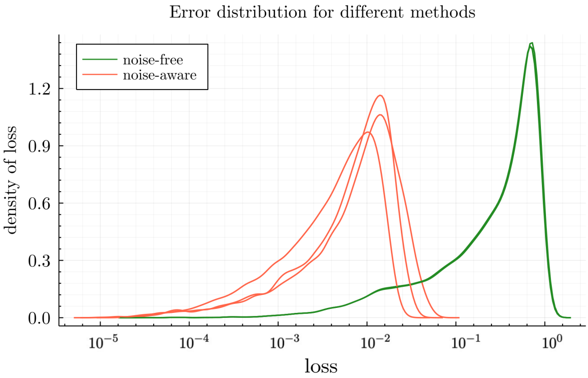
- noise model:

$$\begin{pmatrix} \epsilon_1 & \epsilon_4 & \\ \epsilon_4 & \epsilon_2 & \epsilon_5 \\ & \epsilon_5 & \epsilon_3 \end{pmatrix}$$

$\epsilon_i \sim Unif(-\delta_i, \delta_i)$.

- curse of dimensionality:

–MC method.



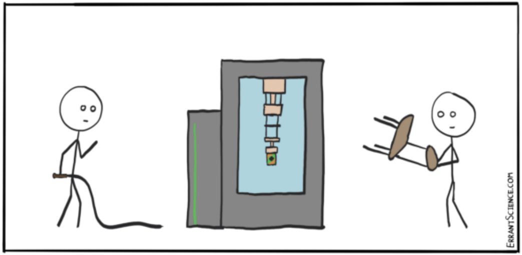
Contribution and future work

contribution:

- implement risk-aware loss functions
- test on 1-dim noise
- test on higher-dim noise

future work:

- N-qubit system
- non-uniform noise
- new loss function
- ...



Two physicists trying to control a qubit in the lab

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