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Problem Chosen

A

2017 Mathematical Contest in Modeling (MCM/ICM) Summary Sheet

Summary

An ant may well destroy an entire dam.

—- The whisper of traditional Chinese wisdom

The Kariba Dam has become a ticking time bomb that could cost lives and plunge the southern African region into darkness at any moment. Its a problem that we should solve without delay. However, dam construction is a controversial issue which contains a variety of aspects, and its essential to assess the dam system in all ways thoroughly before we take action.

First, we give a brief assessment system of the three options listed. We come up with ten factors to evaluate the potential benefit and cost of each option mainly from economic and ecological way. We draw the conclusion below:

1. Repairing the exist dam cost the least, meanwhile brings the least benefit;
2. The cost and benefit of rebuilding the exist dam are both in between;
3. Building a smaller dam system costs the most, but also offers the most benefit.

Second, we set a dam evaluating system model based on the Advanced Analytic Hierarchy Process(AHP), which is the highlight of our work. The process has three steps:

1. We develop the thought in the first task further, sort out six factors, and quantify the factors one by one in a physical, economic or mathematics situation. We achieve this by introducing some parameters and ignoring secondary factors in the problem. Data collection is done by measuring the width of river in the Google Map.
2. Afterwards, we improve the Analytic Hierarchy Process(AHP) by obtaining the weight of each factor not through unreliable supposition but accurate data of the Three Gorges. Through constructing hierarchical structure model, establishing judgment matrix, calculating weight, and testing consistency, which can be achieved in MATLAB, we can get the weight of each factor.
3. Then, we can combine the factors with concerning weight to express the water management capabilities of the dam system. We can use optimization arithmetic to optimize the solution. In the end, we test the sensitivity of it, which proves to be stable.

Third, we establish a Smaller Dams Model based on mathematical foundation and economic principles. However, it only provides a way to calculate and its not that practical due to data shortage. Then we consider the water cycle and flooding. In addition, we give two reasonable suggestions to cope with the extreme situations. Thus, we successfully solve a part of the problem.

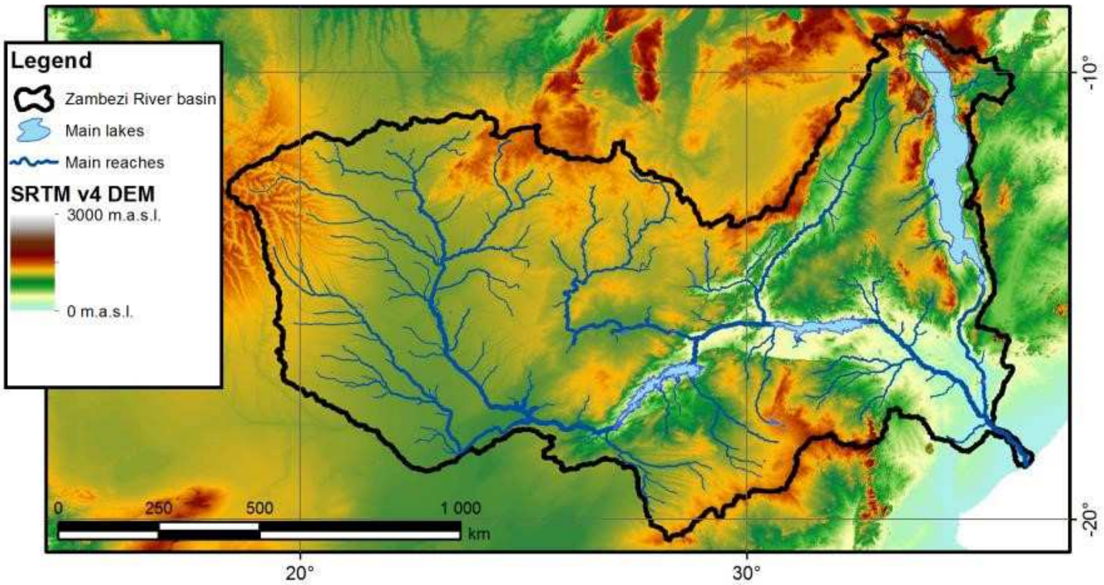
Contents

1 Introduction

1.1 Background of the Problem

1.1.1 The Zambezi River

The Zambezi River is 2,574 kilometers long, rising in Zambia and crosses several countries to empty into the Indian Ocean. The north of the Zambezi basin has mean annual rainfall of 1100 to 1400 mm which declines towards the south, reaching about half that figure in the south-west. The rain falls in a 4-to-6-month summer rainy season between October and March. [1] The Zambezi River can be divided mainly into three major segments, each having a distinctive geomorphological unity. The first of these extends from the headwaters to Victoria Falls; the second from the Falls to the edge of Mozambique coastal plain, which commences below Cahora Bassa Gorge; while the third comprises the stretch traversing the coastal plain. [2]



1.1.2 The Kariba Dam

There are two main sources of hydroelectric power on the Zambezi River, and one of them is the Kariba Dam, which provides power to Zambia and Zimbabwe. The Kariba Dam stands 128 meters tall and 579 meters long. The dam forms Lake Kariba which extends for 280 kilometers and holds 185 cubic kilometers of water. It controls 90% of the total runoff of the Zambezi River, thus changing the downstream ecology dramatically. The construction of dams regulating the flow of the river has had a major effect on wildlife and human populations in the lower Zambezi region. In 2014, the BBC reported that The Kariba Dam is in a dangerous state...engineers are now warning that without urgent repairs, the whole dam will collapse. [3] As its mentioned in the instructions of problem A, the Institute of Risk Management of South Africa warned that the dam is in dire need of maintenance while giving a report in 2015. We can see that the problem of the Kariba Dam is of severe urgency to African people, and it's important to come up with a satisfying solution.

1.2 The Task at Hand

Our first task is to assess the three options listed briefly. Our thought is to evaluate the potential costs and benefits associated with each option from different angels to get an overall assessment of the options. To grasp the essence of the problem and add details to our ideas, collection of relevant literatures and data is also necessary. Its important to refer to the relevant literature for more information about the geographical, ecological, economic and human features of the Zambezi River and the Kariba Dam. We can use analogy method to have an intuitive view of the problem at the same time.

Our second task, which is the main task, is to analyze Option (3) thoroughly and find some ways to modulate water flow for a better balance between safety and costs in the new dam system. We can develop our thought in the first task, using quantitative methods to study the factors in a more accurate way. We can set the variables as the number and placement of the new dams, the condition is that the overall water management capabilities of the whole system remain the same, and the objective function should be the levels of protection and water management options. We can express the factors by variables, and add the factors according to the contribution each makes to the objective function, using optimization arithmetic analysis to settle the problem.

The third task is to take normal water cycles and emergency situations into consideration and provide some practical guidance. This is a specific application of our model set in the second task, which requires us to extend our model to adjust to extreme situations. We can do some numerical test to optimize our strategy, and combine it with real life to make it practical.

1.3 Previous Work

- Interpolation Method(IM)

  Interpolation is a method of constructing new data points within the range of a discrete set of known data points. It is often required to interpolate the value of that function for an intermediate value of the independent variable. This may be achieved by curve fitting or regression analysis. A few known data points from the original function can be used to create an interpolation based on a simpler function.

- Analytic Hierarchy Process(AHP)

  The analytic hierarchy process (AHP) was developed by Thomas L. Saaty in the 1970s and has been extensively studied and refined since it was born. AHP is a structured technique for analyzing complex decisions, based on both mathematics and psychology foundation. It provides a comprehensive and rational framework for structuring a decision problem, for representing and quantifying its elements, for relating those elements to overall goals.[4] The related factors were processed quantitatively through constructing hierarchical structure model, establishing judgment matrix, calculating weight, and testing consistency.

However, it relies on the experience of the decision maker to a great extent, and the subjective factors are of great influence on the result.

- Monte Carlo Methods

  Monte Carlo Methods (or Monte Carlo experiments) are a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results. Their essential idea is using randomness to solve problems that might be deterministic in principle. They are often used in physical and mathematical problems and are most useful when it is difficult or impossible to use other approaches. Monte Carlo methods are mainly used in three distinct problem classes: optimization, numerical integration, and generating draws from a probability distribution.

1.4 Simplifying Assumptions

1.4.1 Narrowing Research Area

The Zambezi River is the fourth-longest river in Africa, and the longest east-flowing river in Africa. It has a large number of branches, and the river covers a vast area. Due to the difficulty of collecting data, its nearly impossible for us to consider the whole area. Thus, **we choose the area near the Kariba reservoir region as an example to consider the distribution of small dams.** The highest elevation of the area is 93m, while the lowest is 29m, and the length is 220km, which means the slope is very small. So its reasonable to assume that **the altitude of terrain has little impact on our model.**

1.4.2 Approximate Ways to Measure Factors

As the height of the dam rises, the water storage capacity increases, which can lead to a decrease of water flow rate. Through the reading materials, we know that the flow rate of the river area we take into consideration is rather slow, and it only changes a little in different sections. Besides, the flux of the reservoir is equal to its width times depth times velocity, so we can suppose that **the flux and rate of each section in the reservoir is the same.** That means the reservoir width is inversely proportional to the depth of the reservoir. In this way, we can get the approximate value of the depth by measuring its width.

When it comes to the construction investment, there are so many factors to consider. To simplify the question, we assume that **construction investment is mainly related to three factors–material expense, labor cost, and reservoir immigrant cost, each of which is considered to be proportional to dam volume.** It is also assumed that **the impact of the dam on the ecological environment is proportional to some features of the dam, including water storage capacity and the number of dams.**

By Ignoring some mechanical and geological factors, we assume that **potential flood control capacity of a dam is only related to the section area of the dams above river surface.** Besides, we suppose that **the probability of safety can be estimated by the capacity index which determines the safety factor.**

It is assumed that **the cross section between the dam and the water is vertical.** In addition, supposing **the depth of the tributaries where the dam will build have the same water depth as the nearest reservoir.** Based on these assumptions, we can compare the comprehensive capacity of each dam.

1.5 Symbol Description

Parameter	Implication
V_i	Volume of the ith dam
d	Construction investments for the dam per volume
h_0	Height of the dam(fixed under our assumptions)
S_i	Upper base area of the ith dam
S_i^1	Bottom base area of the ith dam
W_i	Width of the ith dam(equal to the river width at that place)
l	Thickness of the upper dam base(fixed under our assumptions)
P	Electric energy production
g	Gravitational constant
η	Efficiency of the whole power system
η_h	Efficiency of the hydroturbine
η_t	Efficiency of the transmission
η_g	Efficiency of the generator
Q	Flux of the river
Q_0	Flux of water which flow into the power system
H	Vertical distance between inlet and the river surface
t	Valid work time of the power system
W	Waterway transportation profit
W_i	Waterway transportation profit of the ith dam
k	Waterway transportation constant
e	Waterway transportation constant
S	Safety factor
p_i	Safety factor of the ith dam
q	Safety index
E	Ecological cost of the whole system
B_i	Benefit of the ith dam(water supply ,fish farming, etc.)
N_i	Quantity of the ith item (water supply ,fish farming, etc.)
C_j	Cost of the jth item(Sediment defense, reservoir flood, etc.)
C_3	Cost of flood store of the dam per volume
m	Maintenance cost of the dam per volume
M_i	Maintenance cost of the ith dam per year
b_i	Benefit of the ith item per unit
u_j	Cost of the jth item per unit
D_i	Water storage capacity of the ith dam
A_i	Area of the river per portion(as we divide)
n	Number of dams
T	Revenue from tourism

2 Requirement 1 A Assessment of Three Options

- (Option 1) Repairing the Existing Kariba Dam

  Repairing the existing Kariba Dam has the least potential costs among the three options. Repairing a dam doesnt need as much construction materials as building one. It only requires little construction materials such as cement to patch the leaks of the dam. A few workers are enough for the repairing work, which decreases the cost of labor. Accordingly, the cost of security maintenance is higher. Besides, since the scale of project is rather small, local people dont have to leave their home and sacrifice their own interests. It does less harm to the environment at the same time. Based on these facts, this option can definitely reduce the economic pressures of the local government.

  Repairing seems the most relaxing way to maintain the precarious dam, but also proves to have the least potential benefits. A dam full of patching is apparently less reliable and effective than the brand new dams in Option 2&3. Therefore, its factors like flood control capacity, electric energy production, waterway transportation, water supply, and irrigation are not that satisfying.

- (Option 2) Rebuilding the Existing Kariba Dam

  The indexes we take into consideration in Option 2 are almost all between that of Option 1&3. The fee of security maintenance is lower than Option 1. Concerning the reservoir emigrant index, its formation based on its geographical, historical, political and economic environment, which is complicated to compare.

- (Option 3) Replacing the Dam with Smaller Dams

  The economic cost of Option 3 is higher than the other two options in most cases. The width of the Dam depends on the width of the river, so we can make the assumption that the width of small dams is the same with the Kariba Dam. In this way, we know that they only differ in their height. Its easy to see that the small dams with the same overall water management capabilities with the Kariba Dam will cost more construction materials, labor force and ecological cost than Option 2. It owns a higher cost of security maintenance than Option 1. At the same time, it will increase employment rate, which is not the emphasis of our discussion.

  When it comes to flood control capacity and safety, the series of smaller dams surely play a better role– the flood control system gets more stable when therere more members. Robustness is the property of being strong and healthy in constitution. When it is transposed into a system, it refers to the ability of tolerating perturbations that might affect the systems functional body. [4] Robustness has been widely applied to our life and production. The capacity of flood storage in the whole system has a directly relationship with the alleviation of flood pressure coming from the Zambezi River. Smaller dams can share the pressure with each other, and thus increase the disturbances and robustness of the system.

The series of smaller dams will increase power generation efficiency if given certain circumstances. For one thing, the sum of the contacting area between the dams and the water is obviously raised. For another thing, the expansion of the space provides the chance to make the best of the terrain along the Zambezi River. In addition, since the system is extensive, its easier to get the raw material needed and make transmission of energy than other options.

Smaller dams can improve waterway transportation, too. As we mentioned, dam construction makes the regimen of the river suitable for sailing. More dams, more shipping efficiency. As for water supply and irrigation, smaller dams also hold the advantage. The dams are distributed in a vast area along the river, so its more convenient for the people nearby to get access to the water.

The comparison of the three options mentioned above can be summarized in Table 4.

Factors \ Options			Option 1	Option 2	Option 3
Potential costs	Economic cost	Construction materials cost	Low	Middle	High
		Labor force cost	Low	Middle	High
		Safety maintenance cost	High	Low	Low
		Reservoir immigrant cost	Low	Unclear	Unclear
	Ecological cost		Low	Middle	High
Potential benefits	Flood control capacity		Low	Middle	High
	Electric energy production		Low	Middle	High
	Waterway transportation profit		Low	Middle	High
	Water supply		Low	Middle	High
	Irrigation		Low	Middle	High

Table 4: A horizontal comparison of the three options listed.

3 Requirement 2 A Detailed Analysis of Option (3)

3.1 Model 1 An Evaluation System of Dams Based on AHP

3.1.1 Preparation

Our preparation work has two process. First is to determine the branches of river we need to study further. Since the terrain of the rivers left side is higher, the dams should be distributed to the right side of the river. The branches which are possible for dam construction can be seen in Figure 1.

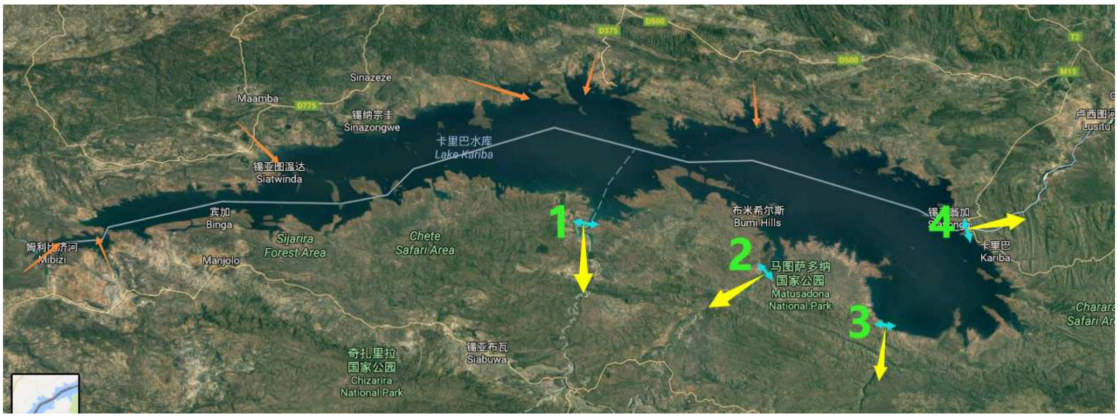


Figure 1: DamLocation

Second work is to collect the concerning data of the river. We have to say its a tough job because of data shortage. All we have is a map of the Zambezi River, by which we measure the width of the main stream of river in the place of the branches we choose and the width of the branches. The result is shown in Table 1. In the figure we get, we find that the width of The Kariba Dam is much smaller than the average width, which means we can only build the dams in the branches of the river. According to our assumptions, the reservoir width equals water flow divided by the depth of the reservoir.

3.1.2 Factors Measurement

To obtain the best plan to distribute the small dams, we consult the related literature and analyze the data, and decided to choose construction investments, electric energy production, waterway transportation profit, ecological impact, flood storage capacity and safety maintenance cost as six vital criteria. Our idea is to first analyze these factors one by one, then use our revised AHP (Analytic Hierarchy Process) to get the weight of each factor, and finally we can evaluate the whole system. The parameters used below can all be referred to in Symbol Description.

- Construction Investment Cost

	Coordinate(mi)	River Width(mi)	River Depth(m)	Dam Width(m)	Dam Height(m)
Dam 1	96.3	20.53	15.55	311.53	15.55 + 50
Dam 2	128	19.9	16.05	321.28	16.05 + 50
Dam 3	158	20.43	15.63	310.58	15.63 + 50
Dam 4	152	17.07	18.71	200.5	18,71 + 50

Table 1:

Construction investments of the project consists of material expense, labor cost and immigrant payment and many other factors. Under our assumption, the quantity of workers and material required for the project are proportional to the volume of the dam. From document [5], we learn that ,

$$\begin{cases} V = \frac{h}{3}(S + S^1 + \sqrt{SS^1}) \\ S^1 = 2.88S \\ C = \sum_{i=1}^{n} V_i \times d \end{cases}$$

- Electric Energy Production

2em Simplified model of river section is shown in figure 4. We know that in physics, the total energy of water(E) is equal to its potential energy plus kinetic energy. Assuming that the flux of the river(Q) is a fixed value in a short time, we select a qualified period of time, then we have:

$$\begin{cases} E_1 = \frac{1}{2}m(gh_1 + V_1^2) \\ E_2 = \frac{1}{2}m(gh_2 + V_2^2) \\ m = \sum_{i=1}^{n} \rho \times Q_i \times t \\ \Delta h = h_1 - h_2 \end{cases}$$

The energy we use to generate electricity comes from the difference between $E_1$and E_2:

$$E_i = \frac{1}{2}\rho \times Q_i \times t \times g[\Delta h + \frac{v_1^2 - v_2^2}{2g}]$$

By calculation, we find that the value of (v12 - v22)/2g is approximately equal to 0,

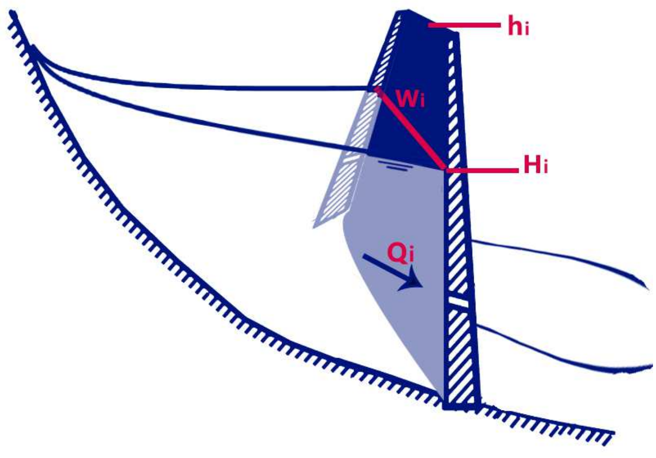


Figure 2: DamDetail

which can be ignore:

$$E_i \approx \frac{1}{2}\rho \times Q_i \times t \times g \times \Delta h$$

Besides, energy loss inevitable in the power generation process, which mainly comes from two aspects. One is from the efficiency of the hydroturbine(η_h), the efficiency of transmission (η_t)and the efficiency of generator(η_g),which can be combined as the mechanical efficiency of the power system(η):

$$\eta = \eta_h \times \eta_t \times \eta_g$$

Another source of energy loss is that the water below the inlet cannot flow into the power system as we wish, which is shown in Figure (2). The vertical distance between inlet and the river surface is presented as h_i^1. For the parameter η,ρg, which are all constant, we use γ to replace it so the equation becomes:

$$P = \sum_{i=1}^{n} Q_i \times t \times h_i^1$$

- Impact on Ecological Environment

Based on literature material, we decide to measure the influence of the dams on ecological environment from three different angles:

1.Benefits come from water supply, fish farming, and decrease of CO_2 and SO_x. Their calculations can be summarized as the following form:

$$B_i = p_i \times N_i$$

For i=1,2, the benefits are related to the section area of the dam. We use bi to represent the proportional coefficient of the section area of the dam above the river. To avoid symbol repetition, we use subscript j to represent the jth dam:

$$\begin{cases} N_i = b_i \times s_i \\ s_i = w_i \times H_i \end{cases}$$

$$\rightarrow B_i = \sum_{j=1}^{n} \rho_i \times b_i \times w_{ij} \times H_{ij}$$

For i=3, the quantity of CO_2 and SO_x reduction is determined by how much electric energy each dam can produce. We choose constant b3 as a proportional coefficient like the former one.

$$N_3 = b_3 * P$$

$$\rightarrow B_3 = \sum_{j=1}^{n} \rho_i \times h_3 \times P$$

2.Costs of sediment defense, reservoir flood and flood store & regulation. Among them, the last one is proportional to the number of dams and the others are related to the volume of the dams. We set coefficient ui in the same way:

$$C_i = \begin{cases} \sum_{j=1}^{n} c_i \times u_i \times V_i, & \text{i = 1,2} \\ c_i \times u_i \times n, & \text{i = 3} \end{cases}$$

3. Revenue from tourism. The construction of dams can promote the development of tourism to a certain extent. We use coefficient T to estimate the impact on the environment:

$$E = \sum_{j=1}^{n} \rho_i * b_i * w_{ij} * H_{ij} + \sum_{j=1}^{n} \rho_i * h_3 * P + \sum_{j=1}^{n} c_i * u_i \times V_i + T$$

- Waterway Transportation Profit

According to our assumptions, the waterway transportation profit of a dam system mainly depends on the width and the flow speed of the river. We simplify this problem in two ways.

1.The wider the river is, more space there is for shipping. Some extreme situations are not taken into consideration such as the case where the river is too narrow or too shallow for transportation. We assume that the waterway transportation profit of the ith dam(Y_i) is in direct proportion to the logarithm value of river width at that place(W_i).

$$Y_i \propto lnW_i$$

2.The faster the river flows, less suitable the river is for shipping. We also ignore other interference factors which are of relatively little importance for our problem solving. We assume that the waterway transportation profit of the ith dam(Y_i) is in inverse proportion to the logarithm value of water speed near the ith dam(v_i).

$$Y_i \propto -lnv_i$$

$$\begin{cases} Y_i \propto ln(\frac{W_i}{v_i}) \\ v_i W_i h_0 = Q \end{cases}$$

$$\rightarrow Y_i \propto lnW_i^2$$

Then we introduce two constants k and e to measure the relationship above.

$$\begin{cases} Y_i = k_i ln W_i^2 + e_i \\ Y = \sum_{i=1}^{n} k_i W_i^2 + e_i \end{cases}$$

- Flood Control Capacity

  Due to the limitation of data sources, we choose to ignore some mechanical and geological factors, assuming that flood control capacity of the dam is only determined by the section area of the dams above the river. Capacity and volume are proportional to the coefficient a so that we have the equation as following:

$$\begin{cases} G = \sum_{i=1}^{n} G_i = \sum_{i=1}^{n} a \times s_i \\ s_i = W_i \times H_i \end{cases}$$

- Safety factor

$$\begin{cases} G_m = max\{G_i\} \\ P_i = G_i * G_m \end{cases}$$

  By normalization, we can regard Pi as a probability that the ith dam stays steady when flood happens. So multiply (1-Pi) can evaluate the overall safety level of the dam system. However, by experimenting with data from the Three Gorges, we find that after the normalization, the result becomes so small, even reach 10-10. So we definite another constant s to adjust its order of magnitude.

$$S = s \prod_{i=1}^{n} (1 - P_i)$$

3.1.3 Advanced Analytic Hierarchy Process(AHP)

Due to shortage of data in the Zambezi River, we cannot stimulate a relatively accurate function to assess the dam system. Analytic Hierarchy Process gives us reference and inspiration for this problem. However, the method largely relies on the experience of the decision maker, and the subjective factors are of great influence on the result, which is not suitable for a prudential decision made by government. So we refer to the statistics of the Three Gorges, and use them to advance the process. Here is our specific process:

1.Calculate the data of the Three Gorges to get the weight of each factor we consider: construction investments, electric energy production, safety maintenance cost, ecological impact, waterway transportation profit and capacity of flood storage. The result is in Table 6.

Construction investments()	Electric energy production	Safety maintenance cost	Waterway transportation profit	Ecological impact	Capacity of flood storage
149.77	211.70	17.94	14.64	68.53	106.84

Table 6: The weight of each factor we consider in the Three Gorges

2.Construct hierarchical structure model(Figure 5)

3.Establishing judgment matrix of criterion layer(Table 2.) and scheme layer(Table 3.)

	B1	B2	B3	B4	B5	B6
B1	1	0.71	8.35	10.23	2.19	1.40
B2	1.41	1	11.80	14.46	3.09	1.98
B3	0.12	0.08	1	1.23	0.26	0.17
B4	0.10	0.069	0.81	1	0.21	0.14
B5	0.46	0.32	3.85	4.76	1	0.64
B6	0.71	0.50	5.88	7.14	1.56	1

Table 3: Judgment matrix of rule layer

4.Calculate weight and test consistency. The result is as the following Table 5. We achieve this process by MATLAB.

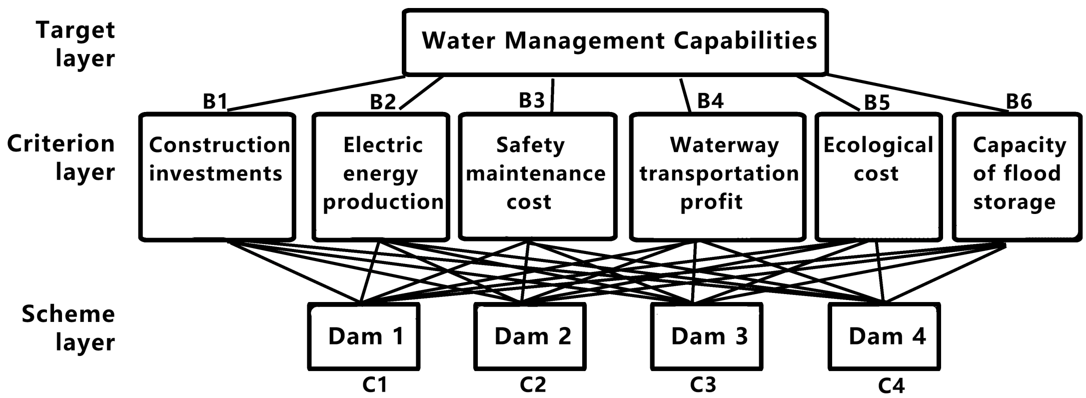


Figure 3: layer

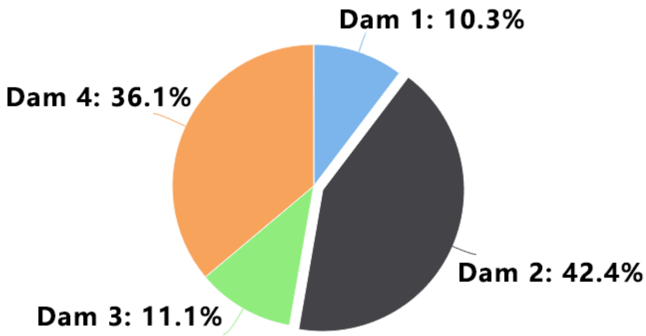
B1	C1	C2	C3	C4	B2	C1	C2	C3	C4	B3	C1	C2	C3	C4
C1	1	1/4	1	9	C1	1	1/6	1/2	1/9	C1	1	4	1	1/9
C2	4	1	4	9	C2	6	1	5	1/8	C2	1/4	1	1/4	1/9
C3	1	1/4	1	9	C3	2	1/5	1	1/9	C3	1	4	1	1/9
C4	1/9	1/9	1/9	1	C4	9	8	9	1	C4	9	9	9	1
B4	C1	C2	C3	C4	B5	C1	C2	C3	C4	B6	C1	C2	C3	C4
C1	1	1/3	1	9	C1	1	1/4	1	9	C1	1	1/4	1	9
C2	3	1	3	9	C2	4	1	4	9 C2	4	1	4	9	
C3	1	1/3	1	9	C3	1	1/4	1	9	C3	1	1/4	1	9
C4	1/9	1/9	1/9	1	C4	1/9	1/9	1/9	1	C4	1/9	1/9	1/9	1

Table 2: judgment matrix of scheme layer

Criterion		Construction investments	Electric energy production	Safety maintenance cost	Waterway transportation profit	Ecological cost	Capacity of flood storage	Weight Sort
Criterion Weight		0.2465	0.4520	0.0414	0.0414	0.0893	0.1295	
Scheme Weight	Dam 2	61281.2	66.05	0.000001	860813	61347.2	61281.2	0.4241
	Dam 4	39783.1	68.71	0.350809	335241	39851.8	39783.1	0.3612
	Dam 3	58863.5	65.63	0.0394487	804430	58929.2	58863.5	0.1113
	Dam 1	58968.8	65.55	0.0377314	809280	59034.3	58968.8	0.1033

Table 5: result

3.2 Conclusion



Little access to the data of the Zambezi River makes it tough to establish an accurate model, so we come up with a relatively better way. We have to say this kind of approximation will cause a certain amount of error, but the weight of the factors in the Three Gorges is far more precise than our personal experience when applied to our model. From the figure above we can see that building a dam in the second branch of river is of the greatest benefit. If we can ignore the influence of some parameters, we can get the results: the number of dams that corresponded to 4 tributaries are 1:4:1:4.

4 Smaller Dams Model

When we first consider the problem of small dams on the same branch, we have the following ideas. We measured the width of the Kariba reservoir region every 2.5miles along the branch on the map. The details in our work are shown in Figure 4.

Then, the original width can be estimated by using the scale approximation, and we can obtain the data several times higher than the original one by using the Interpolation Method mentioned above to simulate the river line. We use four kinds of algorithms: Lagrange polynomial, Linear interpolation, Three spline interpolation, Two spline interpolation. By evaluating, we find that Three spline interpolation was the best way to simulate.

At this point, if the edge is smooth and the depth in the tributaries is within a certain range, we consider the river depth H is a fixed value. Besides, the bottom cross-sectional area S between the two adjacent dam is the same. Under the assumptions above, it can be approximated by a similar area and we can obtain the position. Then, using the evaluation system of the six factors, we can get the function of the average value which takes n as an independent variable. Afterwards, the solving process can be achieved by programming. In the process, we use Monte Carlo methods to calculate the curvilinear integral.

$$\begin{cases} MaxAE = \frac{1}{6}\sum_{i=1}^{6} Evai \\ Evai = h(n) \\ n > 0 \end{cases}$$

Getting the solution n which is corresponding to the maximum value, then we can determine the total number of small dams. Even if the distance between the two dams and the

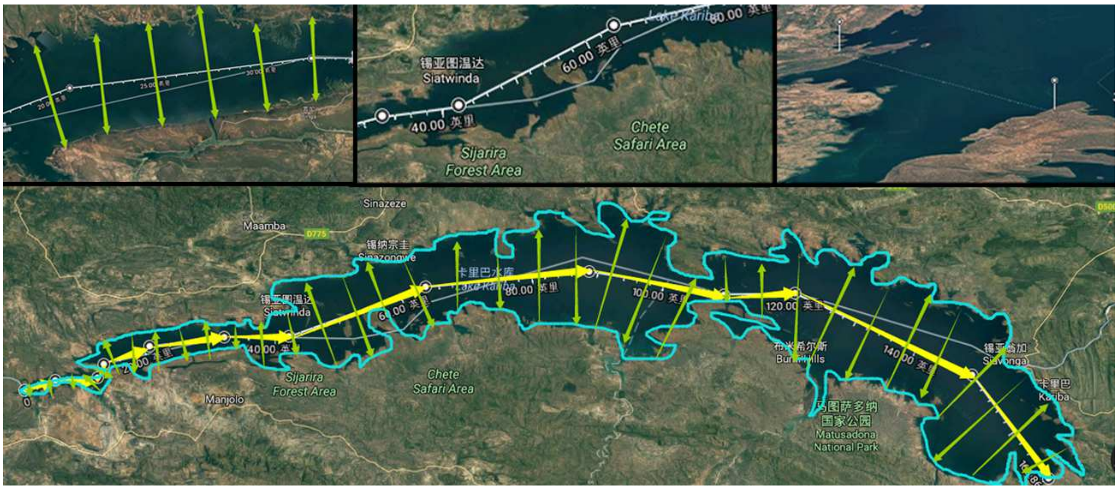


Figure 4: Our method to collect the data

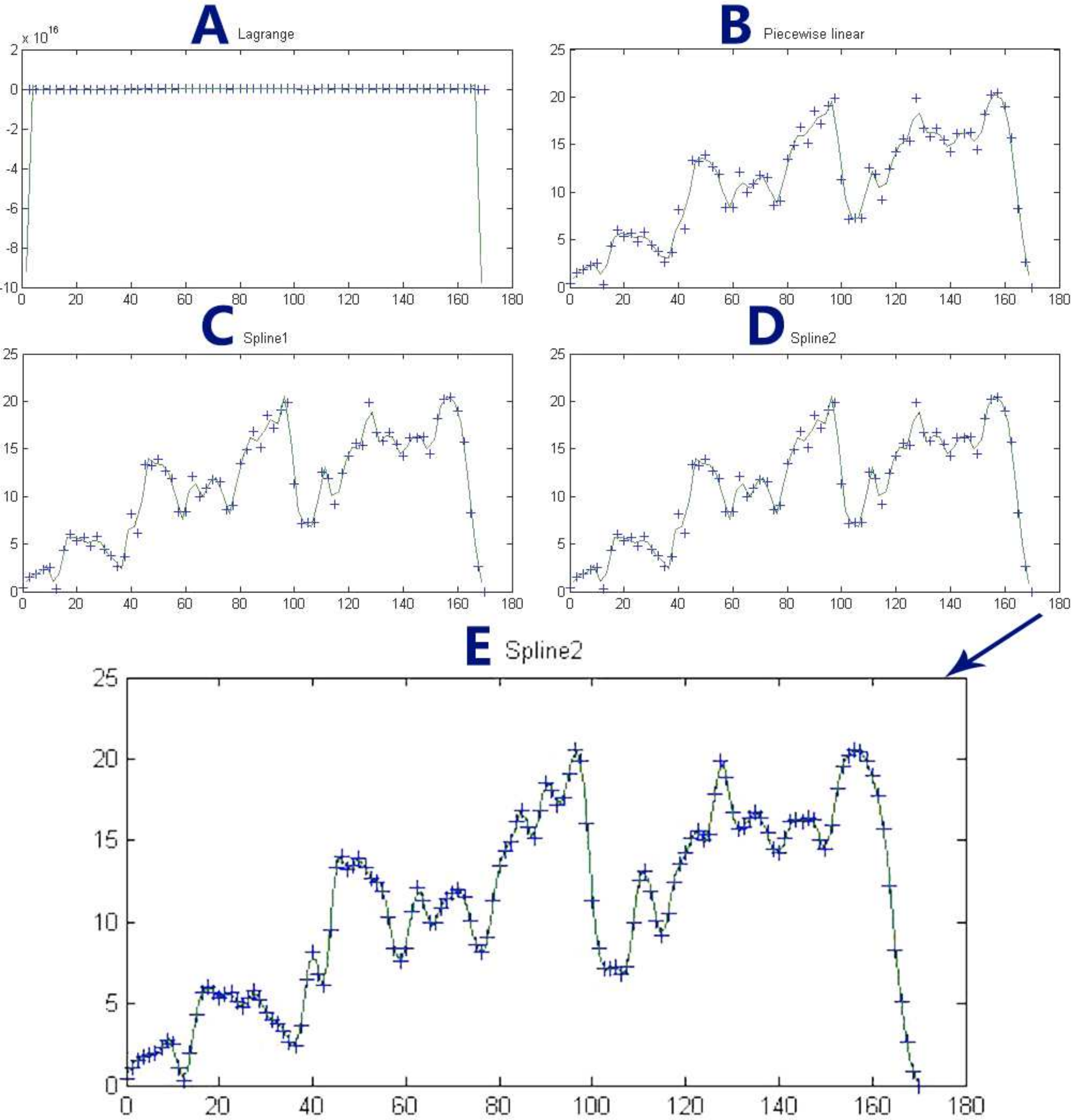


Figure 5: using Interpolation Method(IM) to simulate the river line

bottom area of the tributary river can be determined, and the results can be calculated according to the existing data, the geological data is not perfect, and the changes in the value of S has a great impact on the calculation of the value so that we cannot arbitrarily determine the size of its area.

The function of small dams is to maintain and strengthen the whole system, which means we can use more than one dam in one branch of river to complement the comprehensive ability, in addition to setting one small dam in each different tributary after the removal of the large dam. To determine the number of small dams on each tributary, we propose the following scheme.

From an economic point of view, there is a need to balance the benefits and costs of building dams. The cost here is not limited to physical and monetary factors, but is expanded into opportunity cost. Still, we can use the unit of money to measure each kind of cost. At this time, the cost is not only connected to the construction process of raw materials, the cost of workers, as well as the amount of pollution to the ecological and environmental damage produced in the process of construction of the dam. As for the benefits, they are from the power generation, aquaculture, tourism, etc. So, we select two functions about n:

$$\begin{cases} MC = fi(n) \\ MR = gi(n) \end{cases}$$

n Number of dams in a tributary
i– the ith tributary ;
P Price
MC- Margin cost
MR- Margin revenue

Moreover, in the process of building a small dam on different tributaries, we use the evaluation index mentioned above and some available data to approximate the general trend of its marginal cost, which will either increase or firstly decline and then increase with the number of the small dams increasing. While the marginal benefit and marginal cost are just opposite to each other. Due to some of the limitations of the data, which cannot be accurately determined by n, we take one tributary for example, to make the following cost and profit chart.

Looking at all of the tributaries, the last marginal net income of all branches should be equal, that is, the Pareto Optimality in economics. Otherwise, if considering the comprehensive factors, net income of the first tributary to build a small dam is still larger than the last small dam that has established on the second branch, theres a way to make the benefits of the whole dam system increased, which means the government has a better solution but gives up the choice, so there is irrationality.

The value of n in the point where marginal revenue is equal to marginal cost may be too large to take into consideration because of the limited nature of the environment. So the position of the last small dam must exist at the marginal revenue equal to the marginal cost or at the maximum integer that is closest to the equilibrium point.

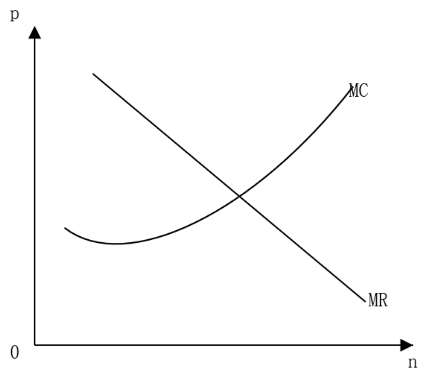


Figure 6: curve

To sum up,

$$\begin{cases} fi(n) \approx gi(n) \approx 0 \\ fi(n), gi(n) >= 0 \end{cases}$$

the number of small dams on each tributary can be determined in this way.

5 Flooding Condition Model

When the precipitation in the nearby area is huge, which may lead to some extreme problems such as flood, therere two measures to be taken. First, when designing a dam, we can draw a flood line by analyzing the river condition, precipitation and flood size, based on historical data. We find a simple introduction of a computing method as follow: **Flood Design Method:**

1. Objectives: Considering costs, we hope to control the height of the dam

Considering safety, we hope to increase the height of the dam properly

2. Parameters:

Qm: Flood peak discharge

W: Flood volume

A: Flood rising point

B: Peak point

C: Flood retreating point

t1: Overflowing last

t2: Back of flood last 3. Procedure:

Regard the series of general flood and the series of the maximum ones as a sample, where discrete series act as representatives. Each item of discrete series can be ranked in the investigation period (N years).

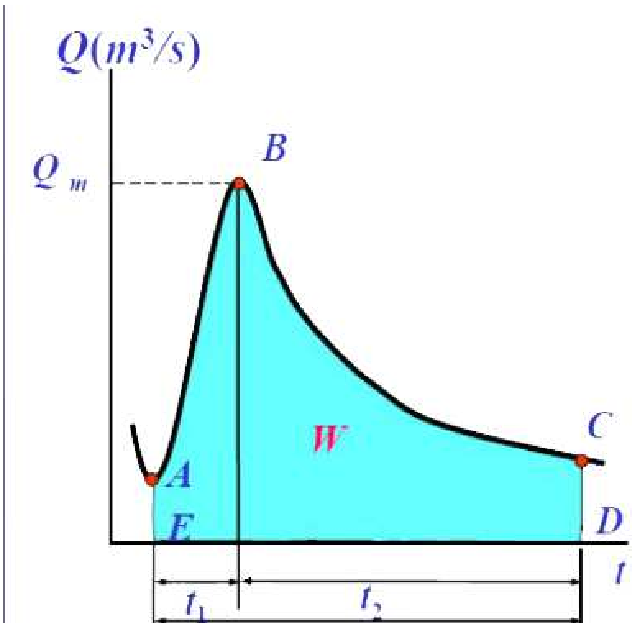


Figure 7: layer

Empirical frequency of catastrophic floods can be presented as:

$$P_m = M \div (N+1), M = 1, 2, , a$$

One of them occurred in the actually investigated period
The empirical frequency of the general flood in the actually investigated period is

$$P_m = P_{ma} + (1 - P_{ma})(m - l)/(n - l + 1)$$

It is assumed that the mean and mean variance of the (n-l) year series are equal to those of the (N-a) year series:

$$\sum_{i=1}^{N-a} Q_{mi} = fracN - an - l \sum_{i=l+1}^{n} Q_{mi}$$
$$\rightarrow Q_m = fracN - an - l$$

N discontinuous series of peak flow s variable coefficient calculation formula:
Qm: Mean value of the discontinuous flood series
Cv: Variation coefficient of the discontinuous flood series
Qmj: The catastrophic flood peak flow and flood volume (j=1,2,,a)
Qmi: The general flood: peak flow and flood volume (i=1,2,,a)
A: The total number of catastrophic floods, including the l terms happening in the measured series.

discontinuous series of peak flows skewness coefficient calculation formula:

We generally dont calculate Cs, and primarily choose Cs=KCv as the first approximation of the line fitting method.

For the basin with smaller variation coefficient

$$C_v <= 0.5C_s = (3\ 4)C_v;$$

For the basin with larger variation coefficient

$$C_v > 1.0C_s = (2\ 3)C_v;$$

6 Normal Water Cycles

The water system main consists of three elements:

1. Surface runoff (including Lake waves) that is originated in the western mountains and flows into the sea;

2. Rich geological resources hidden in the alluvial plain;

3. Water vapor in the atmosphere.

These three parts is a sum of the freshwater resources in the basin, which are interdependent and complementary with each other, keeping a dynamic balance together in the system. The process of water circulation within the basin can also be divided into three parts:

1.Forming precipitation convection when humidity reaches saturation;

2.Forming a part of the surface runoff, which add up to both the surface runoff and groundwater resources. Its the main source of alluvial plain groundwater.

3.Surface runoff, soil evaporation and plant transpiration of water vapor in the atmosphere.

However, the construction of dams and embankments affect the runoff part in water cycle, thereby affecting the flow, velocity, gap, local base level, local water resources, deposition, local weather and so on.

We try to combine the theory with the situation of the Zambezi River to analyze the impact of dam construction on water cycle. Because of the high elevation of the valley below the dam, f water supply in the rivers and ditches is limited. Statistics show that the total amount of runoff decreases, and the soil moisture is relatively lower, both of which will reduce the total amount of water vapor evaporation and atmospheric water vapor. Therefore, we conclude that the critical humidity of the precipitation is not high enough. The climatic characteristics of the basin are the decrease of the rainy season, and even occasional drought. In this case, our proposal is to compensate for the loss through water storage and reducing the extent of ecological damage. For example, construct a drainage channel with the nearest reservoir, to ensure high humidity through small flow.

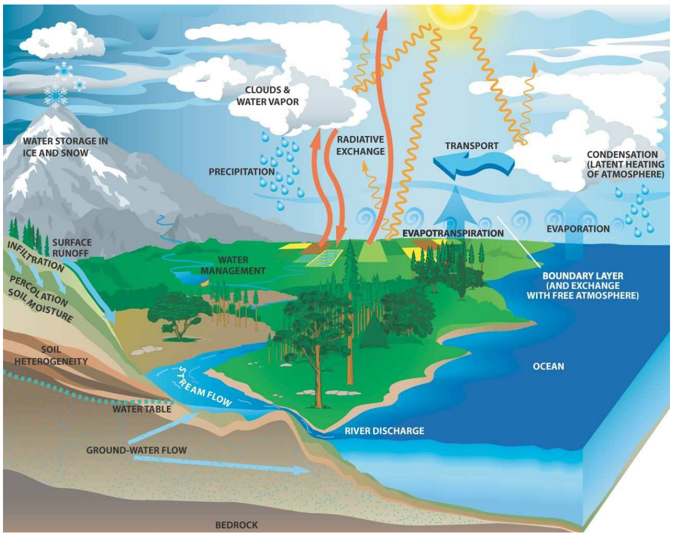


Figure 8: WaterCycle

7 Sensitivity analysis

We get some parameters through the method of least squares fitting in the model to estimate the management capabilities of the Kariba Dam. To test the effect of changing parameters, we will produce a sensitivity analysis that shows whether our model is properly sensitive to these variations.

Since dam height affects most of the factors in criterion layer, and its a key factor in dam construction, we will change the dam height above the water to do our sensitivity analysis. The results of each change are obtained by using the Analytic Hierarchy Process(AHP), and the results are shown in the following Figure 9.

From the figure, we can see that the weight tends to increase slowly with the increase of height and the trend is expected to focus in one point. Generally speaking, our model is stable.

8 Conclusion

Strengths and Weaknesses Now we will analyze the strengths and weaknesses of the models in our paper:

8.1 An Evaluation System of Dams Based on AHP

Strengths

1. We use the real data of the Three Gorges to calculate the judgment matrix of the criterion layer, which eliminates the subjective deviation of psychological factors.

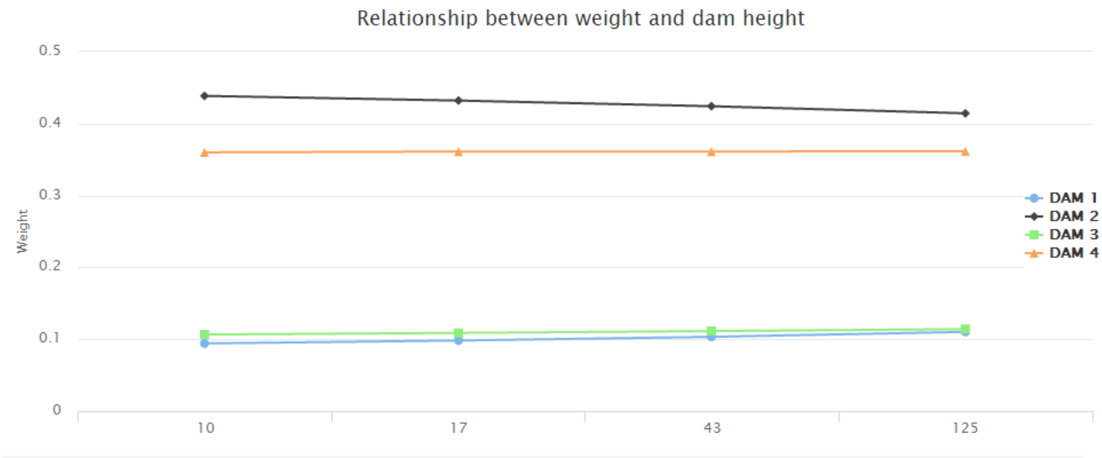


Figure 9: sensitivity analysis

2. After obtaining data from the Google map, we take the terrain topography factors into account to select the location of each dam, making our plan more practical.

Weaknesses

1. Due to the lack of data, the coefficients of some formulas cannot be determined.

2. The Analytic Hierarchy Process is relatively rough, and the precision is relatively low.

8.2 Smaller Dams Model

Strengths

1. Combined with the principles of economics and mathematical foundation, the relationship between costs and benefits is clearly analyzed in principle, and a method for determining the number of small dams can be found.

2. The model is relatively simple but effective to solve the delivery system, which is also connected with our evaluation system.

Weaknesses

Due to the limitation of the data, the formula of the variable i, and the number of dams to be determined cannot be calculated.

8.3 Flooding Condition Model

Strengths

1. The influence mechanism of dam construction on water cycle and ecological environment is analyzed clearly.

2. Two solutions to the two extreme cases are proposed, and the calculation method is found out.

Weaknesses

Because of the limited time and lack of data, we did not calculate the specific flood level line. The model is accurate, but complex, not easy to use and have lots of requirements.

Overall Results

Up to now, we have accomplished all our tasks. And here we give the overall results of our work.

Above all, we develop an assessment system model with six main factors and quantify them. Then, based on Analytic Hierarchy Process(AHP), we improve the model by using the real data of the Three Gorge to calculate the weight of each factor. Afterwards, we combine the weight and the value to compare the benefit of each dam construction plan. Next, we establish a general algorithm to solve the distributions of the small dams by combining it with economic principles. By using this model, if with adequate data, we can determine the numbers of the small dams in each tributary. After that, we consider the mechanism of the water cycle, and study the impact of dams building on the ecological environment and water cycle. In addition, we give two reasonable suggestions to cope with the extreme situations. Eventually, we test the sensitivity of our first model and it shows that they are stable.

9 Recommendation

To: Respected Officials of the Zambezi River Authority (ZRA)
From: Team # 55484 Date: January 24, 2017
Subject: Advice on dam distribution along the Zambezi River

As an old traditional Chinese proverb goes, An ant may well destroy an entire dam. Dam construction including a variety of aspects, making it significant to do an overall assessment of the dam system. To settle the problem, we set a dam evaluating system model based on the Advanced Analytic Hierarchy Process(AHP).

First, we sort out six important factors for the water management capability of the dam, and quantify the factors one by one. Second, we improve the Analytic Hierarchy Process(AHP) by obtaining the weight of each factor not through unreliable supposition but accurate data of the Three Gorges. We can get the weight of each factor in this way. Third, we can combine the factors with concerning weight to express the water management capabilities of the dam system, so we can optimize the solution for a better plan. In the end, we test the sensitivity of our model, and it proves to be stable.

In addition, we also establish a general algorithm to solve the distribution of the small dams using economic principles. The best plan we get can be seen in the figure below.

We also consider the impact of dams on the water cycle and flooding. Here we give an effective way to cope with flooding: drawing a flood line when designing a dam by analyzing river condition, precipitation and flood size, based on historical data.

Besides, we have done other jobs to evaluate the dam system, you can see our paper for details.

Y

ours sincerely Team # 55484

References

[1] Study on the influence of dams on river ecosystem service and its evaluation¿ Jianhong Xiao

[2] Influence of large hydropower projects on chemical composition of river water¿ Zhiwei Han

[3] Analysis and calculation of ecological runoff process in the middle and lower reaches of Yangtze River¿ Zhuqing Chen

[4] Healthy water cycle and new water strategy¿ Feng Yang

[5] Study on flood risk management system ¿ Wenting Hong

[6] Classification of water conservancy and hydropower projects and flood standard¿SL252-2000

[7] pplication of genetic algorithm in dam deformation monitoring model Wang Zhiwei Ma Bin Qi Lanma Yuanhua

[8] tudy on the influence of dams on river ecosystem service and its evaluation Name: Xiao Jianhong

[9] ttps://en.wikipedia.org/wiki/Robustness

[10] ttps://en.wikipedia.org/wiki/Zambezi

Figure 10: our distribution plan

[11] ttps://en.wikipedia.org/wiki/Robustness

Appendices

.1 First appendix: Analytic hierarchy process

Input matlab source:

```
clc
clear  %
a=[1       1/2      6       6       3       2       ;
2       1       9       9       6       4       ;
1/6     1/9     1       1       1/2     1/4     ;
1/6     1/9     1       1       1/2     1/4     ;
1/3     1/6     2       2       1       1       ;
1/2     1/4     4       4       1       1       ;];
[x,y]=eig(a);eigenvalue=diag(y);lamda=eigenvalue(1);
ci1=(lamda-6)/5;cr1=ci1/1.24
w1=x(:,1)/sum(x(:,1))
b1=[1       1/4     1       9       ;
4       1       4       9       ;
1       1/4     1       9       ;
1/9     1/9     1/9     1       ;
];
[x,y]=eig(b1);eigenvalue=diag(y);lamda=eigenvalue(1);
ci21=(lamda-4)/3;cr21=ci21/0.9
w21=x(:,1)/sum(x(:,1))
b2=[1       1/6     1/2     1/9     ;
6       1       5       1/8     ;
2       1/5     1       1/9     ;
9       8       9       1       ;
];
[x,y]=eig(b2);eigenvalue=diag(y);lamda=eigenvalue(1);
ci22=(lamda-4)/3;cr22=ci22/0.9
w22=x(:,1)/sum(x(:,1))
b3=[1       4       1       1/9     ;
1/4     1       1/4     1/9     ;
1       4       1       1/9     ;
9       9       9       1       ;

];
[x,y]=eig(b3);eigenvalue=diag(y);lamda=eigenvalue(1);
ci23=(lamda-4)/3;cr23=ci23/0.9
w23=x(:,1)/sum(x(:,1))
b4=[1       1/3     1       9       ;
3       1       3       9       ;
1       1/3     1       9       ;
1/9     1/9     1/9     1       ;
];
[x,y]=eig(b4);eigenvalue=diag(y);lamda=eigenvalue(1);
ci24=(lamda-4)/3;cr24=ci24/0.9
w24=x(:,1)/sum(x(:,1))
b5=[1       1/4     1       9       ;
4       1       4       9       ;
1       1/4     1       9       ;
1/9     1/9     1/9     1       ;];
[x,y]=eig(b5);eigenvalue=diag(y);lamda=eigenvalue(2);
ci25=(lamda-4)/3;cr25=ci25/0.9
w25=x(:,2)/sum(x(:,2))
```

```
b6=[1       1/4     1       9       ;
4       1       4       9       ;
1       1/4     1       9       ;
1/9     1/9     1/9     1       ;];
[x,y]=eig(b6);eigenvalue=diag(y);lamda=eigenvalue(1);
ci26=(lamda-4)/3;cr26=ci26/0.9
w26=x(:,1)/sum(x(:,1))
w_sum=[w21,w22,w23,w24,w25,w26]*w1
ci=[ci21,ci22,ci23,ci24,ci25,ci26];
cr=ci*w1/sum(0.9*w1)
```

.2 Second appendix: Extremum seeking

Input matlab source:

```
clc; close all; clear;
%
data=[0.44 0.8208 1.121 1.353 1.53 1.665 1.766 1.839 1.89 1.931 1.988 2.094 2.28 2.548 2.782 2.838 2.5
%
IndMin=find(diff(sign(diff(data)))>0)+1;
figure; hold on; box on;
plot(1:length(data),data);
plot(IndMin,data(IndMin),'r^')
legend('curve','wave trough')
title('wave trough of Lake Kariba', 'FontWeight', 'Bold');
```

.3 Thrid appendix: Computing matrix

Input C++ source:

```
// os_memory1.cpp :
//

#include "stdafx.h"
#include <iostream>
#include <string>
#include<iomanip>
using namespace std;
//ctrl+F5:

#define N 273
double RiverWidths[N] = { 0.44, 0.8208, 1.121, 1.353, 1.53, 1.665, 1.766, 1.839, 1.89, 1.931, 1.988, 2.


double sumArea = 0;


struct Dams {
    double horizontalDistance;
    double damWidth;
    double riverDepth;
    double riverWidth;
    double damHeight;
}dam[4];

double Height = 18;

void Init() {
```

```
    dam[0].horizontalDistance = 96.3;
    dam[0].damWidth = 899.6;
    dam[0].riverDepth = 15.55;
    dam[0].riverWidth = 20.53;
    dam[1].horizontalDistance = 128;
    dam[1].damWidth = 927.8;
    dam[1].riverDepth = 16.05;
    dam[1].riverWidth = 19.9;
    dam[2].horizontalDistance = 158;
    dam[2].damWidth = 896.9;
    dam[2].riverDepth = 15.63;
    dam[2].riverWidth = 20.43;
    dam[3].horizontalDistance = 152;
    dam[3].damWidth = 579;
    dam[3].riverDepth = 18.71;
    dam[3].riverWidth = 17.07;

    for (int i = 0; i < 4; i++) {
        dam[i].damHeight = dam[i].riverDepth + Height;
    }

}


double mini[6];
double P[6][4];

void Calculate() {
    for (int i = 0; i < 4; i++) {
        P[0][i] = dam[i].damHeight * dam[i].damWidth;
    }
    for (int i = 0; i < 4; i++) {
        P[1][i] = dam[i].damHeight;
    }
    for (int i = 0; i < 4; i++) {
        P[3][i] = log(dam[i].damWidth);
    }
    for (int i = 0; i < 4; i++) {
        P[4][i] = dam[i].damHeight * dam[i].damWidth + dam[i].damHeight;
    }
    int max = 0;
    for (int i = 0; i < 4; i++) {
        P[5][i] = dam[i].damHeight * dam[i].damWidth;
        if (max < P[5][i]) {
            max = P[5][i];
        }
    }
    for (int i = 0; i < 4; i++) {
        P[2][i] = 1 - (P[5][i] / max);
        if (P[2][i] == 0)
            P[2][i] = 0.001;
    }
    for (int j = 0; j < 6; j++) {
        mini[j] = 9999;
        for (int i = 0; i < 4; i++) {
            cout << P[j][i] << " \t";
        }
        cout << endl;
    }
}
```

```
int M = 4;

void SchemeMatrix() {

    cout << endl << endl;

    for (int m = 0; m < 6; m++) {
        double Min = 9999999999;
        double Max = 0;
        for (int i = 0; i < M; i++) {
            if (Min > P[m][i]) {
                Min = P[m][i];
            }
            if (Max < P[m][i]) {
                Max = P[m][i];
            }
        }
        double MaxScale = Max - Min;

        double haha = 40;

        for (int i = 0; i < M; i++) {
            for (int j = 0; j < M; j++) {
                if (P[m][i] > P[m][j]) {
                    int temp = (int)(haha * ((P[m][i] - P[m][j]) / MaxScale) + 0.5);
                    if (temp == 0)
                        cout << "1\t";
                    else if (temp >= 10)
                        cout << "9\t";
                    else if (temp == 1)
                        cout <<"2\t";
                    else
                        cout << temp << "\t";
                }
                else if (P[m][i] == P[m][j]) {
                    cout <<"1\t";
                }
                else {
                    int temp = (int)(haha * ((P[m][j] - P[m][i]) / MaxScale) + 0.5);
                    if (temp == 0)
                        cout << "1\t";
                    else if (temp >= 10)
                        cout<< "1/9\t";
                    else if (temp == 1)
                        cout << "1/2\t";
                    else
                        cout << "1/" <<temp << "\t";
                }
            }
            cout << ";\n";
        }
    }
    cout << endl << endl << endl;
}

double C[6] = { 149.77, 211.7, 17.94, 14.64, 68.53, 106.84};


int main()
{
    Init();
    Calculate();
```

```
    SchemeMatrix();
    //PrincipalMatrix();
    return 0;
}
```