Portfolio Returns with S&P 500 Constituents

1. Analysis of Individual Stock Returns

Statistics Properties Across the 500 Stocks

Table A below shows the mean, standard deviation, skewness, kurtosis, and autocorrelation of the 500 stocks.

	Mean	Standard Deviation	Skewness	Kurtosis	Autocorrelation
mean	0.000190	0.031972	-0.054544	8.040060	-0.139616
std	0.001068	0.009544	0.659788	4.216372	0.129781
min	-0.003840	0.015514	-2.061322	1.167578	-0.511316
10%	-0.001137	0.022733	-0.804924	3.822205	-0.312779
50%	0.000170	0.029332	-0.035370	7.004850	-0.133142
90%	0.001465	0.044100	0.638686	13.209999	0.023452
max	0.003943	0.074817	3.422080	45.487180	0.220307

***Table A. Distributional properties of mean, standard deviation, skewness, kurtosis, and autocorrelation of the approx. 500 stocks**

Some findings we found might be noteworthy are:

- **Standard Deviation**: Given an average daily standard deviation of 0.031972, the annual standard deviation return is approximately 0.5075 or 50.75%, which is quite high meaning the overall stock market was pretty volatile during the captured time period.
- **Kurtosis:** The average kurtosis is quite high (8.040060), suggesting that the returns distribution for the assets has fat tails and sharp peaks compared to a normal distribution. This indicates a large likelihood of extreme events. The maximum kurtosis of 45.487180 is extremely high, indicating a fat tail for at least one stock.

Stocks with the highest and lowest variance

We found that among the approx. 500 stocks we analyzed, the stock with the highest variance is Norwegian Cruise Line Holdings Ltd (NCLH), and the one with the lowest variance is Verizon Communications Inc. (VZ). **Figure A** shows the plot of daily returns and cumulative returns of each stock.

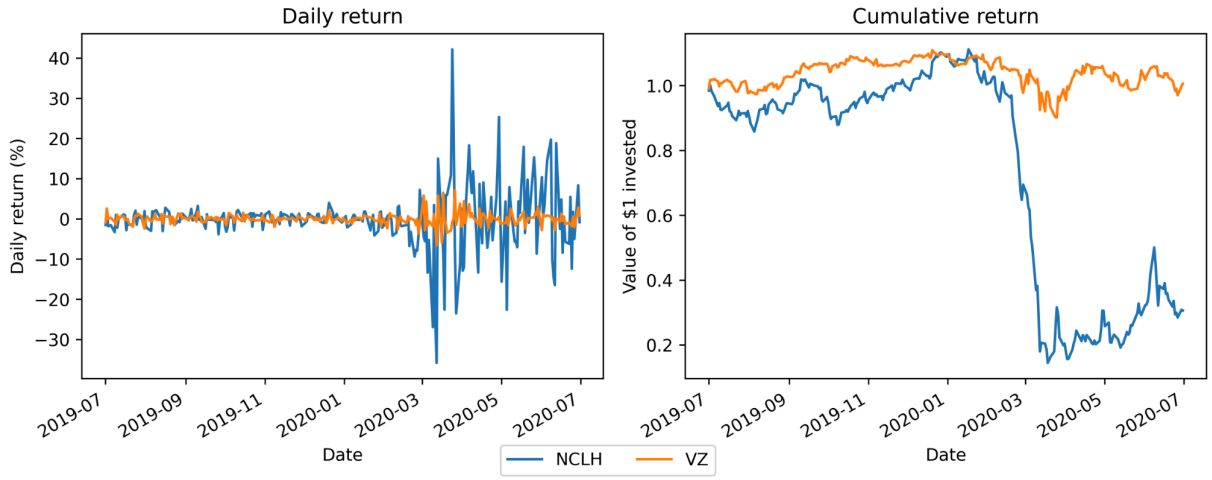

***Figure A. Daily return and cumulative return of the stocks with the highest and the lowest variance**

2. Analysis of value-weighted portfolio returns

Figure B shows the plot of daily portfolio returns and cumulative portfolio returns when adopting a monthly rebalance, value-weighted strategy. **Table B** summarizes the portfolio's mean, variance, standard deviation, skewness, kurtosis, and autocorrelation under this weighting method and the same statistics for the individual stocks.

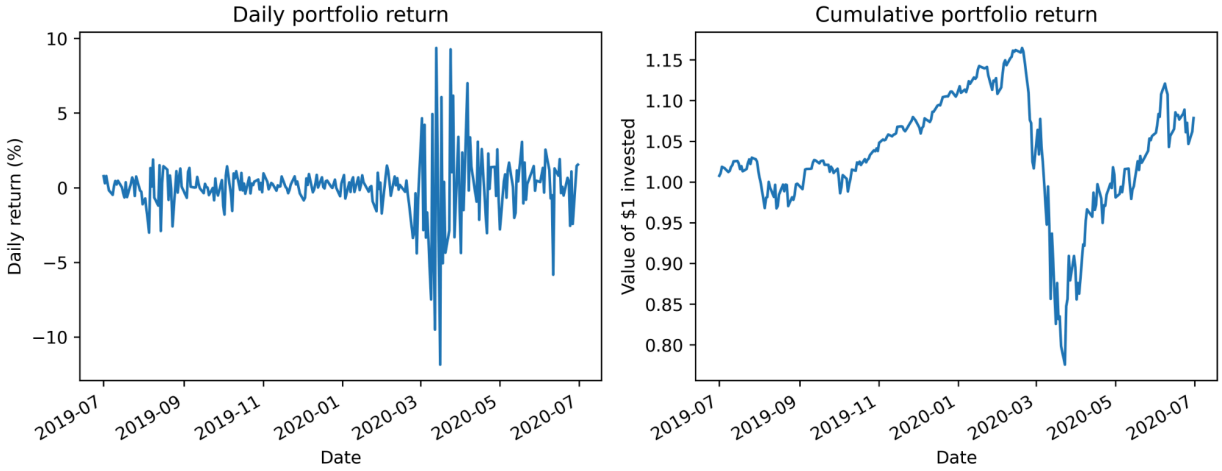

***Figure B. Daily return and cumulative return of a monthly rebalanced & value-weighted portfolio**

Weighting Method	Mean	Standard Deviation	Skewness	Kurtosis	Autocorrelation
Individual Stocks	0.000190	0.031972	-0.054544	8.040060	-0.139616
Value-weighted	0.000521	0.020996	-0.509193	9.129940	-0.370333

***Table B. Mean, variance, standard deviation, skewness, kurtosis and autocorrelation of the portfolio**

Findings:

- **Mean return**: The portfolio yields a higher average daily return of 0.0521% (appx. 14% annual return) than 0.000190 (appx. 4.9% annual return)
- **Standard deviation**: The portfolio shows a lower volatility of appx 33.33% annual standard deviation, compared to 50% of the individual stocks, meaning that the diversification diluted the individual level volatility.
- **Skewness:** Skewness increases from -0.054 to -0.509 indicating that losses are more prominent in the portfolio.
- **Kurtosis:** The portfolio shows a higher kurtosis suggesting even greater potential of extreme events.

3. Analysis of equal-weighted portfolio returns

To calculate the return of a monthly rebalanced, equal-weighted portfolio, we made modifications to the code for Question 2 as shown in the screenshot below.

```
# ================================================================================
#     ## calculate portfolio returns
#     # value weights = invest in each stock with equal weight
#     # weight 1/total number of stocks in the portfolio
#     # grab last market cap of the previous month
    num_of_stock = len(ret.axes[1]) # 471
    equal_weight = 1 / num_of_stock

#     # weight cumulative returns by equal weight, the a *= b means a = a*b, element by element
    retm_cum *= equal_weight
# ================================================================================
```

Figure C shows the plot of daily portfolio returns and cumulative portfolio returns when adopting a monthly rebalance, equal-weighted strategy. **Table C** summarizes the portfolio's mean, variance, standard deviation, skewness, kurtosis, and autocorrelation under the two weighting methods and of individual stocks.

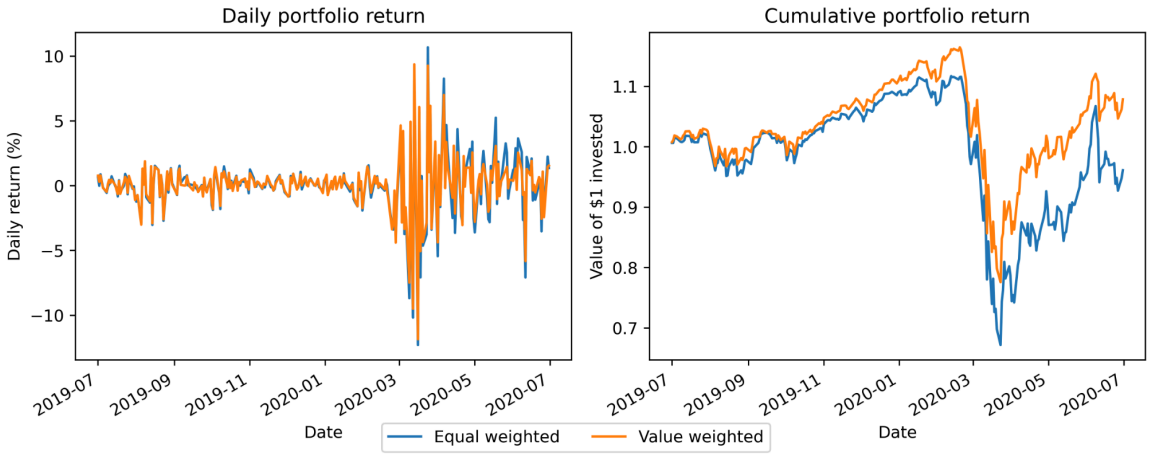

***Figure C. Daily return and cumulative return of a monthly rebalanced & equal-weighted portfolio**

Weighting Method	Mean	Standard Deviation	Skewness	Kurtosis	Autocorrelation
Individual Stocks	0.000190	0.031972	-0.054544	8.040060	-0.139616
Value-weighted	0.000521	0.020996	-0.509193	9.129940	-0.370333
Equal-weighted	0.000122	0.023532	-0.550341	7.136939	-0.213509

***Table C. Mean, variance, standard deviation, skewness, kurtosis and autocorrelation of the portfolio**

Findings:

- **Mean return**: The value-weighted strategy yields a significantly higher average daily return compared suggesting that allocating more weights to higher-value firms yields better returns.
- **Standard deviation**: The value-weighted strategy exhibits lower volatility, indicating that equally distributing capital across assets may introduce more risks to the portfolio.
- **Skewness:** Both strategies exhibit negative skewness, but the equal-weighted strategy has a slightly more pronounced negative skew.
- **Kurtosis:** The value-weighted strategy has higher kurtosis, indicating a higher likelihood of extreme returns compared to the equal-weighted strategy.
- **Autocorrelation:** The value-weighted strategy exhibits more negative autocorrelation than the equal-weighted strategy, suggesting that returns are more inversely related to their previous values.

Comparison:

Method	Value-weighted Method	Equal-weighted Method
Feature	Larger companies with larger market capitalization will be given more weight and have a greater influence on the portfolio	Each stock is equally weighted regardless of company size or market capitalization.
Advantage	This method may generate lower volatility as larger companies are often more stable.	- By giving equal weight to all stocks, this method can avoid the portfolio's exposure to a single stock's negative impact and lead to better diversification. - Equal weighting tends to favor small-cap stocks and may exhibit higher potential growth
Disadvantage	The concentration of portfolio weight in larger companies may increase the portfolio's exposure to certain risks (sector-specific) and reduce diversification.	Equal weighting might introduce more volatility to the portfolio since smaller companies are often more volatile.

4. Capped value weights

Codes Explanation:

The process of constructing a value-weighted portfolio with a cap (e.g., 2%) on individual stock weights involves several steps:

1) **Initial Weight Calculation:** For each month, initial weights are calculated based on the market capitalization of stocks at the end of the previous month. This step determines the proportion of each stock in the portfolio based on its size relative to the total market capitalization.
2) **Applying the Cap:** Weights exceeding the 2% threshold are capped, and the difference between the initial and capped weights represents the excess weight that needs to be redistributed among the other stocks.
3) **Redistributing Excess Weight:** The excess weight from capped stocks is proportionally redistributed among stocks whose weights were below the cap, ensuring their new weights are adjusted in line with their original market capitalizations.
4) **Normalization:** After redistribution, weights are normalized to ensure the total sum of weights equals 100%, maintaining the portfolio's fully invested status.
5) **Calculating Portfolio Returns:** Using the adjusted weights, the portfolio's monthly returns are calculated, and these returns are aggregated to derive the portfolio's overall performance.

The key to the reweighting process lies in maintaining the proportional increase in weights for the uncapped stocks, ensuring the portfolio remains aligned with its value-weighting principle while adhering to the imposed cap. The described reweighting process ensures that the value-weighted portfolio respects the 2% cap on individual stock weights. This method balances risk by preventing overexposure to any single stock while maintaining a diversified portfolio structure. The proportional redistribution of excess weight ensures that the portfolio's value-weighting principle is preserved, aligning with the overall investment strategy.

Comparison:

Table D shows that implementing a cap on individual stock weights in a value-weighted portfolio marginally reduces mean return and slightly increases risk (as indicated by variance and standard deviation), without significantly altering the nature of return distribution (as skewness and kurtosis remain similar). The strategy of capping weights may be more about risk management—specifically, reducing concentration risk—than about optimizing for higher returns. By limiting the weight of any single stock, the portfolio aims to mitigate the impact of any adverse movement in high-weight stocks, trading off some potential return for a more controlled risk profile.

	Uncapped Value-weighted Portfolio	Capped Value-weighted Portfolio
Mean	0.000521	0.000377
Variance	0.000441	0.000445
Standard Deviation	0.020996	0.021101

Skewness	-0.509193	-0.510645
Kurtosis	9.129940	8.972531
Autocorrelation	-0.370333	-0.356423

***Table D. Mean, variance, standard deviation, skewness, kurtosis and autocorrelation of the uncapped & capped value-weighted portfolio**

Pros & Cons:

Imposing a cap on the value weights within a portfolio introduces a nuanced approach to investment strategy, blending both protective measures and potential limitations. **On the positive side,** capping individual stock weights significantly reduces concentration risk, safeguarding the portfolio against the adverse performance of any single, highly weighted stock. Furthermore, it promotes enhanced diversification, broadening the investment across a wider array of portfolio constituents and thereby capturing potential growth from a more extensive range of stocks.

However, this strategy does not come without **its drawbacks**. Capping the weights of the largest stocks could potentially curtail the portfolio's ability to benefit from the significant gains these major players might offer, thus impacting the overall returns unfavorably. Additionally, to uphold these caps, portfolios might necessitate more frequent rebalancing efforts, which could elevate transaction costs and, over time, reduce the returns. This balancing act between mitigating risks and embracing growth opportunities underlines the strategic considerations fund managers must weigh when applying caps to value weights in their portfolios.