

Collection of mapping reductions and pseudocode that prove them.

1. Show a reduction from  $H_{TM} \leq_M B$ , where  $H = \{ \langle M \rangle \mid M \text{ halts on } \epsilon \}$

- Pseudo code

```

def R(<M>):
    def N(x):
        if x == '00':
            M('')
            return Accept
        elif x == '11':
            return Reject
        else:
            return Reject
    return(N)

```

- Commentary:

- If N is in B then N accepts 00 or 11, but not both. By construction N doesn't accept 11, so it must accept 00. The only way it can accept 00 is when  $M(\epsilon)$  either accepts or rejects.
- If N is not in B then it can't accept on the previous conditions: N must accept either both 00 and 11, or neither. By construction N rejects 11, which means it can't also reject 00. Again by construction N can't ever reject 00: it either accepts 00 or it loops.

2. Show a reduction from  $H_{TM} \leq_M B$ , where  $H = \{ \langle M \rangle \mid M \text{ halts on } \epsilon \}$

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def R(<M>):
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- Commentary:

- If N is in B then N accepts 00 or 11, but not both. By construction N doesn't accept 11, so it must accept 00. The only way it can accept 00 is when  $M(\epsilon)$  either accepts or rejects.

- If  $N$  is not in  $B$  then it can't accept on the previous conditions:  $N$  must accept either both  $00$  and  $11$ , or neither. By construction  $N$  rejects  $11$ , which means it can't also reject  $00$ . Again by construction  $N$  can't ever reject  $00$ : it either accepts  $00$  or it loops.