1. Recall that a reduction of language A to language B is a computable function f such that $w \in A$ if and only if $f(w) \in B$. Write out python-like code for the following (rather pointless) reductions.

a. Let A = {‘John’}. Show that $A \leq_M \bar{A}$.

b. Let H_{TM} be the set of Turing machine descriptions that halt on the empty string. Show that $\{'Jane'\} \leq_M H_{TM}$.

c. Let E be the set of binary strings describing even number and let O be the set of binary strings describing odd numbers. Show that $E \leq_M O$.

2.Consider the language {<M> | L(M) is the set of palindromes}. Is this language recognizable? How about its complement?

3. Show that a non-empty language L is recognizable if and only if there is a computable f such that $L = \{f(1), f(11), f(111), \ldots\}$, where f is a computable function with domain $1^+ = \{1, 11, 111, \ldots\}$.

Recall a function is computable if there is a Turing machine halts for every input w in the domain leaving $f(w)$ on the tape.

[Bonus] Show that an infinite language L is decidable if and only if there is a computable function f such that $|f(1^{k+1})| \geq |f(1^k)|$ for all for all $k \geq 1$ and $L = \{f(1), f(11), f(111), \ldots\}$.