# CS 6505 - HOMEWORK 1

KEN BROOKS

For this problem set, you may find it useful to consult Ken Rosens textbook *Discrete Math and Its Applications*.

1. Give the contrapositive of the following statement. "If every bird flies, then there is a hungry cat."

   **Answer:**

   If there is not a hungry cat, then no bird flies.

2. A proposition is a statement that can be true or false but not both. Let A, B, and C be propositions. Let $\wedge$ denote logical AND, let $\vee$ denote logical OR, and let $\neg$ denote logical NOT. Argue that if $(A \vee B) \wedge (\neg B \vee C)$ is true, then $(A \vee C)$ must be true as well.

   **Answer:**
   - $(A \vee B) \wedge (\neg B \vee C) \Rightarrow (A \vee C)$ is true if all possible values in its domain map to "true" in its range.
   - The table below maps each possibility of A, B, and C through each of the terms of the expression $(A \vee B) \wedge (\neg B \vee C) \Rightarrow (A \vee C)$
   - In each case the resulting value is true.
   - Therefore $(A \vee B) \wedge (\neg B \vee C) \Rightarrow (A \vee C)$, or if $(A \vee B) \wedge (\neg B \vee C)$ is true, then $(A \vee C)$ must be true as well.

   | | | |
   |---|---|---|
   | 1 | 2 | 3 |
   | 4 | 5 | 6 |
   | 7 | 8 | 9 |

3. We use the notation $A \Rightarrow B$ to indicate that A implies B. This new proposition $A \Rightarrow B$ is true except when A is true and B is false. We write $A \Leftrightarrow B$ when either both A and B are true or both are false. Argue that $A \Leftrightarrow B$ if and only if $A \Rightarrow B$ and $B \Rightarrow A$.

   **Answer:**

*Date*: August 23, 2014.

4. We will use the notation $|\cdot|$ to indicate the number of elements in the set or its cardinality, e.g. $|A|$ is the number of elements in the set $A$. Consider four sets $A, B, C, D$ such that the intersection of any three is empty. Use the inclusion-exclusion to give an expression for $|A \cup B \cup C \cup D|$ without using any union ($\cup$) symbols.

   **Answer:**

5. State the formal definition of $O(n)$, and show that the function $f(n) = (n^4 + n^2 - 9)/(n^3 + 1)$ is $O(n)$.

   **Answer:**

6. Let $A$ be a set. We use the notation $P(A)$ to indicate the power set of $A$, which consist of all subsets of $A$. For example, if $A = \{0, 1\}$, then $P(A) = \{\{\}, \{0\}, \{1\}, \{0, 1\}\}$. Consider $Q(n) = P(\{1, \ldots, n\}) - \{\{\}\}$ and use an inductive argument to show that the sum

$$\sum_{\{a_1,\ldots,a_k\} \in Q(n)} \frac{1}{a_1 \cdots a_k} = n$$

   (For example, the expansion for $n = 3$ is $\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{1\cdot 2} + \frac{1}{1\cdot 3} + \frac{1}{2\cdot 3} + \frac{1}{1\cdot 2\cdot 3} = 3$.)

   **Answer:**

7. Prove that the set of all languages over $\{0, 1\}$ that have a bounded maximum string length is countable.

   **Answer:**