$L_d = \{< M > \mid M\ does\ not\ accept\ input\ < M >\}$
L_d is not recognizable

A reduction from A to B is a computable function f such that
1. If x is in A then f(x) in B
2. If x is not in A then f(x) is not in B

Basic Theorems about reductions
If A reduces to B ($A \leq B$)
1. If B is recognizable then A is recognizable
2. If B is decidable then A is decidable
3. If A is not recognizable then B is not recognizable
4. If A is not decidable then B is not decidable

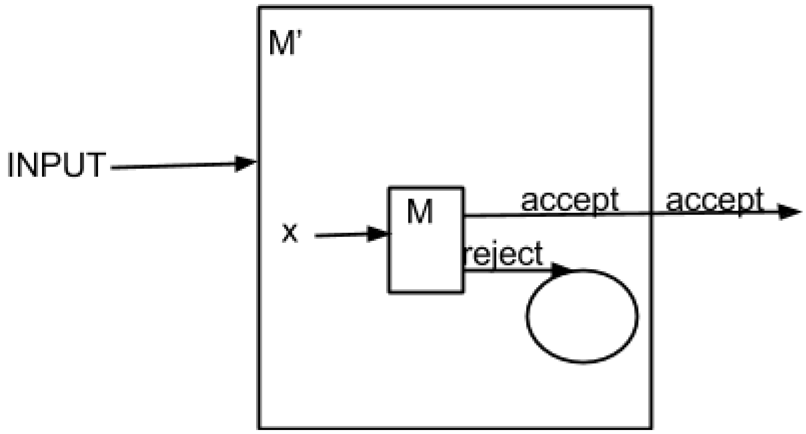
3 and 4 are the contrapositives of 1 and 2

Proof of 2
B is decidable via some Turing Machine M
Need a Turing Machine to decide A
M'(x) simulates M(f(x))
x is in A then f(x) is in B then M(f(x)) accept then M'(x) accepts
x is not in A then f(x) is not in B then M(f(x)) reject then M'(x) rejects
So M'(x) accepts if x is in A and rejects if x is not in A so M' decides A so A is decidable

$L_u = \{(< M >, x) \mid M\ accepts\ input\ x\}$
L_u is recognizable
$\overline{L_d} = \{< M > \ \mid M\ accepts\ input\ M\}$
$\overline{L_d}$ is not decidable
$\overline{L_d} \leq L_u$ via f(<M>) = (<M>,<M>)
M accept <M> if and only if (<M>,<M>) is in L_u if and only if f(<M>) is in L_u

By Theorem #1, $\overline{L_d}$ is recognizable.
By Theorem #4, L_u is not decidable

$L_h = \{< M > \mid M\ does\ halt\ on\ blank\ tape\}$
Give a reduction from L_u to L_h



f(<M>) = <M'>

$L_u = \{(< M >, x) \mid M \textit{ accepts input } x\}$

$L_h = \{< M > \mid M \textit{ does halt on blank tape}\}$

M(x) accepts then M' on blank tape simulate M(x) which accepts so M' accepts on blank tape

M(x) doesn't halt then M' on blank tape simulate M(x) which doesn't halt

M(x) rejects then M' on blank tape simulates M(x) which rejects which causes M' go into infinite loop and doesn't halt

If M accepts x then M' halts on blank tape

If M doesn't accept x then M' doesn't halt on blank tape

Since L_u is not decidable then L_h is not decidable