1. There are many incarnations of the diagonalization paradox. One famous one defines a barber to be "one who shaves all men, and only those men, who do not shave themselves." Identify the paradox and illustrate the problem with a grid. Then explain how one can resolve the paradox, exploring as many solutions as you can think of.

2. Show that the following problem is decidable. Given a Turing Machine M, an input string w and a number k, does M use at most k tape squares on input w?

3. Recall the BB-n game from the last discussion. We restrict ourselves to Turing machines with two tape symbols, blank and 1, and allow the machine a two-way infinite tape. For every n > 0, we have a game where the goal is to find the Turing machine that executes the maximum number of shifts while still halting eventually. We define $S : N \rightarrow N$ to be the mapping between the number of operational (non-halting) states and this maximum numbers of shifts.

Consider the following argument (taken from Wikipedia). Find the contradiction and draw the proper conclusion. Prepare to argue that other step is correct.

1. Let EvalS be the Turing machine that given n 1s on the tape, halts with S(n) 1s on the tape.
2. Let Clean denote a Turing machine cleaning the sequence of 1s initially written on the tape.
3. Let Double denote a Turing machine that given a tape with n 1s it will produce 2n 1s on the tape and then halt.
4. The Turing machines Double | EvalS | Clean can be composed into a single Turing machine having some number of states, say n_0.
5. There is a machine that has exactly n_0 states and simply writes out n_0 1s to the tape. Call this machines Create_n0.
6. Let BadS denote the composition Create_n_0 | Double | EvalS | Clean. This machine has exactly $2n_0$ states.
7. After the EvalS part of BadS runs, the tape will have $S(2n_0)$ 1s on it.
8. The cleaning phase of BadS will require at least $S(2n_0)$ shifts, so BadS, which has $2n_0$ states, will use more than $S(2n_0)$ shifts.