Surplus VaR

- Equity/Surplus = Asset – Liability
- $\text{Surplus} = \text{Asset} - \text{Liability}$
- $\Delta\text{Surplus} = \Delta\text{Asset} - \Delta\text{Liability}$

注意

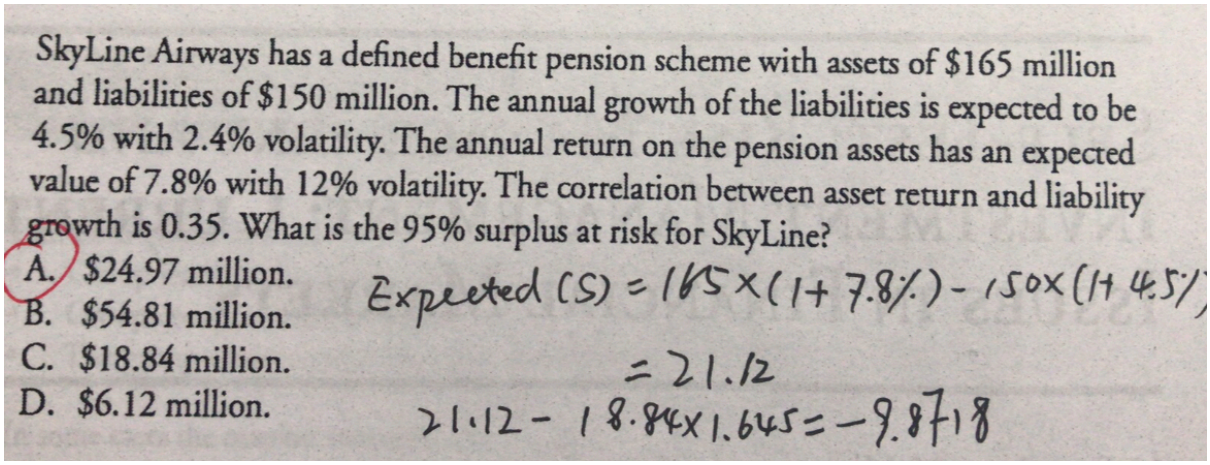
- VaR 的定义有很多种，在投资里的定义就是
  - $\text{VaR} = \text{Z} \times \sigma_\text{p}$
- $UL = VaR = Z \times \sigma_p > 0$ (loss is implied)
- $EL = -\mu => 0$ expected loss (negative is implied)
  - $EL = -Expected\ return$
- $\text{SaR} = \text{Z} \times \sigma_\text{p} - \mu = UL + EL$

投资组合角度

- $\text{Surplus} = \text{Asset} - \text{Liability}$
- Surplus 是 2 个资产组合： A对应资产 asset， L对应 liability 负债。
  - $\text{Surplus} = \text{Asset} + (-\text{Liability})$
- 定义
  - 资产 A：金额是$\text{P}_\text{A} > 0$，回报是$\text{r}_\text{A} > 0$，标准差是s_A
  - 资产 L：金额是$\text{P}_\text{L} > 0$，回报是$\text{r}_\text{L} < 0$（负的，是负债），标准差是s_L
    - 说明：这里把负债当成资产，因此金额是正的，只是回报是负的
  - 总资产： $\text{P} = \text{P}_\text{A} + P_L$
    - 每个资产的权值 $\text{w}_\text{A} = \frac{P_A}{P}, \text{w}_\text{L} = \frac{P_L}{P}$
  - 2 个资产的相关性是$\rho < 0$（用负的，题目给正的）。题目给正的说的是**资产的增长**和**负债的增长**的相关性是正的。这里把负债也看成是资产，负债的增长就是资产 L 的减少，于是组合里的想关性就是负的。
- VaR 的显著性是α，对应的临界值是 Z
- 组合 VaR
  - 每个资产的 VaR
    - $\text{VaR}_\text{A} = P_A \times Z \times s_A$， $\text{VaR}_\text{L} = P_L \times Z \times s_L$
  - 组合 VaR
    - $VaR_p^2 = VaR_A^2 + 2 \times \rho \times VaR_A \times VaR_L + VaR_L^2$
    - **注意$\rho < 0$ 要用负的**
- 期望收益 (expected return = expected surplus **growth**)
  - $R_p = P \times (\text{w}_\text{A} \times r_A + w_L \times r_L) = P_A \times r_A + P_L \times r_L$
- SaR (surplus-at-risk)
  - $\text{SaR}_\text{p} = \text{VaR}_\text{p} - R_p = \text{Z} \times \sigma_\text{p} - \mu$
- Surplus Terms
  - Beginning surplus $\text{S}_0 = \text{P}_\text{A} - P_L$ 初始 Surplus
  - Expected surplus **growth** $\Delta\text{S} = \text{R}_\text{p} = P_A \times r_A - P_L \times |r_L|$ surplus 的增长
  - **Expected** surplus $\text{S}_1 = \text{S}_0 + \text{R}_\text{p} = \text{P}_\text{A} \times (1 + \text{r}_\text{A}) - \text{P}_\text{L} \times (1 + |r_L|)$ 年底 Surplus
  - **Surplus at risk** $\text{SaR}_\text{p} = \text{VaR}_\text{p} - R_p = UL + EL > 0$ (worst case loss)
    - Expected worst case loss

- Surplus **deficit** Deficit $= \mathrm{SaR_p} - \mathrm{S_0} = -(\mathrm{S_0} + \mathrm{R_p} - Z \times \sigma_p) > 0$
  - Expected worst-case surplus

- 原始值：0
- 期望：u
- 方差: z * alpha
- VaR = WCL = u -a*alpha (<0)


SkyLine Airways has a defined benefit pension scheme with assets of $165 million and liabilities of $150 million. The annual growth of the liabilities is expected to be 4.5% with 2.4% volatility. The annual return on the pension assets has an expected value of 7.8% with 12% volatility. The correlation between asset return and liability growth is 0.35. What is the 95% surplus at risk for SkyLine?

A. $24.97 million.
B. $54.81 million.
C. $18.84 million.
D. $6.12 million.

Expected (S) = 165 × (1 + 7.8%) - 150 × (1 + 4.5%)
= 21.12
21.12 - 18.84 × 1.645 = -9.8718

结合例子, 95%VaR 用 1.65

	Principle	**Return**	**Volatility**	**VaR**
Asset	165	7.8%	12%	165*1.65*12%=32.67
Liability	150	**-4.5%**	2.4%	150*1.65*2.4%=5.94
Correlation	-0.35			
平均回报（也就是 surplus 的增长）	165*7.8%+150*(-4.5%) =6.12			
不考虑期望的 VaR	VaR_0 = sqrt(32.67^2+5.94^2 - 2*32.67*5.94*0.35)=31.09			
最终的 VaR	31.09 – 6.12=24.97			

4. A Step 1: Calculate the expected surplus growth.

Expected surplus growth = growth in assets – growth in liabilities

Expected surplus growth = ($165m × 0.078) – ($150m × 0.045)

Expected surplus growth = $12.87m – $6.75m = $6.12m

Step 2: Calculate the variance then the standard deviation of the A&L.

$$Var_{A\&L} = w^2_A\sigma^2_A + w^2_L\sigma^2_L - 2 \times w_A \times w_L \times \sigma_A \times \sigma_L \times Corr_{AL}$$

$$Var_{A\&L} = 165^2 \times 0.12^2 + 150^2 \times 0.024^2 - 2 \times 165 \times 150 \times 0.12 \times 0.024 \times 0.35$$

$$Var_{A\&L} = 392.04 + 12.96 - 49.896$$

$$Var_{A\&L} = 355.104$$

Standard deviation = $\sqrt{355.104}$ = 18.84m

Step 3: Calculate VaR of the assets.

VaR = Z-Score × volatility

VaR = 1.65 × $18.84m

VaR = $31,086,000

Surplus at risk = expected growth in surplus – VaR

Surplus at risk = $6.12m – $31.086m = –$24.97m

Note: Although it is a negative, it is usually expressed as a positive figure as it is assumed that it is a shortfall.

(See Topic 68)

46. SkyLine Airways has a defined benefit pension scheme with assets of $165 million and liability of $150 million. The annual growth of the liabilities is expected to be 4.5% with 2.4% volatility. The annual return on the pension assets has an expected value of 7.8% with 12% volatility. The correlation between asset return and liability growth is 0.35. What is the 95% surplus at risk for SkyLine?

A. $24.97million
✓B. $9.97million
C. $18.84million
D. $6.12million

$E(S) = 165 \times (1+7.8\%) - 150(1+4.5\%) = 21.12$

$\sigma_S = \sqrt{165^2 \times 12\%^2 + 150^2 \times 2.4\%^2 - 2 \times 0.35 \times 165 \times 150 \times 12\% \times 2.4\%}$

$= \sqrt{392.04 + 12.96 - 49.896} = 18.84$

Answer: B

Expected surplus growth = $165m ×(1+ 0.078) - $150m × (1+0.045)=21.12m

Variance of surplus = 355.104， Standard Deviation = 18.84m

Surplus at risk = expected growth in surplus – 1.65 × Standard Deviation of Surplus

Surplus at risk = $21.12m - 1.65*$18.84m = -$9.97m

$21.12 - 1.645 \times 18.84 = -9.87$

Note: Although it is a negative, it is usually expressed as a positive figure as it is assumed that it is a shortfall.