DEVELOPMENT OF RELIABLE CONTROL STRATEGIES FOR UNMANNED AERIAL VEHICLES

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Abstract

With the increasing demand for unmanned aerial vehicles (UAVs) in both military and civilian applications, critical safety issues should be considered significantly in order to make better and wider uses of them. UAVs are usually employed to work in hazardous and complex environment, which may seriously threaten the safety and reliability of UAVs. Therefore, the reliability and survivability of UAVs are becoming imperative for development of advanced intelligent control systems. The key challenge now is the lack of fully autonomous and reliable control techniques in face of different operation conditions and sophisticated environments. Further development of UAV control systems is required to be reliable in the presence of system component faults and insensitive to model uncertainties and external disturbances.

This research aims to design and develop novel control schemes for UAVs with consideration of all the factors which may threaten their safety and reliability. A novel adaptive sliding mode control strategy is proposed to accommodate actuator fault for an unmanned multirotor helicopter. Compared to the existing adaptive sliding mode control strategies in the literature, the proposed adaptive scheme can tolerate larger fault without stimulating control chattering due to the use of adaptation parameters in both continuous and discontinuous control parts. Furthermore, a fuzzy logic based boundary layer is synthesized to further improve the fault-tolerance capability of the control scheme. In the presence of severe fault, the stability of the closed-loop system can be maintained with a little bit sacrifice of tracking accuracy. Then, a control allocation approach is combined with adaptive sliding mode control to achieve the capability of tolerating complete actuator failures with application to a modified octorotor helicopter. The significance of this control scheme is that the stability of the closed-loop system is theoretically guaranteed in the presence of simultaneous actuator faults. Moreover, in order to make the developed control schemes more practical, model uncertainties and external disturbances should be explicitly considered. Thus, an adaptive control scheme with consideration of model uncertainties, actuator faults, and bounded external disturbances is proposed. All the above proposed control schemes are verified through simulation results. Some of them are verified through real flight tests.

The future works include design of a disturbance observer to make UAVs have the capability to actively compensate external disturbances and the development of fault detection and diagnosis scheme to further improve the integrity of the proposed control schemes. More experimental tests should also be carried out to verify the developed control schemes based on the available unmanned multirotor helicopter and fixed-wing aircraft.

Contents

1 Introduction

Over the last decades, unmanned aerial vehicles (UAVs) have been widely used by commercial industries, research institutes and military sectors, such as payload transportation, forest fire detection and fighting, surveillance, border patrol, remote sensing and aerial mapping, etc. More recently, with the development of micro-electro-mechanical system (MEMS) and onboard computer, more and more small scale UAVs are available in the market which further broadens the applications of UAVs. For many applications, in order to accomplish specific tasks, different sensors and measurement systems need to be incorporated with the corresponding UAV to make it become a fully functional system which is usually referred to as an unmanned aerial system (UAS). In this sense, a UAV can be regarded as an aerial sensor or payload carrier, and usually the cost of those onboard instruments can easily exceed the cost of the UAV itself, especially for small size UAVs. Therefore, the reliability and survivability of UAVs are becoming imperative. Critical safety issues should be considered significantly especially for those applications carried out in urban areas, where a UAV failure will easily damage its surroundings and induce catastrophic accidents. Moreover, UAVs are usually employed to work in hazardous and dangerous environment, which may seriously threaten the reliability and survivability of UAVs. As argued in [1], the increasing demands for safety, reliability, and high system performance have stimulated research in the area of fault-tolerant control (FTC) with the development in control theory and computer technology. Fault-tolerant capability is an important feature for safety-critical systems [2], such as aircraft, spacecraft, railways, autonomous cars, etc.

Two major and complementary approaches to deal with system uncertainties are robust and adaptive approaches [3]. Among these technologies, sliding mode control (SMC) is an approach to design robust control system dealing with uncertainties with discontinuous control strategy which can demonstrate invariance to so-called matched uncertainties while on a reduced order sliding surface [4]. However, this kind of traditional robust control algorithm makes trade-off between system performance and robustness in order to accommodate certain faults. In fact, at most time, real systems work under fault-free conditions. The consideration of fault at the system design stage will sacrifice some level of system performance in fault-free conditions. This will also limit the fault-tolerant capability of the system, because if a large fault is considered, the big trade-off may lead the system performance unacceptable in fault-free situations. In [4], Hess and Wells present that SMC approach is intended to serve as an alternative to truly reconfigurable systems while maintaining the desired performance without requiring any fault information. In this regard, some researchers are motivated to integrate adaptive technology into robust SMC to ensure system performance in both fault-free and faulty conditions [5, 6, 7, 8]. However, most of the studies in the literature mainly focus on the adaptation of the discontinuous control part and sliding variable is employed to construct the adaptive law. This may lead to overestimation of the adaptive parameter, and the estimation may never stop due to the cross movement of the sliding variable. Moreover, if large fault occurs in the system, the overuse of the discontinuous control strategy will lead to control chattering effect. This is one of the motivations of this proposed research which aims to design and develop an adaptive robust control strategy to tolerate larger fault without stimulating control chattering.

To ensure safety and reliability of UAVs, the capability of fault-tolerance is a key subject, but not the only one, to be considered. In fact, a lot of uncertain factors always occur in real applications. Among civilian applications such as forest fire fighting and parcel delivery, to name a few, payload transportation is a basic requirement for UAVs. For those applications, due to the added/released payload and external disturbances, the stability of UAVs can be seriously affected which is normally beyond the ability of a conventional flight controller. For controlling UAVs, a lot of control strategies have been proposed to achieve a good tracking performance. Nevertheless, among most of these studies, total take-off weight is assumed constant throughout the entire flight phase. For UAVs, especially small size UAVs, adding or releasing payload will significantly affect the tracking performance of the onboard flight controller. Furthermore, UAVs are expected to be cheap and low cost, and unlike manned aircraft, researchers may not spend a lot of time and money to build a very accurate mathematical model for designing control system. In this case, the control system for UAVs should have the capability of tolerating certain level of model uncertainties and external disturbances. This presents another motivation for the proposed research which aims to accommodate model uncertainties and external disturbances to ensure safety and reliability of UAVs.

It turns out that, it will be beneficial to have a UAS with the capability of tolerating certain faults, parametric uncertainties, and external disturbances without imperiling itself and its surroundings. In the control point of view, it is to design a so-called self-repairing "smart" flight control system to improve reliability and survivability of UAVs which is also the requirement for the next generation flight control system according to UAS roadmap released in 2005 by U.S. Department of Defense.

The rest of this proposal report is organized as follows. In Section 2, a brief literature survey is presented on the field of fault-tolerant flight control. In Section 3, the objectives and scope of the proposed research are described along with the employed methodologies. A timeline table in Section 4 illustrates the time schedule of all the works involved in this research. Finally, the concluding remarks and future works are summarized in Section 5.

2 Literature Survey

2.1 Fault-Tolerant Control

As modern technological systems become complex, their corresponding control systems are designed more and more sophisticated which presents the urgent need to increase reliability of the system. This stimulates the study of reconfigurable control system. Most research in reconfigurable control system focuses on fault detection and diagnosis (FDD) which can serve as a monitoring system by detecting, localizing, and identifying fault in system. FDD is a very important procedure but it is not sufficient to ensure safe operation of the system. For some safety-critical systems, such as aircraft, spacecraft, etc., the continuity of operation is a key feature and the closed-loop system should be capable of maintaining its pre-specified performance in terms of quality, safety, and stability despite the presence of fault. This calls for the appearance of fault-tolerant control system (FTCS). More precisely, FTCS is a control system that can accommodate system component fault and is able to maintain system stability and performance in both fault-free and faulty conditions [9]. Generally speaking, FTCS can be classified into two types known as *passive* (PFTCS) and *active* AFTCS [2]. PFTCS is designed to be robust against a class of presumed faults without requiring on-line detection of fault [10, 11], while AFTCS is based on controller reconfiguration or selection of pre-designed controllers with the help of FDD scheme [12, 13]. A comparative study between AFTCS and PFTCS is presented by Jiang and Yu [14]. From performance perspective, a PFTCS focuses more on the robustness of the control system to accommodate multiple system faults without striving for optimal performance for any specific fault conditions. Since stability is the prior consideration in a passive approach, the designed controller turns to be more conservative from performance viewpoint. An AFTCS typically consists of a FDD scheme, a reconfigurable controller, and a controller reconfiguration mechanism. These three units have to work in harmony to complete successful control tasks, and an optimal solution with certain preset performance criteria can be found. However, there are other issues which can impair an AFTCS to achieve its mission. Usually, system component fault can render the system unstable, and therefore there is only a limited amount of time available for an AFTCS to react to the fault and to make corrective control actions [2].

2.2 Sliding Mode Control

Sliding mode control (SMC) theory was introduced for the first time in the context of the variable structure system (VSS) [15]. It was developed for electric drives by Utkin [16]. During the last 30 years, it has shown to be a very effective approach in robust control area. Various books on SMC have also been pubished recently by Utkin *et. al.* [17], Edwards and Spurgeon [18], Fridman *et. al.* [19]. Studies by Alwi and Edwards [20] and Wang *et al.* [5] show some of the most recent researches in fault-tolerant flight control area using sliding mode control technique. The main advantage of SMC over the other nonlinear control approaches is its robustness to external disturbances, model uncertainties, and system parametric variations. To be more precise, one design parameter is synthesized in the discontinuous control part to deal with uncertainties which leads to significant control performance. The property of insensitivity to parametric uncertainties and external disturbances makes SMC as one of the most promising control approaches. The robust characteristics of SMC provides a natural environment for the use of such techniques on designing PFTCS [20, 21, 22, 23, 24]. There are two stages involved in the design of SMC.

The first step features the construction of a sliding surface, on which the system performance could be maintained as expected. The second step is concerned with the selection of appropriate control law to force the sliding variable reach the designed sliding surface, and thereafter keep the sliding motion within the close neighborhood of the sliding surface. However, in order to use this conventional SMC, the bound of uncertainty is always needed at the design stage [5]. For some applications, fault and parametric variation could occur at any time with unknown magnitude, thus it is hard to obtain the exact uncertainty bound in advance. This stimulates a new control strategy which incorporates adaptive control scheme into SMC to accommodate fault and parametric uncertainty without knowing the exact uncertainty bound [5, 6, 7]. In [5], with the proposed adaptive SMC, it shows a good tracking performance when actuator fault occurs in the system. Nevertheless, as fault becomes larger, a big tracking error occurs, and the system performance cannot be maintained anymore. This is because that the considered actuator fault is related to the uncertainty in the control effectiveness matrix, then after fault occurrence, the post-fault control effectiveness matrix should be changed correspondingly. The existing adaptive SMC strategies in the literature mainly incorporate adaptation parameter into the discontinuous control part which only use the discontinuous robust property for accommodating actuator fault. In fact, since the control effective matrix is used to derive the equivalent continuous control part, if one can incorporate the adaptation parameter in the continuous control part as well, it will significantly reduce the use of discontinuous strategy. In this case, control chattering can be avoided and larger fault can also be tolerated.

2.3 Control Allocation

Complete actuator failure in an aircraft could significantly reduce reliability and even cause catastrophic accidents. Despite the analytical redundancy, the hardware redundancy is also very important for a safety-critical system, such as passenger aircraft [25] and modern fighter aircraft [26]. Over-actuation gives freedom for designing FTCS to maintain stability and acceptable performance in the presence of actuator fault/failure which can also achieve fast system response in fault-free situations. Since such a system has finite number of solutions, the problem is to find at least one that satisfies control input constraints and some additional optimization criterion. Control allocation (CA) approach offers the advantage of a modular design where the design of the high-level control strategy is independent of actuator configuration by introducing virtual control module and control allocation module, respectively. The allocation of virtual control signal to the individual actuators is accomplished whithin control allocation module. Important issues such as input saturation, rate constraints, and actuator fault-tolerance are also handled within this module. The CA problem without considering system fault has been intensively studied following the work of Durham [27]. The simplest control allocation methods are based on the unconstrained least-squares algorithm and modifications of the solution aimed at accounting for position and rate limits [28, 29]. More complex methods formulate control allocation as a constrained optimization problem [30, 31]. Because of the limited number of variables and the convexity of the constraint set, the optimization problems are simple from a modern computational perspective. The redistributed pseudo-inverse is a very simple and effective approach to achieve CA [32]. However, it cannot guarantee full utilization of the actuators. The quadratic programming method seems to be favorable since the solution tends to combine the use of all control surfaces rather than just a few [33]. The fixed-point method can also provide an exact solution to the optimization problem, and it is guaranteed to converge [34]. Its drawback is that convergence of the algorithm can be very slow and strongly depends on the problem [35]. Due to its over-actuated feature, the CA approach has an inherent capability to tolerate certain level of fault. In the presence of actuator fault, an effective re-allocation of the virtual control signal to the remaining healthy actuators is needed to maintain system stability and performance which is referred to as reconfigurable control allocation [36]. In the context of reconfigurable control, Zhang *et. al.* [36, 37] present the concept of control allocation and re-allocation for aircraft with redundant control effectors. For the sake of the overall system stability, a high-level virtual controller is needed to provide the desired virtual control signal for the control allocation module. In [20, 38], a sliding mode based reconfigurable control allocation scheme is proposed to improve the system performance with single actuator fault for a fixed-wing aircraft benchmark model as used in [36] and [37] which presents a good example for making full use of the advantages of both high-level controller and CA. However, most of the reconfigurable control allocation schemes in the literature only focus on the allocation of the virtual control signal over the available actuators and rarely concern the stability of the overall system in the context of FTC. If the control allocation module fails to meet the required virtual control signal, the performance of the overall system will be degraded or even the stability of the overall system cannot be maintained anymore.

3 Objectives and Research Scope

In this section, the objectives and research scope of this proposal are elaborated in section 3.1. The proposed control schemes and strategies are presented step by step in section 3.2.

3.1 Thesis Objectives

The main goal of this research is to design and develop novel control schemes with applications to UAVs to improve safety and reliability of such safety-critical systems. The proposed research plan in particular is organized around the following objectives:

1) Design and develop an adaptive sliding mode control strategy for accommodating actuator fault which can show an advantage of tolerating larger actuator fault without stimulating control chattering. Then, based on this developed adaptive strategy, a fuzzy logic based boundary layer is designed to make a trade-off between tracking performance and system stability which means much larger fault can be compensated with sacrifice of certain level of tracking accuracy.
2) Develop an adaptive sliding mode based control allocation scheme which has the capability

to tolerate complete actuator failure and multiple actuator faults. The stability of the overall closed-loop system is theoretically proven.

3) Design and develop a model based disturbance observer which can effectively estimate environmental disturbance from wind and unstructured model disturbance. The designed disturbance observer is expected to alleviate control chattering effect of sliding mode control.
4) Develop an effective adaptive scheme to estimate model uncertainties, and then combine it with the former designed fault-tolerant control scheme and disturbance observer to construct a complete reliable control scheme for UAVs.
5) Design and develop a neural network based fault detection and diagnosis scheme which can not only set up a cost-effective monitoring framework, but also can be used for integration with fault-tolerant control strategies in active fault-tolerant architectures.

The proposed research in brief is expected to consider all the uncertain factors to design and develop an advanced control scheme for UAVs to significantly improve their reliability and survivability and also to satisfy strict safety and reliability demands by US Federal Aviation Administration (FAA) or other country's licensing & certificating authorities to further broad the commercial uses of developed UAVs. The developed control schemes and strategies from this research proposal will be verified by both simulation and experiment on a multirotor helicopter and a fixed-wing aircraft in the presence of actuator faults, model uncertainties, environmental disturbances, and different realistic fault scenarios.

3.2 Research Methodologies

3.2.1 Problem Statement

Consider a nonlinear affine system with parametric uncertainties, actuator faults and external disturbances:

$$\begin{aligned} \dot{x}(t) &= f(x(t),t) + h(x(t),t)\nu(t) + d(t) \\ \nu(t) &= B_u L(t) u(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, and $\nu(t) \in \mathbb{R}^n$ is the intermediate virtual control input. The vector $f(x(t),t) \in \mathbb{R}^n$ is the nonlinear function containing parametric uncertainties. $d(t)$ represents external disturbances including environmental and unstructured model disturbance which are unknown but bounded, i.e., $|d(t)| \leq D$. $B_u \in \mathbb{R}^{n\times m}$ is the control effectiveness matrix. $L(t) = diag(l_1(t), \cdots, l_m(t))$ representing control effectiveness of the actuators, where $l_j(t)_{(j=1,\ldots,m)}$ is a scalar satisfying $0 \leq l_j(t) \leq 1$. If $l_j(t) = 1$, the jth actuator works perfectly, otherwise, the jth actuator suffers some level of fault with a special case $l_j(t) = 0$ denoting the complete failure of the jth actuator. In order to deal with parametric uncertainties of the system, let $\hat{f}(x(t),t)$ represent the estimated value of the actual vector $f(x(t),t)$. Therefore, the estimated error $\tilde{f}(x(t),t)$ can be obtained as follows:

$$\tilde{f}(x(t),t) = \hat{f}(x(t),t) - f(x(t),t) \tag{2}$$

Assumption 1: The control input $u(t)$ lies in a compact set Ω_u described as:

$$u(t) \in \Omega_u = \{u(t) \in \mathbb{R}^m | u_{\min} \leq u(t) \leq u_{\max}\} \tag{3}$$

where $u_{\min} = \{u_{1\min}, u_{2\min}, \cdots, u_{m\min}\}$ and $u_{\max} = \{u_{1\max}, u_{2\max}, \cdots, u_{m\max}\}$.

For simplicity of expression, the notation (t) is omitted in the following sections, e.g., $x(t)$ is expressed as x.

3.2.2 System Formulation

As applied to an unmanned multirotor helicopter, the state vector is defined as:

$$\begin{aligned} x &= [z_e \quad \dot{z}_e \quad \phi \quad \dot{\phi} \quad \theta \quad \dot{\theta} \quad \psi \quad \dot{\psi}]^T \\ &= [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6 \quad x_7 \quad x_8]^T \end{aligned} \tag{4}$$

With this state vector, the dynamic equations of multirotor helicopter can be summarized in Eq. (5), where m is the total mass of multirotor helicopter, L_d is the distance between center of mass and motor, and g is acceleration of gravity. I_{xx}, I_{yy} and I_{zz} is the moment of inertia along x, y and z axis, respectively. I_r is the inertial moment of rotor. z_e is the altitude of multirotor helicopter. ϕ, θ and ψ are the Euler angles of multirotor helicopter defined in inertial reference frame. $K_{p(p=1,2,3,4)}$ is drag coefficient. Ω is the summary of rotor speed defined as $\Omega = \Omega_1 + \Omega_2 - \Omega_3 - \Omega_4$. U_z, U_ϕ, U_θ, and U_ψ are the corresponding intermediate virtual control inputs.

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \end{bmatrix} = \begin{bmatrix} x_2 \\ -g + \cos\phi\cos\theta U_z/m - K_1\dot{z}_e/m \\ x_4 \\ x_6x_8(I_{yy} - I_{zz})/I_{xx} + U_\phi/I_{xx} - I_r\Omega\dot{\theta}/I_{xx} - K_2L_d\dot{\phi}/I_{xx} \\ x_6 \\ x_4x_8(I_{zz} - I_{xx})/I_{yy} + U_\theta/I_{yy} + I_r\Omega\dot{\phi}/I_{yy} - K_3L_d\dot{\theta}/I_{yy} \\ x_8 \\ x_4x_6(I_{xx} - I_{yy})/I_{zz} + U_\psi/I_{zz} - K_4\dot{\psi}/I_{zz} \end{bmatrix} \tag{5}$$

In order to facilitate the following controller design, the above nonlinear system can be expressed in the following brief manner:

$$\begin{aligned} \dot{x}_{2i-1} &= x_{2i} \\ \dot{x}_{2i} &= f_i(x) + h_i\nu_i + d_i \end{aligned} \tag{6}$$

where $i = 1, 2, 3, 4$ represents each subsystem, namely height, roll, pitch, and yaw.

3.2.3 Integral Sliding Mode Control

The integral sliding surface for the system is defined by the following set:

$$S_i = \{x \in \mathbb{R}^n : \ \sigma_i(x) = 0\} \tag{7}$$

For $\forall i = 1, 2, 3, 4$, denoting x_{2i-1}^d and x_{2i}^d as the desired trajectories, the tracking errors can be defined as $\tilde{x}_{i1} = x_{2i-1} - x_{2i-1}^d$ and $\tilde{x}_{i2} = x_{2i} - x_{2i}^d$. Thus, the switching function can be defined as:

$$\sigma_{i0} = c_i \tilde{x}_{i1} + \tilde{x}_{i2} \tag{8}$$

$$\begin{aligned} \dot{z}_i &= -c_i \tilde{x}_{i2} + k_{i2} \tilde{x}_{i2} + k_{i1} \tilde{x}_{i1} \\ z_i(0) &= -c_i \tilde{x}_{i1}(t_0) - \tilde{x}_{i2}(t_0) \end{aligned} \tag{9}$$

Such that

$$\sigma_i = \tilde{x}_{i2} + k_{i2} \tilde{x}_{i1} + k_{i1} \int_{t_0}^{t} \tilde{x}_{i1}(\tau) d\tau - k_{i2} \tilde{x}_{i1}(t_0) - \tilde{x}_{i2}(t_0) \tag{10}$$

Then, after obtaining the sliding surface, the problem is to design an appropriate control law to make the sliding surface attractive. The design problem can be formulated as that, given $x(t_0) = x^0(t_0)$, the identity $x = x^0$ should be guaranteed all the time $t \geq t_0$. According to this requirement, the control law is designed in the following form:

$$\nu_i = \nu_{i0} + \nu_{i1} \tag{11}$$

where ν_{i0} is the continuous nominal control part to stabilize the ideal system and guide it to a given trajectory with satisfactory accuracy. ν_{i1} is the discontinuous control part for compensating the perturbations and disturbances in order to ensure the sliding motion.

In order to analyze the sliding motion associated with the switching function as shown in Eq. (10), the time derivative of the switching function is computed as follows:

$$\dot{\sigma}_i = \dot{\tilde{x}}_{i2} + k_{i2} \tilde{x}_{i2} + k_{i1} \tilde{x}_{i1} \tag{12}$$

The equivalent control ν_{i0} is designed by equalizing $\dot{\sigma}_i = 0$. In this case, the disturbance d_i is omitted, and the system is given as:

$$\dot{x}_{2i} = f_i(x) + h_i \nu_i \tag{13}$$

Then, in order to reject disturbances, a discontinuous control part is synthesized as shown below:

$$\nu_{i1} = -h_i^{-1} k_{ci} \mathrm{sign}(\sigma_i) \tag{14}$$

where k_{ci} is a positive high gain which makes the sliding surface attractive.

Therefore, the control law can be developed as:

$$\nu_i = h_i^{-1} (\dot{x}_{2i}^d - k_{i2} \tilde{x}_{i2} - k_{i1} \tilde{x}_{i1} - f_i(x)) - h_i^{-1} k_{ci} sign(\sigma_i) \tag{15}$$

However, in order to account for disturbances, control discontinuity is increased which may lead to control chattering. One can remove this condition by smoothing control discontinuity in a thin boundary layer neighboring the sliding surface. The boundary layer is formulated as:

$$\bar{B} = \{\tilde{x}_{i1}, \tilde{x}_{i2}, |\sigma_i| \leq \mathit{\Phi}_i\} \tag{16}$$

where Φ_i is the boundary layer thickness with positive value.

Accordingly, the feedback control law becomes:

$$\nu_i = h_i^{-1}(\dot{x}_{2i}^d - k_{i2}\tilde{x}_{i2} - k_{i1}\tilde{x}_{i1} - f_i(x)) - h_i^{-1}k_{ci}sat(\sigma_i/\Phi_i) \tag{17}$$

where the *sat* function is defined as:

$$sat(\sigma_i/\Phi_i) = \begin{cases} sign(\sigma_i) & if \quad |\sigma_i| \geq \Phi_i \\ \sigma_i/\Phi_i & if \quad |\sigma_i| < \Phi_i \end{cases} \tag{18}$$

3.2.4 Adaptive Sliding Mode Control Allocation

Control allocation problem refers to the distribution of virtual control signals over the available actuators. In a faulty condition where $l_j(t) < 1$, given the desired virtual control signal $\nu_d(t)$, the solution $u(\nu_d(t))$ is searched such that $\nu_d(t) = B_u L(t) u(\nu_d(t))$ is satisfied. The schematic of the proposed control strategy is illustrated in Fig. 1.

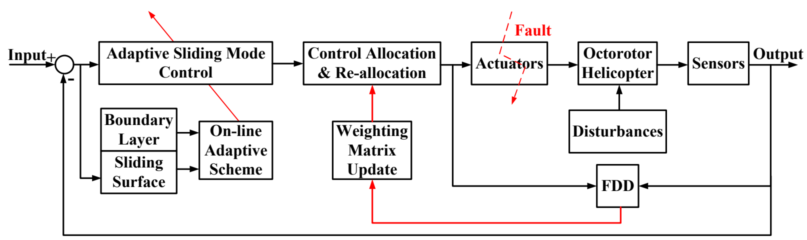

Figure 1: The schematic of the proposed adaptive sliding mode control allocation strategy.

Lemma 1: The quadratic programming (QP) approach based on minimizing control input can be described as:

$$\begin{aligned} J &= arg \min_u u^T W u \\ s.t. \quad \nu_i &= B_{ui} u \end{aligned} \tag{19}$$

and it has an explicit solution as follows:

$$u = W B_{ui}^T (B_{ui} W B_{ui}^T)^{-1} \nu_i \tag{20}$$

where $W = W^T$ is a symmetric positive definite weighting matrix, $B_{ui} \in \mathbb{R}^{n \times m}$ is the control effectiveness matrix directly related to actuators, and ν_i is the virtual control signal derived from the high-level control module.

In the case of single actuator fault, the weighting matrix is updated according to the fault information without affecting the high-level controller, namely, $w_j = 1/l_j$. In this situation, more control efforts will be distributed to the healthier actuator. Specially, when the jth actuator experiences complete failure, the corresponding weighting parameter w_j will become infinity which means there will be no control effort distributed to this actuator.

In the case of simultaneous actuator faults, where control allocation and re-allocation scheme fails to maintain the overall system stability, an adaptive scheme is synthesized to compensate this faulty condition. In this circumstance, conditions described in Eqs. (3) and (19) could not be satisfied at the same time due to the existence of the error between the generated virtual control signal from the control allocation module and the desired one from the high-level sliding mode control module.

Let $\nu_i = \nu_{id} + \tilde{\nu}_i$, the following system dynamics can be obtained:

$$\dot{x}_{2i} = f_i(x) + h_i\nu_{id} + h_i\tilde{\nu}_i + d_i \tag{21}$$

where $\tilde{\nu}_i$ denotes the virtual control error.

In order to maintain the overall system performance, the high-level sliding mode controller needs to be reconfigured. For this reason, an adaptive approach is employed. Observed from Eq. (21), in order to maintain the tracking performance of the high-level controller when there is error between ν_i and ν_{id}, the parameter h_i should be adjusted accordingly to eliminate this error. In this case, the term $h_i\tilde{\nu}_i$ in Eq. (21) can be expressed as $\tilde{h}_i\nu_{id}$. Therefore, Eq. (21) can be rewritten as:

$$\begin{aligned}\dot{x}_{2i} &= f_i(x) + (h_i + \tilde{h}_i)\nu_{id} + d_i \\ &= f_i(x) + \hat{h}_i\nu_{id} + d_i\end{aligned} \tag{22}$$

In this case, considering the sliding surface in Eq. (10), the high-level sliding mode control law is redesigned using the estimated $\hat{h}_i$ as follows:

$$\nu_i = \hat{h}_i^{-1}(\dot{x}_{2i}^d - k_{i2}\tilde{x}_{i2} - k_{i1}\tilde{x}_{i1} - f_i(x)) - \hat{h}_i^{-1}k_{ci}sat(\sigma_i/\Phi_i) \tag{23}$$

In order to develop the adaptive scheme to update the estimated parameter $\hat{h}_i$, a new variable is defined based on the switching function and boundary layer as follows:

$$\sigma_{\Delta i} = \sigma_i - \Phi_i sat(\sigma_i/\Phi_i) \tag{24}$$

where $\sigma_{\Delta i}$ is the measurement of the algebraic distance between the current state and the boundary layer. It features $\dot{\sigma}_{\Delta i} = \dot{\sigma}_i$ outside the boundary layer and $\sigma_{\Delta i} = 0$ inside the boundary layer.

Based on this newly-designed parameter, the on-line adaptive scheme is formulated as:

$$\dot{\hat{h}}_i^{-1} = (-\dot{x}_{2i}^d + k_{i2}\tilde{x}_{i2} + k_{i1}\tilde{x}_{i1} + f_i(x) + k_{ci}sat(\sigma_i/\Phi_i))\sigma_{\Delta i} \tag{25}$$

Theorem 1: Consider a nonlinear system with simultaneous actuator faults and bounded external disturbances. Given the sliding surface in (10) and control input constraints in (3), by employing the feedback control laws in (20) and (23) and the on-line adaptive scheme in (25), the sliding motion will be achieved and maintained on the sliding surface to ensure the overall system tracking performance with the discontinuous gain chosen as $k_{ci} \geq \eta_i + D_i$ regardless of the virtual control error, i.e., $\tilde{\nu}_i = \nu_i - \nu_{id} \neq 0$.

3.2.5 Adaptive Control for Parametric Uncertainty and Actuator Fault

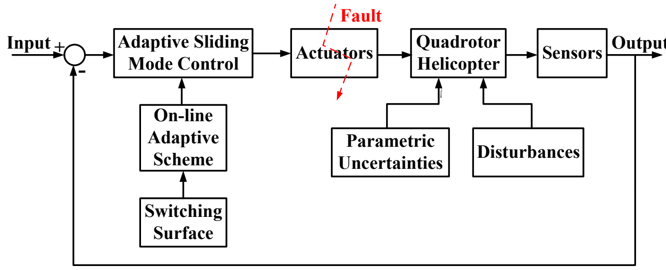

Figure 2: The schematic of the proposed adaptive control strategy.

In the case of both parametric uncertainty and actuator fault, all the related parameters $\hat{f}_i(x)$ and $\hat{h}_i$ need to be estimated on-line, and the corresponding control scheme is described in Fig. 2. Then, the afore-designed baseline sliding mode controller could be rewritten as:

$$\begin{aligned} \nu_i &= \hat{h}_i^{-1}(\dot{x}_{2i}^d - k_{i2}\tilde{x}_{i2} - k_{i1}\tilde{x}_{i1} - \hat{f}_i(x)) - \hat{h}_i^{-1}k_{ci}sat(\sigma_i/\Phi_i) \\ u &= B_u^+\nu_i = B_u^T(B_uB_u^T)^{-1}\nu_i \end{aligned} \tag{26}$$

Denoting $\hat{\chi}_i = h_i^{-1}\hat{f}_i(x)$ and $\hat{\Upsilon}_i = \hat{h}_i^{-1}$, Eq. (26) can be rearranged as:

$$u = B_u^T(B_uB_u^T)^{-1}[-\hat{\chi}_i + \hat{\Upsilon}_i(\dot{x}_{2i}^d - k_{i2}\tilde{x}_{i2} - k_{i1}\tilde{x}_{i1} - k_{ci}sat(\sigma_i/\Phi_i))] \tag{27}$$

The corresponding on-line adaptive schemes for estimating $\hat{\chi}_i$ and $\hat{\Upsilon}_i$ are designed as follows:

$$\dot{\hat{\chi}}_i = \sigma_{\Delta i} \tag{28}$$

$$\dot{\hat{\Upsilon}}_i = (-\dot{x}_{2i}^d + k_{i2}\tilde{x}_{i2} + k_{i1}\tilde{x}_{i1} + k_{ci}sat(\sigma_i/\Phi_i))\sigma_{\Delta i} \tag{29}$$

Theorem 2: Consider a nonlinear affine system with parametric uncertainty, actuator fault and bounded external disturbance. Given the designed integral sliding surface (10), by employing the feedback control law (27) and updated by designed on-line adaptive schemes (28) and (29), the sliding motion will be achieved and maintained inside the boundary layer (16) regardless of the parametric uncertainty, actuator fault and external disturbance with the discontinuous gain chosen as $k_{ci} \geq D_i + \eta$.

3.2.6 Fuzzy Adaptive Sliding Mode Control

As described in the previous section, in order to avoid control chattering caused by control discontinuity, the boundary layer approach is employed. The thickness of the boundary layer is an important design parameter which determines system tracking accuracy. Moreover, it is also an important element to ensure system stability in the presence of actuator fault. In some cases when actuator fault becomes excessively large, the afore-designed adaptive control law will not bring the sliding variable within the boundary layer, and the system stability will not be maintained anymore. In this case, the thickness of boundary layer should be enlarged to sacrifice some level of tracking accuracy and maintain system stability. This section introduces a fuzzy logic based approach for adjusting the boundary layer thickness to obtain a good trade-off between tracking accuracy and system stability.

To this end, the thickness of boundary layer Φ_i is the output of the fuzzy logic control scheme, which is determined by two parameters: the absolute value of the sliding variable $|\sigma_i|$ and the rate of adaptation $\dot{\hat{\Upsilon}}_i$, namely,

$$\Phi_i = f(|\sigma_i|, \dot{\hat{\Upsilon}}_i) \tag{30}$$

The main idea of the fuzzy logic used in this thesis is summarized as follows:

i) When the rate of adaptation becomes big, the boundary layer thickness should be increased to smooth the control input.

ii) The boundary layer thickness should be decreased if the rate of adaptation is low or zero. This is because, in order to maintain the best tracking accuracy, the thickness of the boundary layer is needed to be as small as possible.

iii) When the absolute value of the sliding variable is high, the trajectory of sliding motion is far away from the nominal sliding surface, thus a steep saturation function is desired to decrease the duration of the reaching phase.

iv) When the absolute value of the sliding variable is low, the trajectory of sliding motion is close to the nominal sliding surface, thus a steep saturation function is likely to introduce control chattering.

4 Timelines

The proposed research started on September 2014. The progress for accomplishing the objectives of the proposed research study is outlined in Table 1.

Table 1: Timeline Table

Main Tasks	**2014-2015**			**2015-2016**			**2016-2017**			**2017-2018**		
	F	**W**	**S**	**F**	**W**	**S**	**F**	**W**	**S**	**F**	**W**	**S**
Course Study												
Literature Review												
Comprehensive Exam												
Development of ASMC Scheme												
Fuzzy Logic Based Boundary Layer												
Development of ASMCA Scheme												
Model Uncertainty & Actuator Fault												
Proposal												
Disturbance Observer												
FDD with Neural Networks												
Dissertation												
Publications												

Note: **F**: fall; **W**: winter; **S**: summer semesters; ASMC: adaptive sliding mode control.

5 Anticipated Significance of The Work

Although tremendous efforts have been dedicated to control of UAVs, there still exist significant challenges to improve the performance of the onboard controllers to further ensure safety and reliability of UAVs. Many technical issues must be solved to increase the level of automation required for more sophisticated and hazardous applications.

In this proposal, an adaptive sliding mode control scheme is developed to accommodate actuator faults in a UAV system. Compared to the conventional and existing adaptive sliding mode control in the literature, the proposed control scheme can tolerate larger fault in the system without stimulating control chattering. Further, with combination of fuzzy logic based boundary layer, the fault-tolerance capability is further improved. Then, with consideration of multiple actuator faults and even complete actuator failure, a control allocation based adaptive sliding mode control scheme is proposed. The low-level control allocation module is responsible for redistributing control signals to the remaining healthy actuators in the presence of actuator faults, whereas the high-level adaptive sliding mode control module is employed to compensate virtual control error and ensure stability of the closed-loop system in the presence of multiple actuator faults. The significant feature of this control scheme is that the stability of the closed-loop system is guaranteed theoretically compared to the existing reconfigurable control allocation schemes. Finally, in order to make the control scheme more practical, modeling uncertainty and actuator fault are both considered in the system. The designed adaptive control scheme can accurately estimate the mass variation and keep the original tracking performance of the system while accommodating actuator fault.

The future works include the design of disturbance observer to be combined with the developed adaptive control scheme to make UAVs actively compensate external disturbances without stimulating control chattering. Moreover, a neural networks based fault detection and diagnosis scheme should also be developed to improve the integrity of the developed adaptive sliding mode control allocation scheme. Finally, more experimental tests on the developed control schemes based on the available unmanned multirotor helicopter and fixed-wing aircraft should also be addressed.

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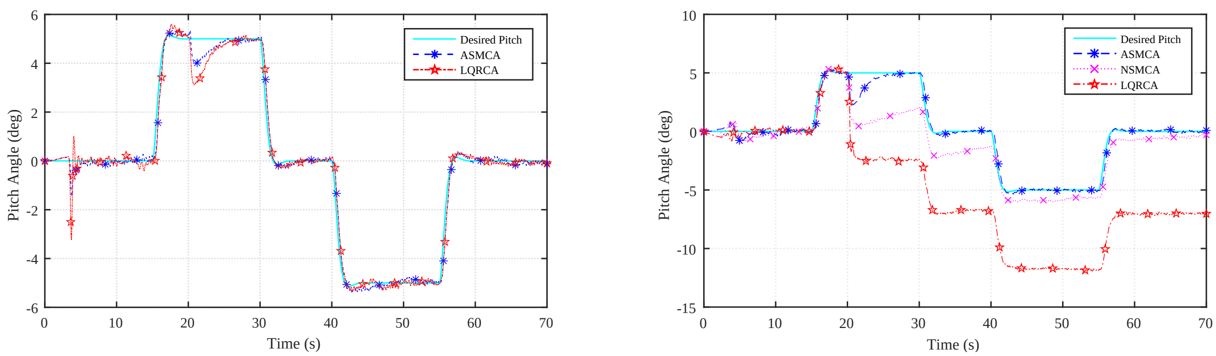
6 Appendix: Progress to Date

6.1 Adaptive Sliding Mode Control Allocation

In order to verify the performance of the proposed adaptive sliding mode control allocation (ASMCA) in the presence of single actuator complete failure and simultaneous actuator faults (actuator 1: complete failure; actuator 5: 40% loss of control effectiveness), normal sliding mode control (NSMCA) and linear quadratic regulator (LQRCA) are presented as a high-level virtual controller combined with the same control allocation scheme for comparison as shown in Fig. 3. The proposed ASMCA presents better performance compared to the other two control strategies.



(a) (b)

Figure 3: (a) Tracking performance of pitch motion in the presence of single actuator complete failure in real flight test; (b) Tracking performance of pitch motion in the presence of simultaneous actuator faults in real flight test.

6.2 Adaptive Control for Parametric Uncertainty and Actuator Fault

In this example, the results based on a quadrotor helicopter subject to mass variation and actuator fault are demonstrated. The quadrotor helicopter is firstly lifted to 0.8 m to pick up a payload with 0.6 kg. With this payload, it is lifted to 2 m and commanded to follow a set of pre-designed trajectory. During this phase, an actuator fault with 40% loss of control effectiveness is injected at 50 sec. At the end, the payload is dropped from the air to the targeted area. During the whole flight phase, the mass of the payload and the level of actuator fault are both unknown to the onboard controller. With comparison to the conventional linear quadratic regulator (LQR) controller, the tracking performances of height and pitch motion are presented in Figs. 4(a) and 4(b). The proposed adaptive control strategy shows a better tracking performance with the ability to estimate the total mass of the quadrotor helicopter as shown in Fig. 4(c) and to compensate actuator fault.

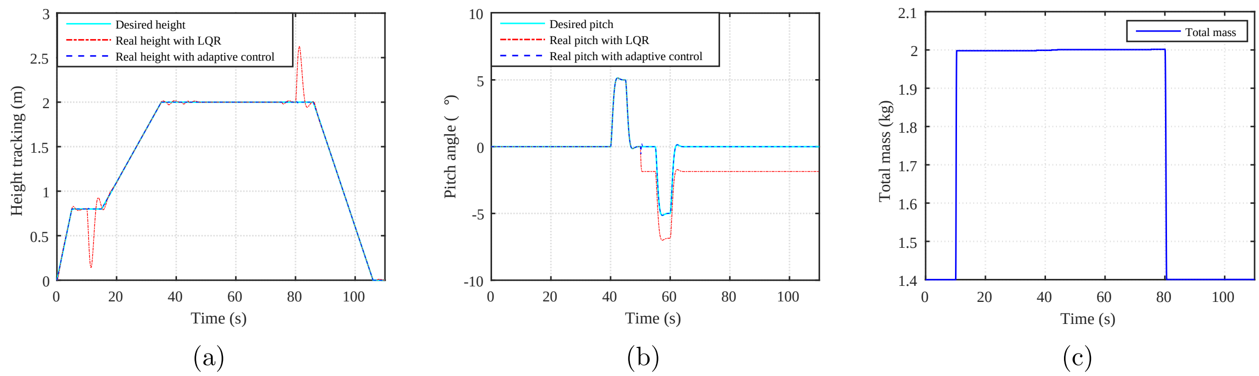


Figure 4: (a) Tracking performance of height in the presence of both parametric uncertainty and actuator fault; (b) Tracking performance of pitch motion in the presence of both parametric uncertainty and actuator fault. (c) Estimation of total mass of the unmanned quadrotor helicopter.

6.3 Fuzzy Adaptive Sliding Mode Control

In this scenario, a 70% loss of control effectiveness fault is introduced to the actuator. As observed from Fig. 5(a), the adaptive sliding mode control scheme fails to maintain system tracking performance and control chattering is stimulated in the presence of large actuator fault. On the contrary, with a little bit sacrifice of tracking accuracy, the system stability is maintained with the proposed fuzzy adaptive sliding mode control as shown in Fig. 5(b).

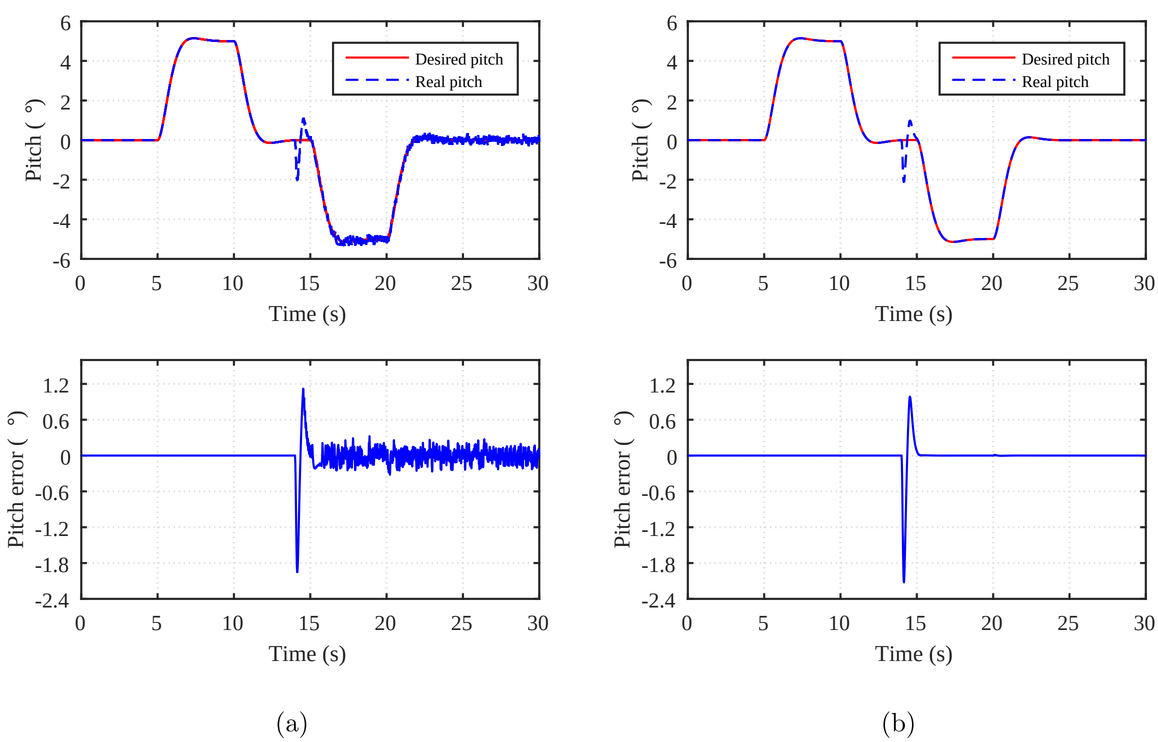


Figure 5: (a) Tracking performance of pitch motion using adaptive sliding mode control with 70% loss of control effectiveness in actuator; (b) Tracking performance of pitch motion using fuzzy adaptive sliding mode control with 70% loss of control effectiveness in actuator.

6.4 Publications

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6.5 Proof

Proof of Theorem 1: Consider the following Lyapunov candidate function:

$$V_i = \frac{1}{2}[\sigma_{\Delta i}^2 + h_i(\tilde{h}_i^{-1})^2] \tag{31}$$

Then, the derivative of the selected Lyapunov candidate function would be:

$$\begin{aligned}
\dot{V}_i =& \sigma_{\Delta i}\dot{\sigma}_{\Delta i} + h_i(\hat{h}_i^{-1} - h_i^{-1})\dot{\hat{h}}_i^{-1} \\
=& \sigma_{\Delta i}(f_i(x) + h_i\hat{h}_i^{-1}(\dot{x}_{2i}^d - k_{i2}\tilde{x}_{i2} - k_{i1}\tilde{x}_{i1} - f_i(x) - k_{ci}sat(\sigma_i/\Phi_i)) \\
& + d_i - \dot{x}_{2i}^d + k_{i2}\tilde{x}_{i2} + k_{i1}\tilde{x}_{i1}) + h_i(\hat{h}_i^{-1} - h_i^{-1})\dot{\hat{h}}_i^{-1} \\
=& (h_i\hat{h}_i^{-1} - 1)(\dot{x}_{2i}^d - k_{i2}\tilde{x}_{i2} - k_{i1}\tilde{x}_{i1} - f_i(x))\sigma_{\Delta i} \\
& + (h_i\hat{h}_i^{-1} - 1)\dot{\hat{h}}_i^{-1} - h_i\hat{h}_i^{-1}k_{ci}sat(\sigma_i/\Phi_i)\sigma_{\Delta i} + d_i\sigma_{\Delta i} \\
=& (h_i\hat{h}_i^{-1} - 1)(\dot{x}_{2i}^d - k_{i2}\tilde{x}_{i2} - k_{i1}\tilde{x}_{i1} - f_i(x))\sigma_{\Delta i} + (h_i\hat{h}_i^{-1} - 1)\dot{\hat{h}}_i^{-1}
\end{aligned} \tag{32}$$

$$
\begin{aligned}
&- (h_i\hat{h}_i^{-1} - 1)k_{ci}sat(\sigma_i/\Phi_i)\sigma_{\Delta i} - k_{ci}sat(\sigma_i/\Phi_i)\sigma_{\Delta i} + d_i\sigma_{\Delta i} \\
=&(h_i\hat{h}_i^{-1} - 1)[\dot{\hat{h}}_i^{-1} + (\dot{x}_{2i}^d - k_{i2}\tilde{x}_{i2} - k_{i1}\tilde{x}_{i1} - f_i(x) \\
&- k_{ci}sat(\sigma_i/\Phi_i))\sigma_{\Delta i}] - k_{ci}sat(\sigma_i/\Phi_i)\sigma_{\Delta i} + d_i\sigma_{\Delta i}
\end{aligned}
$$

Substituting Eq. (25) into Eq. (32) leads to,

$$
\begin{aligned}
\dot{V}_i =& - k_{ci}sat(\sigma_i/\Phi_i)\sigma_{\Delta i} + d_i\sigma_{\Delta i} \\
\leq& - (\eta_i + D_i)sat(\sigma_i/\Phi_i)\sigma_{\Delta i} + D_i\sigma_{\Delta i} \\
\leq& - \eta_i|\sigma_{\Delta i}|
\end{aligned}
\tag{33}
$$

Therefore, with the proposed control scheme the performance of the overall system is maintained in the presence of simultaneous actuator faults. □

Proof of Theorem 2: Consider the following Lyapunov candidate function:

$$
V_i = \frac{1}{2}\left[\sigma_{\Delta i}^2 + \Upsilon_i^{-1}(\hat{\chi}_i - \chi_i)^2 + \Upsilon_i^{-1}(\hat{\Upsilon}_i - \Upsilon_i)^2\right] \tag{34}
$$

Then, the derivative of the selected Lyapunov candidate function would be:

$$
\begin{aligned}
\dot{V}_i =&\sigma_{\Delta i}\dot{\sigma}_{\Delta i} + \Upsilon_i^{-1}(\hat{\chi}_i - \chi_i)\dot{\hat{\chi}}_i + \Upsilon_i^{-1}(\hat{\Upsilon}_i - \Upsilon_i)\dot{\hat{\Upsilon}}_i \\
=&\sigma_{\Delta i}(\Upsilon_i^{-1}\chi_i - \Upsilon_i^{-1}\hat{\chi}_i + \Upsilon_i^{-1}\hat{\Upsilon}_i(\dot{x}_{2i}^d - k_{i2}\tilde{x}_{i2} - k_{i1}\tilde{x}_{i1} - k_{ci}sat(\sigma_i/\Phi_i)) \\
&+ d_i - \dot{x}_{2i}^d + k_{i2}\tilde{x}_{i2} + k_{i1}\tilde{x}_{i1}) + \Upsilon_i^{-1}(\hat{\chi}_i - \chi_i)\dot{\hat{\chi}}_i + \Upsilon_i^{-1}(\hat{\Upsilon}_i - \Upsilon_i)\dot{\hat{\Upsilon}}_i \\
=&\Upsilon_i^{-1}(\hat{\chi}_i - \chi_i)(\dot{\hat{\chi}}_i - \sigma_{\Delta i}) + (\Upsilon_i^{-1}\hat{\Upsilon}_i - 1)(\dot{x}_{2i}^d - k_{i2}\tilde{x}_{i2} - k_{i1}\tilde{x}_{i1})\sigma_{\Delta i} \\
&+ (\Upsilon_i^{-1}\hat{\Upsilon}_i - 1)\dot{\hat{\Upsilon}}_i + d_i\sigma_{\Delta i} - \Upsilon_i^{-1}\hat{\Upsilon}_i k_{ci}sat(\sigma_i/\Phi_i)\sigma_{\Delta i} \\
=&\Upsilon_i^{-1}(\hat{\chi}_i - \chi_i)(\dot{\hat{\chi}}_i - \sigma_{\Delta i}) + (\Upsilon_i^{-1}\hat{\Upsilon}_i - 1)[\dot{\hat{\Upsilon}}_i + (\dot{x}_{2i}^d - k_{i2}\tilde{x}_{i2} \\
&- k_{i1}\tilde{x}_{i1} - k_{ci}sat(\sigma_i/\Phi_i))\sigma_{\Delta i}] + d_i\sigma_{\Delta i} - k_{ci}sat(\sigma_i/\Phi_i)\sigma_{\Delta i}
\end{aligned}
\tag{35}
$$

Substituting Eqs. (28) and (29) into Eq. (35) leads to,

$$
\begin{aligned}
\dot{V}_i =& - k_{ci}sat(\sigma_i/\Phi_i)\sigma_{\Delta i} + d_i\sigma_{\Delta i} \\
\leq& - (D_i + \eta)sat(\sigma_i/\Phi_i)\sigma_{\Delta i} + D_i\sigma_{\Delta i} \\
\leq& - \eta|\sigma_{\Delta i}|
\end{aligned}
\tag{36}
$$

Thus, the system satisfies the standard $\eta-$reachability condition, and the performance of the system can be maintained with the proposed control law in the presence of parametric uncertainty and actuator fault. □