Delaunay Triangulation Demo

Laboratory Report

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1 Introduction

Delaunay triangulation for a set P of points in a plane is a triangulation DT(P) such that no point in P is inside the circumcircle of any triangle in DT(P). Delaunay triangulations maximize the minimum angle of all the angles of the triangles in the triangulation, thus to avoid skinny triangles.

Define angle-vector $A(\mathcal{T})$ of a triangulation $\mathcal{T}$ as vector of angles sorted in non-decreasing order, use lexicographic comparison on the angle vectors to compare skinness of triangulations. Let $A(\mathcal{T}) = \{\alpha_1, \alpha_2,...\}$ and $A(\mathcal{T}') = \{\alpha'_1, \alpha'_2,...\}$, we say that $A(\mathcal{T}) > A(\mathcal{T}')$ if for some i ,

$$\forall j < i, \alpha_j = \alpha_j' \text{ and } \alpha_i > \alpha_j'$$

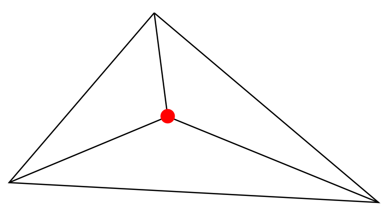
We say that a triangulation $\mathcal{T}$ is angle-optimal if $\exists\mathcal{T}'$ such that $A(\mathcal{T}) > A(\mathcal{T}')$.

2 Randomized Incremental Algorithm

We implemented the randomized expected-$O(n\log n)$ time algorithm, this idea is insert the points in random order, once at a time; and update the triangulation with each new addition.

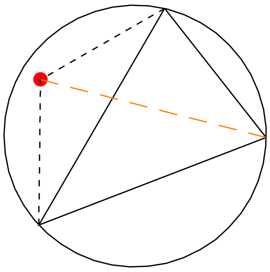
2.1 Spliting

When the incrementing vertex is inserted, we obtain the face it maps to by searching from the VertexToFace property, which is maintained by the rebucking process, then the face will be split. Similar procedure can be performed through the VertexToHEdge property if it lies on an edge.



2.2 Flipping and In-Circle test

After splitting, we must legalize each triangle around the incrementing vertex. The criteria that adopted here is the in-circle test. If the test failed, a flipping will be conducted.



2.3 Rebucketing

The uninserted vertices were kept in the following properties as mentioned earlier, VertexToFace/VertexToHEdge/FaceToVertices. When a face was split, the vertices mapped to the former face will be rebucked, by finding the new faces they belong to. Similar actions will be performed if a flipping happens, thus the vertex-face relations are maintained in this way, effectively reducing the time locating its position when a new vertex inserted. The total expected time spent on rebucketing is $O(n\log n)$.

3 Geometric Transformation

Consider the paraboloid of revolution of equation $z = x^2 + y^2$. A point $p = (x, y)$ is lifted to the point $l(p) = (X, Y, Z)$ in $\mathbb{E}^3$, where $X = x$, $Y = y$, and $Z = x^2 + y^2$. Therefore, we have shown that the projection of the part of the convex hull of the lifted set $l(p) = (X, Y, Z)$ consisting of the downward-facing faces is the Delaunay triangulation of P. Thus A flip in P corresponds to the concave-convex transformation. Futhermore, we changed the $z = x^2 + y^2$ to other convex functions, so generalized delaunay algorithms were obtained.