

Figure 1 (line plot of all variables)

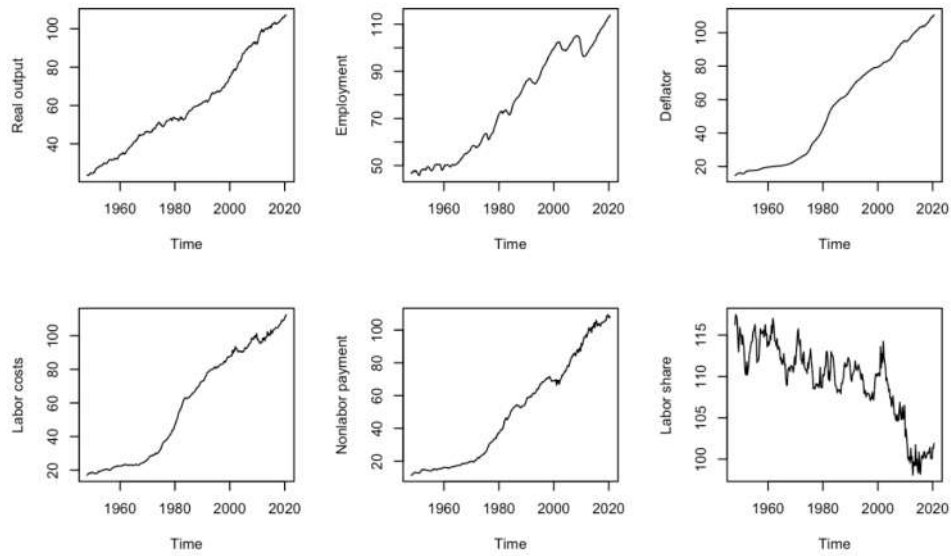
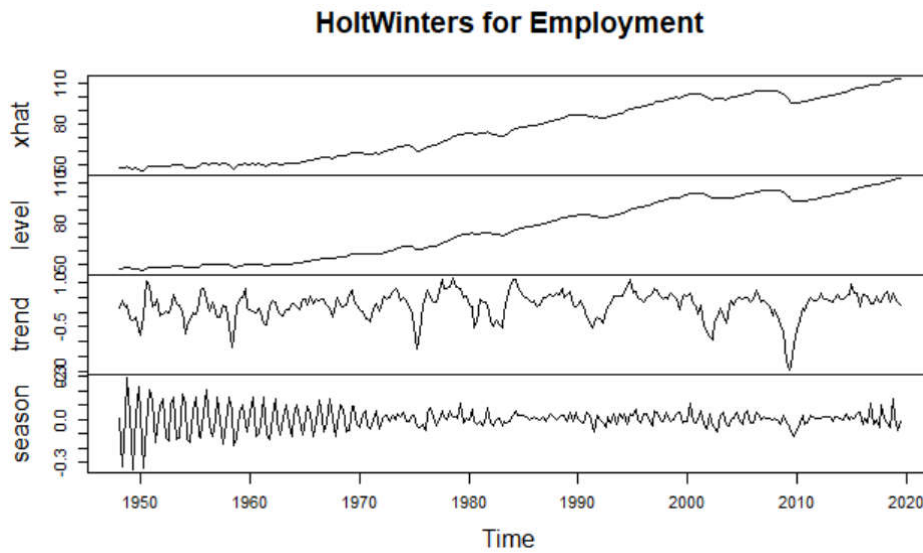
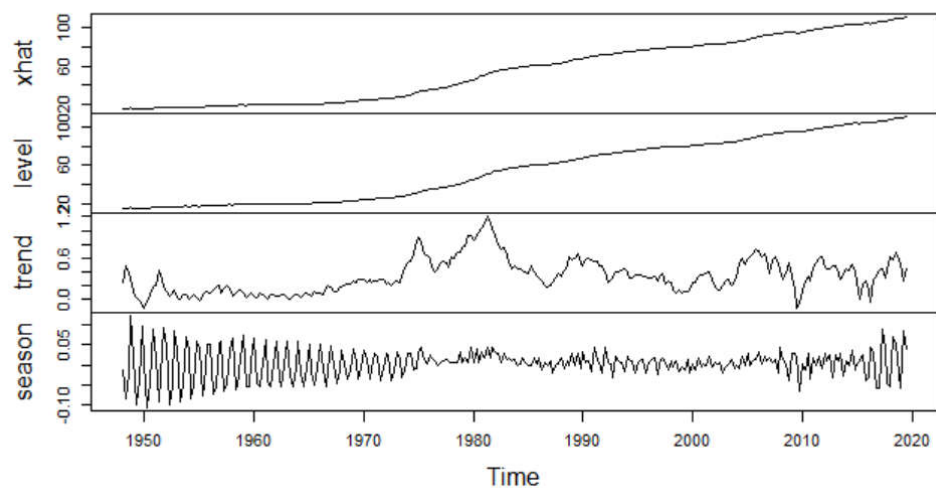


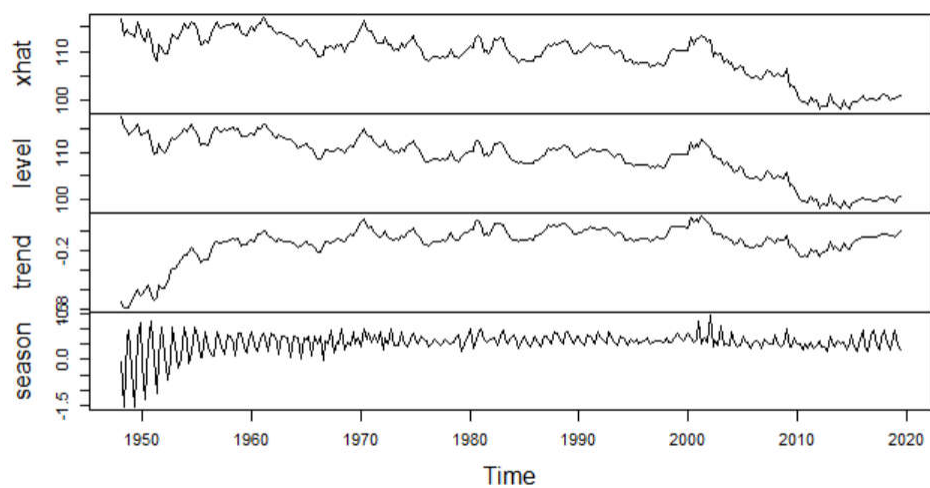
Figure 2-6 (HW plot for 5 predictors)



HoltWinters for Implicit.Price.Deflator



HoltWinters for labor.share



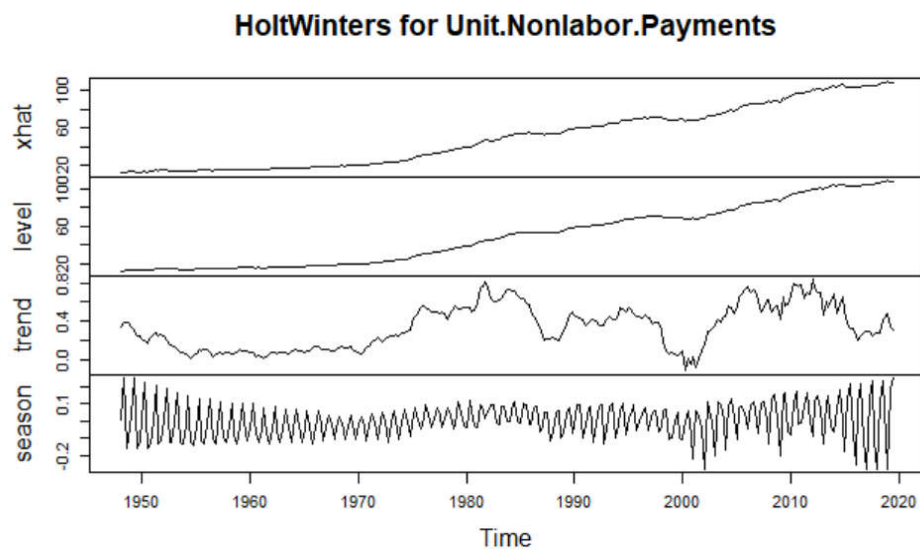
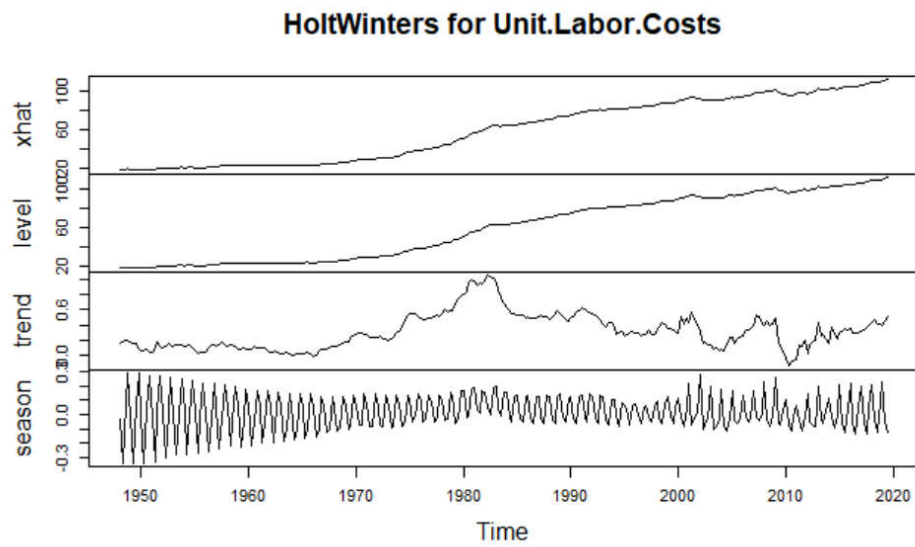


Figure 7 (ACF of All variables)

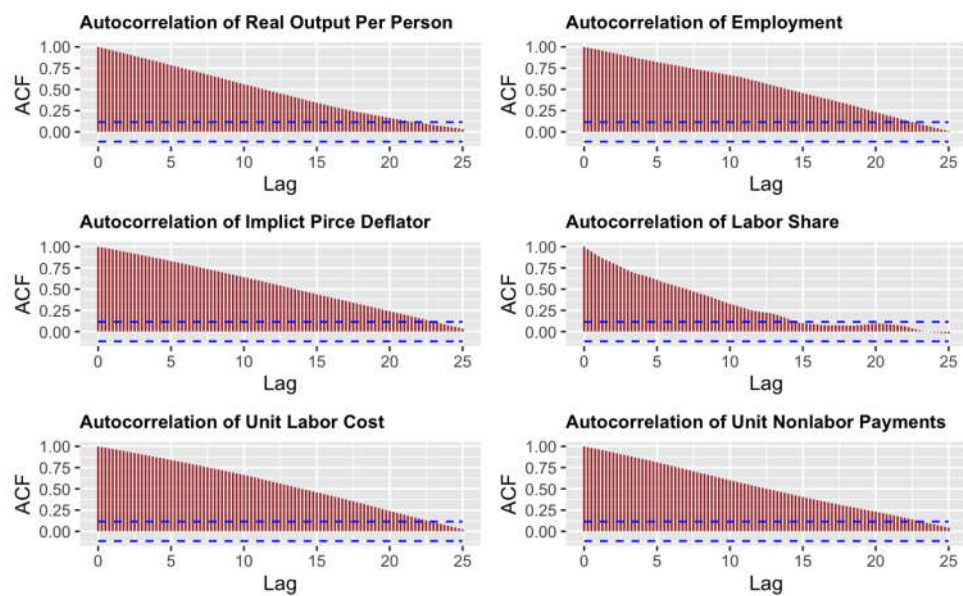


Figure 8 (First difference ACF)

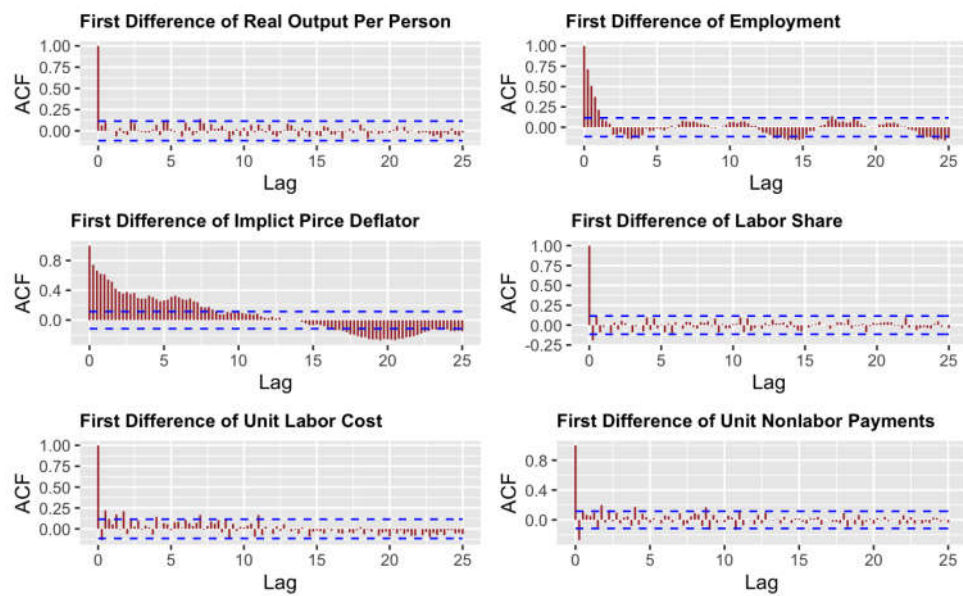


Figure 9-10 (CCFs of all variables)

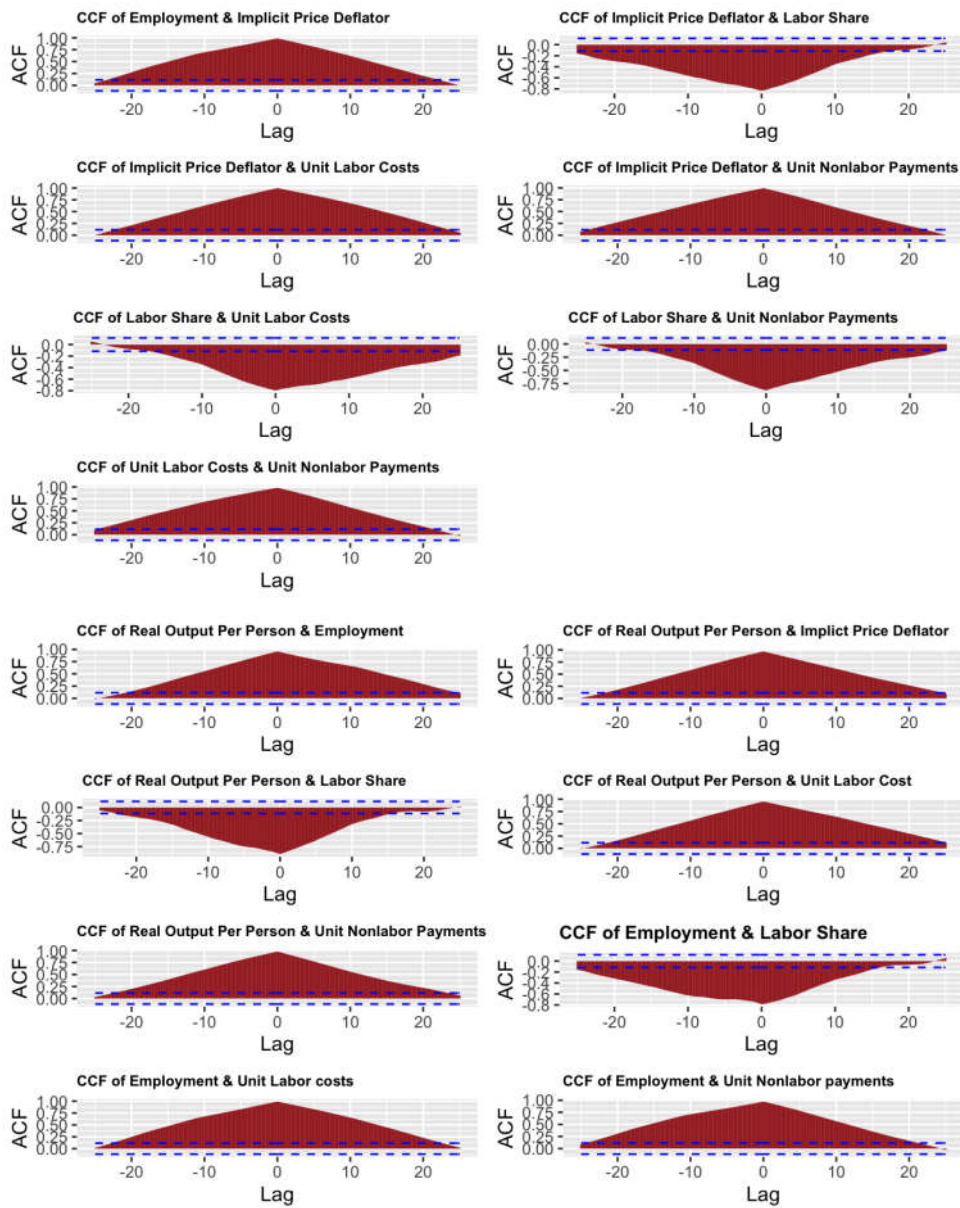


Figure 11 (correlation matrix)

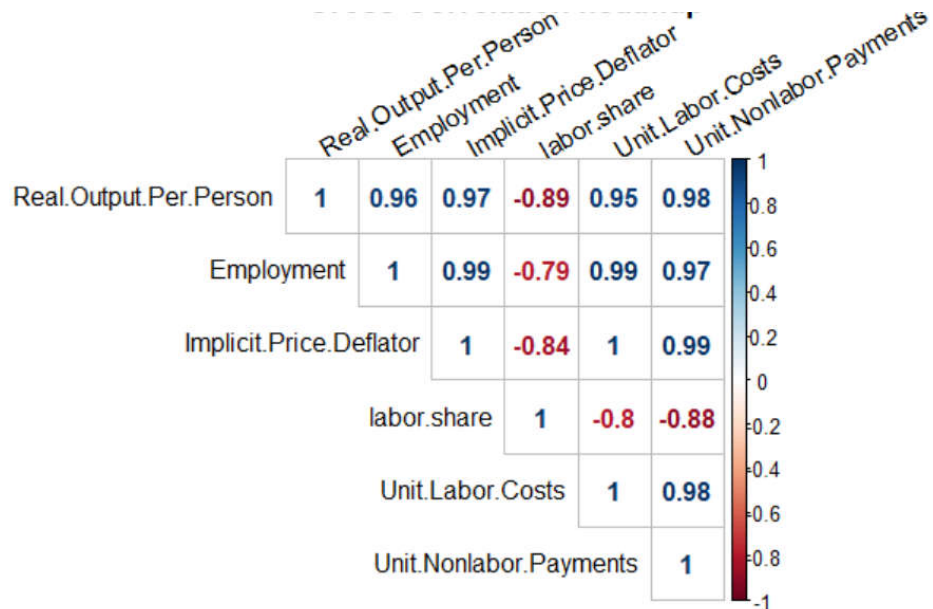


Figure 12 (scatterplot of all variables)

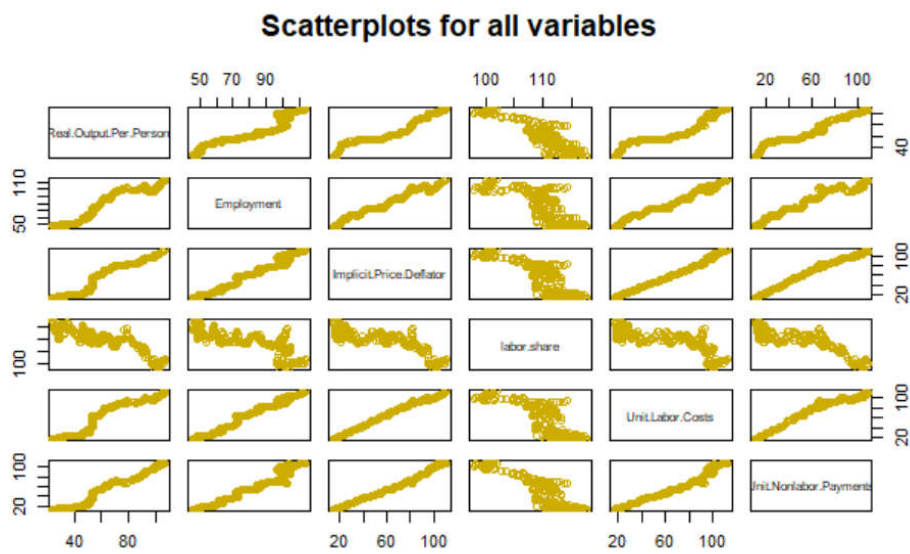


Table 1:

Vairables	Model 1	Model 2	Model 3
Labor share	-1.88315***	-1.01056***	-1.126e-01***
Unit Labor Cost	0.50485***	-0.06745*	-2.548e-01***
Time		0.26352***	1.125e-01***
Time^2			4.183e-03***
Time^3			-4.250e-05***
Time^4			1.945e-07***
Time^5			-3.025e-10***
Constant	237.27067***	136.93295***	4.157e+01***
Adjusted R^2	0.9593	0.9833	0.9983
DW test	p-value < 2.2e-16	p-value < 2.2e-16	p-value < 2.2e-16
Signif. codes: 0 '***' 0.001 '**' 0.01 '*'			

Figure 13-15 (plot of residuals)

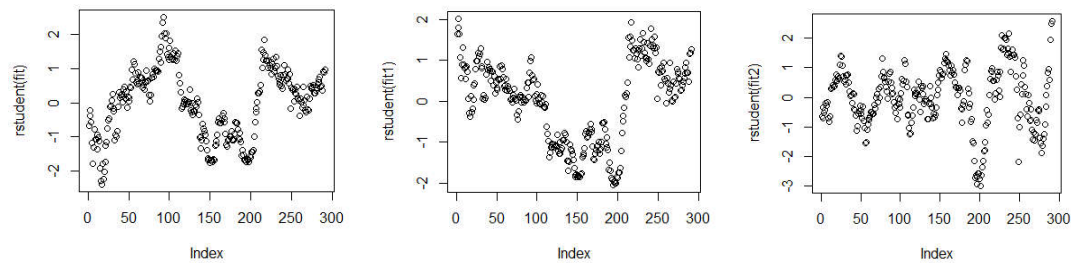


Table 2:

Vairables	Model 4 (Hildreth-Lu)	Model 4 (Cochrane-Orcutt)	Model 4 (First Difference)
Labor share	-0.44119***	-0.66443***	-0.43392***
Unit Labor Cost	0.22068***	0.58471***	0.20605***
Constant	1.15268***	4.00612***	
Adjusted R^2	0.3703	0.7574	0.3637
DW test	p-value = 2.671e-10	p-value =7.559e-14	p-value = 2.045e-10
Signif. codes: 0 '***' 0.001 '**' 0.01 '*'			

Figure 16-18 (plot of residuals for transformed model)

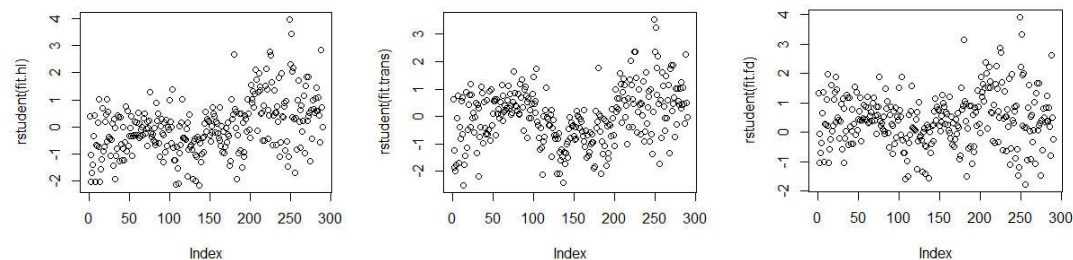


Figure 19-21 (ACF for residuals of transformed model)

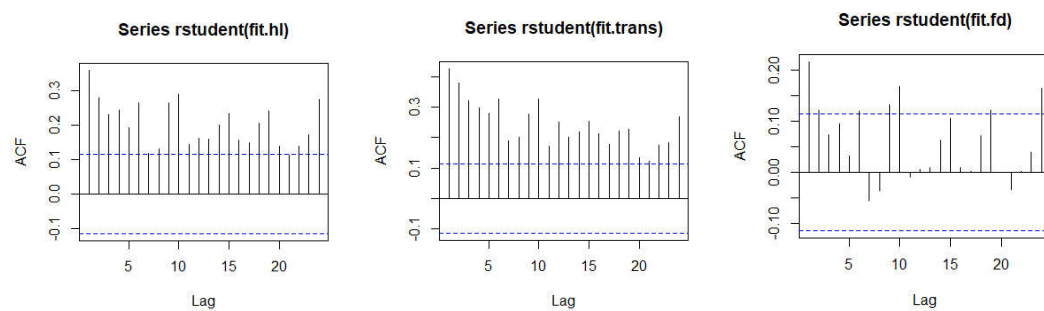


Figure 22-24 (Histogram of transformed model)

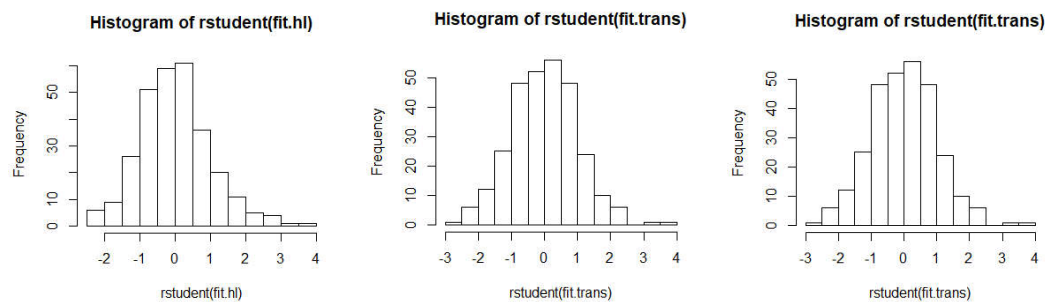


Table 3

Vairables	Model 5 (Hildreth-Lu)	Model 5 (Cochrane-Orcutt)	Model 5 (First Difference)
Labor share	-0.40616	-0.17249***	-0.25562***
Unit Labor Cost	0.03074***	0.08293***	-0.07256
Constant	0.02044	0.05966***	
Adjusted R^2	0.3338	0.459	0.4742
DW test (positive)	p-value = 1	p-value = 1	p-value = 1
DW test (negative)	p-value < 2.2e-16	p-value = 1.719e-14	p-value = 2.143e-14
Signif. codes: 0 '***' 0.001 '**' 0.01 '*'			

Figure 25, 26

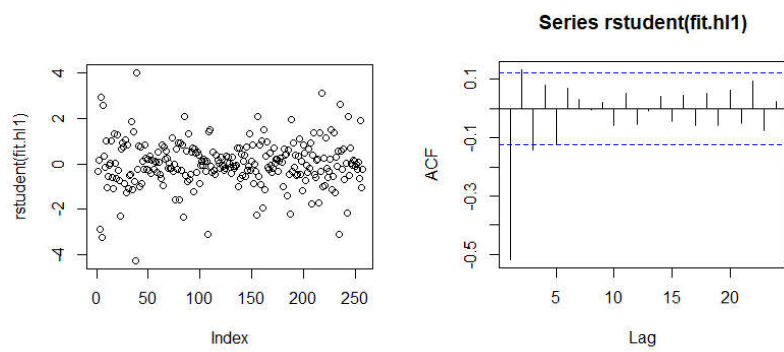


Figure 27, 28

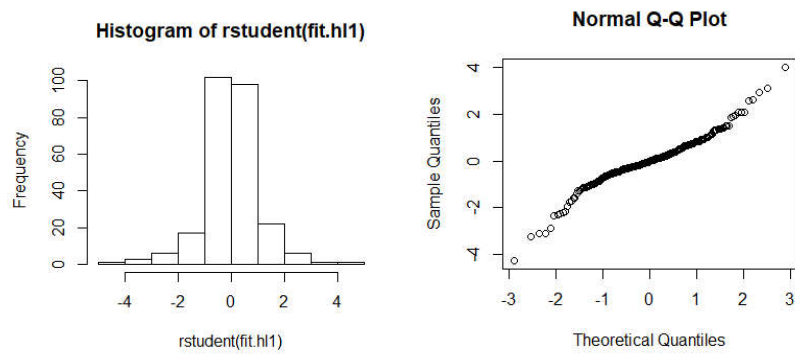


Figure 29, 30

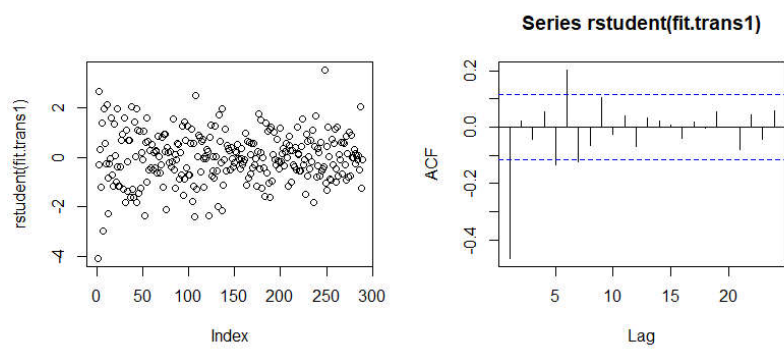


Figure 31, 32

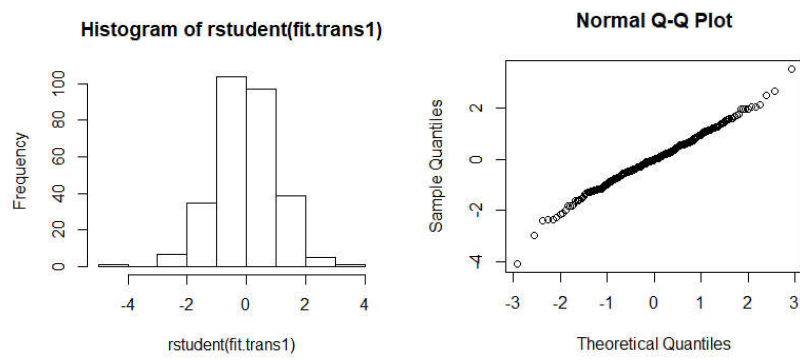


Figure 33, 34

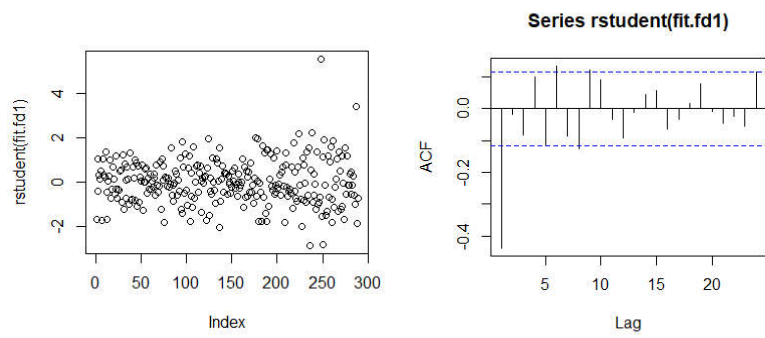


Figure 35, 36

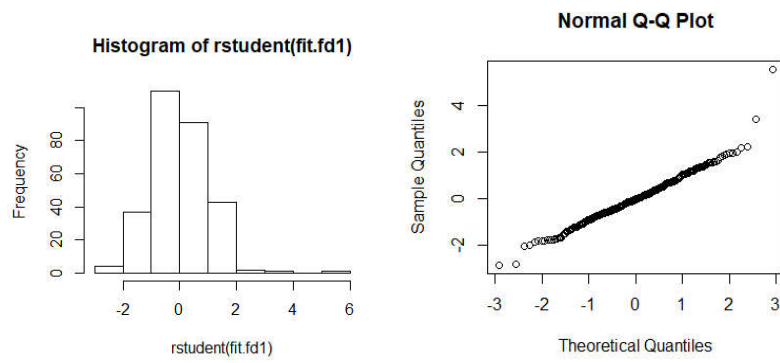


Figure 37: (ACF of *Real Output Per Person* after first differencing)

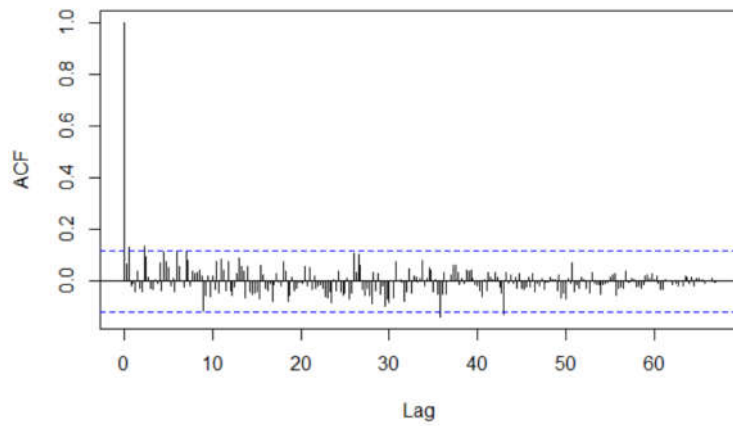


Figure 38: (PACF of *Real Output Per Person* after first differencing)

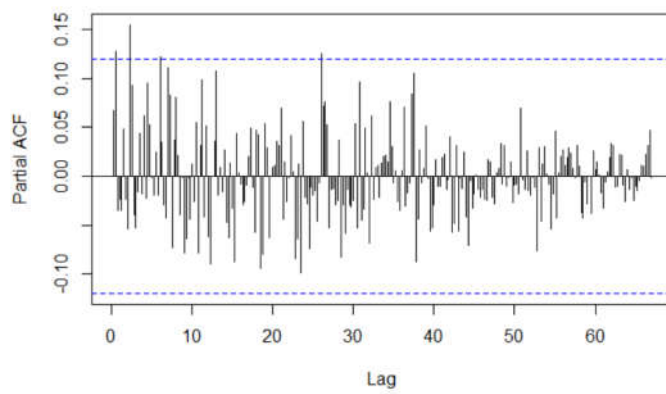


Figure 39: (ACF of *Real Output Per Person* after second differencing)

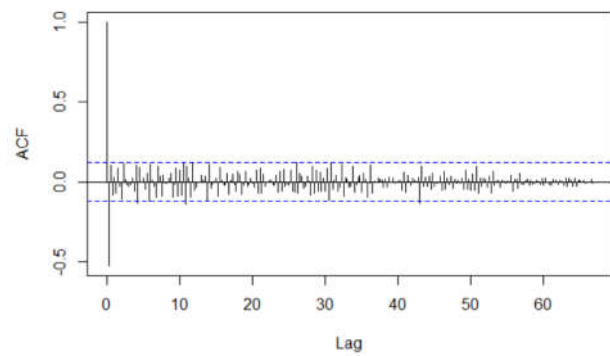


Figure 40: (PACF of *Real Output Per Person* after second differencing)

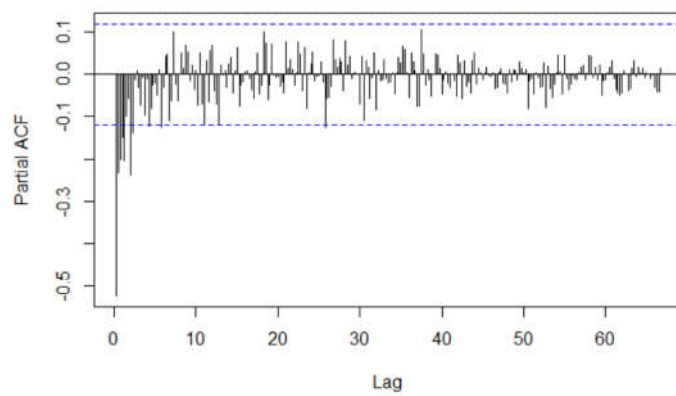


Table 4: Model Selection for the ARIMA Model

order (AR order, differencing, MA order)	Estimate of σ^2	AIC
(0,1,2)	0.2918	438.27
(0,1,3)	0.2873	436.07
(2,1,0)	0.2796	426.84
(1,1,0)	0.3048	447.87
(0,2,1)	0.251	396.94
(0,2,2)	0.2505	398.53

(0,2,3)	0.2468	396.53
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Table 5: Model Selection for the ARIMA Model and ARIMA Model with predictors

Model	Estimate of σ^2	AIC
Order (0,2,1)	0.251	396.94
Order (0,2,1) + <i>labor.share</i> + <i>Unit.Labor.Costs</i>	0.1383	240.87

Figure 41: Residual plots of ARIMA (0,2,1) after adding *labor.share* and *Unit.Labor.Costs*

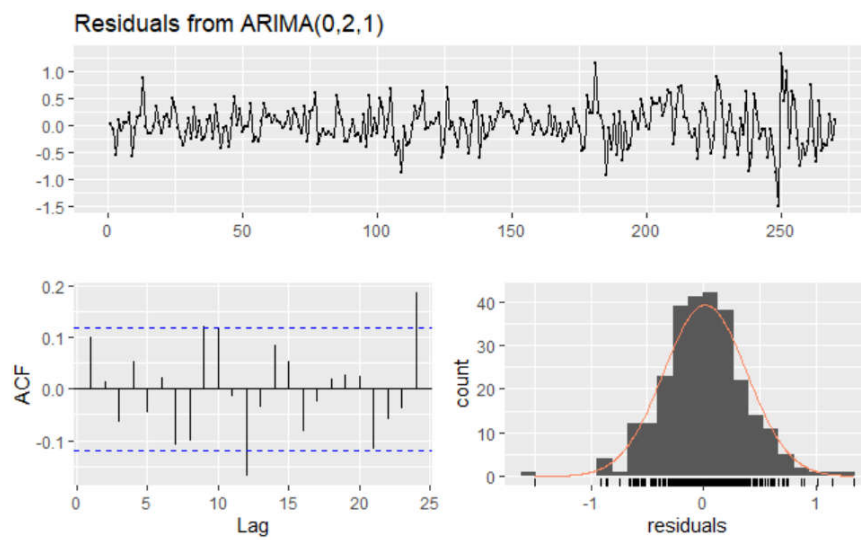


Figure 42: Ljung-Box Q Test results of ARIMA (0,2,1) after adding *labor share* and *Unit Labor Costs*

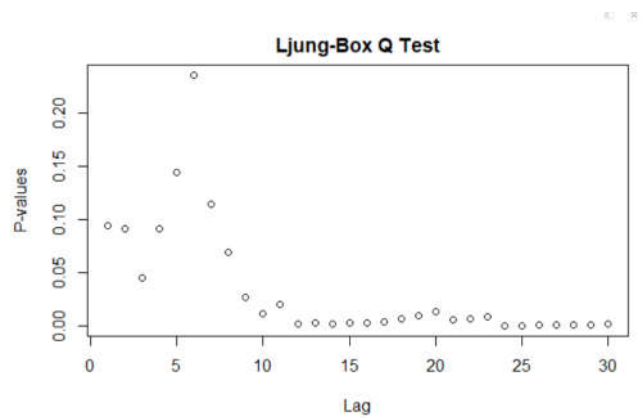


Table 6 (Order of VAR)

```

$selection
AIC(n)  HQ(n)  SC(n)  FPE(n)
      3      2      2      3

$criteria
      1      2      3      4      5      6      7
AIC(n) -2.326976e+01 -2.496805e+01 -2.501698e+01 -2.492464e+01 -2.491065e+01 -2.481723e+01 -2.467272e+01
HQ(n)  -2.302509e+01 -2.451366e+01 -2.435287e+01 -2.405081e+01 -2.382709e+01 -2.352395e+01 -2.316973e+01
SC(n)  -2.266245e+01 -2.384019e+01 -2.336857e+01 -2.275568e+01 -2.222113e+01 -2.160717e+01 -2.094211e+01
FPE(n)  7.836355e-11 1.434778e-11 1.368094e-11 1.504239e-11 1.531717e-11 1.691967e-11 1.971665e-11
      8      9      10      11      12      13      14
AIC(n) -2.454768e+01 -2.457196e+01 -2.449972e+01 -2.432732e+01 -2.429176e+01 -2.434541e+01 -2.424616e+01
HQ(n)  -2.283496e+01 -2.264952e+01 -2.236756e+01 -2.198545e+01 -2.174017e+01 -2.158410e+01 -2.127512e+01
SC(n)  -2.029651e+01 -1.980025e+01 -1.920745e+01 -1.851451e+01 -1.795840e+01 -1.749149e+01 -1.687169e+01
FPE(n)  2.259717e-11 2.237959e-11 2.450267e-11 2.978089e-11 3.172095e-11 3.107328e-11 3.568546e-11
      15      16      17      18      19      20
AIC(n) -2.412853e+01 -2.410957e+01 -2.404575e+01 -2.413515e+01 -2.411066e+01 -2.410993e+01
HQ(n)  -2.094778e+01 -2.071910e+01 -2.044556e+01 -2.032523e+01 -2.009103e+01 -1.988057e+01
SC(n)  -1.623351e+01 -1.569401e+01 -1.510964e+01 -1.467848e+01 -1.413345e+01 -1.361216e+01
FPE(n)  4.202929e-11 4.519413e-11 5.126274e-11 5.036372e-11 5.604239e-11 6.162787e-11

```

Table 7 (VAR model summary)

```

VAR Estimation Results:
=====
Endogenous variables: Real.Output.Per.Person, Employment, Implicit.Price.Deflator, labor.share, Unit.Labor.Costs,
Unit.Nonlabor.Payments
Deterministic variables: const
Sample size: 288
Log Likelihood: 1158.125
Roots of the characteristic polynomial:
1.001 0.9876 0.9473 0.9278 0.798 0.798 0.5899 0.5899 0.4366 0.4366 0.4171 0.3576 0.3576 0.348 0.348 0.2941 0.2902 0.2902
Call:
VAR(y = data2, p = 3)

```

Table 8 (Granger causality)

```

$Granger

Granger causality H0: Real.Output.Per.Person do not Granger-cause Employment Implicit.Price.Deflator
labor.share Unit.Labor.Costs Unit.Nonlabor.Payments

data: VAR object var
F-Test = 3.3006, df1 = 15, df2 = 1614, p-value = 1.778e-05

$Instant

H0: No instantaneous causality between: Real.Output.Per.Person and Employment Implicit.Price.Deflator
labor.share Unit.Labor.Costs Unit.Nonlabor.Payments

data: VAR object var
Chi-squared = 88.29, df = 5, p-value < 2.2e-16

```

Figure 43 (stability test of VAR model)

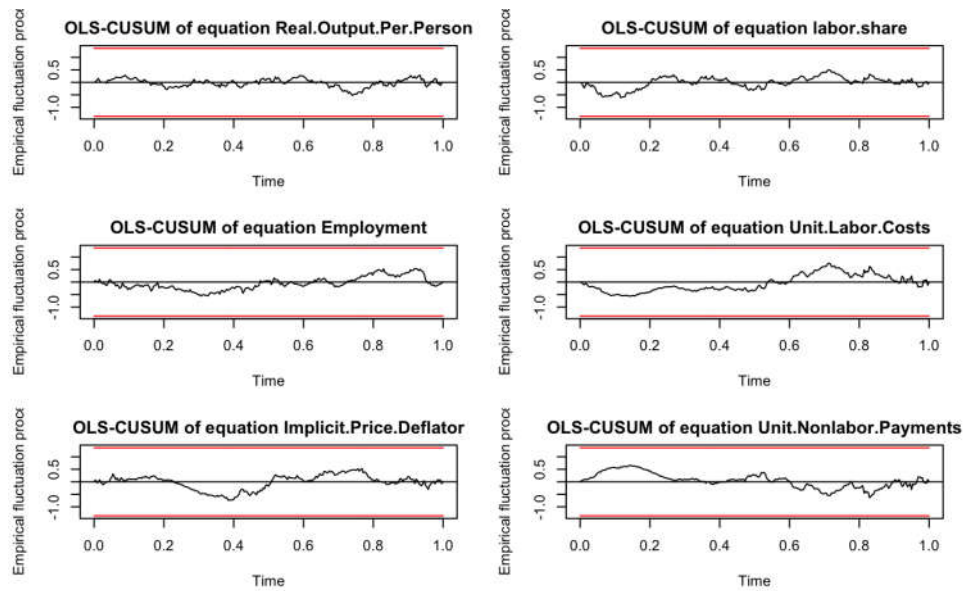


Figure 44: Drop the last 21 of the data to do forecasting (red: real data; black: predictions using the model)

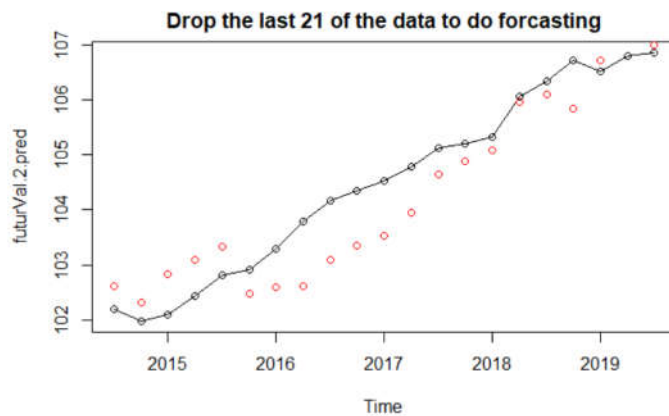


Figure 45: Normal Q-Q Plot of the residuals

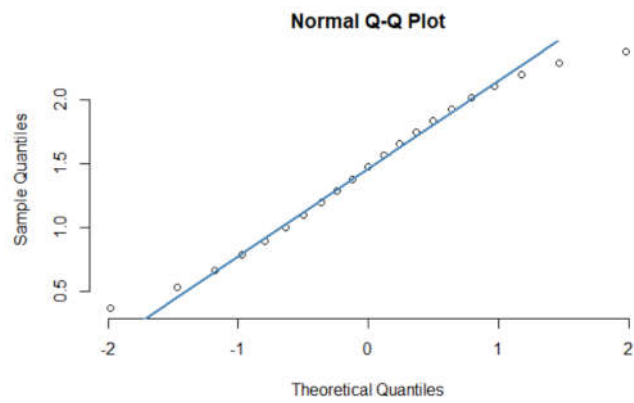


Figure 46: Predictions results into the future (3 years)

	Qtr1	Qtr2	Qtr3	Qtr4
2019			108.7716	109.1550
2020	109.5384	109.9218	110.3052	110.6886
2021	111.0720	111.4554	111.8388	112.2222
2022	112.6056	112.9890		

Figure 47: Predictions into the future (3 years)

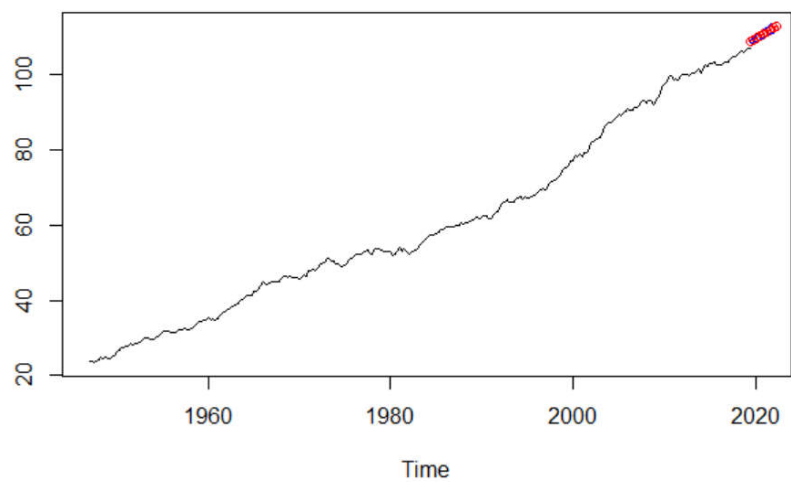


Figure 48 (fanchart of VAR predictions)

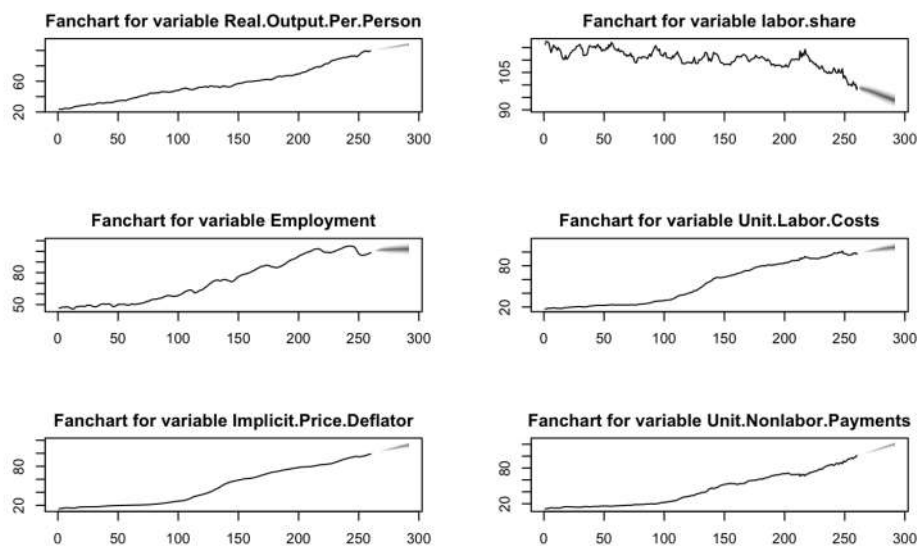


Table 9 (VAR prediction outcome)

\$Real.Output.Per.Person

	fcst	lower	upper	CI
[1,]	107.4526	106.5568	108.3484	0.8958224
[2,]	107.6935	106.4356	108.9515	1.2579208
[3,]	107.9986	106.4439	109.5532	1.5546845
[4,]	108.3150	106.5235	110.1065	1.7915010
[5,]	108.6667	106.6755	110.6579	1.9911896
[6,]	109.0233	106.8576	111.1890	2.1657210
[7,]	109.3913	107.0675	111.7151	2.3237896
[8,]	109.7639	107.2930	112.2349	2.4709480
[9,]	110.1407	107.5311	112.7503	2.6096081
[10,]	110.5191	107.7777	113.2606	2.7414646
[11,]	110.8984	108.0311	113.7657	2.8673194
[12,]	111.2774	108.2896	114.2653	2.9878464
[13,]	111.6557	108.5521	114.7593	3.1035820
[14,]	112.0328	108.8177	115.2479	3.2150604
[15,]	112.4084	109.0856	115.7311	3.3227754
[16,]	112.7823	109.3551	116.2095	3.4271775
[17,]	113.1545	109.6259	116.6832	3.5286514
[18,]	113.5251	109.8976	117.1526	3.6275103
[19,]	113.8940	110.1700	117.6180	3.7239947
[20,]	114.2615	110.4432	118.0798	3.8182797
[21,]	114.6277	110.7172	118.5382	3.9104853
[22,]	114.9926	110.9919	118.9933	4.0006888
[23,]	115.3566	111.2676	119.4455	4.0889360
[24,]	115.7197	111.5444	119.8949	4.1752524
[25,]	116.0820	111.8224	120.3417	4.2596516
[26,]	116.4438	112.1016	120.7859	4.3421423
[27,]	116.8051	112.3824	121.2278	4.4227329
[28,]	117.1661	112.6646	121.6675	4.5014354
[29,]	117.5268	112.9486	122.1051	4.5782672
[30,]	117.8874	113.2342	122.5407	4.6532520

\$Employment				
	fcst	lower	upper	CI
[1,]	114.2929	113.6460	114.9398	0.6469184
[2,]	114.6380	113.3711	115.9049	1.2669126
[3,]	114.9209	112.9863	116.8555	1.9346428
[4,]	115.1511	112.5719	117.7303	2.5792430
[5,]	115.3523	112.1797	118.5249	3.1726151
[6,]	115.5402	111.8339	119.2465	3.7062919
[7,]	115.7245	111.5461	119.9028	4.1783489
[8,]	115.9107	111.3196	120.5018	4.5911124
[9,]	116.1025	111.1527	121.0523	4.9497945
[10,]	116.3020	111.0411	121.5628	5.2608885
[11,]	116.5100	110.9788	122.0412	5.5312007
[12,]	116.7270	110.9597	122.4943	5.7672702
[13,]	116.9528	110.9778	122.9279	5.9750530
[14,]	117.1870	111.0272	123.3468	6.1597686
[15,]	117.4290	111.1031	123.7548	6.3258615
[16,]	117.6781	111.2010	124.1551	6.4770325
[17,]	117.9335	111.3172	124.5499	6.6163091
[18,]	118.1947	111.4486	124.9409	6.7461324
[19,]	118.4610	111.5925	125.3294	6.8684472
[20,]	118.7316	111.7469	125.7164	6.9847873
[21,]	119.0062	111.9098	126.1025	7.0963517
[22,]	119.2841	112.0801	126.4882	7.2040700
[23,]	119.5651	112.2564	126.8737	7.3086583
[24,]	119.8486	112.4380	127.2593	7.4106638
[25,]	120.1345	112.6240	127.6450	7.5105022
[26,]	120.4224	112.8139	128.0308	7.6084875
[27,]	120.7121	113.0072	128.4169	7.7048552
[28,]	121.0034	113.2036	128.8032	7.7997814
[29,]	121.2962	113.4028	129.1896	7.8933972
[30,]	121.5903	113.6045	129.5761	7.9857999

	\$Implicit.Price.Deflator	fcst	lower	upper	CI
[1,]	110.9592	110.6217	111.2966	0.3374274	
[2,]	111.4125	110.7952	112.0298	0.6172972	
[3,]	111.8430	110.9082	112.7779	0.9348501	
[4,]	112.2815	111.0207	113.5423	1.2607994	
[5,]	112.7177	111.1263	114.3091	1.5913971	
[6,]	113.1530	111.2337	115.0723	1.9192626	
[7,]	113.5873	111.3457	115.8288	2.2415715	
[8,]	114.0214	111.4648	116.5780	2.5566020	
[9,]	114.4556	111.5927	117.3186	2.8629332	
[10,]	114.8905	111.7307	118.0503	3.1597877	
[11,]	115.3264	111.8797	118.7730	3.4466532	
[12,]	115.7636	112.0402	119.4869	3.7233268	
[13,]	116.2024	112.2126	120.1922	3.9897861	
[14,]	116.6431	112.3969	120.8893	4.2461697	
[15,]	117.0858	112.5931	121.5785	4.4927249	
[16,]	117.5306	112.8008	122.2603	4.7297816	
[17,]	117.9775	113.0197	122.9352	4.9577231	
[18,]	118.4265	113.2495	123.6035	5.1769677	
[19,]	118.8777	113.4897	124.2656	5.3879521	
[20,]	119.3308	113.7397	124.9219	5.5911198	
[21,]	119.7859	113.9990	125.5728	5.7869123	
[22,]	120.2429	114.2671	126.2186	5.9757628	
[23,]	120.7016	114.5435	126.8596	6.1580921	
[24,]	121.1618	114.8275	127.4961	6.3343053	
[25,]	121.6236	115.1188	128.1284	6.5047903	
[26,]	122.0868	115.4169	128.7567	6.6699167	
[27,]	122.5513	115.7213	129.3813	6.8300349	
[28,]	123.0169	116.0315	130.0024	6.9854765	
[29,]	123.4837	116.3471	130.6202	7.1365543	
[30,]	123.9514	116.6679	131.2350	7.2835625	

	\$labor.share	fcst	lower	upper	CI
[1,]	101.37728	99.57061	103.1840	1.806672	
[2,]	101.34822	99.15390	103.5425	2.194316	
[3,]	101.26867	98.73291	103.8044	2.535758	
[4,]	101.20187	98.48207	103.9217	2.719799	
[5,]	101.11677	98.27824	103.9553	2.838528	
[6,]	101.04389	98.12606	103.9617	2.917830	
[7,]	100.96037	97.97464	103.9461	2.985726	
[8,]	100.87446	97.82156	103.9274	3.052902	
[9,]	100.78249	97.65814	103.9068	3.124350	
[10,]	100.68717	97.48768	103.8867	3.199488	
[11,]	100.58888	97.31282	103.8649	3.276057	
[12,]	100.48902	97.13774	103.8403	3.351281	
[13,]	100.38854	96.96563	103.8114	3.422904	
[14,]	100.28836	96.79896	103.7777	3.489393	
[15,]	100.18914	96.63920	103.7391	3.549940	
[16,]	100.09138	96.48707	103.6957	3.604315	
[17,]	99.99541	96.34272	103.6481	3.652697	
[18,]	99.90141	96.20589	103.5969	3.695519	
[19,]	99.80946	96.07611	103.5428	3.733347	
[20,]	99.71955	95.95275	103.4863	3.766799	
[21,]	99.63160	95.83512	103.4281	3.796484	
[22,]	99.54553	95.72256	103.3685	3.822970	
[23,]	99.46118	95.61441	103.3079	3.846768	
[24,]	99.37843	95.51010	103.2467	3.868322	
[25,]	99.29712	95.40910	103.1851	3.888014	
[26,]	99.21712	95.31095	103.1233	3.906163	
[27,]	99.13828	95.21525	103.0613	3.923034	
[28,]	99.06050	95.12165	102.9993	3.938844	
[29,]	98.98365	95.02988	102.9374	3.953770	
[30,]	98.90762	94.93967	102.8756	3.967955	

\$Unit.Labor.Costs				
	fcst	lower	upper	CI
[1,]	112.8740	111.6766	114.0714	1.197419
[2,]	113.3715	111.9172	114.8258	1.454293
[3,]	113.8664	112.0724	115.6604	1.794024
[4,]	114.3364	112.2103	116.4624	2.126046
[5,]	114.7967	112.3356	117.2579	2.461158
[6,]	115.2560	112.4427	118.0693	2.813317
[7,]	115.7034	112.5269	118.8798	3.176450
[8,]	116.1447	112.5965	119.6930	3.548291
[9,]	116.5792	112.6560	120.5024	3.923227
[10,]	117.0089	112.7120	121.3059	4.296953
[11,]	117.4349	112.7695	122.1003	4.665407
[12,]	117.8586	112.8328	122.8843	5.025761
[13,]	118.2810	112.9050	123.6570	5.376006
[14,]	118.7031	112.9881	124.4180	5.714960
[15,]	119.1256	113.0836	125.1677	6.042031
[16,]	119.5493	113.1922	125.9064	6.357088
[17,]	119.9744	113.3141	126.6347	6.660314
[18,]	120.4013	113.4492	127.3534	6.952100
[19,]	120.8302	113.5972	128.0631	7.232965
[20,]	121.2611	113.7576	128.7646	7.503495
[21,]	121.6942	113.9299	129.4585	7.764301
[22,]	122.1294	114.1134	130.1453	8.015991
[23,]	122.5665	114.3074	130.8257	8.259155
[24,]	123.0057	114.5113	131.5000	8.494351
[25,]	123.4467	114.7246	132.1688	8.722101
[26,]	123.8894	114.9465	132.8323	8.942888
[27,]	124.3337	115.1766	133.4909	9.157158
[28,]	124.7796	115.4143	134.1449	9.365321
[29,]	125.2268	115.6591	134.7946	9.567752
[30,]	125.6754	115.9106	135.4402	9.764795

	\$Unit.Nonlabor.Payments				
	fcst	lower	upper	CI	
[1,]	108.4993	106.9217	110.0769	1.577637	
[2,]	108.8961	107.0926	110.6995	1.803420	
[3,]	109.2439	107.1933	111.2946	2.050650	
[4,]	109.6419	107.3596	111.9241	2.282264	
[5,]	110.0471	107.5356	112.5585	2.511484	
[6,]	110.4515	107.7096	113.1935	2.741969	
[7,]	110.8689	107.8929	113.8448	2.975952	
[8,]	111.2938	108.0838	114.5038	3.209972	
[9,]	111.7277	108.2862	115.1693	3.441563	
[10,]	112.1692	108.5015	115.8369	3.667726	
[11,]	112.6177	108.7313	116.5042	3.886454	
[12,]	113.0723	108.9761	117.1685	4.096212	
[13,]	113.5323	109.2362	117.8284	4.296107	
[14,]	113.9968	109.5112	118.4825	4.485675	
[15,]	114.4654	109.8005	119.1302	4.664808	
[16,]	114.9373	110.1037	119.7710	4.833644	
[17,]	115.4122	110.4197	120.4047	4.992493	
[18,]	115.8897	110.7479	121.0315	5.141782	
[19,]	116.3695	111.0875	121.6515	5.282005	
[20,]	116.8512	111.4375	122.2649	5.413699	
[21,]	117.3346	111.7972	122.8720	5.537419	
[22,]	117.8195	112.1658	123.4732	5.653725	
[23,]	118.3058	112.5426	124.0689	5.763168	
[24,]	118.7933	112.9270	124.6595	5.866288	
[25,]	119.2818	113.3182	125.2454	5.963607	
[26,]	119.7713	113.7156	125.8269	6.055625	
[27,]	120.2616	114.1188	126.4044	6.142819	
[28,]	120.7527	114.5270	126.9783	6.225645	
[29,]	121.2444	114.9399	127.5490	6.304532	
[30,]	121.7368	115.3569	128.1167	6.379888	

