批处理反向传播推导

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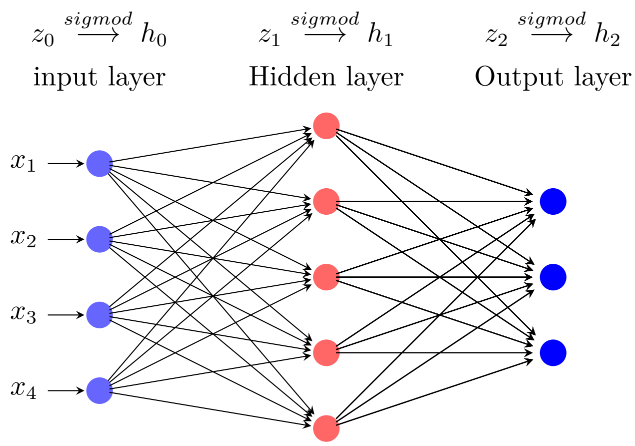

$$z_0 = \begin{bmatrix} z_{11} & z_{12} & z_{13} & z_{14} \\ \vdots & \vdots & \vdots & \vdots \\ z_{n1} & z_{n2} & z_{n3} & z_{n4} \end{bmatrix}_{n\times 4}$$

$$h_0 = sigmod(z_0)$$

$$z_1 = \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ \vdots & \vdots & \vdots & \vdots \\ h_{n1} & h_{n2} & h_{n3} & h_{n4} \end{bmatrix}_{n\times 4} \times \begin{bmatrix} w_{11} & w_{12} & w_{13} & w_{14} & w_{15} \\ w_{21} & w_{22} & w_{23} & w_{24} & w_{25} \\ w_{31} & w_{32} & w_{33} & w_{34} & w_{35} \\ w_{41} & w_{42} & w_{43} & w_{44} & w_{45} \end{bmatrix}_{4\times 5} + \begin{bmatrix} b_{11}^{(1)} & b_{12}^{(1)} & b_{13}^{(1)} & b_{14}^{(1)} & b_{15}^{(1)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{n1}^{(1)} & b_{n2}^{(1)} & b_{n3}^{(1)} & b_{n4}^{(1)} & b_{n4}^{(1)} \end{bmatrix}_{n\times}$$

$b_{1i} = b_{ni}$

$$h_1 = sigmod(z_1)$$

$$z_2 = \begin{bmatrix} h_{11}^{(1)} & h_{12}^{(1)} & h_{13}^{(1)} & h_{14}^{(1)} & h_{15}^{(1)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ h_{n1}^{(1)} & h_{n2}^{(1)} & h_{n3}^{(1)} & h_{n4}^{(1)} & h_{n4}^{(1)} \end{bmatrix}_{n\times 5} \times \begin{bmatrix} w_{11}^{(2)} & w_{12}^{(2)} & w_{13}^{(2)} \\ w_{21}^{(2)} & w_{22}^{(2)} & w_{23}^{(2)} \\ w_{31}^{(2)} & w_{32}^{(2)} & w_{33}^{(2)} \\ w_{41}^{(2)} & w_{42}^{(2)} & w_{43}^{(2)} \\ w_{41}^{(2)} & w_{42}^{(2)} & w_{43}^{(2)} \end{bmatrix}_{5\times 3} + \begin{bmatrix} b_{11}^{(2)} & b_{12}^{(2)} & b_{13}^{(2)} \\ \vdots & \vdots & \vdots \\ b_{n1}^{(2)} & b_{n2}^{(2)} & b_{n3}^{(2)} \end{bmatrix}_{n\times 3}$$

$$h^{(2)} = sigmod(z_2) = \begin{bmatrix} h_{11}^{(2)} & h_{12}^{(2)} & h_{13}^{(2)} \\ \vdots & \vdots & \vdots \\ h_{n1}^{(2)} & h_{n2}^{(2)} & h_{n3}^{(2)} \end{bmatrix}_{n\times 3}$$

$$L = loss = 0.5 \times \sum_{i=1}^{n} \sum_{j=1}^{3} (t_{ij} - h_{ij}^{(2)})^2$$

t 代表训练集标签

$$\frac{\partial L}{\partial h^{(2)}} = \begin{bmatrix} h_{11}^{(2)} - t_{11} & h_{12}^{(2)} - t_{12} & h_{13}^{(2)} - t_{13} \\ \vdots & \vdots & \vdots \\ h_{n1}^{(2)} - t_{n1} & h_{n2}^{(2)} - t_{n2} & h_{n3}^{(2)} - t_{n3} \end{bmatrix}_{n\times 3}$$

$\frac{\partial L}{\partial z^{(2)}}$ 的表示

$$\frac{\partial L}{\partial z^{(2)}} = \frac{\partial L}{\partial h^{(2)}} \frac{\partial h^{(2)}}{\partial z^{(2)}} = \begin{bmatrix} \frac{\partial L}{\partial z_{11}^{(2)}} & \frac{\partial L}{\partial z_{12}^{(2)}} & \frac{\partial L}{\partial z_{13}^{(2)}} \\ \vdots & \vdots & \vdots \\ \frac{\partial L}{\partial z_{n1}^{(2)}} & \frac{\partial L}{\partial z_{n2}^{(2)}} & \frac{\partial L}{\partial z_{n3}^{(2)}} \end{bmatrix}_{n\times 3}$$

$$= \begin{bmatrix} h_{11}^{(2)} - t_{11} & h_{12}^{(2)} - t_{12} & h_{13}^{(2)} - t_{13} \\ \vdots & \vdots & \vdots \\ h_{n1}^{(2)} - t_{n1} & h_{n2}^{(2)} - t_{n2} & h_{n3}^{(2)} - t_{n3} \end{bmatrix}_{n\times 3} \cdot \begin{bmatrix} z_{11}^{(2)}(1 - z_{11}^{(2)}) & z_{12}^{(2)}(1 - z_{12}^{(2)}) & z_{13}^{(2)}(1 - z_{13}^{(2)}) \\ \vdots & \vdots & \vdots \\ z_{n1}^{(2)}(1 - z_{n1}^{(2)}) & z_{n2}^{(2)}(1 - z_{n2}^{(2)}) & z_{n3}^{(2)}(1 - z_{n3}^{(2)}) \end{bmatrix}_{n\times 3}$$

$z^{(2)}$ 的线性表示

$$z^{(2)} = \begin{bmatrix} h_{11}^{(1)} & h_{12}^{(1)} & h_{13}^{(1)} & h_{14}^{(1)} & h_{15}^{(1)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ h_{n1}^{(1)} & h_{n2}^{(1)} & h_{n3}^{(1)} & h_{n4}^{(1)} & h_{n4}^{(1)} \end{bmatrix}_{n\times 5} \times \begin{bmatrix} w_{11}^{(2)} & w_{12}^{(2)} & w_{13}^{(2)} \\ w_{21}^{(2)} & w_{22}^{(2)} & w_{23}^{(2)} \\ w_{31}^{(2)} & w_{32}^{(2)} & w_{33}^{(2)} \\ w_{41}^{(2)} & w_{42}^{(2)} & w_{43}^{(2)} \\ w_{41}^{(2)} & w_{42}^{(2)} & w_{43}^{(2)} \end{bmatrix}_{5\times 3} + \begin{bmatrix} b_{11}^{(2)} & b_{12}^{(2)} & b_{13}^{(2)} \\ \vdots & \vdots & \vdots \\ b_{n1}^{(2)} & b_{n2}^{(2)} & b_{n3}^{(2)} \end{bmatrix}_{n\times 3}$$

$$= \begin{bmatrix} z_{11}^{(2)} & z_{12}^{(2)} & z_{13}^{(2)} \\ \vdots & \vdots & \vdots \\ z_{n1}^{(2)} & z_{n2}^{(2)} & z_{n3}^{(2)} \end{bmatrix}_{n\times 3}$$

$$\frac{\partial L}{\partial w^{(2)}} = \begin{bmatrix} \frac{\partial L}{\partial w_{11}^{(2)}} & \frac{\partial L}{\partial w_{12}^{(2)}} & \frac{\partial L}{\partial w_{13}^{(2)}} \\ \frac{\partial L}{\partial w_{21}^{(2)}} & \frac{\partial L}{\partial w_{22}^{(2)}} & \frac{\partial L}{\partial w_{23}^{(2)}} \\ \frac{\partial L}{\partial w_{31}^{(2)}} & \frac{\partial L}{\partial w_{32}^{(2)}} & \frac{\partial L}{\partial w_{33}^{(2)}} \\ \frac{\partial L}{\partial w_{41}^{(2)}} & \frac{\partial L}{\partial w_{42}^{(2)}} & \frac{\partial L}{\partial w_{43}^{(2)}} \\ \frac{\partial L}{\partial w_{41}^{(2)}} & \frac{\partial L}{\partial w_{42}^{(2)}} & \frac{\partial L}{\partial w_{43}^{(2)}} \end{bmatrix}_{5\times 3}$$

$$\frac{\partial L}{\partial w_{11}^{(2)}} = \sum_{i=1}^{n} \frac{\partial L}{\partial z_{i1}^{(2)}} h_{i1}^{(1)}$$

以此类推

$$\frac{\partial L}{\partial w^{(2)}} = \begin{bmatrix} h_{11}^{(1)} & \cdots & h_{n1}^{(1)} \\ h_{12}^{(1)} & \cdots & h_{n2}^{(1)} \\ h_{13}^{(1)} & \cdots & h_{n3}^{(1)} \\ h_{14}^{(1)} & \cdots & h_{n4}^{(1)} \\ h_{15}^{(1)} & \cdots & h_{n5}^{(1)} \end{bmatrix}_{5\times n} \times \begin{bmatrix} \frac{\partial L}{\partial z_{11}^{(2)}} & \frac{\partial L}{\partial z_{12}^{(2)}} & \frac{\partial L}{\partial z_{13}^{(2)}} \\ \vdots & \vdots & \vdots \\ \frac{\partial L}{\partial z_{n1}^{(2)}} & \frac{\partial L}{\partial z_{n2}^{(2)}} & \frac{\partial L}{\partial z_{n3}^{(2)}} \end{bmatrix}_{n\times 3} = (h^{(1)})^T \times \frac{\partial L}{\partial z^{(2)}}$$

L 对 b 的求导

$$\frac{\partial L}{\partial b^{(2)}} = \begin{bmatrix} \frac{\partial L}{\partial b_1^{(2)}} \\ \frac{\partial L}{\partial b_2^{(2)}} \\ \frac{\partial L}{\partial b_3^{(2)}} \end{bmatrix}_{3\times 1} = \begin{bmatrix} \sum_{i=1}^{n} \frac{\partial L}{\partial z_{i1}^{(2)}} \\ \sum_{i=1}^{n} \frac{\partial L}{\partial z_{i2}^{(2)}} \\ \sum_{i=1}^{n} \frac{\partial L}{\partial z_{i3}^{(2)}} \end{bmatrix}_{3\times 1}$$

L 对 $h(1)$ 的导数, 可由 L 对 $w(2)$ 的导数联想得到（观察 $z^{(2)}$ 的线性表示）

$$\frac{\partial L}{\partial h^{(1)}} = \begin{bmatrix} \frac{\partial L}{\partial z_{11}^{(2)}} & \frac{\partial L}{\partial z_{12}^{(2)}} & \frac{\partial L}{\partial z_{13}^{(2)}} \\ \vdots & \vdots & \vdots \\ \frac{\partial L}{\partial z_{n1}^{(2)}} & \frac{\partial L}{\partial z_{n2}^{(2)}} & \frac{\partial L}{\partial z_{n3}^{(2)}} \end{bmatrix}_{n\times 3} \times \begin{bmatrix} w_{11}^{(2)} & w_{21}^{(2)} & w_{31}^{(2)} & w_{41}^{(2)} & w_{51}^{(2)} \\ w_{12}^{(2)} & w_{22}^{(2)} & w_{32}^{(2)} & w_{42}^{(2)} & w_{52}^{(2)} \\ w_{13}^{(2)} & w_{23}^{(2)} & w_{33}^{(2)} & w_{43}^{(2)} & w_{53}^{(2)} \end{bmatrix}_{3\times 5} = \frac{\partial L}{\partial z^{(2)}} \times (w^{(2)})^T$$