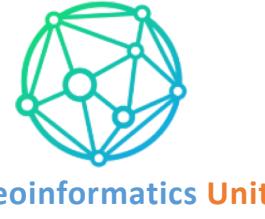


Total-Variation-Regularized Tensor Ring Completion for Remote Sensing Image Reconstruction

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Introduction

Task: Time-series remote sensing (RS) image reconstruction



Background: Because of poor weather conditions and sensor failure, RS images often miss information due to clouds and dead pixels (right figure) which significantly influence the subsequent applications. Therefore, it is essential to predict and reconstruct the missing information of RS images [2].

Previous solution: Low-rank tensor completion (LRTC) has the advantage to simultaneously explore prior spatial, spectral, and temporal information. However, Tucker rank components are the ranks of the matrices based on an unbalanced matricization scheme (one mode versus the remaining).

TVTR

Tensor ring (TR) decomposition

Decompose $\mathcal{X} \in \mathbb{R}^{I_1 \times \dots \times I_N}$ in TR-format:

$$\mathcal{X}(i_1, i_2, \dots, i_N) = \text{trace}(\prod_{n=1}^N \mathbf{G}_{i_n}^{(n)}), \quad i_n \in 1, \dots, I_n$$

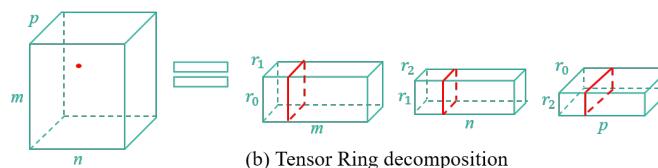
where $\mathbf{G}^{(n)} \in \mathbb{R}^{R_n \times I_n \times R_{n+1}}$, $n = 1, \dots, N$ stands for TR core tensor, the TR-rank is $\{R_1, R_2, \dots, R_{N+1}\}$, $R_1 = R_{N+1}$,

Proposed method:
clean RS image $\mathcal{X} \in \mathbb{R}^{I_1 \times \dots \times I_N}$, observed image \mathcal{Y} , observation set Ω

Proposed total-variation-regularized TR completion (TVTR):

$$\min_{\mathcal{X}, \mathcal{G}} \lambda \|\mathcal{X}\|_{TV} + \|\mathcal{X} - \Phi(\mathcal{G})\|_F^2, \quad s.t., \quad \mathcal{X}_\Omega = \mathcal{Y}_\Omega$$

Advantage: spatial total-variation (TV) is adopted to explore the spatial smoothness, and TR decomposition is utilized to explore the low-rank property of the images along different matrix unfolding.



Algorithm

We propose an efficient augmented Lagrange multiplier (ALM) algorithm to solve the model (2).

Objective function:

By introduction latent variable \mathcal{U} , the augmented function is:

$$\begin{aligned} & \min_{\mathcal{X}, \mathcal{G}, \mathcal{U}} \lambda \|\mathcal{U}\|_1 + \|\mathcal{X} - \Phi(\mathcal{G})\|_F^2 + \langle \Lambda, \mathcal{U} - D\mathcal{X} \rangle \\ & \quad + \frac{\mu}{2} \|\mathcal{U} - D\mathcal{X}\|_F^2, \quad s.t. \quad \mathcal{X}_\Omega = \mathcal{Y}_\Omega. \end{aligned}$$

where λ is the Language parameter, and D is the difference operator along spatial directions.

Alternative minimization

Update \mathcal{U} : (Lemma 1 and Lemma 2)

$$\begin{aligned} \text{Update } \mathcal{G}^{(n)} &= \text{fold}_2(\mathcal{X}_{<n>} (\mathbf{G}_{<2>}^{(\neq n)})^{T,\dagger}), \\ \mathcal{X} &= \mathcal{F}^{-1} \left[\frac{\mathcal{F}(\Phi(\mathcal{G})) + D^T(\mu\mathcal{U} + \Lambda)/2}{1 + (\mathcal{F}(\mu D_x/2))^2 + (\mathcal{F}(\mu D_y/2))^2} \right] \end{aligned}$$

$$\text{Update } \mathcal{U} = \min_{\mathcal{U}} \lambda \|\mathcal{U}\|_1 + \frac{\mu}{2} \|\mathcal{U} - \mathcal{X} + \Lambda/\mu\|_F^2,$$

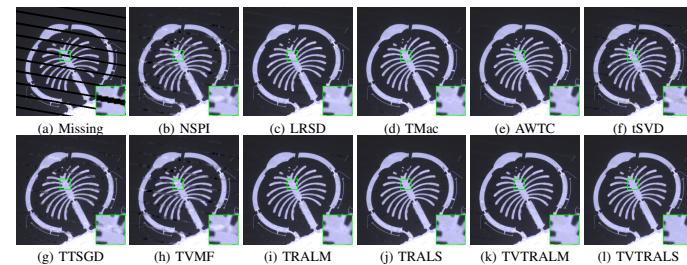
$$\text{Update } \Lambda = \Lambda + \mu(\mathcal{U} - \mathcal{X}),$$

Experiments

Real experiment: Washington DC (WDC) Mall dataset from the HYDICE sensor, of size $256 \times 256 \times 191$. Real experiments: time-series Landsat-7 images from Dubai area, of size $300 \times 300 \times 6 \times 8$ (from year 2004 to 2011)

Quantitative evaluation results of the different completion methods for HSI random missing:

Missing ratio	Index	LRCM	Tmac	AWTC	t-SVD	TTSGD	TMF	TRALM	TRALS	TVTRALM	TVTRALS
0.5	MPSNR	53	53.34	54.23	50.42	37.12	32.35	53.79	54.89	55.88	54.91
	MSSIM	0.9983	0.9982	0.998	0.9969	0.957	0.8933	0.998	0.999	0.9991	0.999
	SAM	1.97	2.02	1.63	2.89	7.65	9.75	2.21	1.57	1.5	1.55
0.7	MPSNR	48.77	47.38	48.32	46.08	35.4	30.28	48.66	50.21	50.82	50.29
	MSSIM	0.9961	0.9951	0.9965	0.9929	0.937	0.8307	0.9961	0.9972	0.9975	0.9973
	SAM	2.6	2.89	2.72	4.23	8.4	10.27	2.61	2.4	2.2	2.31
0.9	MPSNR	30.23	29.72	33.61	35.83	32.24	28.47	34.73	35.63	36.4	36.21
	MSSIM	0.8806	0.8693	0.9433	0.9466	0.8791	0.7486	0.919	0.9339	0.9504	0.9438
	SAM	9.63	7.95	7.06	8.25	10.57	10.95	9.05	8.78	5.67	8.36



Real time-series Landsat-7 images strips inpainting results

Conclusion

In this study, the TR decomposition was used to simultaneously explore the spatial, spectral, and temporal information and TV was adopted to further explore the spatial smoothness in the RS images. The proposed algorithm was proven in the simulated and real data experiments to achieve the best or near best results.

Reference

- [1] Q. Zhao, G. Zhou, S. Xie, L. Zhang, and A. Cichocki, "Tensor ring decomposition," arXiv preprint arXiv:1606.05535, 2016.
- [2] M. K.-P. Ng, et.al., An adaptive weighted tensor completion method for the recovery of remote sensing images with missing data, TGRS, 2017.