

Quantum Noise and error mitigation

Quantum Computing Minicourse ICTP-SAIFR

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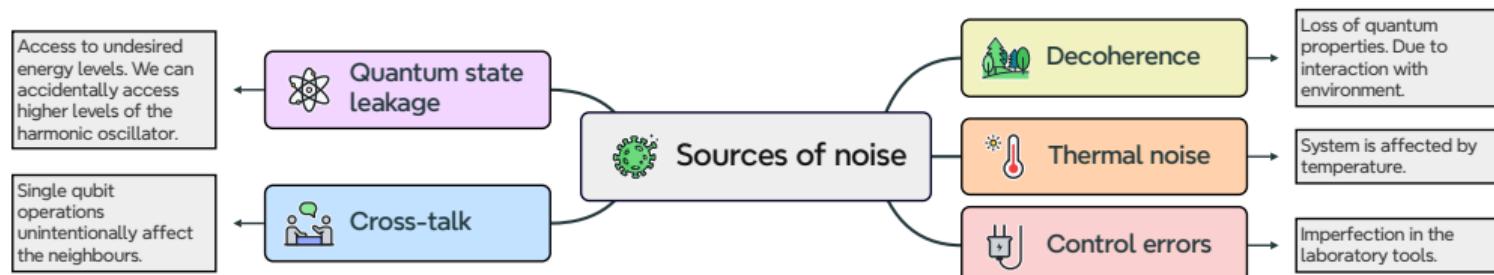
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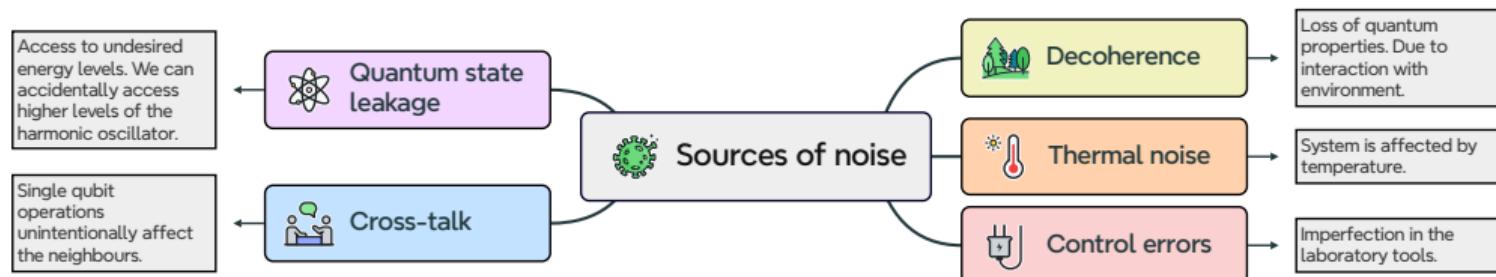
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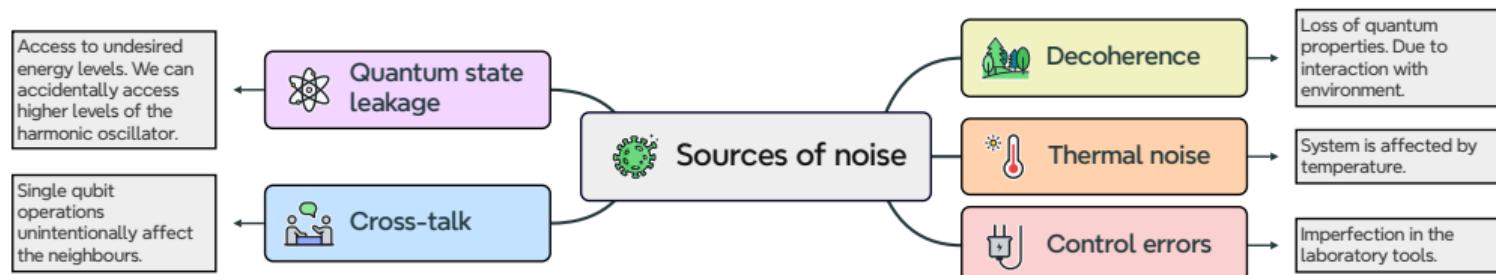


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What about the shot-noise? Is it similar to these noises?

Without noise we can use state vectors

For now we talked about state vector manipulation, where unitary gates \mathcal{U} are used to evolve quantum states:

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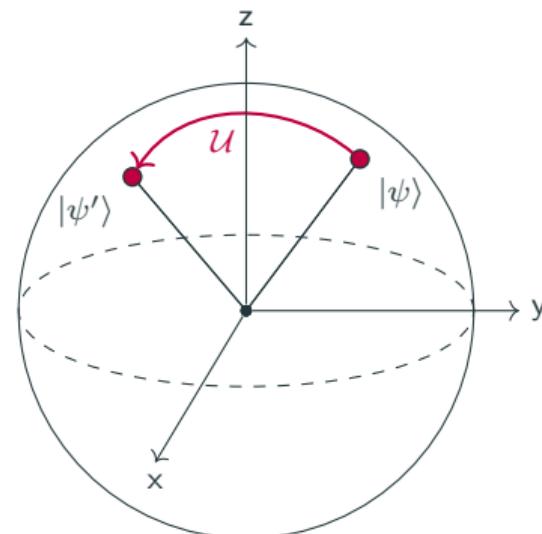
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About pure states

- Pure states are preserved until the quantum system is isolated, thus doesn't interact with the environment.
- A pure state of a single qubit can be visualized as a point on the surface of the Bloch sphere.
- Using unitaries \mathcal{U} we can move any point of the sphere into another point in a reversible way.



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$$\rho = \sum_{i=0}^n p_i |\psi_i\rangle \langle\psi_i|,$$

where p_i is the statistical weight of the pure state $|\psi_i\rangle$ in the mix. If $n = 1$, then $\rho = |\psi\rangle \langle\psi|$ and $|\psi\rangle$ is pure.

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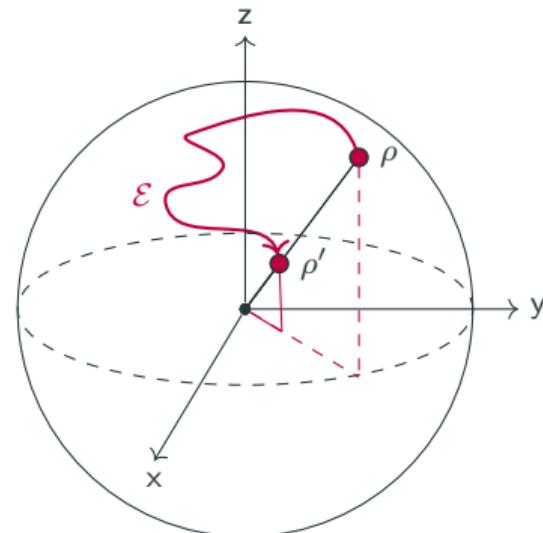
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About mixed states

- The evolution of a density matrix is described in terms of superoperators (or *channels*) $\rho' = \mathcal{E}(\rho)$, which, in the case of pure states act like unitaries $\rho' = \mathcal{E}^\dagger \rho \mathcal{E}$.
- A mixed state of a single qubit can be visualized as a point within the surface of the Bloch sphere.
- The effect of the noise can be represented by a superoperator which can map points on the surface into points located within the Bloch sphere.



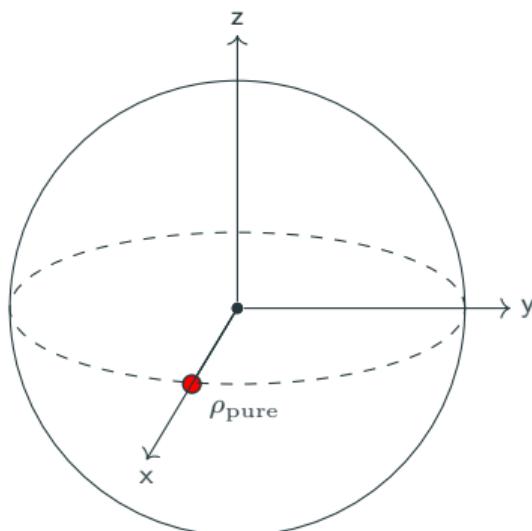
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The maximally entangled state is a pure state

The density matrix of $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ is

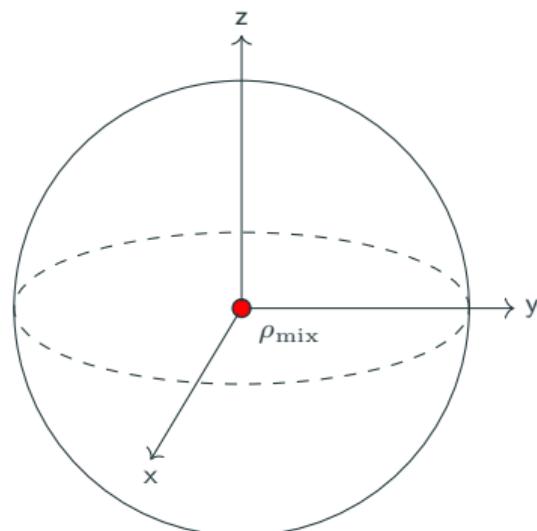
$$\begin{aligned}\rho_{\text{pure}} &= |+\rangle \langle +| = \\ &= [1/\sqrt{2} \quad 1/\sqrt{2}] \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}\end{aligned}$$



The state which maximally mix $|0\rangle$ and $|1\rangle$

The density matrix of $\frac{1}{2}(|0\rangle \langle 0| + |1\rangle \langle 1|)$ is

$$\begin{aligned}\rho_{\text{mix}} &= \frac{1}{2}(|0\rangle \langle 0| + |1\rangle \langle 1|) = \\ &= \frac{1}{2} [1 \quad 0] \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{2} [0 \quad 1] \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}\end{aligned}$$



A dummy noise representation

We can now define a channel \mathcal{N} , which represents the action of the noise on a state: $\rho_{\text{noisy}} = \mathcal{N}(\rho)$.

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We can exploit the Pauli's operator to write a noise model:

- we use X to apply random bitflip $|0\rangle = X|1\rangle$ and $|1\rangle = X|0\rangle$ with probability p_x ;
- we use Z to apply random phase flip $|0\rangle = Z|0\rangle$ and $-|1\rangle = Z|1\rangle$ with probability p_z ;
- we use Y to apply more complex manipulations since $Y = iXZ$ with probability p_y .

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Then the combined effect of these Pauli components is:

$$\mathcal{N}(\rho) = \left(1 - \sum_k p_k\right)\rho + \sum_k p_k P_k \rho P_k$$

where $\sum_k p_k \leq 1$, P_k are the Paulis $\{X, Y, Z\}$ and the **first term** describe when ρ remains unchanged.

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Effect of the noise when computing expectation values

1. The effect of this noise model is to push any state ρ closer to the maximally mixed state $\rho_{\text{mix}} = \frac{1}{2^N}I$, where N is the number of qubits of the system and I is the identity.
2. The expectation value of a Pauli string (X, Y, Z or combinations P) over ρ_{mix} zero. In fact, since $E[P] = \text{Tr}[P] = 0$:

$$E_{\rho_{\text{mix}}}[P] = \text{Tr}[P\rho_{\text{mix}}] = \frac{1}{2^N}\text{Tr}[P I] = \frac{1}{2^N}\text{Tr}[P] = 0.$$

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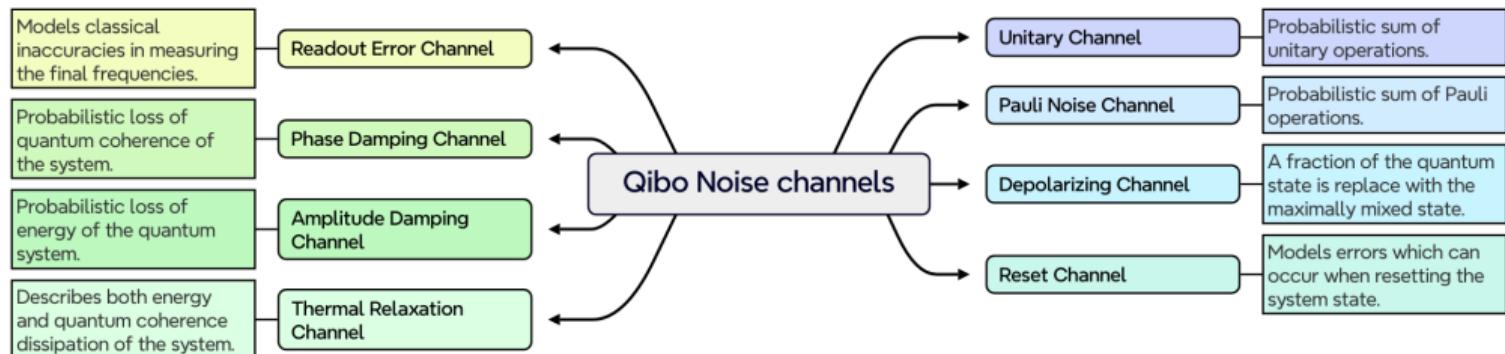
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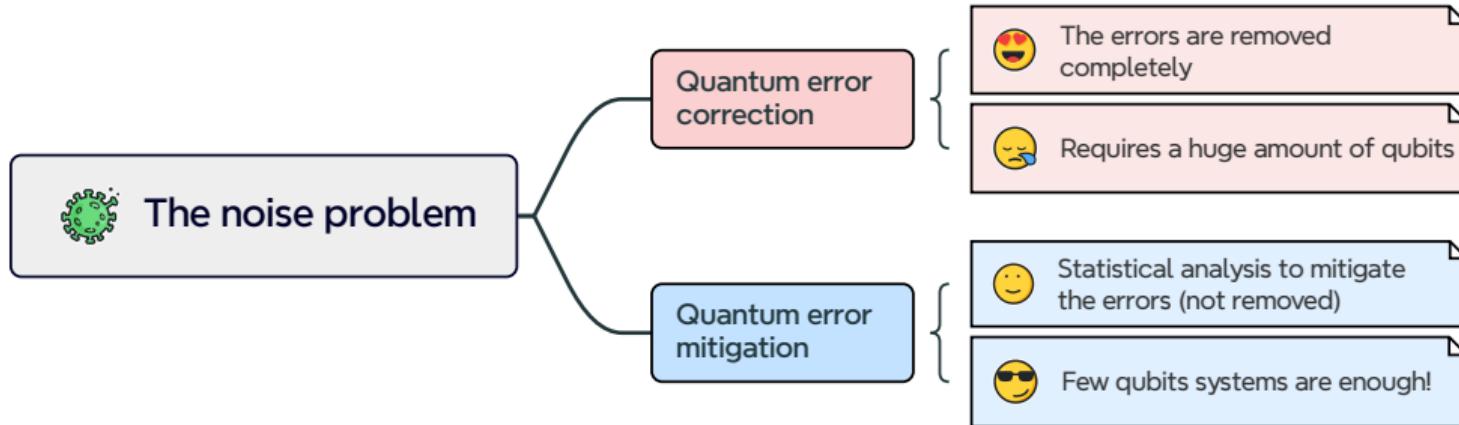
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How can we face this problem?



Some references on Quantum Error Correction

- Quantum Error Correction for Beginners
- Quantum Error Correction: An Introductory Guide
- Bosonic quantum error correction codes in superconducting quantum circuits
- Experimental Quantum Error Correction

Some references on Quantum Error Mitigation

- Quantum Error Mitigation
- Efficient variational quantum simulator incorporating active error minimisation
- Error mitigation for short-depth quantum circuits
- Error mitigation with Clifford quantum-circuit data

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Suppose we are interested in computing the expectation value $\langle \mathcal{O} \rangle^0$ of an observable \mathcal{O} over the state we get applying the quantum circuit \mathcal{C}^0 to a state ρ .

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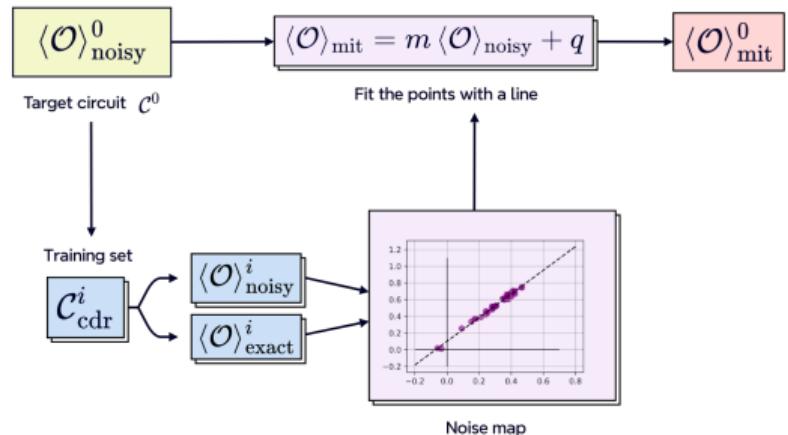
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Clifford Data Regression algorithm

1. Sample a set of circuits $\mathcal{C}_{\text{cdr}}^i$, of the same size of the target \mathcal{C}^0 , but fast to simulate¹.
2. For each $\mathcal{C}_{\text{cdr}}^i$ calculate exact and noisy exp value.
3. Make a scatter plot of exact versus noisy expectation values and fit the points with a line.
4. This line is actually a noise map, which can be used to map our $\mathcal{C}_{\text{noisy}}^0$ into a mitigated $\mathcal{C}_{\text{mit}}^0$.
5. The obtained map can be used for any new circuit of the same size of \mathcal{C}^0 .



This method assumes the noise is affecting the gates!

¹We generate these circuits by randomly replacing some gates of the circuit with Clifford gates. When dealing with rotations, one can replace the angles with multiples of $\pi/2$. For more info on Clifford simulators: arXiv:quant-ph/0406196.

Let's code!

