Ancillary qubits

Quantum Computing Minicourse ICTP-SAIFR

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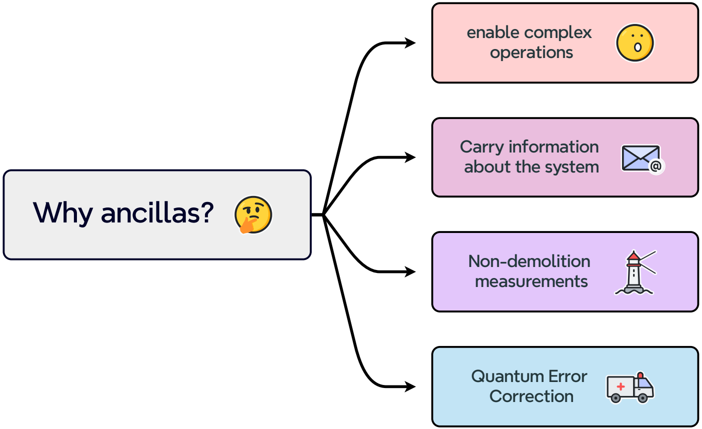
Ancillary qubits

Ancilla qubits are extra qubits, which help a qubit system in some computations. For example:



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Important

This happens if the ancilla is prepared into a state which is eigenvector of the controlled operation.

Is the phase kickback useful?

Some of the most powerful and famous quantum computing algorithms make use of the phase kickback

Figure 1: Deutsch-Josza

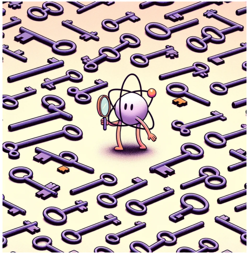

Figure 2: Grover

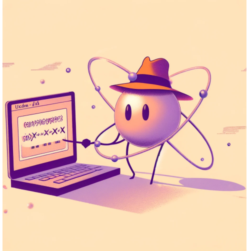

Figure 3: Shor

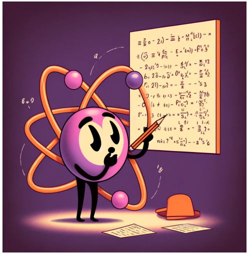

Figure 4: HHL

All of them are proved to outperform any classical algorithm, leading to a theoretical **quantum advantage**!

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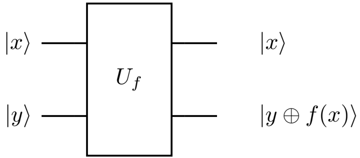

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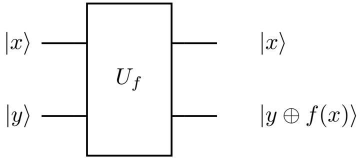

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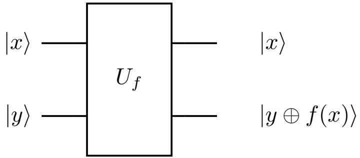

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Extending it to a multi-dimensional case, we can write: $U_f |x\rangle |-\rangle = (-1)^{f(x)} |x\rangle |-\rangle$, whith $f(x) \in \{0, 1\}$.

Let's code!