Quantum Noise and error mitigation

Quantum Computing Minicourse ICTP-SAIFR

Stefano Carrazza[‡] and Matteo Robbiati[†]

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‡ Associate Professor & Researcher, University of Milan and INFN Milan, Italy.
† PhD candidate, University of Milan, Italy and CERN, Switzerland.

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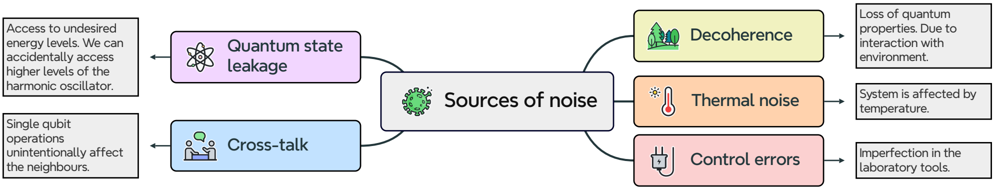

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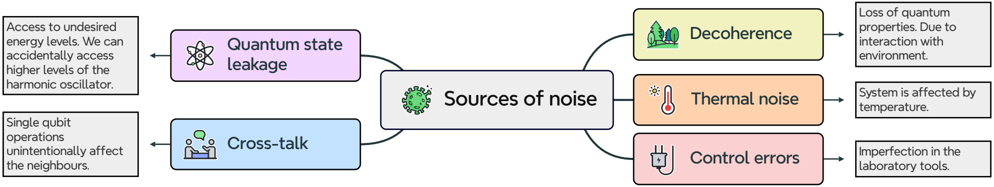

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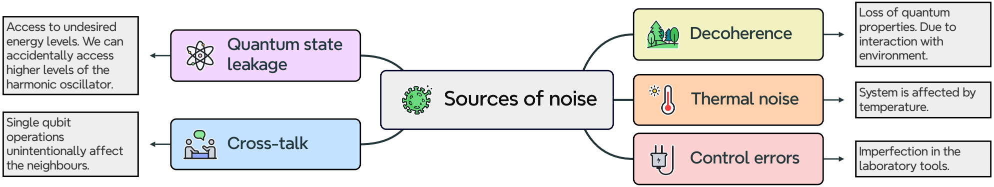

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What about the shot-noise? Is it similar to these noises?

Without noise we can use state vectors

In the previous lectures we introduced the state vector manipulation, where unitary operators $\mathcal{U}$ have been used to evolve quantum states:

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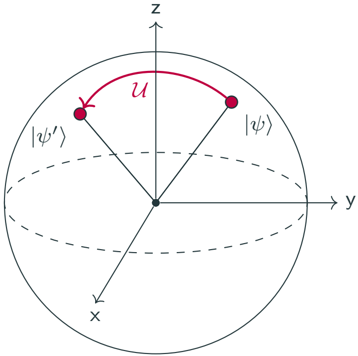
About pure states

- Pure states are preserved until the quantum system is isolated, thus doesn't interact with the environment.
- A pure state of a single qubit can be visualized as a point on the surface of the Bloch sphere.
- Using unitaries $\mathcal{U}$ we can move any point of the sphere into another point in a reversible way.



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Noise corrupts the quantum states, building **mixed states**, namely classical mixture of pure states. An useful tool to represent both pure and mixes states is the **density matrix**, which in general is defined as:

$$\rho = \sum_{i=0}^{n} p_i \left|\psi_i\right\rangle \left\langle\psi_i\right| ,$$

where p_i is the statistical weight of the pure state $\left|\psi_i\right\rangle$ in the mix. If $n = 1$, then $\rho = \left|\psi\right\rangle \left\langle\psi\right|$ and $\left|\psi\right\rangle$ is pure.

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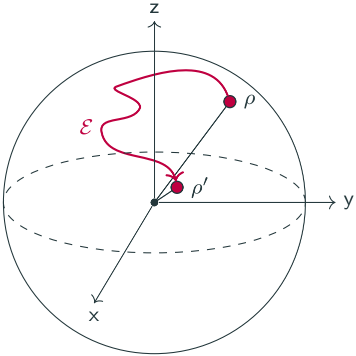
About mixed states

- The evolution of a density matrix is described in terms of superoperators $\rho' = \mathcal{E}(\rho)$, which is a generalization of the unitary case: $\rho' = U^\dagger \rho U$.
- A mixed state of a single qubit can be visualized as a point within the surface of the Bloch sphere.
- The effect of the noise can be seen as moving a point from the surface of the sphere to the inner region. This is not reversible!



Pure and mixed states in the Bloch sphere

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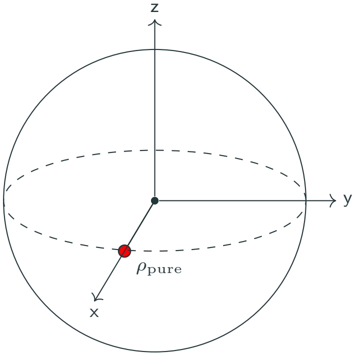
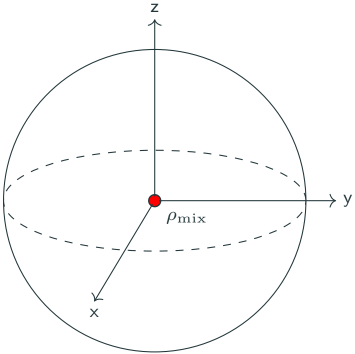
Example of simple pure state
The density matrix of $|+\rangle$ is

$$\begin{aligned}\rho_{\text{pure}} &= |+\rangle\langle+| = \\ &= \begin{bmatrix}1/\sqrt{2} & 1/\sqrt{2}\end{bmatrix}\begin{bmatrix}1/\sqrt{2}\\1/\sqrt{2}\end{bmatrix} = \begin{bmatrix}1/2 & 1/2\\1/2 & 1/2\end{bmatrix}\end{aligned}$$



Example of simple mixed state
The density matrix of $\frac{1}{2}\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right)$ is

$$\begin{aligned}\rho_{\text{mix}} &= \frac{1}{2}\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right) = \\ &\frac{1}{2}\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} + \frac{1}{2}\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}1/2 & 0\\0 & 1/2\end{bmatrix}\end{aligned}$$



The Pauli representation of the noise

We can now define a superoperator $\mathcal{N}$, which represents the action of the noise on a state: $\rho_{\text{noisy}} = \mathcal{N}(\rho)$.

We can exploit the Pauli's operator to write a noise model:

- we use X to apply random bitflip $|0\rangle = X\,|1\rangle$ and $|1\rangle = X\,|0\rangle$;
- we use Z to apply random phase flip $|0\rangle = Z\,|0\rangle$ and $-\,|1\rangle = Z\,|1\rangle$;
- we use Y to apply more complex manipulations since $Y = iXZ$.

Then the combined effect of these Pauli components is:

$$\mathcal{N}(\rho) = p_I I \rho I + p_x X \rho X + p_y Y \rho Y + p_x Z \rho Z,$$

where $p_I + p_x + p_y + p_z = 1$ and the first term of the sum represent the case in which ρ remains unchanged.

Final comments

1. The effect of this noise model is to push any state ρ closer to the maximally mixed state $\rho_{\text{mix}} = \frac{1}{2^N} I$.
2. The expectation value of a Pauli observable $(X, Y, Z$ or a combination $P)$ over ρ_{mix} is zero.
3. From 1. and 2., the more intense is the noise, the more $\langle \rho | P | \rho \rangle \to 0$

How can we face this problem?

- **The noise problem**
  - **Quantum error correction**
    - The errors are removed completely
    - Requires a huge amount of qubits
  - **Quantum error mitigation**
    - Statistical analysis to mitigate the errors (not removed)
    - Few qubits systems are enough!

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Suppose we are interested in computing the expectation value $\langle P \rangle^0$ of an observable P over the state we get applying the quantum circuit $\mathcal{U}$ to a state ρ_0:

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$$\rho = \mathcal{U}(\rho_0).$$

But our system is noisy! Thus what we get is a noisy expectation value $\langle P \rangle^0_{\text{noisy}}$.