

# Grover search algorithm

Quantum Computing Minicourse ICTP-SAIFR

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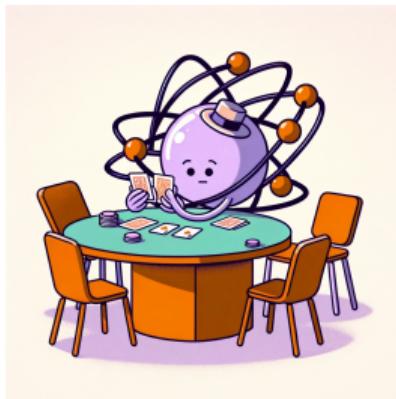
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# Motivation

The Grover algorithm is powerful when searching an item among an unordered set of candidates.

Extract the jack of clubs from a Poker deck



Find a passcode composed of 10 numbers



Find an antidote to the Cobra poison, exploring  $10^{20}$  molecules



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How many attempts could you need, in the worst scenario, to explore all the possibilities?

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In the worst scenario, you will need to check 52 cards,  $10^{10}$  passcodes and  $10^{20}$  molecules.

# Quadratic speedup

If we consider a time cost of  $\delta = 10^{-8}$  seconds for any algorithmic call (quantum or classical) we would wait:

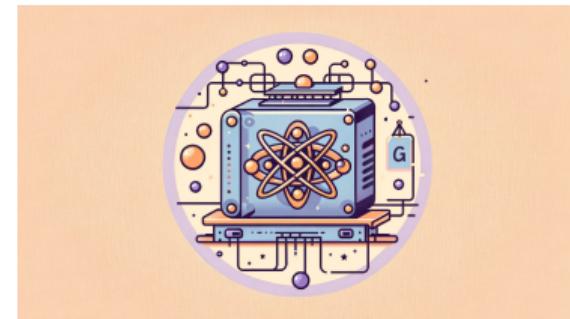
## On a classical computer

- $0.52 \mu\text{s}$  to find the jack of clubs;
- 100 seconds to find the passcode;
- $\sim 31688$  years to find the Cobra antidote.



## On a quantum computer

- $0.0721 \mu\text{s}$  to find the jack of clubs;
- 0.001 seconds to find the passcode;
- 100 seconds to find the Cobra antidote.



The Grover algorithm solves this kind of search with a number of algorithmic calls proportional to  $\sqrt{N}$ , where  $N$  is the dimension of the search space.

## The Grover algorithm

The key steps of the Grover algorithm:

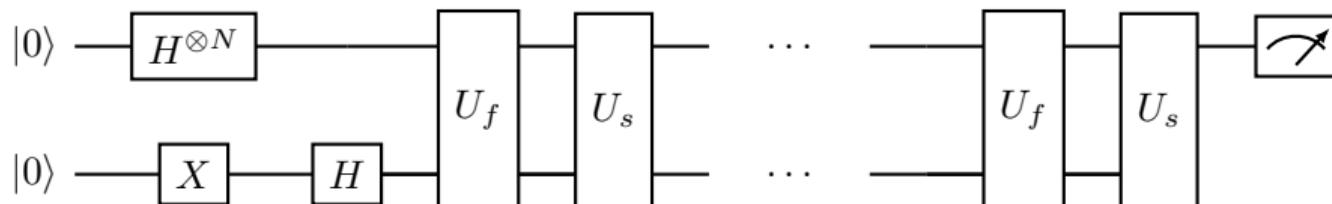
1. prepare a system of  $N$  qubits into a maximally superposed state;
2. prepare an ancilla qubit into the  $|-\rangle$  state;
3. apply an oracle operator  $U_f$  which can mark the correct solution;
4. apply a diffusion operator  $U_s$  which amplifies the correct solution;
5. repeat 3. and 4. for the optimal number of times.

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In terms of quantum circuit:



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$$\begin{bmatrix} \text{item}_1 \\ \text{item}_2 \\ \dots \\ \text{item}_{2^N} \end{bmatrix} \rightarrow |\psi\rangle = \begin{bmatrix} \psi_{00\dots 0} \\ \psi_{00\dots 1} \\ \dots \\ \psi_{11\dots 1} \end{bmatrix} \equiv \begin{bmatrix} x_0 \\ x_1 \\ \dots \\ x_{2^N-1} \end{bmatrix}.$$

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In total, we will use  $N$  qubits of the system and one ancilla. Then, the full quantum state will be  $|\psi\rangle_N |a\rangle$ .

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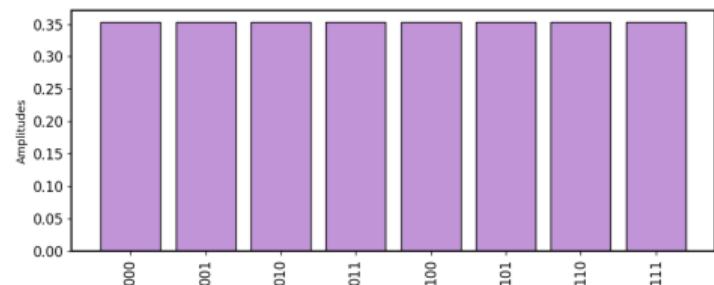
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The first step of the algorithm is the state preparation into the following superposed state:

$$H^{\otimes N+1} |0\rangle_N |1\rangle = \left[ \frac{1}{2^{N/2}} \sum_{i=0}^{2^N-1} |x_i\rangle \right] \otimes |-\rangle.$$



### Step 3: the oracle $U_f$

We consider now a function  $f : \{0, 1\}^N \rightarrow \{0, 1\}$ , whose goal is to mark the correct solution:

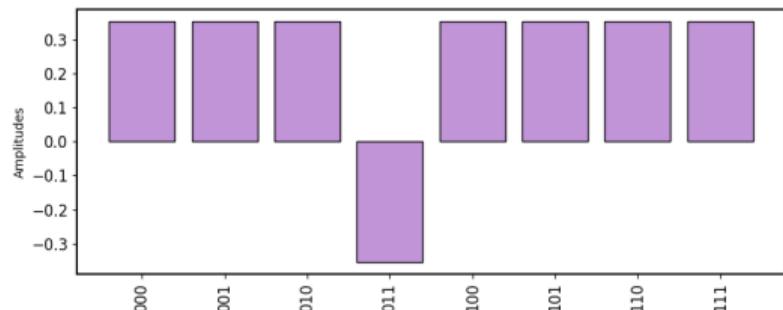
$$f(x) = \begin{cases} 1 & \text{if correct,} \\ 0 & \text{otherwise.} \end{cases}$$

This function is typically embedded into a quantum oracle  $U_f$  which is able to recognize the solution for us. It's important to underline that even if the oracle can detect the solution, may don't know its exact value.

The oracle marks the solution (one of the amplitudes) by flipping its sign thanks to a phase-kickback procedure.

$$U_f |x\rangle |-\rangle = (-1)^{f(x)} |x\rangle |-\rangle .$$

This can be done using a multi-controlled operation which triggers the kickback only when the system state corresponds to the target one.



## Step 4: the diffusion operator $U_s$

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# Let's code!

