Grover search algorithm

Quantum Computing Minicourse ICTP-SAIFR

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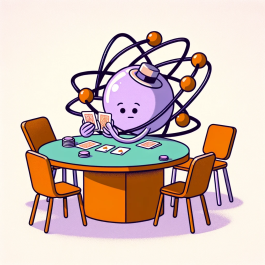
Motivation

The Grover algorithm is powerful when searching an item among an unordered set of candidates.

Extract the jack of clubs from a Poker deck



Find a passcode composed of 10 numbers

Find an antidote to the Cobra poison, exploring 10^{20} molecules

? How many attempts could you need, in the worst scenario, to explore all the possibilities?

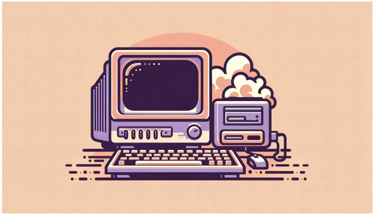
⚠ In the worst scenario, you will need to check 52 **cards**, 10^{10} **passcodes** and 10^{20} **molecules**.

Quadratic speedup

If we consider a time cost of $\delta = 10^{-8}$ seconds for any algorithmic call (quantum or classical) we would wait:

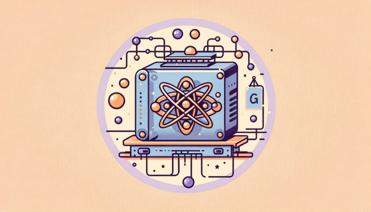
On a classical computer

- 0.52 μs to find the jack of clubs;
- 100 seconds to find the passcode;
- $\sim$ 31688 years to find the Cobra antidote.



On a quantum computer

- 0.0721 μs to find the jack of clubs;
- 0.001 seconds to find the passcode;
- 100 seconds to find the Cobra antidote.



The Grover algorithm solves this kind of search with a number of algorithmic calls proportional to $\sqrt{N}$, where N is the dimension of the search space.

The Grover algorithm

The key steps of the Grover algorithm:

1. prepare a system of N qubits into a maximally superposed state;
2. prepare an ancilla qubit into the $|-\rangle$ state;
3. apply an oracle operator U_f which can mark the correct solution;
4. apply a diffusion operator U_s which amplifies the correct solution;
5. repeat 3. and 4. for the optimal number of times.

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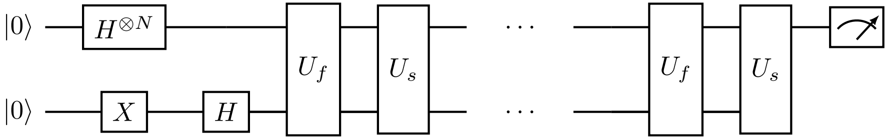
In terms of quantum circuit:



step 1 and 2: the state preparation

Lets consider 2^N elements composing our search space.

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We can encode them into the amplitudes of a quantum state of N qubits:

$$\begin{bmatrix} \text{item}_1 \\ \text{item}_2 \\ \dots \\ \text{item}_{2^N} \end{bmatrix} \quad \rightarrow \quad \begin{bmatrix} \psi_{00\dots0} \\ \psi_{00\dots1} \\ \dots \\ \psi_{11\dots1} \end{bmatrix}$$

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Let us use an **ancilla**, which will help us in manipulating the system's qubits through a kickback-like procedure.

In the complete framework, we are going to use $N+1$ qubits, whose state is $|\psi\rangle\,|a\rangle$.

The first step of the algorithm is the state preparation into one maximally superposed state:

$$H^{\otimes N+1}\,|00\dots0\rangle\,|1\rangle = |+ + \dots +\rangle\,|-\rangle\,.$$

Step 3: the oracle U_f

Step 4: the diffusion operator U_s