Dedicated Algorithm based on Discrete Cosine Transform for the Analysis of Industrial Processes Using Ultrasound Tomography

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The basic problem of all non-invasive tomography is a small number of measurements in relation to the number of reconstructed image pixels. From a mathematical point of view, it leads to extremely underdetermined systems of equations. Due to the above feature, the solution that minimizes the mean square reconstruction error is not unique, and moreover, the rare matrices obtained from the equations are very poor conditioned numerically, what results in the difficulties in computing of the inverse operator.

The basic idea of the presented method is the observation that instead of trying to reconstruct inaccurately the whole picture pixel by pixel, the number of variables could be reduced by using lossy image compression. In this work we show how to transform the original system (reconstruction of image pixels) into a smaller system (reconstruction of a compressed image) so that the compression process is incorporated in the reconstruction process. Therefore, at the stage of reconstruction, there is no need to reconstruct an uncompressed image.

The compression technique is based on dividing the image into square blocks and applying a transform (allowing compression) on each block separately. For the purposes of this work, a discrete cosine transform (DCT) was used for compression. In this work, tomograms with a resolution of 64x64 pixels, divided into blocks of 8x8 pixels, were the starting point. Which gives a total of 64 blocks spread over an eight by eight grid. It has been verified that the use of block transform allows for effective compression and decompression of typical tomograms, which allowed for the tests on real measurement data.

The results obtained in this way from the real data from transmission ultrasound tomography do not differ in quality from reconstruction with other, more classic methods. In the method used, we can operate on smaller systems of equations what significantly reducing the unknowns in the equations, and also simplify process regularization. In addition, it saves memory because you can archive results directly in a compressed form, without need of compression done on the server side, which allows to speed up the archiving process, due to the reduced system of equations. These features make the presented method ideally suited for real-time imaging in distributed systems, reducing the amount of data necessary to transmit from measuring systems: we locally solve a smaller system of equations locally and we can send it to a server without additional compression, where it will also not need further compression neither.
A part of single-channel image (8x8 pixels) can be treated as an 8x8 size matrix, and such a matrix can be treated as a linear combination of matrix with ones in individual pixels:

$$I_m = \sum_{i,j=1}^{64} w_{ij}\, F_{ij} \quad (1)$$

where I_m is an image block,w_{ij} numerical factors,F_{ij}base images.

DCT compression is based on a changing the canonical base to, an orthogonal base, which instead of encoding the information of individual pixel encodes the image content in frequency domain.

Initial tests and algebraic analysis proves that the application of compression to tomographic images is possible and should allow to obtain satisfying results. It is crucial to analyze the process of solving the inverse problem.

We start from the forward model equation, (with resolution of 64 by 64 pixels):

$$m = Jr \quad (2)$$

where: $J \in \mathbb{R}^{M\times4096}, r \in \mathbb{R}^{4096\times1}$ and:
r –reconstructed image reshaped into a column vector,
M –number of measurements (210 from 21 sensors),
m –measurement vector,
J –matrix representing the linearization of the tomographic system.
According to equation (2), the measurement vector can be represented as the sum of individual influences from each 8x8 pixel block:

$$m = \sum_{i=1}^{8}\sum_{j=1}^{8} J^{ij} r_{ij} \quad (3)$$

where $J^{ij}$is a minor matrix of J with size [M x 64], created by the columns of *J* corresponding to pixels of block *i,j*.
r_{ij} - part of the vector r corresponding to the pixels of the block *i,j*.

We can reformulate it as:

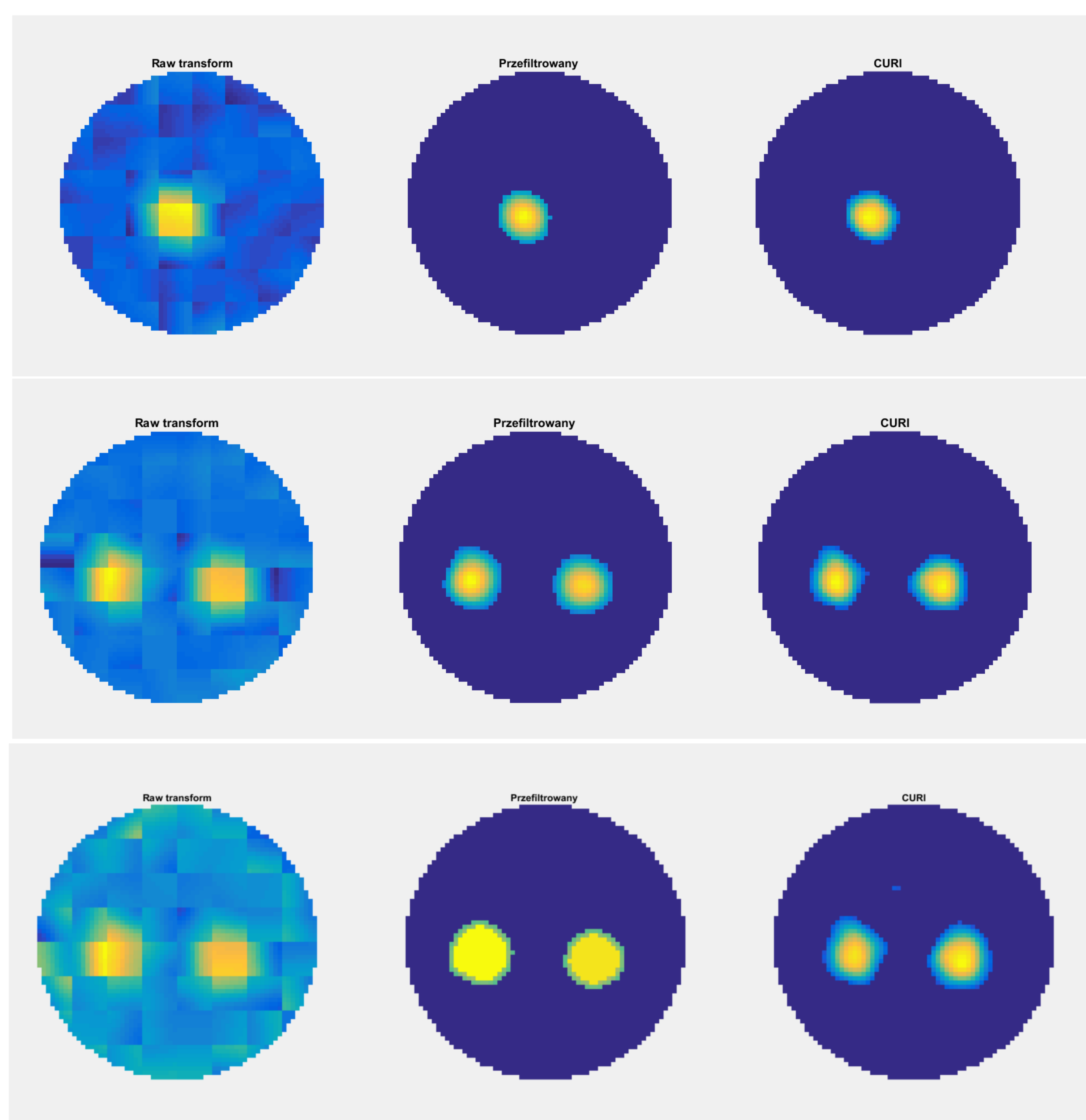
$$m = J_{DCT}\boldsymbol{b}, \qquad (4)$$

If, for the formulation convenience, we change the indexing from 2-index to 1-index manner, we can represent the parts of eq. (4) as:

$$J_{DCT} = \left[J_{DCT}^{1}\ J_{DCT}^{2} \cdots J_{DCT}^{8^2}\right], \qquad \boldsymbol{b} = \begin{bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \\ \vdots \\ \boldsymbol{b}_{8^2} \end{bmatrix}.$$

Where:
$\boldsymbol{b}_i, i \in \{1 \dots 8^2\}$ are the column vectors of coefficients of DCT transform for a block of index i.
$J_{DCT}^{i}$are the transformations of the matrices J^{ij} from the canonical base to the DCT base.
With such a model we can reconstruct the $\boldsymbol{b}$ vector. The reconstructed vector is a direct representation of the compressed image.

a)
b)
c)



Examples of reconstruction using UST:
(a) for 1 object, (b) for 2objects, (c) for 2 objects with segmentation