Chapter2 Perfectly-Secret Encryption

本章问题

1. perfectly-secret的定义是什么?
2. 什么是一次一密OTP，OTP的性质是?
3. Perfect secrecy的局限性?

问题答案:

Question 1

Definition 1:
$Pr[M=m|C=c]=Pr[M=m]$
Definition 2:
$Pr[C=c|M=m]=Pr[C=c]$
Definition 3:
$Pr[C=c|M=m_1]=Pr[C=c|M=m_0]$
Definition 4:
定义实验$PrivK^{eav}_{A,\Pi}$:

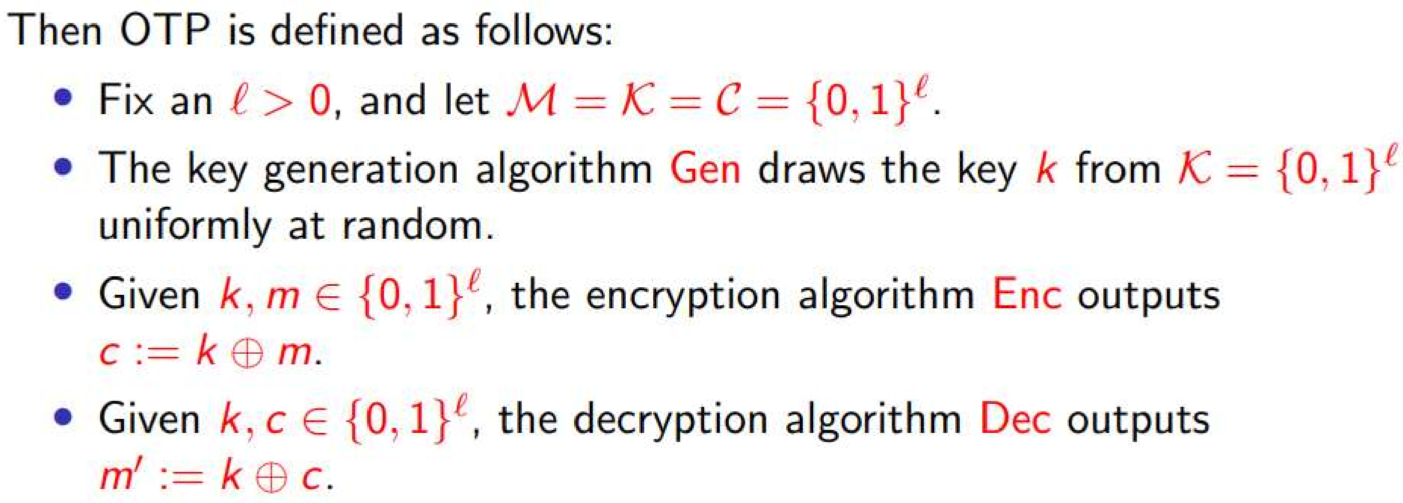
1. A outputs $m_0,m_1\in M$;
2. $k\leftarrow Gen(\cdot)$;$b\leftarrow\{0,1\}$;$c\leftarrow Enc_k(m_b)$;
3. Given c, A outputs $b'\in\{0,1\}$;
4. if $b=b'$,output 1, ohterwise 0;

$Pr[PrivK^{eav}_{A,\Pi}=1]=\frac{1}{2}$
证明定义等价

> 条件概率相关公式:
>
> 1. $P(B|A)=\frac{P(AB)}{P(A)}$
> 2. $P(A_1A_2\cdots A_n)=P(A_1)P(A_2|A_1)\cdots P(A_n|A_1A_2\cdots A_{n-1})$
> 3. (全概率公式) $P(A)=\sum_{i=1}^{n}P(B_i)P(A|B_i)$
> 4. (贝叶斯公式) $P(B_i|A)=\frac{P(AB^i)}{P(A)}=\frac{P(B_i)P(A|B_i)}{\sum_i P(B_i)P(A|B_i)}$

Question 2

Definition



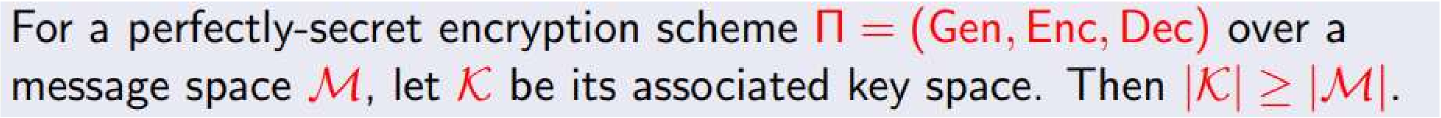
Then OTP is defined as follows:

- Fix an $\ell > 0$, and let $\mathcal{M} = \mathcal{K} = \mathcal{C} = \{0,1\}^\ell$.
- The key generation algorithm Gen draws the key k from $\mathcal{K} = \{0,1\}^\ell$ uniformly at random.
- Given $k, m \in \{0,1\}^\ell$, the encryption algorithm Enc outputs $c := k \oplus m$.
- Given $k, c \in \{0,1\}^\ell$, the decryption algorithm Dec outputs $m' := k \oplus c$.

Theorem

OTP is perfectly-secret

Question 3



For a perfectly-secret encryption scheme $\Pi = (\mathsf{Gen}, \mathsf{Enc}, \mathsf{Dec})$ over a message space $\mathcal{M}$, let $\mathcal{K}$ be its associated key space. Then $|\mathcal{K}| \geq |\mathcal{M}|$.