

Homework 2*Handed Out: January 25**Due: February 8***Name:** Qihang Dai**PennKey:** ahgdyyycc**PennID:** 78803164

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1 Multiple Choice & Written Questions

1. (a) decrease bias and increase variance
(b) increase bias and decrease variance
(c) decrease bias and increase variance

This means the model is overfitting: low bias and high variance. thus we want a more generalized model and a more simple prediction. Thus we increase n , decrease λ , and decrease d .

2. (A) global maximum
3. (a)

$$\frac{\partial R_1}{\partial B_j} = \lambda \text{sgn}(B_j)$$

$$\frac{\partial R_2}{\partial B_j} = 2\lambda B_j^2$$

- (b) since L1 regularization's gradient is independent of w , it will do a better job to push weight to zero when weight is small.
4. (a) since $x > 0$, we have $a = 1$ and $b = 0$. $y = x$

for x distribution on $[-1, 0]$, we have:

$$MSE = \frac{1}{n} \sum_{i=1}^n (x_i - 0)^2 \quad (1)$$

$$= \frac{1}{n} \sum_{i=1}^n x_i^2 \quad (2)$$

$$= \int_{-1}^0 x^2 dx \quad (3)$$

$$= \frac{1}{3} \quad (4)$$

(b) the learned model should be $y = 0$ ($a = 0$ and $b = 0$). for MSE on $X \sim U[0, 1]$, we have: $y = x$ and MSE is still $\frac{1}{3}$

5.

$$f_{\hat{\beta}}(x) = \hat{\beta}^T x = x^T \hat{\beta}$$

$$\hat{\beta} = (X^T X)^{-1} X^T Y$$

$$f_{\hat{\beta}}(x) = x^T (X^T X)^{-1} X^T Y$$

$$Y = (y_1, y_2, \dots, y_n)^T, \text{ we have}$$

$$f_{\hat{\beta}}(x) = x^T (X^T X)^{-1} X^T (y_1, y_2, \dots, y_n)^T$$

$$= \sum_{i=1}^n x^T (X^T X)^{-1} X^T y_i$$

$$k_i = x^T (X^T X)^{-1} X^T I_i$$

I_i represent $(d \times 1)$ vector where only i th element is 1 and others are 0.

2 Python Programming Questions

TODO: Please add the resulting plots for Question 1.3 and Question 1.4.

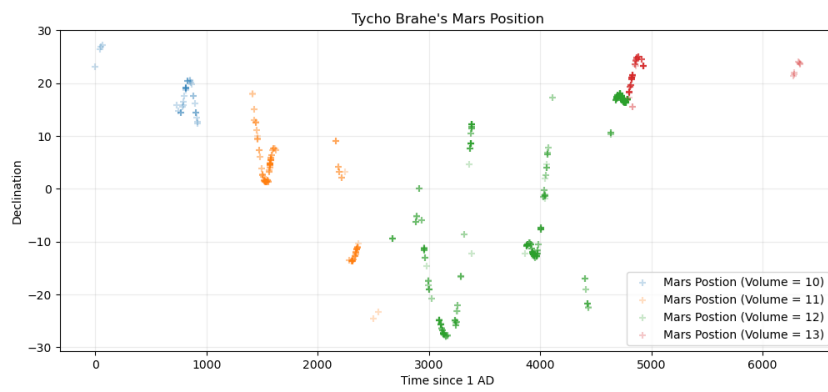


Figure 1: Figure