

**Homework 2***Handed Out: January 25**Due: February 8***Name:** Qihang Dai**PennKey:** ahgdyyycc

## 1 Multiple Choice & Written Questions

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1. (a) decrease bias and increase variance  
(b) increase bias and decrease variance  
(c) decrease bias and increase variance

This means the model is overfitting: low bias and high variance. thus we want a more generalized model and a more simple prediction. Thus we increase  $n$ , decrease  $\lambda$ , and decrease  $d$ .

2. (A) global maximum
3. (a)

$$\frac{\partial R_1}{\partial B_j} = \lambda \text{sgn}(B_j)$$

$$\frac{\partial R_2}{\partial B_j} = 2\lambda B_j^2$$

- (b) since L1 regularization's gradient is independent of  $w$ , it will do a better job to push weight to zero when weight is small.
4. (a) since  $x > 0$ , we have  $a = 1$  and  $b = 0$ .  $y = x$

for x distribution on  $[-1, 0]$ , we have:

$$MSE = \frac{1}{n} \sum_{i=1}^n (x_i - 0)^2 \quad (1)$$

$$= \frac{1}{n} \sum_{i=1}^n x_i^2 \quad (2)$$

$$= \int_{-1}^0 x^2 dx \quad (3)$$

$$= \frac{1}{3} \quad (4)$$

(b) the learned model should be  $y = 0$  ( $a = 0$  and  $b = 0$ ). for MSE on  $X \sim U[0, 1]$ , we have:  $y = x$  and MSE is still  $\frac{1}{3}$

5.

$$\begin{aligned} f_{\hat{\beta}}(x) &= \hat{\beta}^T x = x^T \hat{\beta} \\ \hat{\beta} &= (X^T X)^{-1} X^T Y \\ f_{\hat{\beta}}(x) &= x^T (X^T X)^{-1} X^T Y \\ Y &= (y_1, y_2, \dots, y_n)^T, \text{ we have} \\ f_{\hat{\beta}}(x) &= x^T (X^T X)^{-1} X^T (y_1, y_2, \dots, y_n)^T \\ &= \sum_{i=1}^n x^T (X^T X)^{-1} X^T y_i \\ k_i &= x^T (X^T X)^{-1} X^T I_i \end{aligned}$$

$I_i$  represent  $(d \times 1)$  vector where only  $i$ th element is 1 and others are 0.

## 2 Python Programming Questions

As we can see from Figure 2, learning rate matters a lot. When learning rate is too small, the model will take a long time to converge. When learning rate is too large, the model will not converge, and the loss will keep increasing. a proper learning rate is 0.1 here

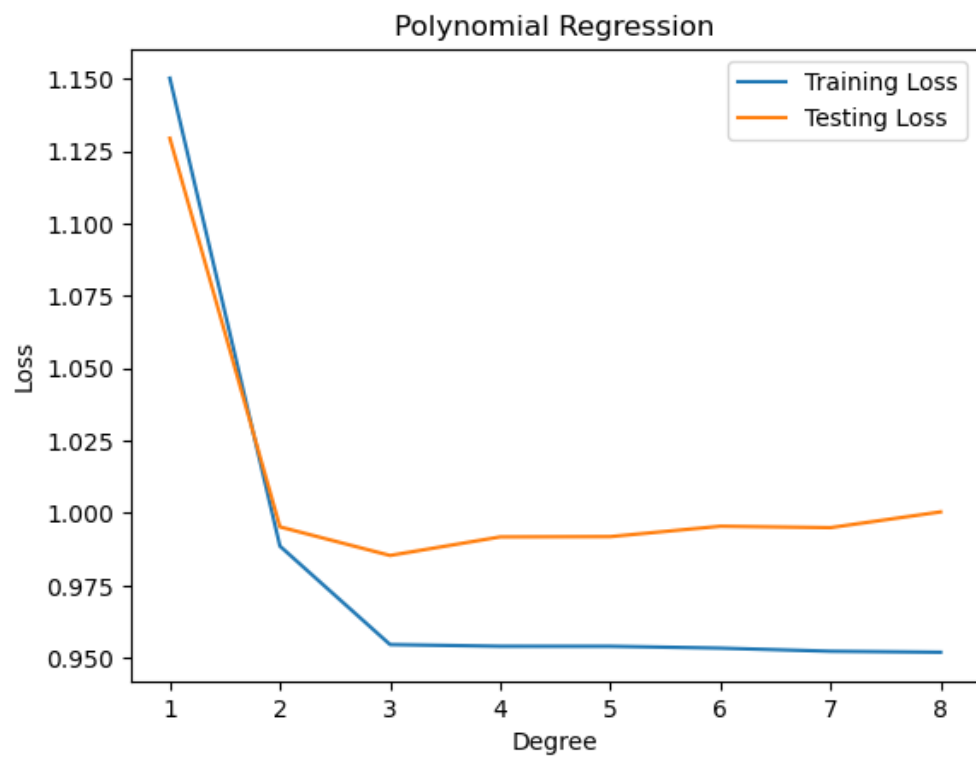


Figure 1: Figure

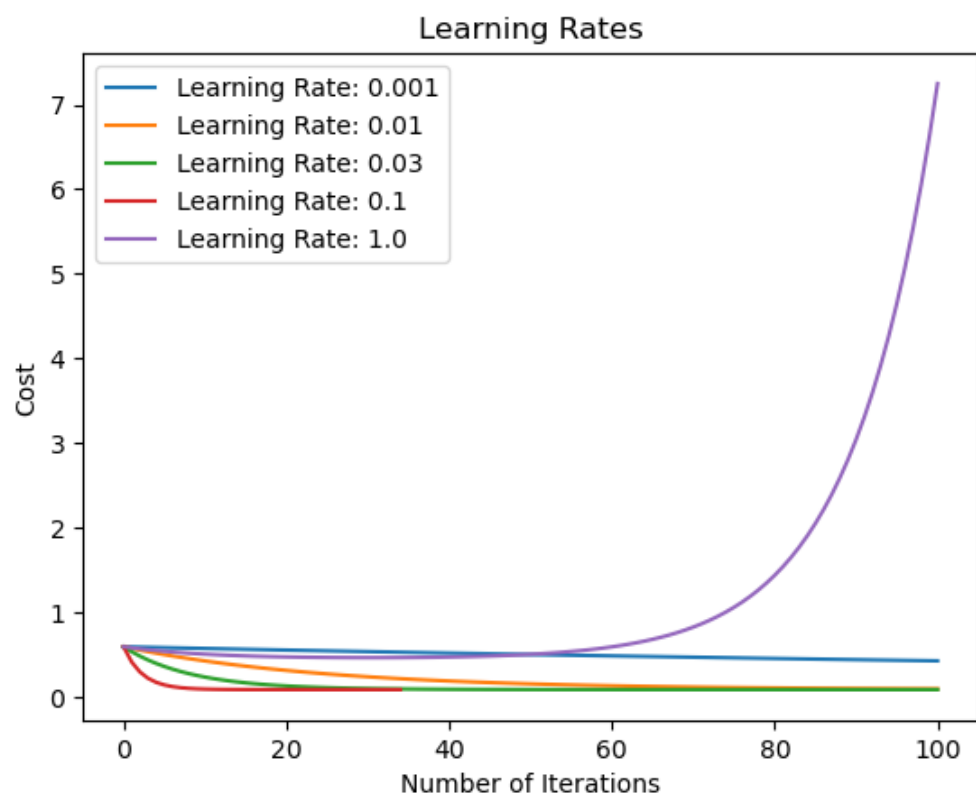


Figure 2: Figure