ECE 176 Assignment 2: Linear Regression

For this part of assignment, you are tasked to implement a linear regression algorithm for multiclass classification and test it on the CIFAR10 dataset.

You sould run the whole notebook and answer the questions in the notebook.

CIFAR 10 dataset contains 32x32x3 RGB images of 10 distinct cateogaries, and our aim is to predict which class the image belongs to

TO SUBMIT: PDF of this notebook with all the required outputs and answers.

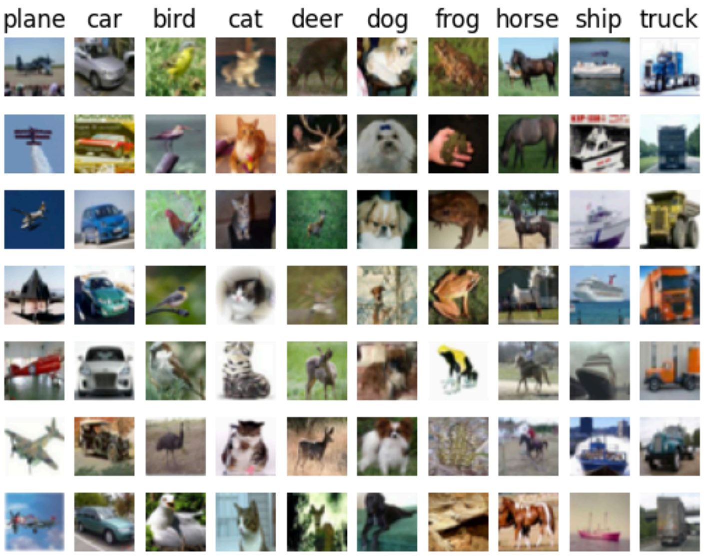
In [1]:
```python
# Prepare Packages
import numpy as np
import matplotlib.pyplot as plt

from utils.data_processing import get_cifar10_data

# Use a subset of CIFAR10 for the assignment
dataset = get_cifar10_data(
    subset_train=5000,
    subset_val=250,
    subset_test=500,
)

print(dataset.keys())
print("Training Set Data  Shape: ", dataset["x_train"].shape)
print("Training Set Label Shape: ", dataset["y_train"].shape)
print("Validation Set Data  Shape: ", dataset["x_val"].shape)
print("Validation Set Label Shape: ", dataset["y_val"].shape)
print("Test Set Data  Shape: ", dataset["x_test"].shape)
print("Test Set Label Shape: ", dataset["y_test"].shape)
```

```
dict_keys(['x_train', 'y_train', 'x_val', 'y_val', 'x_test', 'y_test'])
Training Set Data  Shape:  (5000, 3072)
Training Set Label Shape:  (5000,)
Validation Set Data  Shape:  (250, 3072)
Validation Set Label Shape:  (250,)
Test Set Data  Shape:  (500, 3072)
Test Set Label Shape:  (500,)
```

In [2]:
```python
x_train = dataset["x_train"]
y_train = dataset["y_train"]
x_val = dataset["x_val"]
y_val = dataset["y_val"]
x_test = dataset["x_test"]
y_test = dataset["y_test"]
```

In [3]:
```python
# Visualize some examples from the dataset.
# We show a few examples of training images from each class.
classes = [
    "plane",
    "car",
    "bird",
    "cat",
    "deer",
    "dog",
    "frog",
    "horse",
    "ship",
    "truck",
]
samples_per_class = 7


def visualize_data(dataset, classes, samples_per_class):
    num_classes = len(classes)
    for y, cls in enumerate(classes):
        idxs = np.flatnonzero(y_train == y)
        idxs = np.random.choice(idxs, samples_per_class, replace=False)
        for i, idx in enumerate(idxs):
            plt_idx = i * num_classes + y + 1
            plt.subplot(samples_per_class, num_classes, plt_idx)
            plt.imshow(dataset[idx])
            plt.axis("off")
            if i == 0:
```

```
                plt.title(cls)
    plt.show()


visualize_data(
    x_train.reshape(5000, 3, 32, 32).transpose(0, 2, 3, 1), classes, samples_per_class
)
```



Linear Regression for multi-class classification

A Linear Regression Algorithm has these hyperparameters:

- **Learning rate** - controls how much we change the current weights of the classifier during each update. We set it at a default value of 0.5, and later you are asked to experiment with different values. We recommend looking at the graphs and observing how the performance of the classifier changes with different learning rate.
- **Number of Epochs** - An epoch is a complete iterative pass over all of the data in the dataset. During an epoch we predict a label using the classifier and then update the weights of the classifier according the linear classifier update rule for each sample in the training set. We evaluate our models after every 10 epochs and save the accuracies, which are later used to plot the training, validation and test VS epoch curves.
- **Weight Decay** - Regularization can be used to constrain the weights of the classifier and prevent their values from blowing up. Regularization helps in combatting overfitting. You will be using the 'weight_decay' term to introduce regularization in the classifier.

Implementation (50%)

You first need to implement the Linear Regression method in `algorithms/linear_regression.py`. The formulations follow the lecture (consider binary classification for each of the 10 classes, with labels -1 / 1 for not belonging / belonging to the class). You need to fill in the training function as well as the prediction function.

In [4]:
```
# Import the algorithm implementation (TODO: Complete the Linear Regression in algorithms/linear_regression.py)
from algorithms import Linear
from utils.evaluation import get_classification_accuracy

num_classes = 10  # Cifar10 dataset has 10 different classes

# Initialize hyper-parameters
learning_rate = 0.0001  # You will be later asked to experiment with different learning rates and report results
num_epochs_total = 200  # Total number of epochs to train the classifier
epochs_per_evaluation = 10  # Epochs per step of evaluation; We will evaluate our model regularly during training
N, D = dataset[
    "x_train"
].shape  # Get training data shape, N: Number of examples, D:Dimensionality of the data
weight_decay = 0.0

# Insert additional scalar term 1 in the samples to account for the bias as discussed in class
x_train = np.insert(x_train, D, values=1, axis=1)
x_val = np.insert(x_val, D, values=1, axis=1)
x_test = np.insert(x_test, D, values=1, axis=1)
```

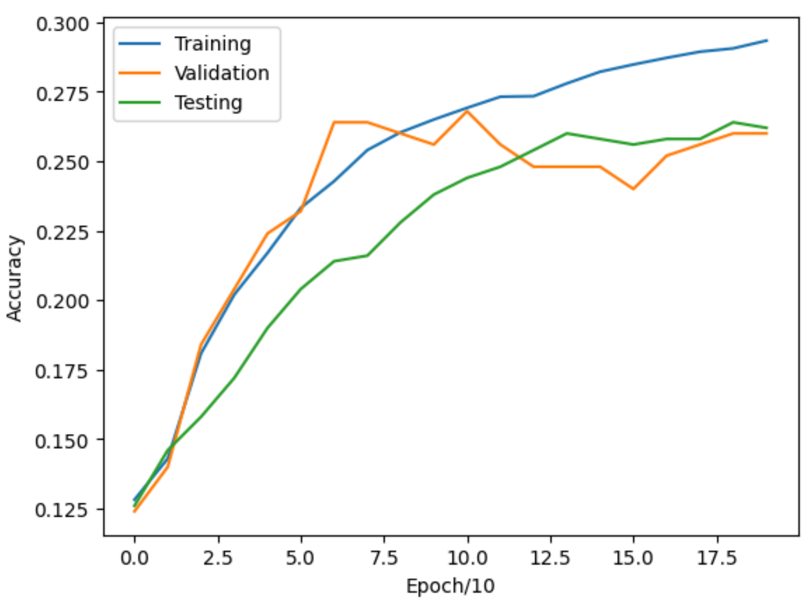
In [5]:
```python
# Training and evaluation function -> Outputs accuracy data
def train(learning_rate_, weight_decay_):
    # Create a linear regression object
    linear_regression = Linear(
        num_classes, learning_rate_, epochs_per_evaluation, weight_decay_
    )

    # Randomly initialize the weights and biases
    weights = np.random.randn(num_classes, D + 1) * 0.0001

    train_accuracies, val_accuracies, test_accuracies = [], [], []

    # Train the classifier
    for _ in range(int(num_epochs_total / epochs_per_evaluation)):
        # Train the classifier on the training data
        weights = linear_regression.train(x_train, y_train, weights)

        # Evaluate the trained classifier on the training dataset
        y_pred_train = linear_regression.predict(x_train)
        train_accuracies.append(get_classification_accuracy(y_pred_train, y_train))

        # Evaluate the trained classifier on the validation dataset
        y_pred_val = linear_regression.predict(x_val)
        val_accuracies.append(get_classification_accuracy(y_pred_val, y_val))

        # Evaluate the trained classifier on the test dataset
        y_pred_test = linear_regression.predict(x_test)
        test_accuracies.append(get_classification_accuracy(y_pred_test, y_test))

    return train_accuracies, val_accuracies, test_accuracies, weights
```

Plot the Accuracies vs epoch graphs

In [6]:
```python
import matplotlib.pyplot as plt


def plot_accuracies(train_acc, val_acc, test_acc):
    # Plot Accuracies vs Epochs graph for all the three
    epochs = np.arange(0, int(num_epochs_total / epochs_per_evaluation))
    plt.ylabel("Accuracy")
    plt.xlabel("Epoch/10")
    plt.plot(epochs, train_acc, epochs, val_acc, epochs, test_acc)
    plt.legend(["Training", "Validation", "Testing"])
    plt.show()
```

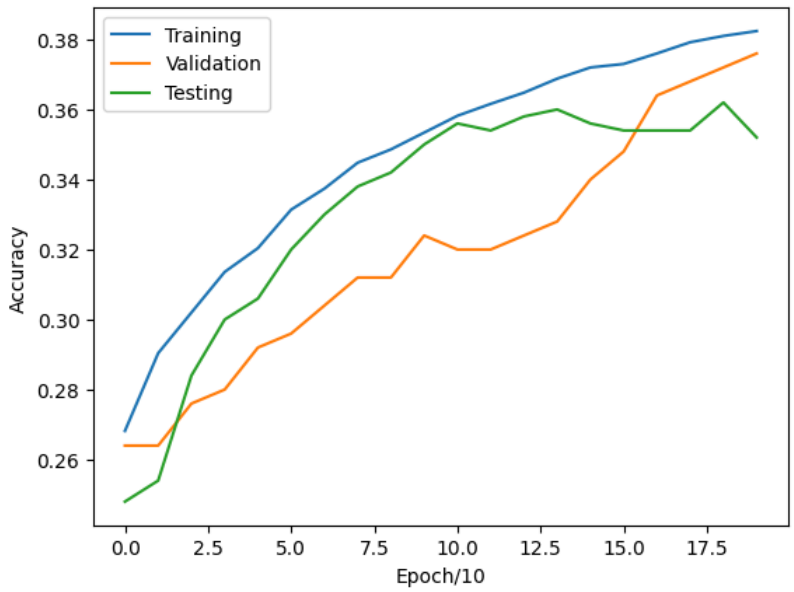
In [7]:
```python
# Run training and plotting for default parameter values as mentioned above
t_ac, v_ac, te_ac, weights = train(learning_rate, weight_decay)
```

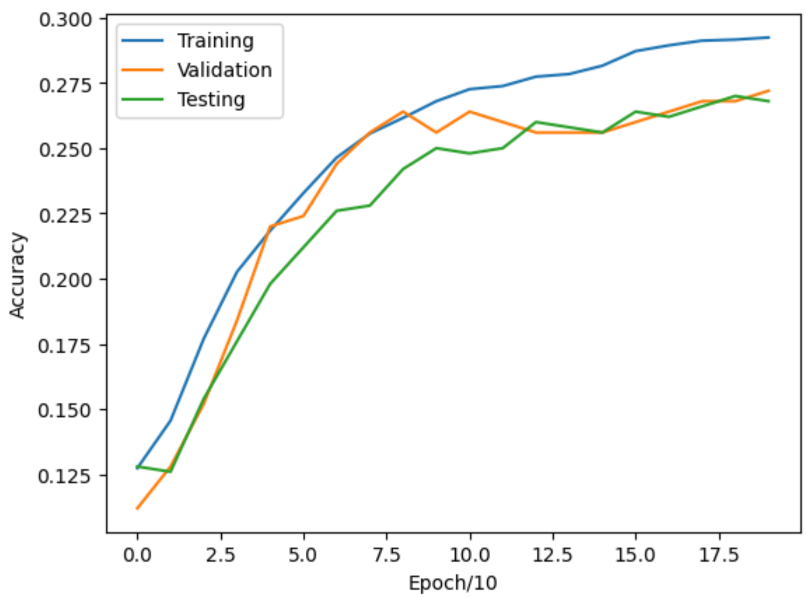
In [8]:
```python
plot_accuracies(t_ac, v_ac, te_ac)
```

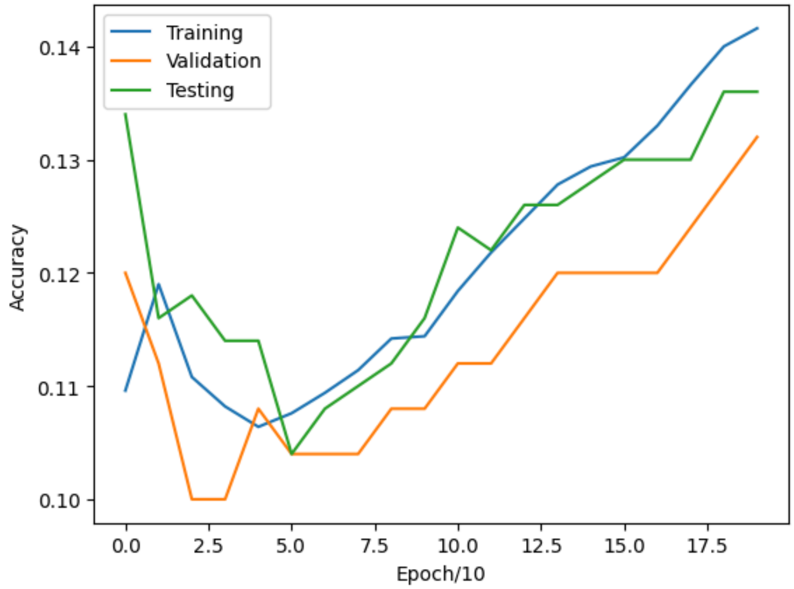


Try different learning rates and plot graphs for all (20%)

In [9]:
```
# Initialize the best values
best_weights = weights
best_learning_rate = learning_rate
best_weight_decay = weight_decay

# TODO
# Repeat the above training and evaluation steps for the following learning rates and plot graphs
# You need to try 3 learning rates and submit all 3 graphs along with this notebook pdf to show your learning rate experiments
learning_rates = [1e-3, 1e-4, 1e-5]
weight_decay = 0.0  # No regularization for now

# FEEL FREE TO EXPERIMENT WITH OTHER VALUES. REPORT OTHER VALUES IF THEY ACHIEVE A BETTER PERFORMANCE

# for lr in learning_rates: Train the classifier and plot data
# Step 1. train_accu, val_accu, test_accu = train(lr, weight_decay)
# Step 2. plot_accuracies(train_accu, val_accu, test_accu)

for learning_rate in learning_rates:
    train_accu, val_accu, test_accu, weights = train(learning_rate, weight_decay)
    plot_accuracies(train_accu, val_accu, test_accu)
```







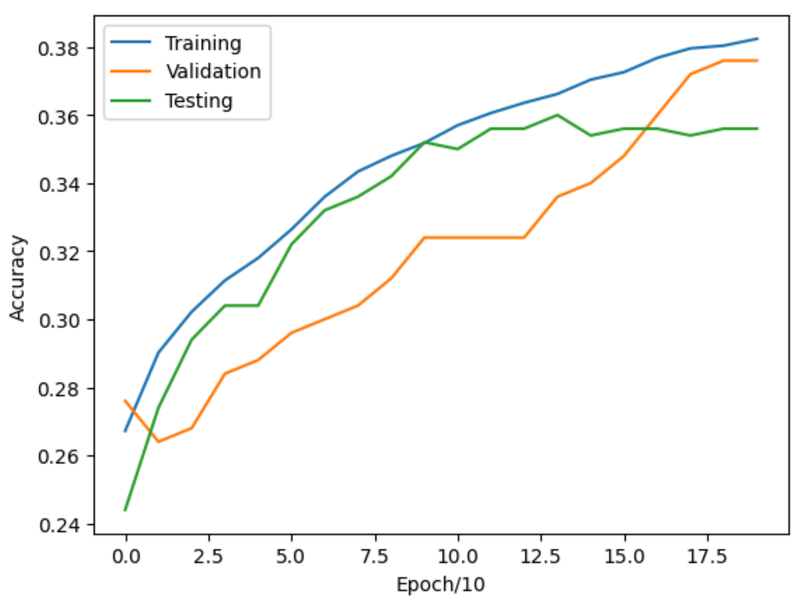
Inline Question 1.

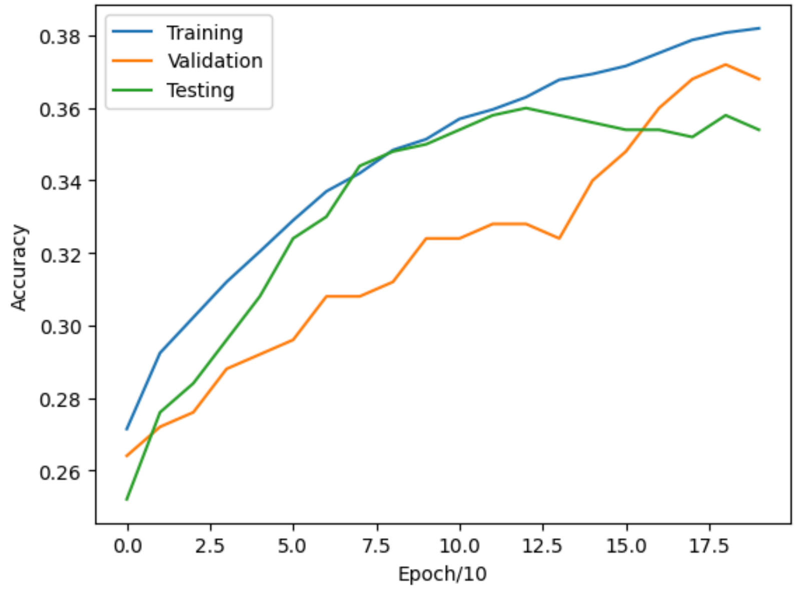
Which one of these learning rates (best_lr) would you pick to train your model? Please Explain why.

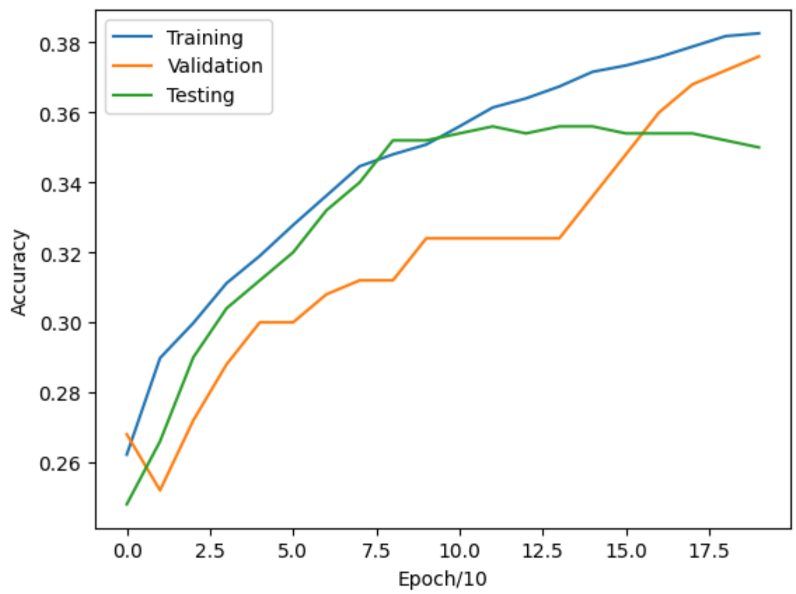
Your Answer:

Regularization: Try different weight decay and plot graphs for all (20%)

In [10]:
```
# Initialize a non-zero weight_decay (Regularization constant) term and repeat the training and evaluation
# Use the best learning rate as obtained from the above exercise, best_lr

# You need to try 3 learning rates and submit all 3 graphs along with this notebook pdf to show your weight decay experiments
weight_decays = [1e-1, 1e-2, 1e-3]
best_lr = 1e-3

# FEEL FREE TO EXPERIMENT WITH OTHER VALUES. REPORT OTHER VALUES IF THEY ACHIEVE A BETTER PERFORMANCE

# for weight_decay in weight_decays: Train the classifier and plot data
# Step 1. train_accu, val_accu, test_accu = train(best_lr, weight_decay)
# Step 2. plot_accuracies(train_accu, val_accu, test_accu)

for weight_decay in weight_decays:
    train_accu, val_accu, test_accu, weights = train(best_lr, weight_decay)
    plot_accuracies(train_accu, val_accu, test_accu)
```







Inline Question 2.

Discuss underfitting and overfitting as observed in the 3 graphs obtained by changing the regularization. Which weight_decay term gave you the best classifier performance? HINT: Do not just think in terms of best training set performance, keep in mind that the real utility of a machine learning model is when it performs well on data it has never seen before

Your Answer:

Visualize the filters (10%)

In [12]:
```
# These visualizations will only somewhat make sense if your learning rate and weight_decay parameters were
# properly chosen in the model. Do your best.

# TODO: Run this cell and Show filter visualizations for the best set of weights you obtain.
# Report the 2 hyperparameters you used to obtain the best model.
best_learning_rate = 1e-3
best_weight_decay = 1e-1
_, _, _, best_weights = train(best_lr, best_weight_decay)
# NOTE: You need to set `best_learning_rate` and `best_weight_decay` to the values that gave the highest accuracy

print("Best LR:", best_learning_rate)
print("Best Weight Decay:", best_weight_decay)

# NOTE: You need to set `best_weights` to the weights with the highest accuracy
```

```
w = best_weights[:, :-1]
w = w.reshape(10, 3, 32, 32).transpose(0, 2, 3, 1)

w_min, w_max = np.min(w), np.max(w)

fig = plt.figure(figsize=(20, 20))
classes = [
    "plane",
    "car",
    "bird",
    "cat",
    "deer",
    "dog",
    "frog",
    "horse",
    "ship",
    "truck",
]
for i in range(10):
    fig.add_subplot(2, 5, i + 1)

    # Rescale the weights to be between 0 and 255
    wimg = 255.0 * (w[i, :, :, :].squeeze() - w_min) / (w_max - w_min)
    # plt.imshow(wimg.astype('uint8'))
    plt.imshow(wimg.astype(int))
    plt.axis("off")
    plt.title(classes[i])
plt.show()
```

```
Best LR: 0.001
Best Weight Decay: 0.1
```