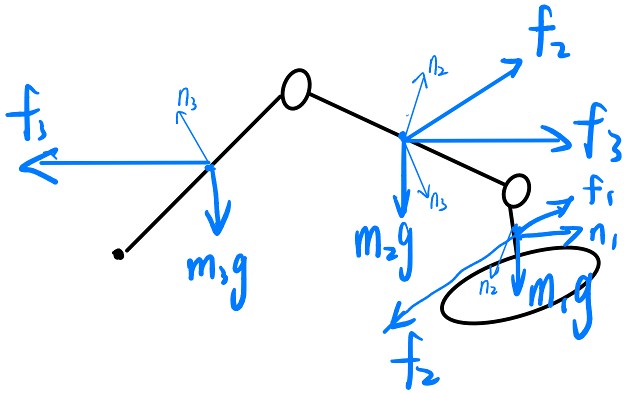
1.



2. $m_i = \rho \pi r_i^2 l_i \quad (i = 1, 2, 3)$

(a) $$I_1^c = \begin{bmatrix} \frac{1}{12} m_1 (l_1^2 + 3r_1^2) & 0 & 0 \\ 0 & \frac{1}{12} m_1 (l_1^2 + 3r_1^2) & 0 \\ 0 & 0 & \frac{1}{2} m_1 r_1^2 \end{bmatrix}$$

$$I_i^c = \begin{bmatrix} \frac{1}{2} m_i r_i^2 & 0 & 0 \\ 0 & \frac{1}{12} m_i (l_i^2 + 3r_i^2) & 0 \\ 0 & 0 & \frac{1}{12} m_i (l_i^2 + 3r_i^2) \end{bmatrix} \quad (i = 2, 3)$$

(b) COM of each link

$$r_{c_1} = \begin{bmatrix} 0 \\ 0 \\ \frac{l_1}{2} \end{bmatrix} \quad r_{c_2} = \begin{bmatrix} \frac{l_2}{2} \\ 0 \\ 0 \end{bmatrix} \quad r_{c_3} = \begin{bmatrix} \frac{l_3}{2} \\ 0 \\ 0 \end{bmatrix}$$

Joint coordinates $\theta \equiv q = [q_1, q_2, q_3]^T$, rates $\dot{\theta} \equiv \dot{q}$

Gravity: ${}^0g = [0, 0, -g]^T$

rotate to base when used in energies: $I_i^{C(0)}(q) = R_{0i}(q) I_i^C R_{0i}^T(q)$

For each link i, $k_i = \frac{1}{2} m_i {}^0v_{ci}^T \, {}^0v_{ci} + \frac{1}{2} {}^0w_i^T I_i^{C(0)} {}^0w_i$

where ${}^0w_i = \sum_{j=1}^{i} \dot{q}_j \, {}^0\hat{z}_j$ and ${}^0v_{c_i} = \sum_{j=1}^{i} \dot{q}_j \, {}^0\hat{z}_j \times ({}^0p_{c_i} - {}^0p_{0_j})$

with Jacobians: ${}^0w_i = J_{w,i}(q)\dot{q}$ ${}^0v_{c_i} = J_{v,i}(q)\dot{q}$

$$k_i = \frac{1}{2} \dot{q}^T (m_i J_{v,i}^T J_{v,i} + J_{w,i}^T I_i^{C(0)} J_{w,i}) \dot{q}$$

Choose the zero reference so $U_{ref_i} = 0$, then for each link

$$U_i = -m_i \, ({}^0 g)^T \, {}^0 P_{C_i}(q) = m_i g \cdot (\text{vertical coordinate of } {}^0 P_{C_i})$$

(c) $$J_{w,i}^{(j)} = \begin{cases} \hat{z}_j, & j \le i \\ 0, & j > i \end{cases} , \quad J_{v,i}^{(j)} = \begin{cases} \hat{z}_j \times (p_{C_i} - {}^0 p_j) & j \le i \\ 0 & j > i \end{cases}$$

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = U$$

$$D(q) = \sum_{i=1}^{3} [m_i J_{v,i}^T J_{v,i} + J_{w,i}^T I_i^{C(0)} J_{w,i}]$$

$$T_{kjl}(q) = \frac{1}{2} \left(\frac{\partial D_{kj}}{\partial q_l} + \frac{\partial D_{kl}}{\partial q_j} + \frac{\partial D_{jl}}{\partial q_k} \right) \quad C_{kj}(q,\dot{q}) = \sum_{l=1}^{3} T_{kjl}(q)\dot{q}_l$$

$$V(q) = \sum_{i=1}^{3} m_i g z(P_{C_i}(q)) \quad , \quad G(q) = \frac{\partial V}{\partial q}$$

$$U = \tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix}$$