CSI 2110 Tutorial (Section A)

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Office Hour: Fri 13:00-14:00
Place: STE 5000G

Review: Shortest Path vs Minimum Spanning Tree

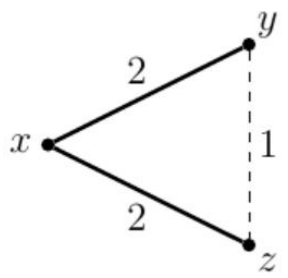
Common (Spanning Tree):

- A tree structure in a graph G
- Covers all the vertices of G
- There may contain multiple shortest path/MST in a graph G

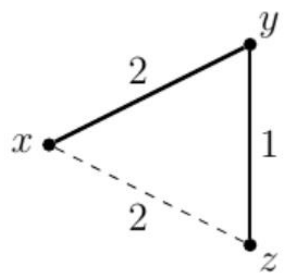
Different:

- Shortest Path: the shortest path is only guaranteed between the start vertex and the rest. The sum of weights of the shortest path is not always minimum

- MST: the sum of weights is guaranteed as the minimum.



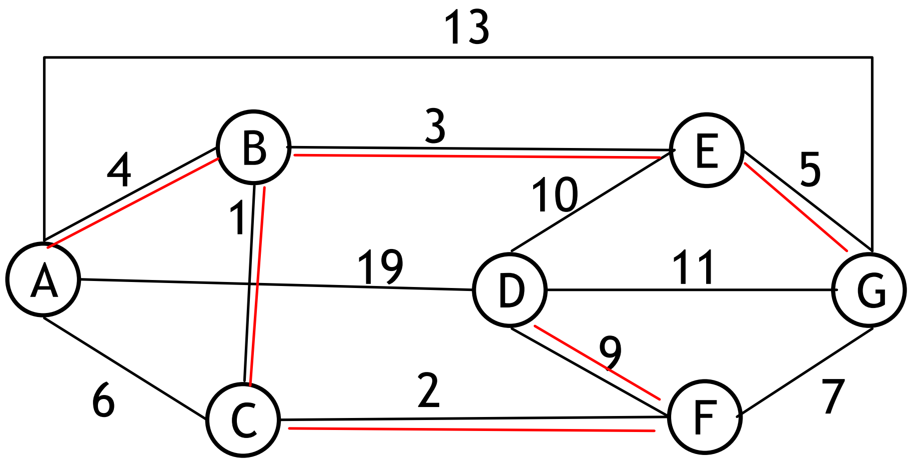
Shortest Path
starts at x



MST

Exercise

1. For the graph given in the figure below, use the Dijkstra algorithm to find the shortest path spanning tree of the graph starting from node A.

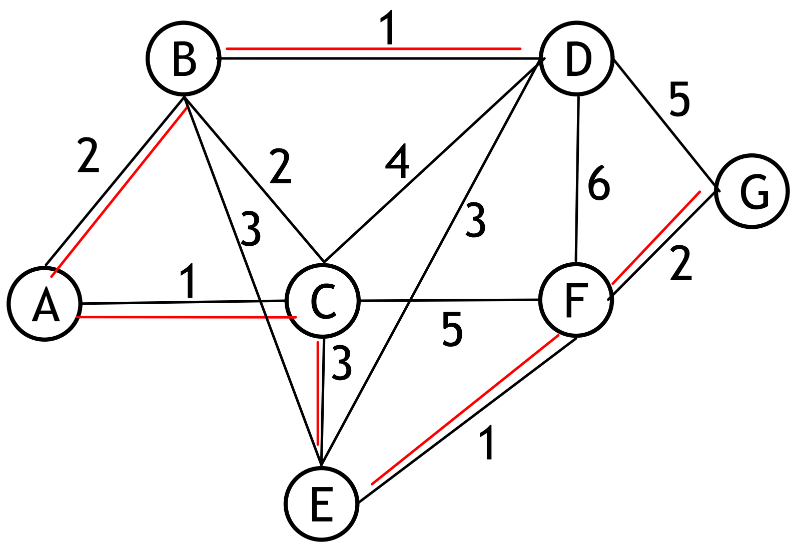


Fill the table below to keep track of the changes of the distance labels after including each new node to the cloud. The first line of the chart is already filled with the initial distance labels.

New Vertex	A	B	C	D	E	F	G	New Edge
A	0	4	6	19	∞	∞	13	-
B	0	4	5	19	7	∞	13	AB
C	0	4	5	19	7	7	13	BC
E	0	4	5	17	7	7	12	BE
F	0	4	5	16	7	7	12	CF
G	0	4	5	16	7	7	12	EG
D	0	4	5	16	7	7	12	DF

Exercise

2. Use Dijkstra's algorithm to obtain the shortest path tree starting from vertex A. Fill the following table to show the updates of the distances associated to the vertices along its execution. You may add rows to the table.



New Vertex	A	B	C	D	E	F	G	New Edge
A	0	2	1	∞	∞	∞	∞	-
C	0	2	1	5	4	6	∞	AC
B	0	2	1	3	4	6	∞	AB
D	0	2	1	3	4	6	8	BD
E	0	2	1	3	4	5	8	CE
F	0	2	1	3	4	5	7	EF
D	0	2	1	3	4	5	7	FG

Exercise

3. There are eight small islands in a lake, and the state wants to build seven bridges to connect them so that each island can be reached from any other one via one or more bridges. The cost of constructing a bridge is proportional to its length. The distances between pairs of islands are given in the following table.

	1	2	3	4	5	6	7	8
1	-	240	210	340	280	200	345	120
2	-	-	265	175	215	180	185	155
3	-	-	-	260	115	350	435	195
4	-	-	-	-	160	330	295	230
5	-	-	-	-	-	360	400	170
6	-	-	-	-	-	-	175	205
7	-	-	-	-	-	-	-	305
8	-	-	-	-	-	-	-	-

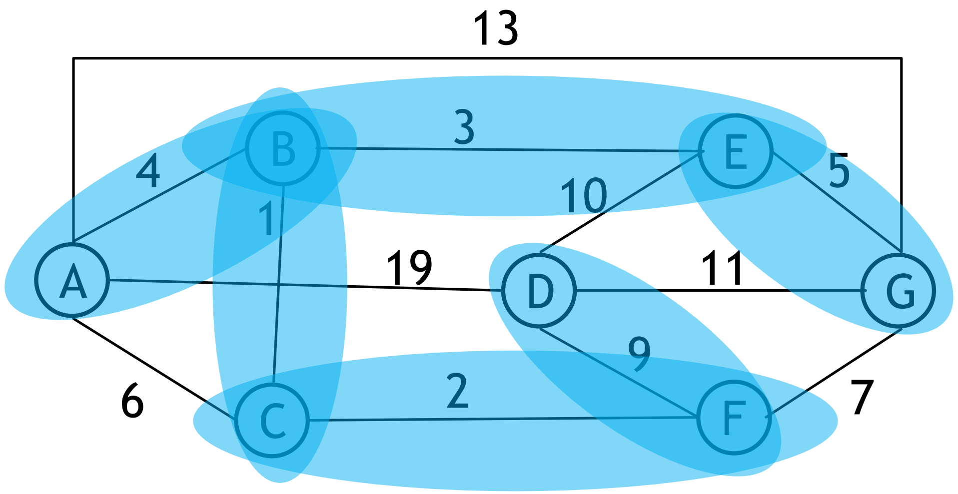
Show the steps of Prim-Jarnik's for finding a MST completing the following table:

Vertex added to the cloud	1	2	3	4	5	6	7	8	Edge added to the MST	edge cost
initializing	0	∞	∞	∞	∞	∞	∞	∞		-
starting from 1	(0)	240	210	340	280	200	340	120	-	0
8	(0)	155	195	230	170	200	305	(120)	(1,8)	120
2	(0)	(155)	195	175	170	180	185	(120)	(2,8)	275
5	(0)	(155)	115	160	(170)	180	185	(120)	(5,8)	445
3	(0)	(155)	(115)	160	(170)	180	185	(120)	(3,5)	560
4	(0)	(155)	(115)	(160)	(170)	180	185	(120)	(4,5)	720
6	(0)	(155)	(115)	(160)	(170)	(180)	175	(120)	(2,6)	900
7	(0)	(155)	(115)	(160)	(170)	(180)	(175)	(120)	(6,7)	1075

Exercise

4. For the graph below, use Kruskal’s algorithm for finding the minimum spanning tree of the connected graph:

1) Write the list of edges in the order they are chosen to be part of the spanning tree.



1) sort the edges from the small to large:
BC, CF, BE, AB, EG, AC, FG, DF, DE, DG, AG, AD

2) Choose the edge with two conditions:
- smallest in the list
- joins 2 distinct clusters

3) Remove the chosen edges or edges in the same cluster from the list

4) Repeat step 2) to 3) until list is empty

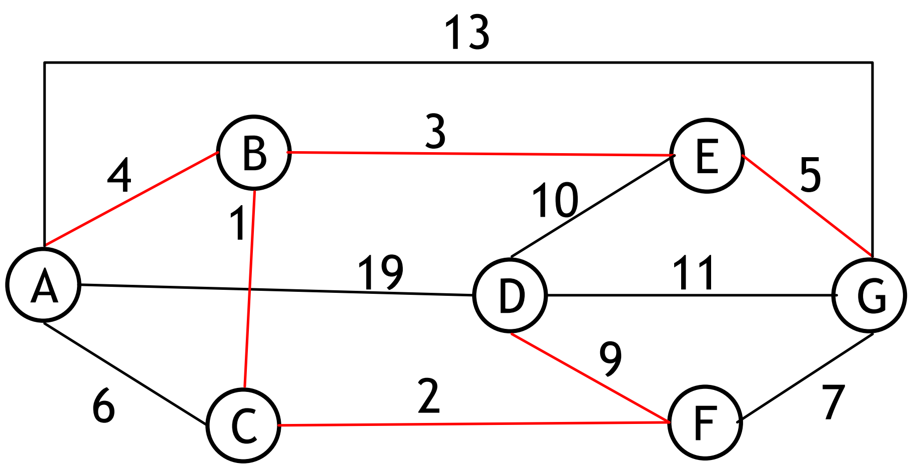
Solution: BC, CF, BE, AB, EG, DF

Exercise

4. For the graph below, use Kruskal’s algorithm for finding the minimum spanning tree of the connected graph:

2) What is the total weight of the spanning tree?

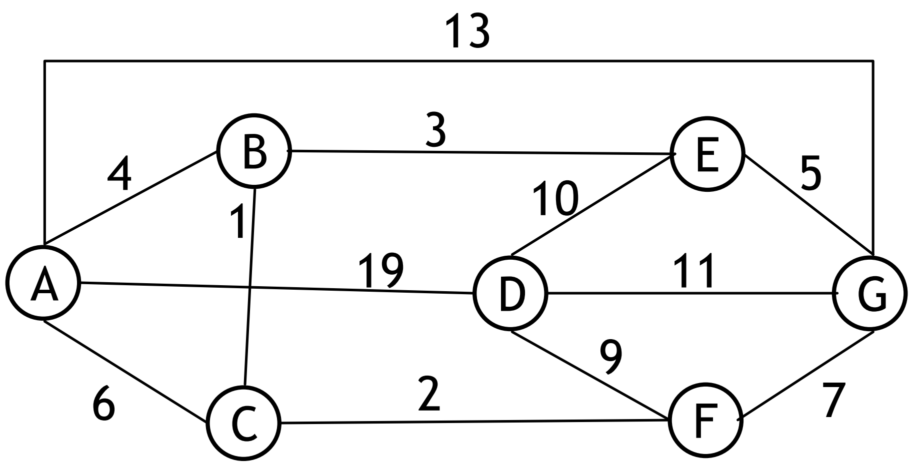
Solution: 24



Exercise

4. For the graph below, use Kruskal's algorithm for finding the minimum spanning tree of the connected graph:

3) What is the complexity of the algorithm as a function of n and m?

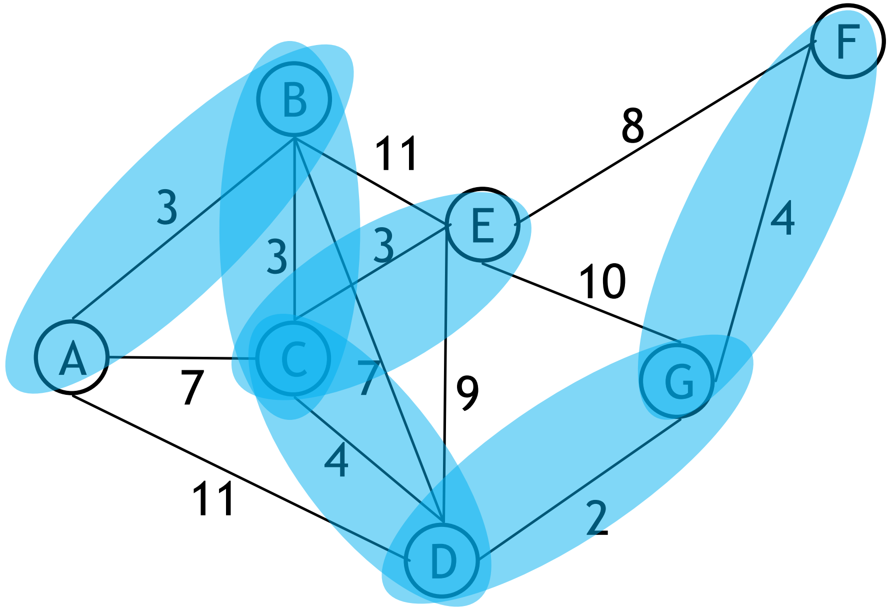


$O(m log(n))$ if not consider priority queue
Otherwise $O((m+n)log(n))$

Exercise

5. Use Kruskal's algorithm in order to find the Minimum Spanning Tree of the graph below.

1) Write the list of edges in the order they are chosen to be part of the spanning tree.



1) sort the edges from the small to large:
DG, AB, BC, CE, CD, FG, AC, BD, EF, DE, EG, AD, BE

2) Choose the edge with two conditions:
- smallest in the list
- joins 2 distinct clusters

3) Remove the chosen edges or edges in the same cluster from the list

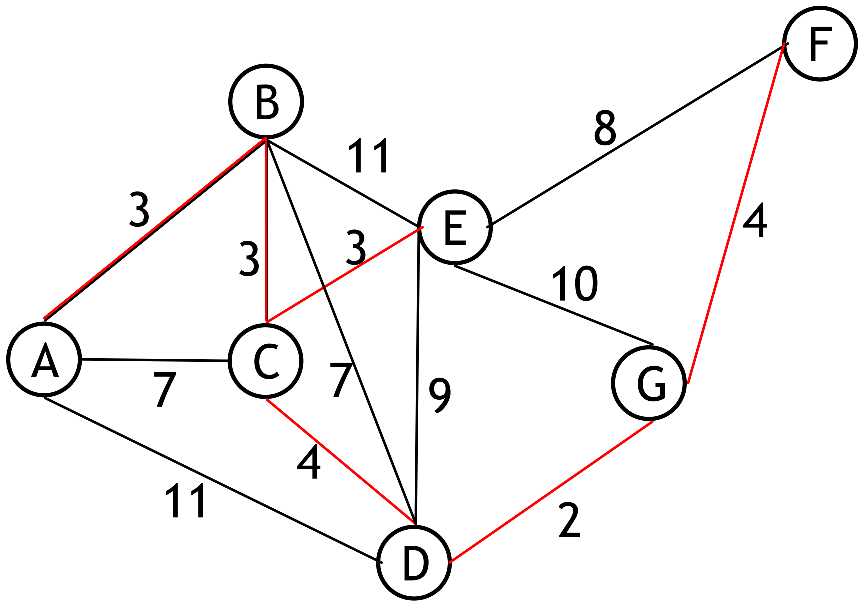
4) Repeat step 2) to 3) until list is empty

Solution: DG, AB, BC, CE, CD, FG

Exercise

5. Use Kruskal's algorithm in order to find the Minimum Spanning Tree of the graph below.

2) What is the total weight of this spanning tree.



Solution: 19