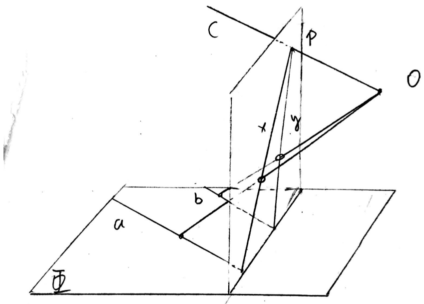
Homework #1

Problem 1.

Vanishing Line:

Geometrically Prove:



Consider we have two parallel line ***a, b*** in plane **Φ**. And there is a line ***c*** passing through the pinhole **O** which is parallel to line ***a, b.*** Line ***c*** intersects with the image plane **Π** at point ***p***. Also line ***a*** and line ***c*** form a plane, **A**, and line ***b*** and line ***c*** form a plane, **B**. The intersection line ***x*** between plane **A** and image plane **Π** is the projection of line ***a*** on image plane. The intersection line ***y*** between plane **B** and image plane **Π** is the projection of line ***b*** on image plane. Line ***x*** and ***y*** converge at point ***p***, meanwhile ***p*** is on line ***c***. Consider we form a line ***h*** passing through p on plane **Π** which is parallel with the intersection line ***n*** of plane **Φ** and plane **Π**. And line ***c*** is parallel with line ***b***, and ***b*** is not parallel with ***n***, so the plane formed by ***h*** and **O** is parallel with plane **Φ**. In this case, we can say that the vanishing point ***p*** lies on the intersection line of image plane and the parallel plane of **Φ** passing through the pinhole. The intersection line is the vanishing line. Also, we can do the same thing with other pair of parallel lines in plane **Φ**. Since there is only one plane passing through a point which is parallel to the same plane, every ***h*** is a same line. So we can conclude that all the parallel line on a plane ***k*** have a same vanishing line which is the intersection line of image plane and the plane passing through the pinhole and parallel with plane ***k***.

Algebraically Prove:

Φ : $y = c$.

Π : $z = n$

a line on plane Φ $ax + bz = d$

From perspective projection equation.

$$\begin{cases} x' = n\frac{x}{z} \\ y' = n\frac{y}{z} \end{cases}$$

where x', y' is the location of point on image plane Π.

plug in the variables:

$$\begin{cases} x' = n\frac{d - bz}{az} \\ y' = n\frac{c}{z}. \end{cases}$$

when $z \to \infty$, projection line will vanish.

$$\lim_{z\to\infty} x' = \lim_{z\to\infty} n\frac{d-bz}{az} = -n\frac{b}{a}$$

$$\lim_{z\to\infty} y' = \lim_{z\to\infty} n\frac{c}{z} = 0$$

$\therefore$ vanishing point $(-n\frac{b}{a}, 0)$

$\therefore$ It's on plane $y=0$, which is the parllel plane through O.

Vanishing point:

We can use perspectie projection function to get p_1, q_1, p_2 and q_2 in homogeneous. cordinate from P_1. Q_1 P_2 and Q_2.

Then the line p_1q_1, $line_1 = p_1 \times q_1$

p_2q_2, $line_2 = p_2 \times q_2$

vanishing point $h = line_1 \times line_2$.

$\therefore \quad h = (p_1 \times q_1) \times (p_2 \times q_2)$

Problem 2

1.

$$\begin{pmatrix} I_1(p) \\ I_2(p) \\ \vdots \\ I_n(p) \end{pmatrix} = \begin{pmatrix} V_1^T \\ V_2^T \\ \vdots \\ V_n^T \end{pmatrix} S(p)$$

in this case $I_i(p) = V_i^T \cdot S(p) = S^T(p)\, V_i$

$I_i(p)$ is the image Intensity at p with light source V_i

V_i is the light source i.

$S^T(p)$ is the transpose of the normal vector at p.

2. Because there are three veriables in $S(p)$ to be determined, we need 3 function to determine each ~~variab~~ variables. So we need at least 3 light source.

3.

$$I = \begin{pmatrix} I(p_1) \\ I(p_2) \\ \vdots \\ I(p_n) \end{pmatrix} = \begin{pmatrix} S^T(p_1) \\ S^T(p_2) \\ \vdots \\ S^T(p_n) \end{pmatrix} \cdot V \qquad S = \begin{pmatrix} S^T(p_1) \\ S^T(p_2) \\ \vdots \\ S^T(p_n) \end{pmatrix}$$

$\because$ There are at least 3 not coplane ~~noncoplanar~~ $S(p_i)$

$\therefore$ rank$(S) = 3$

$\therefore$ There are at least 3 $S^T(p_i)$ that are linearly independent.

So S is a linear basis.

And V can serve as a coeffient for 3 independent $S^T(p)$

$\therefore$ S is a linear basis for all I.

4.

a. General surface have some mirror-like reflections

b. General surface will absorb some light intensity.

c. The reflection brightness from different angle will be different.