Efficient Inference in Bayesian Networks

Exercise T12.1: Junction trees (tutorial)

(a) What are *cliques* and *separators*?

(b) What is a *decomposable graph*?

(c) What is the *running intersection property*?

(d) Generate a junction tree from a directed acyclic graph (DAG).

(e) How is evidence introduced to the junction tree?

Exercise T12.2: Message passing (tutorial)

(a) What is a tree and what is a bipartite graph?

(b) How can a conditional marginal be expressed by a bipartite tree?

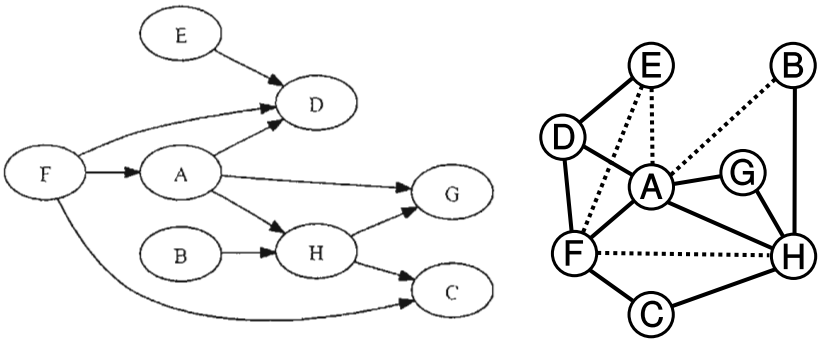
(c) Explain at an example (Lecture slides 3.2) how to perform marginalization.

(d) How can all marginals be computed simultaneously by three message passes?

(e) Formulate the *sum-product algorithm* for message passing.

Exercise H12.1: Construct a junction tree (homework, 4 points)

Given is a DAG (below left) and the corresponding moral graph (below right). The dotted edges in the latter are added during moralization.



(a) (1 point) Determine the cliques of the moral graph.

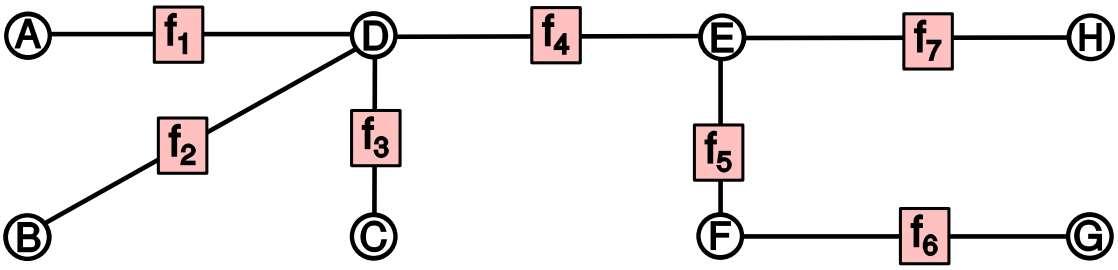
(b) (1 point) Draw the corresponding bipartite graph of cliques and separators.

(c) (1 point) Construct one possible junction tree from the bipartite graph.

(d) (1 point) Does the *running intersection property* hold for your tree?

Exercise H12.2: Message Passing (homework, 4 points)

Given is the following junction tree:



(a) (1 point) Describe the order in which messages are generated in the "request", the "collect" and the "distribute" pass, starting at node B as the root node.

(b) (1 point) Write out the computation performed for the message $\mu_{f_4 \to D}(D)$, e.g. $\mu_{f_3 \to D}(D) = \sum_C f_3(C, D)$.

(c) (1 point) Write out the message $\mu_{f_4 \to D}(D)$ as in (b), after the evidence $F = f^*$ and $H = h^*$ has been observed.

(d) (1 point) Give respectively short expressions for evaluating of the following (conditional marginal) distributions.

(i) $P(D|F = f^*, H = h^*)$

(ii) $P(A|F = f^*, H = h^*)$

Pay attention to the normalization coefficient. Note that it can be assumed that all necessary messages have been passed, i.e., $\mu_{f_j \to X}(X)$ is available for all cliques j and respective variables X.

Total 8 points.