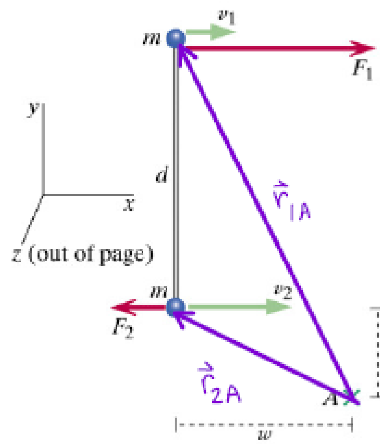
Physics 2211 GPS Week 14

Problem #1

In the figure two small objects each of mass $m = 0.235$ kg are connected by a lightweight rod of length $d = 1.20$ m. At a particular instant they have speeds $v_1 = 25$ m/s and $v_2 = 58$ m/s and are subjected to external forces $F_1 = 41$ N and $F_2 = 16$ N. A point is located distances $w = 0.80$ m and $h = 0.32$ m from the bottom object. No other external forces are acting on this system.



(a) What is the velocity of the center of mass?

$$\vec{P}_{total} = M_{total}\vec{V}_{cm} \Longrightarrow \vec{V}_{cm} = \frac{\vec{P}_{total}}{M_{total}} = \frac{\vec{p}_1 + \vec{p}_2}{m_1 + m_2} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} =$$

$$= \frac{(0.235)(25)\,\hat{x} + (0.235)(58)\,\hat{x}}{0.235 + 0.235} = \frac{\cancel{0.235}\,(25+58)\,\hat{x}}{2\,(\cancel{0.235})} =$$

$$= \boxed{41.5 \text{ m/s } \hat{x}}$$

(b) What is the total angular momentum of the system relative to point A?

$$\vec{L}_A = \vec{L}_{1A} + \vec{L}_{2A} = (\vec{r}_{1A} \times \vec{p}_1) + (\vec{r}_{2A} \times \vec{p}_2) = (h+d)(m_1 v_1)(-\hat{z}) + (h)(m_2 v_2)(-\hat{z}) =$$

$$= (-\hat{z})(m)\left[(h+d)v_1 + hv_2\right] = (-\hat{z})(0.235)\left[(0.32+1.20)(25) + (0.32)(58)\right] =$$

$$= (-\hat{z})(0.235)(38 + 18.56) = \boxed{13.29 \text{ kg m}^2\text{/s } (-\hat{z})}$$

(c) What is the rotational angular momentum of the system?

$$\vec{L}_{total} = \vec{L}_{trans} + \vec{L}_{rot} \Rightarrow \vec{L}_{rot} = \vec{L}_{total} - \vec{L}_{trans}$$

$$\checkmark\ \vec{L}_{trans} = \vec{r}_{cm} \times \vec{p}_{total} = \left(h + \frac{d}{2}\right)(\hat{y}) \times \left(M_{total}\vec{v}_{cm}\right) =$$

$$= \left(0.32 + \frac{1.2}{2}\right)(0.235 + 0.235)\,\hat{y} \times (41.5)\,\hat{x} = 17.9\ \text{kgm}^2/\text{s}\ (-\hat{z})$$

$$\Rightarrow \vec{L}_{rot} = 13.29\,(-\hat{z}) - 17.9\,(-\hat{z}) = (-13.29 - -17.9)\,\hat{z} =$$

$$= (-13.29 + 17.9)\,\hat{z} = \boxed{4.61\ \text{kgm}^2/\text{s}\ (\hat{z})}$$

(d) After a short time interval $\Delta t = 0.035$ s, determine the total (linear) momentum of the system?

$$\vec{P}_f = \vec{P}_i + \vec{F}_{net}\,\Delta t = (2)(0.235)(41.5)\,\hat{x} + (41 - 16)(\hat{x})(0.035) =$$

$$= \boxed{20.38\ \text{kgm/s}\ \hat{x}}$$

(e) Calculate the new rotational angular momentum of the system?

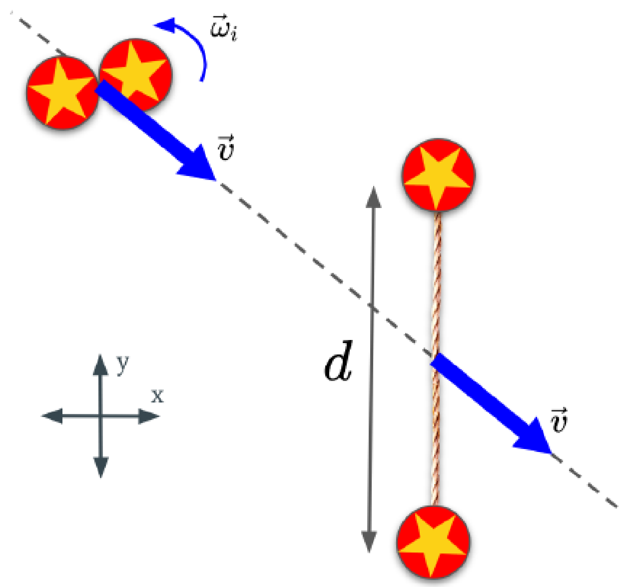
$$\vec{\tau}_{net} = \vec{\tau}_1 + \vec{\tau}_2 = (\vec{r}_1 \times \vec{F}_1) + (\vec{r}_2 \times \vec{F}_2) = \frac{d}{2}F_1\,(-\hat{z}) + \frac{d}{2}F_2\,(-\hat{z}) =$$

$$= (-\hat{z})\,\frac{d}{2}(F_1 + F_2) = (-\hat{z})\left(\frac{1.2}{2}\right)(41 + 16) = 34.2\ \text{Nm}\ (-\hat{z})$$

$$\vec{L}_{rot,f} = \vec{L}_{rot,i} + \vec{\tau}_{net}\,\Delta t = 4.61\,(\hat{z}) + (34.2)(0.035)(-\hat{z}) = \boxed{3.4\ \text{kgm}^2/\text{s}\ (\hat{z})}$$

Problem #2

A yoyo can be approximated as a solid cylinder of mass m, radius R and thickness d. Two identical such yoyos have their strings tied together and are wound so that the two yoyos are touching each other. These stuck together yoyos are ejected into deep space far from any other objects. Shortly after being ejected, the center of mass of the yoyos have an initial velocity $\vec{v}$ as indicated in the diagram. At this instant, the stuck together yoyos are rotating about the center of mass counterclockwise with an angular speed $|\vec{\omega}_i|$. As the yoyos fly through space the strings unwind so that at some later time all of the string has unwound from each yoyo. At this time, the velocity of the center of mass is $\vec{v}$ and the distance between the center of the yoyos is d. Determine the unknown angular velocity (magnitude and direction) of the center of mass for the tied together yoyos. You can neglect the mass of the string and you can assume that the yoyos are tied to the string so that the string is not slipping on the axle of the yoyo.



Key insight: the angular frequency that the yoyo rotates about its own enter of mass is the same as the angular frequency that the center of mass of the yoyo rotates about the center of mass of the system.
The net external torque is zero, so the angular momentum is conserved,

$$\Delta\vec{L} = 0$$

$$\begin{aligned}\vec{L}_i &= \vec{L}_{cm} + \vec{L}_{rot,i} \\ &= \vec{L}_{cm} + 2(\vec{L}_{yo,cm} + \vec{L}_{yo,rot}),\end{aligned}$$

where $\vec{L}_{cm}$ is the angular momentum of the system's center of mass about some point (it doesn't matter which), $\vec{L}_{yo,cm}$ is the angular momentum of the center of mass of the yoyo about the system's center of mass, and $\vec{L}_{yo,rot}$ is the angular momentum of the yoyo about its own center of mass.

$$\begin{aligned}\vec{L}_{yo,cm} &= \vec{r}_{yo,cm} \times \vec{p}_{yo,cm} \\ &= Rm\vec{\omega}_i R = mR^2\vec{\omega}_i\end{aligned}$$

$$\begin{aligned}\vec{L}_{yo,rot} &= I\vec{\omega} \\ &= \frac{1}{2}mR^2\vec{\omega}_i\end{aligned}$$

$$\vec{L}_i = \vec{L}_{cm} + 3mR^2\vec{\omega}_i$$

By the same logic for the final time,

$$\begin{aligned}\vec{L}_{yo,cm} &= \vec{r}_{yo,cm} \times \vec{p}_{yo,cm} \\ &= \frac{d}{2}m\vec{\omega}_f\frac{d}{2} = \frac{md^2}{4}\vec{\omega}_f\end{aligned}$$

$$\begin{aligned}\vec{L}_{yo,rot} &= I\vec{\omega} \\ &= \frac{1}{2}mR^2\vec{\omega}_f\end{aligned}$$

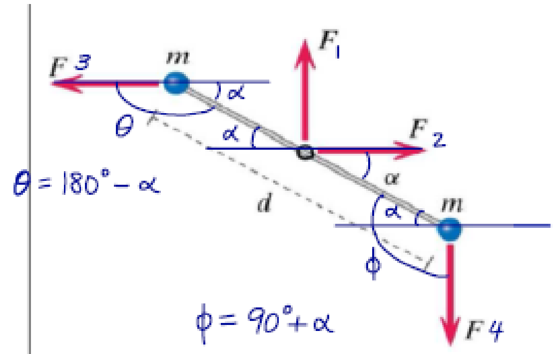
$$\vec{L}_f = \vec{L}_{cm} + \left(\frac{md^2}{2} + mR^2\right)\vec{\omega}_f$$

Equating the initial and final, the angular momentum of the system's center of mass cancels and therefore

$$\vec{\omega}_f = \frac{3R^2}{\frac{d^2}{2} + R^2}\vec{\omega}_i$$

Problem #3

A barbell is mounted on a nearly frictionless axle through its center of mass. The rod has negligible mass and a length d. Each ball has a mass m. At the instant shown, there are four forces of equal magnitude F applied to the system, with the directions indicated. At this instant, the angular velocity is ω_i, counterclockwise (positive), and the bar makes an angle α (which is less than 45 degrees) with the horizontal.



(a) Calculate the magnitude of the net torque on the barbell about the center of mass.

$$\vec{\tau}_1 = \vec{r}_1 \times \vec{F}_1 = 0 \quad \text{b/c } \vec{r}_1 = 0$$

$$\vec{\tau}_2 = \vec{r}_2 \times \vec{F}_2 = 0 \quad \text{b/c } \vec{r}_2 = 0$$

$$\vec{\tau}_3 = \vec{r}_3 \times \vec{F}_3 = r_3 F_3 \sin\theta \, (\hat{z}) = \frac{d}{2} F \sin(180° - \alpha) \, (\hat{z}) = \frac{dF}{2} \sin\alpha \, (\hat{z})$$

$$\vec{\tau}_4 = \vec{r}_4 \times \vec{F}_4 = r_4 F_4 \sin\phi \, (-\hat{z}) = \frac{d}{2} F \sin(90° + \alpha) \, (-\hat{z}) = \frac{dF}{2} \cos\alpha \, (-\hat{z})$$

$$\vec{\tau}_{Net} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + \vec{\tau}_4 = \frac{dF}{2} \sin\alpha \, (\hat{z}) + \frac{dF}{2} \cos\alpha \, (-\hat{z})$$

$$\Rightarrow \boxed{|\vec{\tau}_{Net}| = \frac{dF}{2} \left| \sin\alpha - \cos\alpha \right|}$$

Since $\alpha < 45°$, then $\cos\alpha > \sin\alpha$, which means $|\vec{\tau}_4| > |\vec{\tau}_3|$

$$\Rightarrow \boxed{\text{direction of } \vec{\tau}_{Net} \text{ is } (-\hat{z}), \text{ into the page}}$$

(b) Select the statement that accurately describes the situation in the figure:

A. α is less than 45 degrees, so the net torque on the system is in the clockwise direction. This means the torque is out of the page.

B. α is less than 45 degrees, so the net torque on the system is in the counterclockwise direction. This means the torque is into the page.

C. α is less than 45 degrees, so the net torque on the system is in the counterclockwise direction. This means the torque is out of the page.

D. α is less than 45 degrees, so the net torque on the system is in the clockwise direction. This means the torque is into the page.

(c) Determine the moment of inertia, about the center of mass, for the barbell.

$$I_{CM} = m_1 r_1^2 + m_2 r_2^2 = m\left(\frac{d}{2}\right)^2 + m\left(\frac{d}{2}\right)^2 = \frac{md^2}{4} + \frac{md^2}{4} =$$

$$= \frac{\cancel{2}md^2}{\cancel{4}_2} = \boxed{\frac{1}{2}md^2}$$