Quantum Teleportation

PyDay BCN

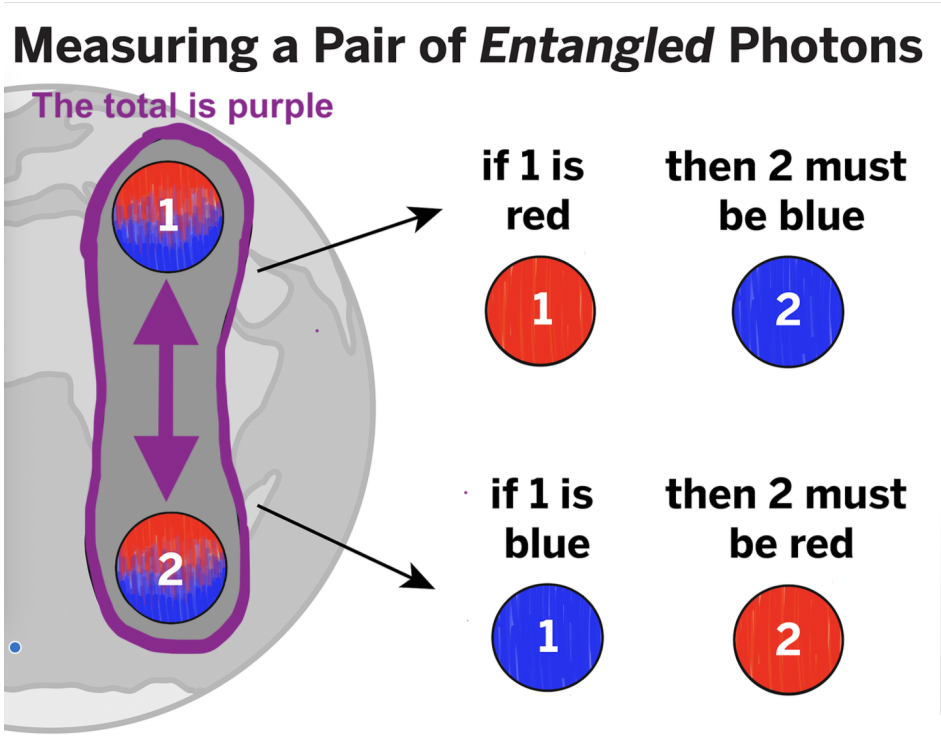
11 November 2023

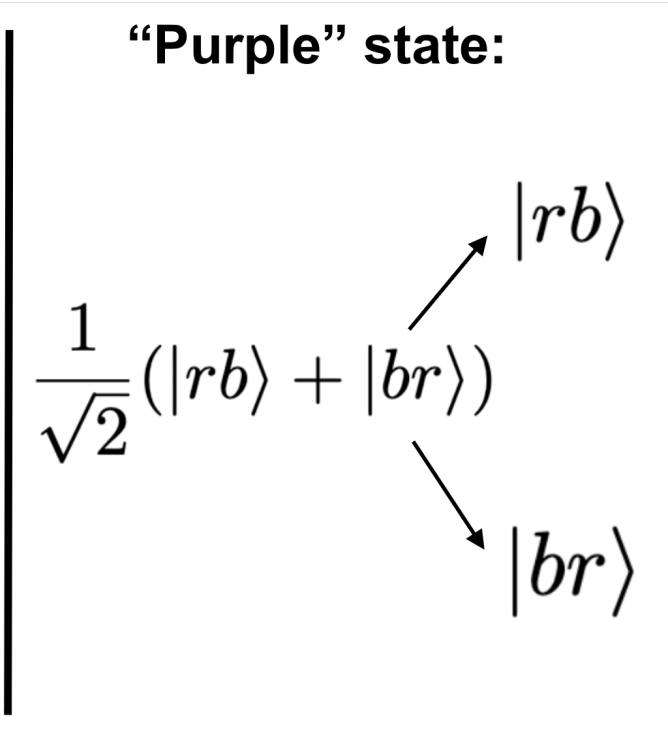
Outline

1. The concept of entanglement
2. The concept of quantum teleportation
3. Coding Teleportation and Quantum Networks

Table of Contents

Quantum entanglement





So, if we have the following state and we measure the 1 in the first qubit:

$$\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \xrightarrow{\text{q1} = 1} |q2 = 0\rangle$$

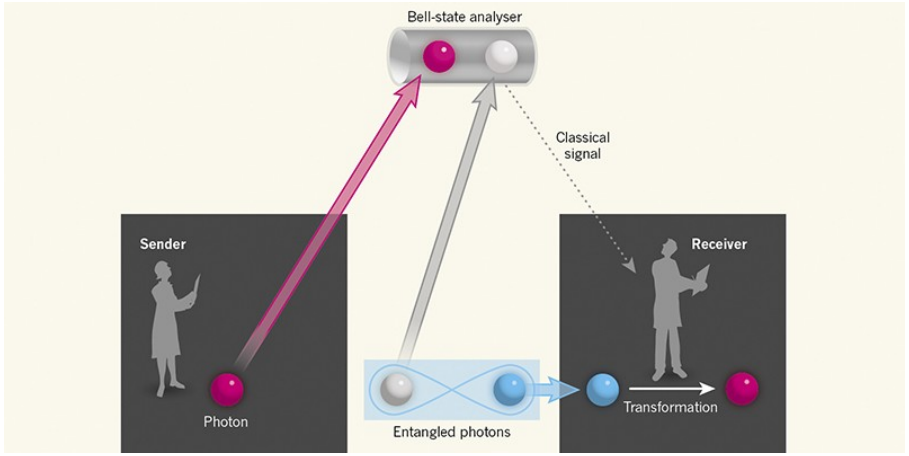
$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \xrightarrow{\text{q1} = 1} |q2 = ?\rangle$$

Table of Contents

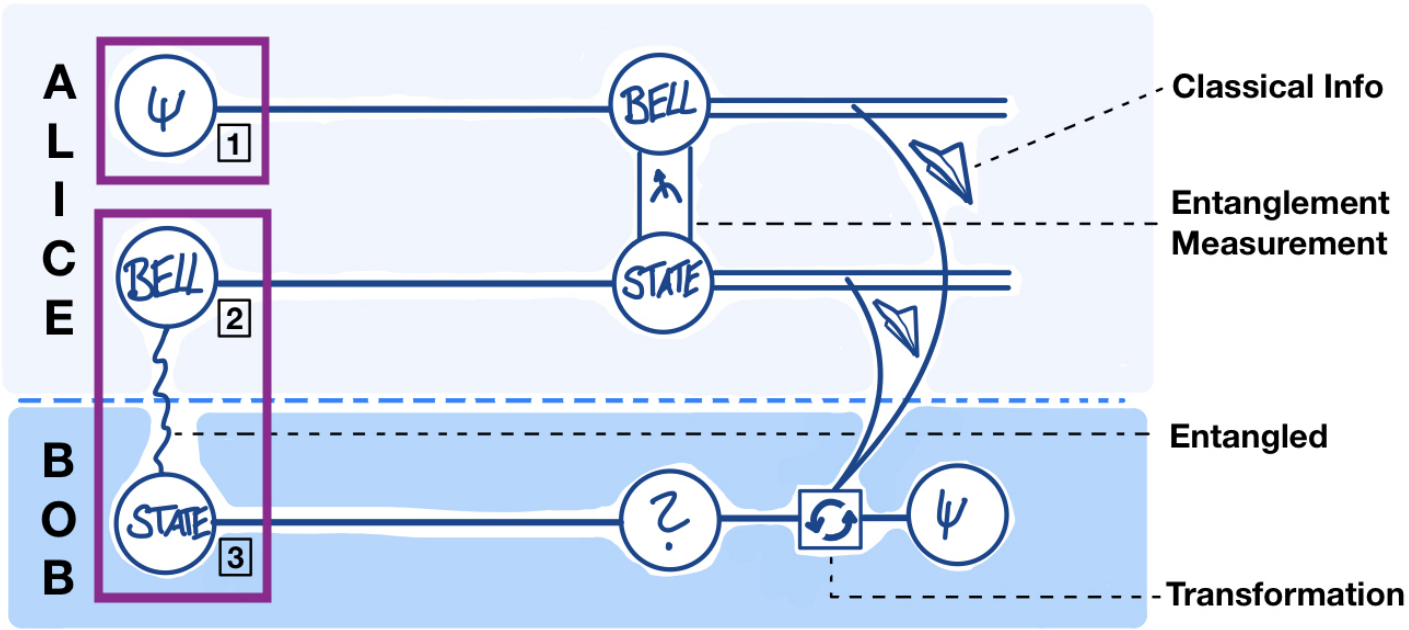
Introduction to Quantum Teleportation

Quantum teleportation:

- **IS NOT:** Instantaneous travel like depicted in films.
- **IS:** A way to transfer actual quantum states.



I. Initial set up for Quantum Teleportation



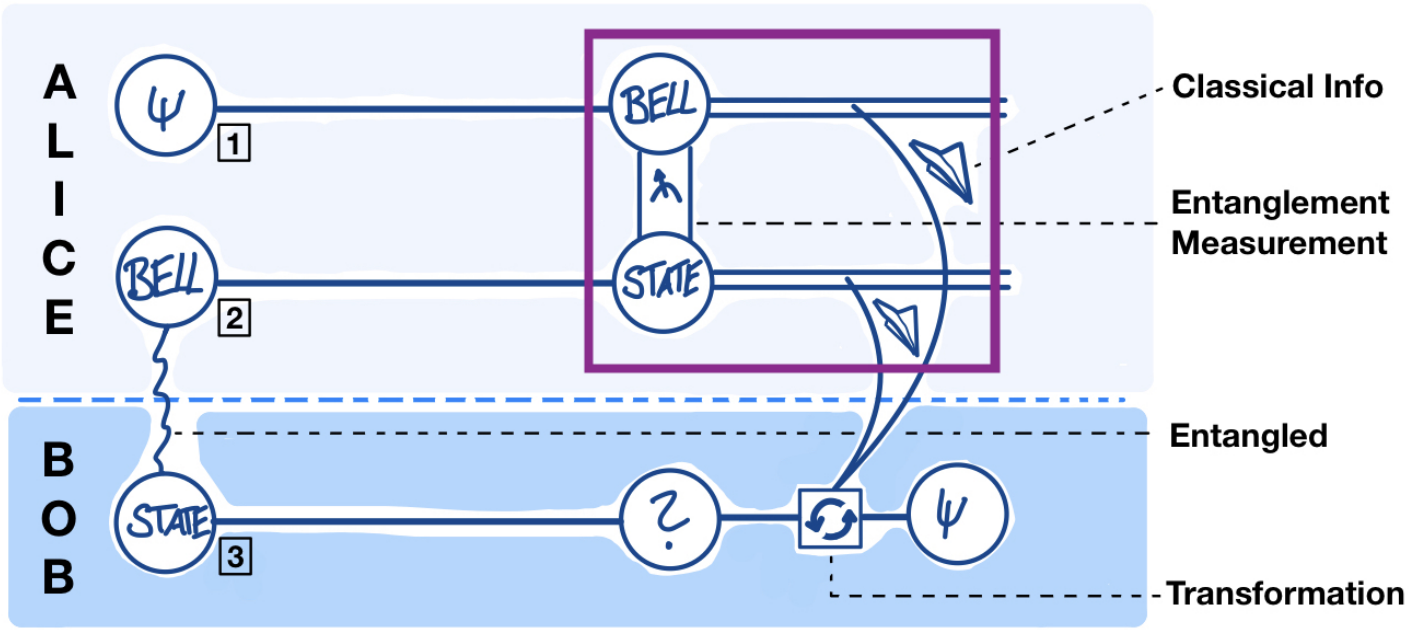
Alice has a **qubit 1** in an **UNKNOWN** state $|\psi\rangle_1$:

$$|\psi\rangle_1 = \alpha\,|0\rangle + \beta\,|1\rangle$$

Qubits **2 (Alice)** and **3 (Bob)** are **entangled**:

$$|\psi\rangle_{23} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

II. Bob states, depending on Alice's result

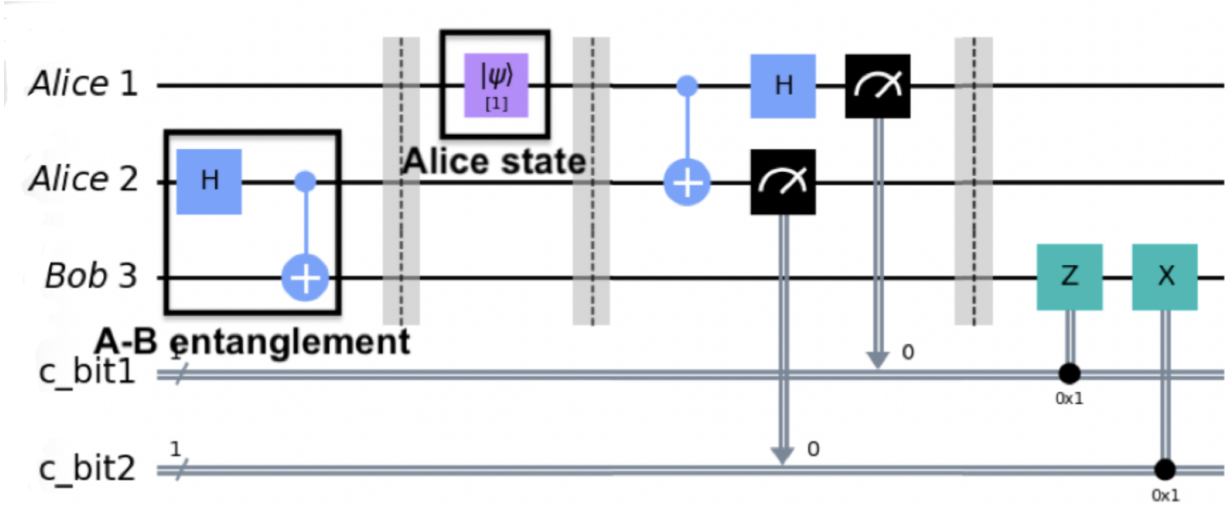


$$
\left.
\begin{aligned}
|A\rangle_{12} &\longrightarrow |\psi\rangle_3 = \alpha\,|0\rangle_3 + \beta\,|1\rangle_3 \\
|B\rangle_{12} &\longrightarrow |\psi\rangle_3 = \alpha\,|0\rangle_3 - \beta\,|1\rangle_3 \\
|C\rangle_{12} &\longrightarrow |\psi\rangle_3 = \alpha\,|1\rangle_3 + \beta\,|0\rangle_3 \\
|D\rangle_{12} &\longrightarrow |\psi\rangle_3 = \alpha\,|1\rangle_3 - \beta\,|0\rangle_3
\end{aligned}
\right\}
\xrightarrow{\hat{U}}
\underbrace{|\psi\rangle_3 = \alpha\,|0\rangle + \beta\,|1\rangle}_{\text{qubit 3 } \textbf{transforms} \text{ into the original state of qubit 1!!!}}
$$

Teleportation happened!

Table of Contents

I. Set up, for basic Quantum Teleportation circuit




```
# Define teleport circuit
c = Circuit(3)

# Initial Alice random state
c.add(gates.U1q(q=0, theta=theta, phi=phi))

# Initial Alice-Bob entangled state
c.add(gates.H(1))
c.add(gates.CNOT(1,2))
```

```
Circuit:
q0: -U1q-o-H-M-o-
q1: -H---o-X-M---|-o-
q2: -----X-------Z-X-M-
```

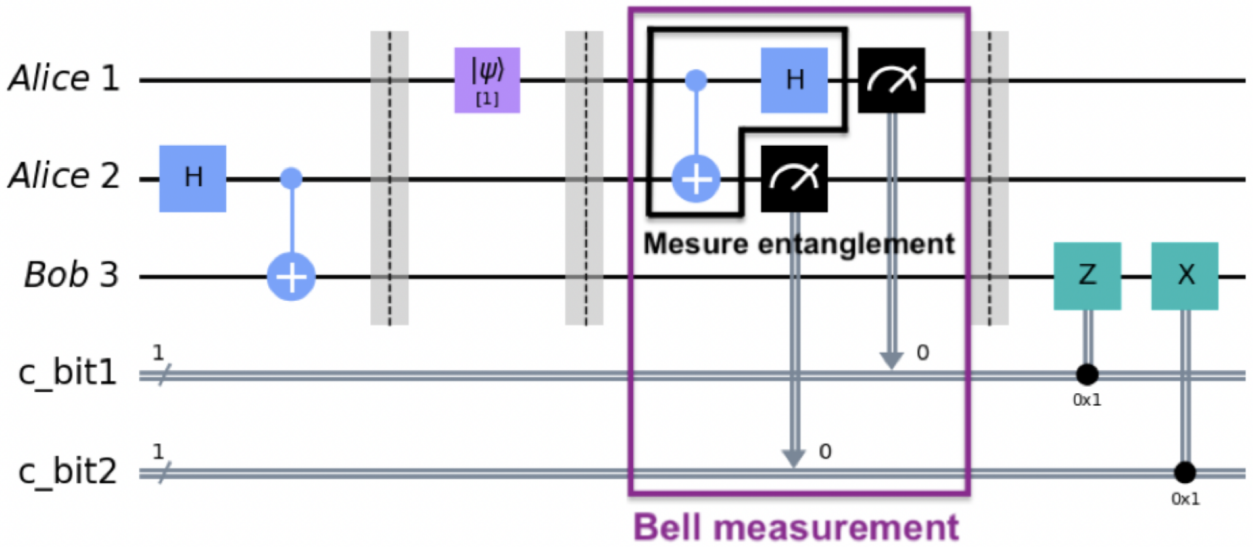
Alice's state, to teleport:

$$|\psi\rangle_1 = \alpha\,|0\rangle + \beta\,|1\rangle$$

And the initial Alice-Bob entangled state:

$$|\psi\rangle_{23} = CNOT(2,3)\ H(2)\ |00\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

II. Bell measurement in a Quantum Teleportation circuit



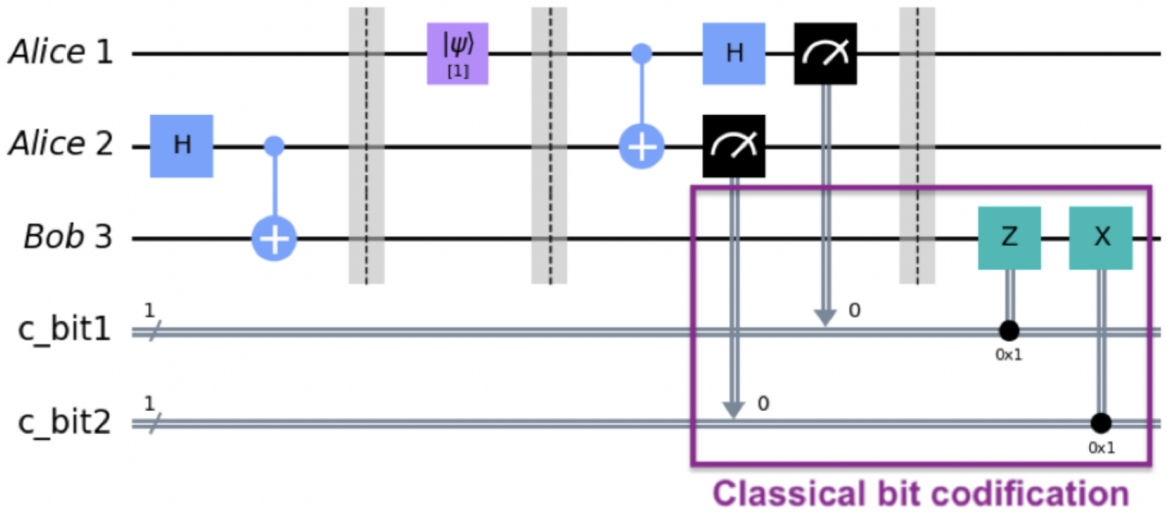

```
# Entanglement measure of Alice
c.add(gates.CNOT(0,1))
c.add(gates.H(0))
c.add(gates.M(0))
c.add(gates.M(1))
```

```
Circuit:
q0: ─U1q──o─H─M─o──────
q1: ─H──o─X─M──┆─o──────
q2: ──────X──────Z─X─M─
```

Since we can only measure individual qubits in their Z axis:

We translate Alice's double measurement with this "reverse" entanglement measure.

III. C-bits encoding in a Quantum Teleportation circuit




```
# From Alice's results, Bob does control Z's & X's:
c.add(gates.CZ(0,2))
c.add(gates.CNOT(1,2))
c.add(gates.M(2, register_name="measure"))

# Plot the circuit:
print(f"Circuit:\n{c.draw()}\n")
```

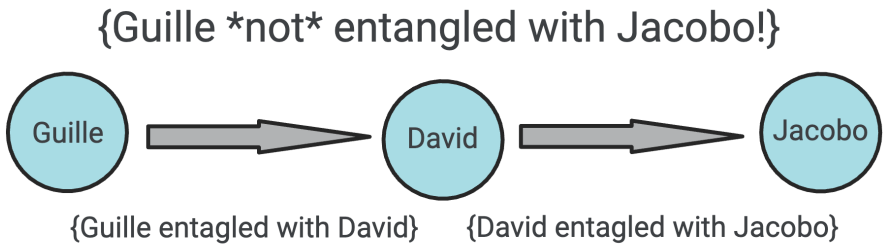
```
Circuit:
q0: ─U1q──o─H─M─o──
q1: ─H──o─X─M──|─o──
q2: ────X──────Z─X─M─
```

Depending on what Alice measures, we have these 4 possibilities:

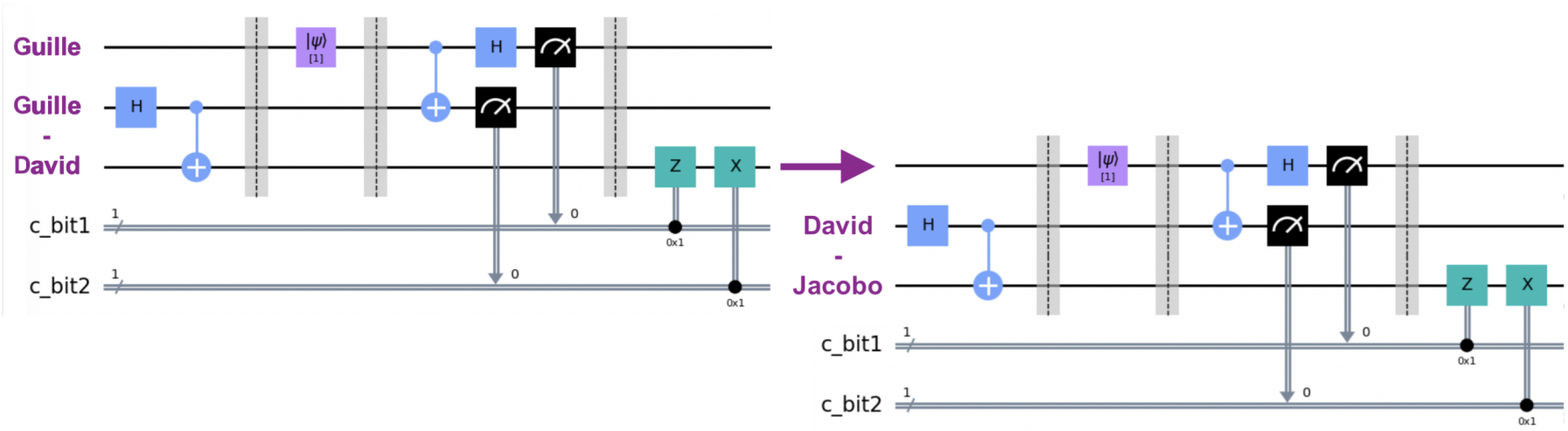
Alice measure	Bob state (ψ_3)	Gates to get ψ_1 on ψ_3
00	$\alpha\|0\rangle + \beta\|1\rangle$	$\mathbb{I}$
01	$\alpha\|1\rangle + \beta\|0\rangle$	$\mathbb{X}$
10	$\alpha\|0\rangle - \beta\|1\rangle$	$\mathbb{Z}$
11	$\alpha\|1\rangle - \beta\|0\rangle$	$\mathbb{ZX}$

IV. Consecutives Quantum Teleportations

Using previous output as input for a new teleportation:

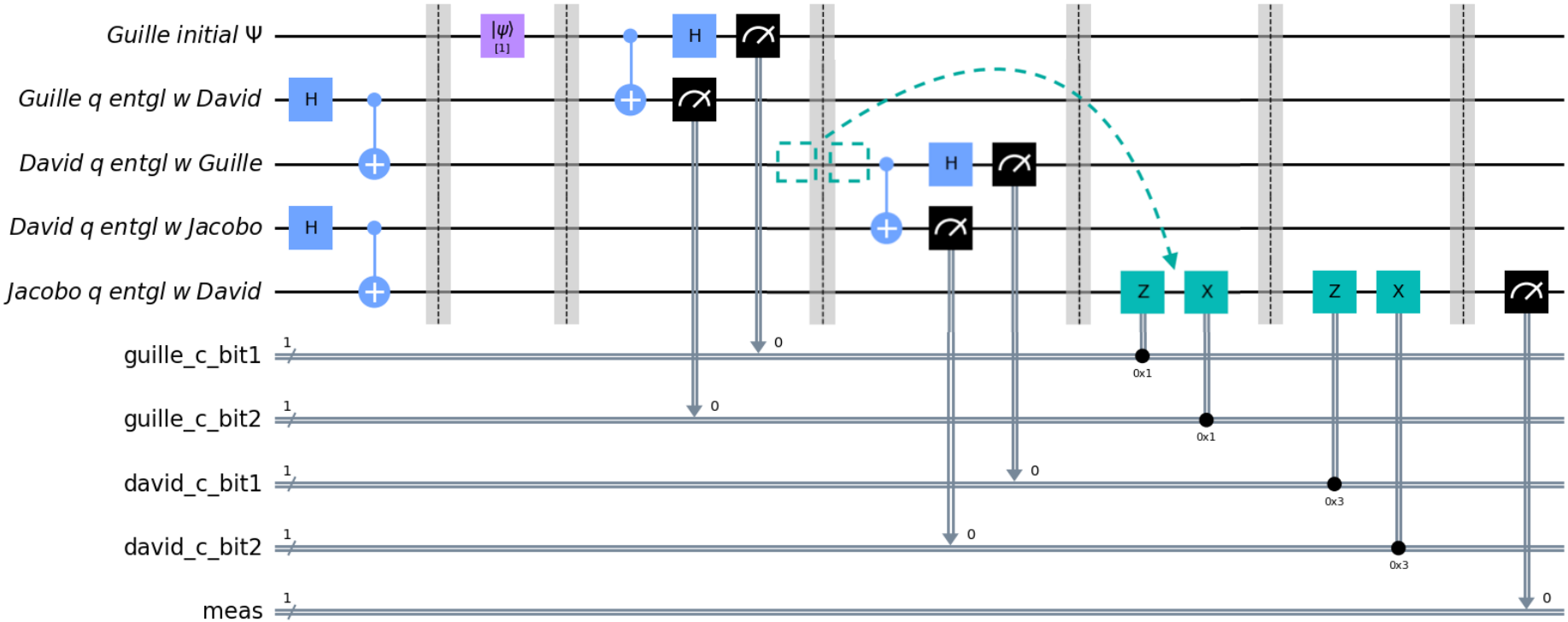


Notice that this will "consume" two entanglements!



V. Secure consecutive Quantum Teleportations

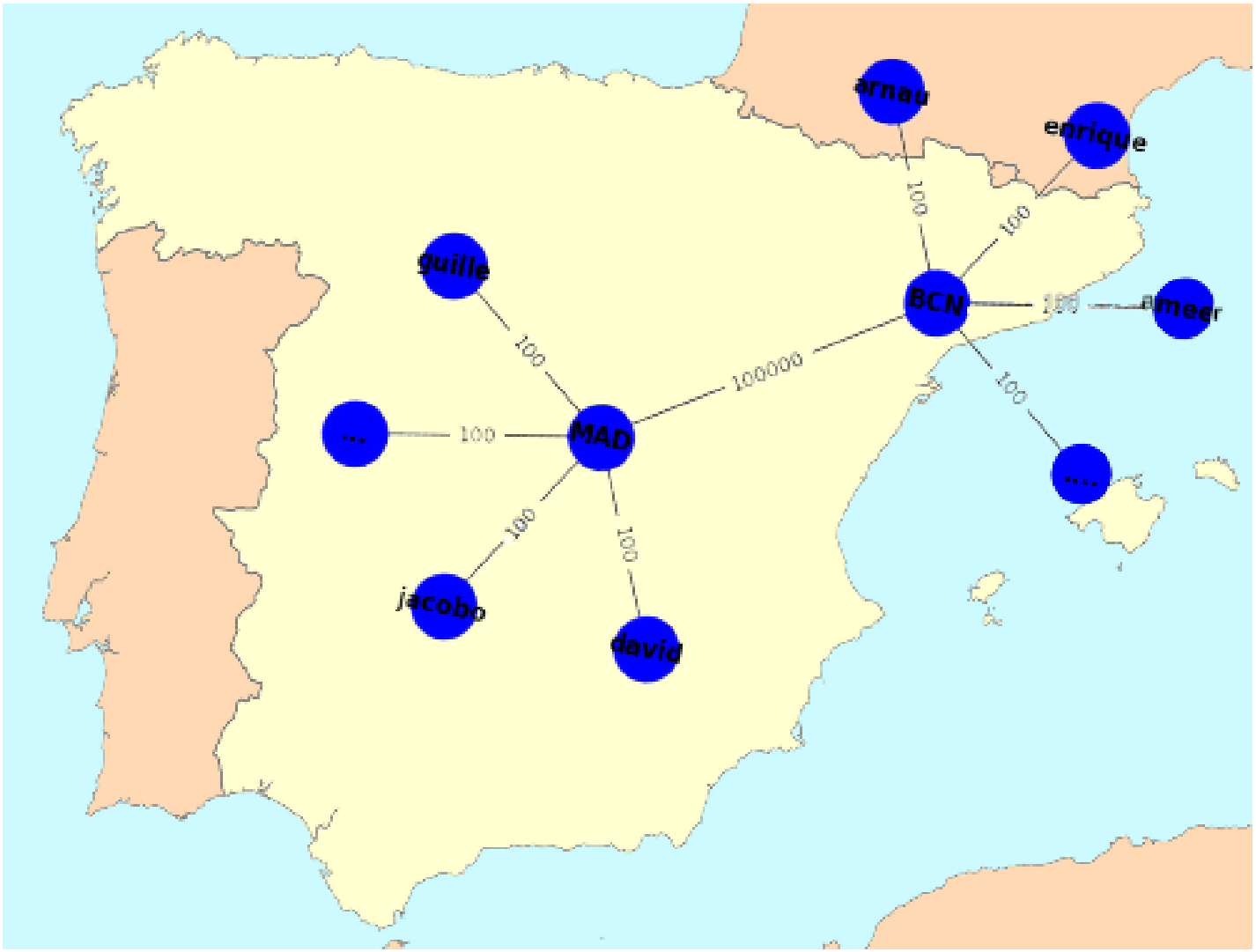
Consecutive teleportations can be secure, if we move all the decoding to the end:



the intermediaries can interrupt such communication, but never obtain it!

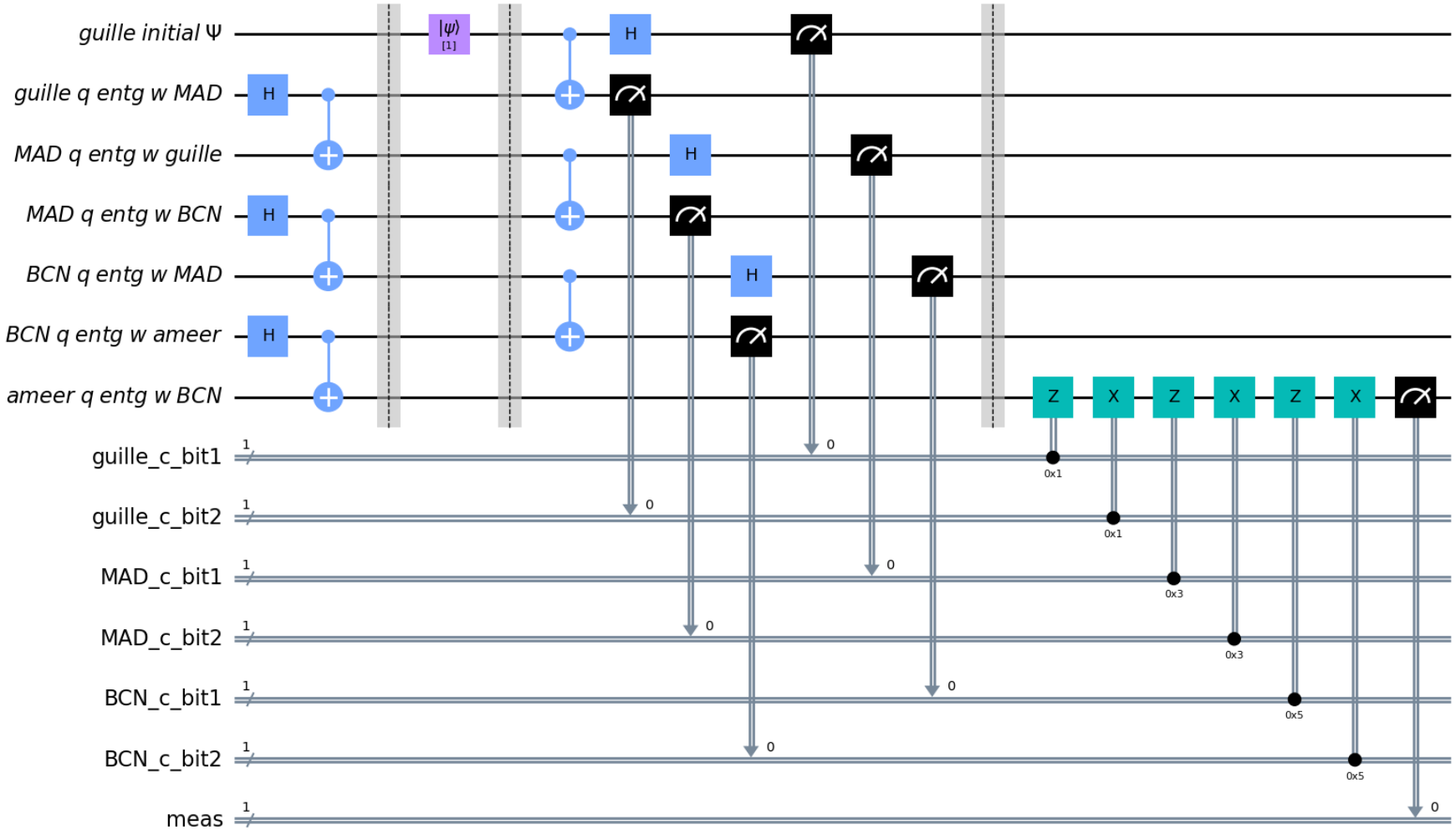
V. Quantum teleportation networks

Quantum teleportation networks, with entanglement highways:



VI. Quantum teleportation network algorithm

In the case, Guille in MAD, wanted to send a state to his friend Ameer in BCN, for the above network, the algorithm automatically generates:



Thank you for your time!