MA684_homework_08

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Getting to know stan

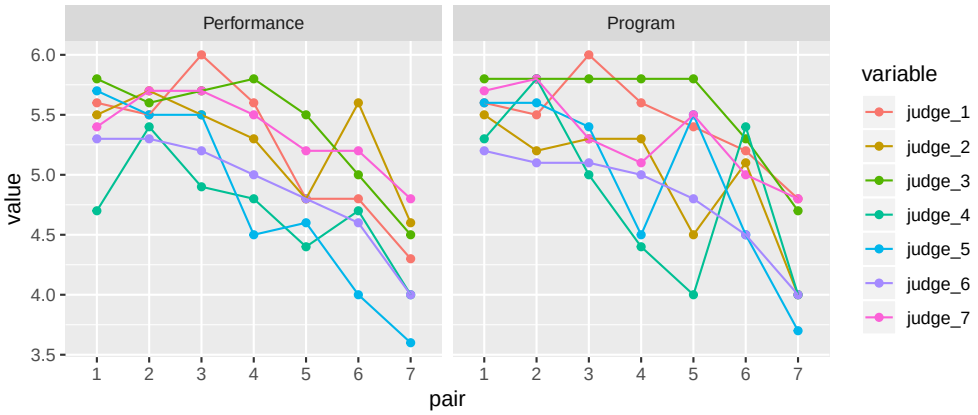
Read through the tutorial on Stan https://github.com/stan-dev/rstan/wiki/RStan-Getting-Started

- Explore Stan website and Stan reference manual and try to connect them with Gelman and Hill 16 - 17.

Data analysis

Using stan:

The folder olympics has seven judges' ratings of seven figure skaters (on two criteria: "technical merit" and "artistic impression") from the 1932 Winter Olympics. Take a look at http://www.stat.columbia.edu/~gelman/arm/examples/olympics/olympics1932.txt




```
##    Program Performance pair   Judge
## 1:     5.6         5.6    1 judge_1
## 2:     5.5         5.5    1 judge_2
## 3:     5.8         5.8    1 judge_3
## 4:     5.3         4.7    1 judge_4
## 5:     5.6         5.7    1 judge_5
## 6:     5.2         5.3    1 judge_6
```

use stan to fit a non-nested multilevel model (varying across skaters and judges) for the technical merit ratings.

$$
\begin{aligned}
y_i &\sim N(\mu + \gamma_{j[i]} + \delta_{k[i]}, \sigma_y^2), \text{ for } i = 1, \ldots, n && (1)\\
\gamma_j &\sim N(0, \sigma_\gamma^2) j = 1, \ldots, 7 && (2)\\
\delta_k &\sim N(0, \sigma_\delta^2) k = 1, \ldots, 7 && (3)
\end{aligned}
$$

https://github.com/stan-dev/example-models/blob/master/ARM/Ch.17/17.3_flight_simulator.stan https://github.com/stan-dev/example-models/blob/master/ARM/Ch.17/17.3_non-nested_models.R

```
fit_program<-lmer(Program~1+(1|pair) +  (1|Judge),olympics_long)
```

```
dataList.1 <- list(N=49, n_judges=7, n_pairs=7,  judge=as.integer(olympics_long$Judge), pair=as.integer

skating_stan<-"
data {
  int<lower=0> N;
  int<lower=0> n_judges;
  int<lower=0> n_pairs;
  int<lower=0,upper=n_judges> judge[N];
  int<lower=0,upper=n_pairs> pair[N];
  vector[N] y;
}
parameters {
  real<lower=0> sigma;
  real<lower=0> sigma_gamma;
  real<lower=0> sigma_delta;
  vector[n_judges] gamma;
  vector[n_pairs] delta;
  real mu;
}
model {
  vector[N] y_hat;

  sigma ~ uniform(0, 100);
  sigma_gamma ~ uniform(0, 100);
  sigma_delta ~ uniform(0, 100);

  mu ~ normal(0, 100);

  gamma ~ normal(0, sigma_gamma);
  delta ~ normal(0, sigma_delta);

  for (i in 1:N)
    y_hat[i] = mu + gamma[judge[i]] + delta[pair[i]];
  y ~ normal(y_hat, sigma);
}
"
```

pilots <- read.table ("http://www.stat.columbia.edu/~gelman/arm/examples/pilots/pilots.dat", header=TRUE)

flight_simulator.sf1 <- stan(model_code=skating_stan , data=dataList.1, iter=2000, chains=4)

Multilevel logistic regression

The folder `speed.dating` contains data from an experiment on a few hundred students that randomly assigned each participant to 10 short dates with participants of the opposite sex (Fisman et al., 2006). For each date, each person recorded several subjective numerical ratings of the other person (attractiveness, compatibility, and some other characteristics) and also wrote down whether he or she would like to meet the other person again. Label $y_{ij} = 1$ if person i is interested in seeing person j again 0 otherwise. And $r_{ij1}, \ldots, r_{ij6}$ as person i's numerical ratings of person j on the dimensions of attractiveness, compatibility, and so forth. Please look at http://www.stat.columbia.edu/~gelman/arm/examples/speed.dating/Speed%20Dating%20Data%20Key.doc for details.

```
dating<-fread("http://www.stat.columbia.edu/~gelman/arm/examples/speed.dating/Speed%20Dating%20Data.csv
```

1. Fit a classical logistic regression predicting $Pr(y_{ij} = 1)$ given person i's 6 ratings of person j. Discuss the importance of attractiveness, compatibility, and so forth in this predictive model.

```
dating_reg1 <- glm(match~attr_o +sinc_o +intel_o +fun_o +amb_o +shar_o,data=dating,family=binomial)
summary(dating_reg1)
```

```
##
## Call:
## glm(formula = match ~ attr_o + sinc_o + intel_o + fun_o + amb_o +
##     shar_o, family = binomial, data = dating)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -1.5300  -0.6362  -0.4420  -0.2381   3.1808
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept) -5.62091    0.21859 -25.714  < 2e-16 ***
## attr_o       0.22047    0.02388   9.233  < 2e-16 ***
## sinc_o      -0.01996    0.03067  -0.651   0.5152
## intel_o      0.07176    0.03716   1.931   0.0535 .
## fun_o        0.25315    0.02922   8.665  < 2e-16 ***
## amb_o       -0.12099    0.02838  -4.264 2.01e-05 ***
## shar_o       0.21225    0.02209   9.608  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 6466.6  on 7030  degrees of freedom
## Residual deviance: 5611.0  on 7024  degrees of freedom
##   (1347 observations deleted due to missingness)
## AIC: 5625
##
## Number of Fisher Scoring iterations: 5
```

Fitted model: $logit(P(match = 1)) = -5.6 + 0.22X_{attractiveness} - 0.019X_{sincere} + 0.071X_{intellligent} + 0.253X_{fun} - 0.12X_{ambitious} + 0.212X_{sharedinterests}$

The attractiveness, fun, ambitious and shared interests are statistically significant.

Attractiveness: this predictor has positive effects on the match of two person, for every unit increase in attractiveness, the log odds of match will increase by 0.22. In other words, the probability of match will be increased.

Fun: this predictor has positive effects on the match of two person, for every unit increase in attractiveness, the log odds of match will increase by 0.25. In other words, the probability of match will be increased.

Ambitious: this predictor has negative effects on the match of two person, for every unit increase in attractiveness, the log odds of match will decrease by 0.12. In other words, the probability of match will be decreased.

Shared Interests: this predictor has positive effects on the match of two person, for every unit increase in attractiveness, the log odds of match will increase by 0.21. In other words, the probability of match will be increased.

Overall, since all four variables are statistically significant, we can explain their effects based on their coefficients. Therefore, I would say "fun" has relatively greater effetcs on the probability of match, following by attractiveness, shared interests and ambitious.

2. Expand this model to allow varying intercepts for the persons making the evaluation; that is, some people are more likely than others to want to meet someone again. Discuss the fitted model.

```
dating_reg2 <- glmer(match~gender+attr_o+sinc_o+intel_o+fun_o+amb_o+shar_o+(1|iid), data=dating, family=
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.038348
## (tol = 0.001, component 1)
```

```
summary(dating_reg2)
```

```
## Generalized linear mixed model fit by maximum likelihood (Laplace
##   Approximation) [glmerMod]
##  Family: binomial  ( logit )
## Formula: match ~ gender + attr_o + sinc_o + intel_o + fun_o + amb_o +
##     shar_o + (1 | iid)
##    Data: dating
##
##      AIC      BIC   logLik deviance df.resid
##   5543.3   5605.0  -2762.6   5525.3     7022
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -1.7416 -0.4458 -0.2883 -0.1459 10.3426
##
## Random effects:
##  Groups Name        Variance Std.Dev.
##  iid    (Intercept) 0.4268   0.6533
## Number of obs: 7031, groups:  iid, 551
##
## Fixed effects:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept) -6.02085    0.24369 -24.707  < 2e-16 ***
## gender       0.15329    0.09314   1.646   0.0998 .
## attr_o       0.23540    0.02648   8.888  < 2e-16 ***
## sinc_o      -0.01372    0.03261  -0.421   0.6740
## intel_o      0.07019    0.03967   1.770   0.0768 .
## fun_o        0.26270    0.03140   8.366  < 2e-16 ***
## amb_o       -0.13138    0.03025  -4.343  1.4e-05 ***
## shar_o       0.22389    0.02325   9.629  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##         (Intr) gender attr_o sinc_o intel_ fun_o  amb_o
## gender  -0.192
## attr_o  -0.264  0.109
## sinc_o  -0.163  0.048 -0.120
## intel_o -0.298 -0.055 -0.039 -0.466
## fun_o   -0.112  0.017 -0.246 -0.151 -0.128
## amb_o   -0.038 -0.092 -0.061 -0.015 -0.372 -0.187
## shar_o  -0.056  0.010 -0.099 -0.053 -0.003 -0.265 -0.201
```

```
## convergence code: 0
## Model failed to converge with max|grad| = 0.038348 (tol = 0.001, component 1)
```

```
ranef(dating_reg2)$'iid'[1:5,]
```

```
## [1]  0.4914645 -0.1811325 -0.4660992 -0.1073909  0.1229939
```

Fitted model: $P(match = 1) = logit^{-1}(\alpha_0 + \alpha_{j[i]} + 0.153X_{gender} + 0.235X_{attr} - 0.013X_{sinc} + 0.07X_{intel} + 0.262X_{fun} - 0.131X_{amb} + 0.223X_{shar})$

$\alpha_j \sim N(\mu_\sigma, \sigma^2_{iid})$

Each observtion(person) shares same fixed effects, but the intercept will be varied for different person. For example, the fitted model for the first person is: $P(match = 1) = logit^{-1}(-6.02 + 0.491 + 0.153X_{gender} + 0.235X_{attr} - 0.013X_{sinc} + 0.07X_{intel} + 0.262X_{fun} - 0.131X_{amb} + 0.223X_{shar})$

The interpretation will be similar to classic logistic regression.

3. Expand further to allow varying intercepts for the persons being rated. Discuss the fitted model.

```
dating_reg3 <- glmer(match~gender+attr_o+sinc_o+intel_o+fun_o+amb_o+shar_o+(1|iid)+(1|pid), data=dating
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.676505
## (tol = 0.001, component 1)
```

```
summary(dating_reg3)
```

```
## Generalized linear mixed model fit by maximum likelihood (Laplace
##   Approximation) [glmerMod]
##  Family: binomial  ( logit )
## Formula: match ~ gender + attr_o + sinc_o + intel_o + fun_o + amb_o +
##     shar_o + (1 | iid) + (1 | pid)
##    Data: dating
##
##      AIC      BIC   logLik deviance df.resid
##   5257.8   5326.4  -2618.9   5237.8     7021
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6948 -0.3825 -0.2195 -0.0921  9.2298
##
## Random effects:
##  Groups Name        Variance Std.Dev.
##  iid    (Intercept) 0.6041   0.7773
##  pid    (Intercept) 1.2467   1.1166
## Number of obs: 7031, groups:  iid, 551; pid, 537
##
## Fixed effects:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept) -8.25538    0.38237 -21.590  < 2e-16 ***
## gender       0.17148    0.14935   1.148  0.25088
## attr_o       0.33695    0.03280  10.272  < 2e-16 ***
## sinc_o       0.02001    0.03896   0.514  0.60749
## intel_o      0.10517    0.04741   2.218  0.02655 *
## fun_o        0.30042    0.03636   8.263  < 2e-16 ***
## amb_o       -0.09309    0.03601  -2.585  0.00973 **
## shar_o       0.26005    0.02844   9.144  < 2e-16 ***
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##         (Intr) gender attr_o sinc_o intel_ fun_o  amb_o
## gender  -0.195
## attr_o  -0.336  0.093
## sinc_o  -0.220  0.036 -0.064
## intel_o -0.301 -0.045 -0.025 -0.438
## fun_o   -0.154  0.009 -0.215 -0.123 -0.099
## amb_o   -0.125 -0.071 -0.051  0.011 -0.334 -0.168
## shar_o  -0.100  0.004 -0.075 -0.059 -0.021 -0.237 -0.159
## convergence code: 0
## Model failed to converge with max|grad| = 0.676505 (tol = 0.001, component 1)
```

```
ranef(dating_reg3)$'iid'[1:5,]
```

```
## [1]  0.461945363 -0.469330425 -0.803753031 -0.321265950  0.002052033
```

Comparing to the previous model, the difference is that the intercept for each observation(person) will be varied further by the random effects of the person being rated.

Fitted model: $P(match = 1) = logit^{-1}(-8.25 + \alpha^{iid}_{j[i]} + \alpha^{pid}_{j[i]} + 0.171X_{gender} + 0.336X_{attr} + 0.02X_{sinc} + 0.105X_{intel} + 0.3X_{fun} - 0.093X_{amb} + 0.26X_{shar})$

$\alpha^{iid}_j \sim N(\mu_\sigma, \sigma^2_{iid})$

$\alpha^{pid}_j \sim N(\mu_\sigma, \sigma^2_{pid})$

4. You will now fit some models that allow the coefficients for attractiveness, compatibility, and the other attributes to vary by person. Fit a no-pooling model: for each person i, fit a logistic regression to the data y_{ij} for the 10 persons j whom he or she rated, using as predictors the 6 ratings $r_{ij1}, \ldots, r_{ij6}$. (Hint: with 10 data points and 6 predictors, this model is difficult to fit. You will need to simplify it in some way to get reasonable fits.)

```
# No pooling model
dating_reg4 <- glm(match~attr_o +sinc_o +intel_o +fun_o +amb_o +shar_o+factor(iid)-1,data=dating,family
summary(dating_reg4)
```

```
##
## Call:
## glm(formula = match ~ attr_o + sinc_o + intel_o + fun_o + amb_o +
##     shar_o + factor(iid) - 1, family = binomial, data = dating)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -2.45659  -0.58492  -0.29618  -0.00006   3.15918
##
## Coefficients:
##                  Estimate Std. Error z value Pr(>|z|)
## attr_o            0.23849    0.03177   7.506 6.08e-14 ***
## sinc_o           -0.01234    0.03680  -0.335 0.737389
## intel_o           0.08502    0.04496   1.891 0.058606 .
## fun_o             0.28628    0.03628   7.891 3.00e-15 ***
## amb_o            -0.14347    0.03391  -4.231 2.33e-05 ***
## shar_o            0.24842    0.02562   9.697  < 2e-16 ***
## factor(iid)1     -5.32218    0.76043  -6.999 2.58e-12 ***
## factor(iid)2     -6.96922    0.93687  -7.439 1.02e-13 ***
## factor(iid)3    -24.60089 3170.24298  -0.008 0.993809
```

```
## factor(iid)4     -6.75713    0.91366  -7.396 1.41e-13 ***
## factor(iid)5     -6.06921    0.94262  -6.439 1.21e-10 ***
## factor(iid)6     -6.26160    0.91833  -6.818 9.20e-12 ***
## factor(iid)7     -6.77550    0.92286  -7.342 2.11e-13 ***
## factor(iid)8     -3.86139    0.90831  -4.251 2.13e-05 ***
## factor(iid)9     -4.05234    0.76048  -5.329 9.89e-08 ***
## factor(iid)10    -5.93231    0.98840  -6.002 1.95e-09 ***
## factor(iid)11   -23.75415 3322.65848  -0.007 0.994296
## factor(iid)12    -6.65217    0.94044  -7.073 1.51e-12 ***
## factor(iid)13    -5.04049    0.77755  -6.483 9.02e-11 ***
## factor(iid)14    -3.20281    1.11581  -2.870 0.004100 **
## factor(iid)15    -5.59256    0.82871  -6.749 1.49e-11 ***
## factor(iid)16    -5.13385    0.94411  -5.438 5.39e-08 ***
## factor(iid)17    -6.98546    1.12073  -6.233 4.58e-10 ***
## factor(iid)18    -5.91106    1.12453  -5.256 1.47e-07 ***
## factor(iid)19    -3.35428    1.11329  -3.013 0.002587 **
## factor(iid)20    -7.68615    1.11660  -6.884 5.84e-12 ***
## factor(iid)21   -23.60047 2689.59436  -0.009 0.992999
## factor(iid)22    -5.07941    0.84659  -6.000 1.97e-09 ***
## factor(iid)23    -5.50085    0.77749  -7.075 1.49e-12 ***
## factor(iid)24   -24.24492 2829.96018  -0.009 0.993164
## factor(iid)25   -24.66505 2946.60375  -0.008 0.993321
## factor(iid)26   -24.97065 2904.90060  -0.009 0.993141
## factor(iid)27    -7.77581    1.10426  -7.042 1.90e-12 ***
## factor(iid)28    -7.32263    1.11656  -6.558 5.45e-11 ***
## factor(iid)29    -5.76808    0.83165  -6.936 4.04e-12 ***
## factor(iid)30    -6.82684    1.15190  -5.927 3.09e-09 ***
## factor(iid)31    -7.25200    1.11419  -6.509 7.58e-11 ***
## factor(iid)32   -24.25599 2766.14312  -0.009 0.993004
## factor(iid)33   -23.99561 2791.35153  -0.009 0.993141
## factor(iid)34    -6.54508    0.84222  -7.771 7.77e-15 ***
## factor(iid)35    -5.12685    0.70972  -7.224 5.06e-13 ***
## factor(iid)36    -5.94506    0.82748  -7.185 6.74e-13 ***
## factor(iid)37    -5.34891    0.85102  -6.285 3.27e-10 ***
## factor(iid)38    -6.63406    0.87013  -7.624 2.46e-14 ***
## factor(iid)39    -6.78859    1.09129  -6.221 4.95e-10 ***
## factor(iid)40   -23.62085 2579.31106  -0.009 0.992693
## factor(iid)41   -22.41728 2697.23258  -0.008 0.993369
## factor(iid)42   -23.66439 2468.71015  -0.010 0.992352
## factor(iid)43    -6.12449    1.10345  -5.550 2.85e-08 ***
## factor(iid)44    -4.70247    0.65757  -7.151 8.59e-13 ***
## factor(iid)45    -4.40151    0.66914  -6.578 4.77e-11 ***
## factor(iid)46    -6.23758    0.74715  -8.349  < 2e-16 ***
## factor(iid)47    -6.75781    1.08433  -6.232 4.60e-10 ***
## factor(iid)48    -7.60341    1.07402  -7.079 1.45e-12 ***
## factor(iid)49    -5.22649    0.62077  -8.419  < 2e-16 ***
## factor(iid)50    -5.87030    0.67327  -8.719  < 2e-16 ***
## factor(iid)51    -6.53571    1.08917  -6.001 1.97e-09 ***
## factor(iid)52    -6.78444    1.18276  -5.736 9.69e-09 ***
## factor(iid)53    -7.04273    1.09140  -6.453 1.10e-10 ***
## factor(iid)54   -23.19537 2379.68458  -0.010 0.992223
## factor(iid)55    -6.10291    0.72418  -8.427  < 2e-16 ***
## factor(iid)56    -6.66784    1.12065  -5.950 2.68e-09 ***
## factor(iid)57    -6.04702    1.10369  -5.479 4.28e-08 ***
```

```
## factor(iid)58    -5.63219    0.92560  -6.085 1.17e-09 ***
## factor(iid)59   -22.73505 6130.80302  -0.004 0.997041
## factor(iid)60    -6.15649    1.11432  -5.525 3.30e-08 ***
## factor(iid)61    -4.83287    0.90556  -5.337 9.46e-08 ***
## factor(iid)62    -6.44171    0.97281  -6.622 3.55e-11 ***
## factor(iid)63    -5.80285    1.12428  -5.161 2.45e-07 ***
## factor(iid)64    -6.43263    1.17043  -5.496 3.89e-08 ***
## factor(iid)65   -24.55660 3277.75563  -0.007 0.994022
## factor(iid)66   -23.56514 3584.38629  -0.007 0.994754
## factor(iid)67    -5.32862    0.93119  -5.722 1.05e-08 ***
## factor(iid)68   -23.56823 3154.44195  -0.007 0.994039
## factor(iid)69    -6.20079    0.90267  -6.869 6.45e-12 ***
## factor(iid)70    -7.04513    1.15782  -6.085 1.17e-09 ***
## factor(iid)71    -4.56396    0.86936  -5.250 1.52e-07 ***
## factor(iid)72   -24.29702 2905.27923  -0.008 0.993327
## factor(iid)73   -24.45394 3423.46257  -0.007 0.994301
## factor(iid)74    -5.93933    0.98476  -6.031 1.63e-09 ***
## factor(iid)75    -5.11068    0.96146  -5.316 1.06e-07 ***
## factor(iid)76    -5.38715    0.66531  -8.097 5.62e-16 ***
## factor(iid)77    -5.15527    0.63918  -8.065 7.30e-16 ***
## factor(iid)78    -5.75968    0.82961  -6.943 3.85e-12 ***
## factor(iid)79    -6.13275    0.73480  -8.346  < 2e-16 ***
## factor(iid)80    -5.42208    0.75640  -7.168 7.60e-13 ***
## factor(iid)81    -6.55072    0.73914  -8.863  < 2e-16 ***
## factor(iid)82    -5.33868    0.65346  -8.170 3.09e-16 ***
## factor(iid)83    -7.03389    0.86630  -8.119 4.68e-16 ***
## factor(iid)84    -6.72107    0.84307  -7.972 1.56e-15 ***
## factor(iid)85    -7.26294    1.10949  -6.546 5.90e-11 ***
## factor(iid)86    -4.70697    0.63475  -7.415 1.21e-13 ***
## factor(iid)87    -6.50040    0.84372  -7.704 1.31e-14 ***
## factor(iid)88   -24.38213 2655.85946  -0.009 0.992675
## factor(iid)89    -6.14345    0.84391  -7.280 3.34e-13 ***
## factor(iid)90    -6.74913    1.26536  -5.334 9.62e-08 ***
## factor(iid)91    -5.44170    0.61579  -8.837  < 2e-16 ***
## factor(iid)92    -5.17743    0.66225  -7.818 5.37e-15 ***
## factor(iid)93    -6.05370    0.67932  -8.911  < 2e-16 ***
## factor(iid)94    -5.56173    0.90388  -6.153 7.60e-10 ***
## factor(iid)95    -6.72158    1.09143  -6.159 7.34e-10 ***
## factor(iid)96   -23.17627 2520.12122  -0.009 0.992662
## factor(iid)97    -6.04721    0.75679  -7.991 1.34e-15 ***
## factor(iid)98    -5.56394    0.81262  -6.847 7.54e-12 ***
## factor(iid)99    -5.45447    0.62295  -8.756  < 2e-16 ***
## factor(iid)100   -5.99761    0.89314  -6.715 1.88e-11 ***
## factor(iid)101  -23.12848 2436.56844  -0.009 0.992426
## factor(iid)102   -6.54898    1.12144  -5.840 5.23e-09 ***
## factor(iid)103  -24.04851 2574.08126  -0.009 0.992546
## factor(iid)104   -5.99428    0.68960  -8.692  < 2e-16 ***
## factor(iid)105   -5.41751    0.61127  -8.863  < 2e-16 ***
## factor(iid)106  -23.74298 2715.34175  -0.009 0.993023
## factor(iid)107   -4.88183    0.63535  -7.684 1.55e-14 ***
## factor(iid)108   -6.27723    0.67776  -9.262  < 2e-16 ***
## factor(iid)109   -5.15322    0.61288  -8.408  < 2e-16 ***
## factor(iid)110   -6.42980    0.84847  -7.578 3.51e-14 ***
## factor(iid)111  -23.84716 2697.10043  -0.009 0.992945
```

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## factor(iid)112   -4.59918    0.83281  -5.522 3.34e-08 ***
## factor(iid)113   -5.62716    0.82035  -6.859 6.91e-12 ***
## factor(iid)114   -6.16107    1.12809  -5.461 4.72e-08 ***
## factor(iid)115   -6.22481    0.90055  -6.912 4.77e-12 ***
## factor(iid)116   -5.56617    0.93873  -5.929 3.04e-09 ***
## factor(iid)117   -7.95238    1.16437  -6.830 8.50e-12 ***
## factor(iid)119   -4.92535    0.74499  -6.611 3.81e-11 ***
## factor(iid)120   -5.54927    0.74639  -7.435 1.05e-13 ***
## factor(iid)121  -23.27119 3124.31901  -0.007 0.994057
## factor(iid)122   -6.56751    0.86357  -7.605 2.85e-14 ***
## factor(iid)123  -23.61609 3571.73806  -0.007 0.994724
## factor(iid)124  -21.06282 3636.96909  -0.006 0.995379
## factor(iid)125   -3.80378    0.93271  -4.078 4.54e-05 ***
## factor(iid)126   -5.61988    0.88409  -6.357 2.06e-10 ***
## factor(iid)127   -4.99355    0.89771  -5.563 2.66e-08 ***
## factor(iid)128   -4.11707    0.82083  -5.016 5.28e-07 ***
## factor(iid)129   -5.43706    0.90982  -5.976 2.29e-09 ***
## factor(iid)130   -6.58176    1.13999  -5.774 7.76e-09 ***
## factor(iid)131  -22.18563 4156.88335  -0.005 0.995742
## factor(iid)132   -6.08609    1.20970  -5.031 4.88e-07 ***
## factor(iid)133  -23.73504 5172.85188  -0.005 0.996339
## factor(iid)134   -5.19452    1.46725  -3.540 0.000400 ***
## factor(iid)135   -4.27246    1.22062  -3.500 0.000465 ***
## factor(iid)136   -5.81197    1.27741  -4.550 5.37e-06 ***
## factor(iid)137   -6.82161    1.19573  -5.705 1.16e-08 ***
## factor(iid)138   -6.98615    1.27904  -5.462 4.71e-08 ***
## factor(iid)139  -23.13495 5112.62139  -0.005 0.996390
## factor(iid)140   -6.65724    1.19753  -5.559 2.71e-08 ***
## factor(iid)141  -24.11690 5239.61760  -0.005 0.996328
## factor(iid)142   -4.90583    0.65589  -7.480 7.45e-14 ***
## factor(iid)143  -23.36792 2757.63443  -0.008 0.993239
## factor(iid)144   -7.89659    1.10447  -7.150 8.70e-13 ***
## factor(iid)145  -24.00022 2633.34217  -0.009 0.992728
## factor(iid)146   -6.96996    1.12915  -6.173 6.71e-10 ***
## factor(iid)147   -6.62649    0.83603  -7.926 2.26e-15 ***
## factor(iid)148  -24.05518 3019.95016  -0.008 0.993645
## factor(iid)149   -6.17548    0.73746  -8.374  < 2e-16 ***
## factor(iid)150   -6.64144    0.83376  -7.966 1.64e-15 ***
## factor(iid)151   -7.02697    0.84220  -8.344  < 2e-16 ***
## factor(iid)152   -5.84457    0.84308  -6.932 4.14e-12 ***
## factor(iid)153   -5.31217    0.73278  -7.249 4.19e-13 ***
## factor(iid)154   -5.17630    0.64050  -8.082 6.39e-16 ***
## factor(iid)155   -7.41980    1.10881  -6.692 2.21e-11 ***
## factor(iid)156   -5.78476    0.68584  -8.435  < 2e-16 ***
## factor(iid)157   -7.63028    1.10752  -6.890 5.60e-12 ***
## factor(iid)158  -22.83183 2887.91697  -0.008 0.993692
## factor(iid)159   -6.12992    1.17698  -5.208 1.91e-07 ***
## factor(iid)160   -6.48200    0.73425  -8.828  < 2e-16 ***
## factor(iid)161   -7.32231    0.84171  -8.699  < 2e-16 ***
## factor(iid)162   -6.16225    0.88538  -6.960 3.40e-12 ***
## factor(iid)163   -6.36929    0.72363  -8.802  < 2e-16 ***
## factor(iid)164   -7.77623    1.13486  -6.852 7.27e-12 ***
## factor(iid)165   -5.88104    0.86036  -6.836 8.17e-12 ***
## factor(iid)166   -5.60387    0.74828  -7.489 6.94e-14 ***
```

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## factor(iid)167   -5.89268    0.74472  -7.913 2.52e-15 ***
## factor(iid)168   -6.73033    1.16911  -5.757 8.57e-09 ***
## factor(iid)169   -6.00153    0.67552  -8.884  < 2e-16 ***
## factor(iid)170  -23.52581 2759.48571  -0.009 0.993198
## factor(iid)171   -5.55115    0.92354  -6.011 1.85e-09 ***
## factor(iid)172   -5.35063    0.70209  -7.621 2.52e-14 ***
## factor(iid)173   -5.60054    0.77187  -7.256 3.99e-13 ***
## factor(iid)174   -6.14547    1.19457  -5.144 2.68e-07 ***
## factor(iid)175   -5.24570    0.81844  -6.409 1.46e-10 ***
## factor(iid)176   -4.74859    0.83935  -5.657 1.54e-08 ***
## factor(iid)177  -24.15001 3399.86638  -0.007 0.994332
## factor(iid)178  -23.77375 3928.66101  -0.006 0.995172
## factor(iid)179   -7.56790    1.12111  -6.750 1.47e-11 ***
## factor(iid)180   -6.08266    0.91802  -6.626 3.45e-11 ***
## factor(iid)181   -5.08170    0.86401  -5.882 4.06e-09 ***
## factor(iid)182  -23.59602 3892.09453  -0.006 0.995163
## factor(iid)183   -7.09332    1.21182  -5.853 4.81e-09 ***
## factor(iid)184   -4.85739    0.77147  -6.296 3.05e-10 ***
## factor(iid)185   -5.94902    1.19696  -4.970 6.69e-07 ***
## factor(iid)186   -5.62903    0.87890  -6.405 1.51e-10 ***
## factor(iid)187   -6.07843    1.12814  -5.388 7.12e-08 ***
## factor(iid)188   -6.02424    1.10413  -5.456 4.87e-08 ***
## factor(iid)189  -23.75516 3415.78046  -0.007 0.994451
## factor(iid)190   -5.59379    0.78015  -7.170 7.49e-13 ***
## factor(iid)191   -6.06665    0.88764  -6.835 8.22e-12 ***
## factor(iid)192   -7.46502    1.12294  -6.648 2.98e-11 ***
## factor(iid)193   -5.89563    0.86673  -6.802 1.03e-11 ***
## factor(iid)194   -6.40121    1.08426  -5.904 3.55e-09 ***
## factor(iid)195   -6.29961    0.64383  -9.785  < 2e-16 ***
## factor(iid)196   -6.31269    0.87429  -7.220 5.18e-13 ***
## factor(iid)197   -6.52775    0.72731  -8.975  < 2e-16 ***
## factor(iid)198  -23.30955 2146.35994  -0.011 0.991335
## factor(iid)199   -5.61368    0.61658  -9.104  < 2e-16 ***
## factor(iid)200   -6.61358    0.71988  -9.187  < 2e-16 ***
## factor(iid)201   -7.89687    1.12400  -7.026 2.13e-12 ***
## factor(iid)202   -7.37595    1.10598  -6.669 2.57e-11 ***
## factor(iid)203  -24.43625 2336.19840  -0.010 0.991654
## factor(iid)204  -23.78855 2297.89316  -0.010 0.991740
## factor(iid)205   -6.79679    0.84733  -8.021 1.05e-15 ***
## factor(iid)206   -4.92973    0.58876  -8.373  < 2e-16 ***
## factor(iid)207   -7.23875    0.71849 -10.075  < 2e-16 ***
## factor(iid)208   -5.01772    0.55928  -8.972  < 2e-16 ***
## factor(iid)209  -23.77271 2294.34929  -0.010 0.991733
## factor(iid)210   -6.38203    0.72581  -8.793  < 2e-16 ***
## factor(iid)211   -6.93836    0.86738  -7.999 1.25e-15 ***
## factor(iid)212   -5.29520    0.56808  -9.321  < 2e-16 ***
## factor(iid)213   -7.28981    0.82660  -8.819  < 2e-16 ***
## factor(iid)214   -7.12112    1.09484  -6.504 7.81e-11 ***
## factor(iid)215   -6.03513    0.64319  -9.383  < 2e-16 ***
## factor(iid)216  -23.28209 2509.98835  -0.009 0.992599
## factor(iid)217   -5.28744    0.61227  -8.636  < 2e-16 ***
## factor(iid)218   -6.53681    1.09049  -5.994 2.04e-09 ***
## factor(iid)219   -5.58457    0.66319  -8.421  < 2e-16 ***
## factor(iid)220   -6.22269    0.75090  -8.287  < 2e-16 ***
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## factor(iid)221   -5.02960    0.64805  -7.761 8.42e-15 ***
## factor(iid)222  -22.90580 2468.76315  -0.009 0.992597
## factor(iid)223   -6.88522    1.09882  -6.266 3.70e-10 ***
## factor(iid)224   -6.15109    0.80001  -7.689 1.49e-14 ***
## factor(iid)225   -6.51738    0.81683  -7.979 1.48e-15 ***
## factor(iid)226   -6.66700    0.69891  -9.539  < 2e-16 ***
## factor(iid)227   -6.39709    0.72969  -8.767  < 2e-16 ***
## factor(iid)228   -6.85783    1.10164  -6.225 4.81e-10 ***
## factor(iid)229   -6.93765    0.83129  -8.346  < 2e-16 ***
## factor(iid)230   -5.52080    0.69116  -7.988 1.37e-15 ***
## factor(iid)231   -6.39344    0.75643  -8.452  < 2e-16 ***
## factor(iid)232   -7.29116    1.18404  -6.158 7.37e-10 ***
## factor(iid)233   -5.25822    0.65530  -8.024 1.02e-15 ***
## factor(iid)234  -24.36918 4179.26985  -0.006 0.995348
## factor(iid)235   -6.90523    1.17985  -5.853 4.84e-09 ***
## factor(iid)236  -23.39858 4064.72424  -0.006 0.995407
## factor(iid)237   -5.11233    1.19605  -4.274 1.92e-05 ***
## factor(iid)238   -5.80750    0.99223  -5.853 4.83e-09 ***
## factor(iid)239   -5.46264    1.13380  -4.818 1.45e-06 ***
## factor(iid)240   -6.77914    1.18334  -5.729 1.01e-08 ***
## factor(iid)241   -5.48930    0.95659  -5.738 9.56e-09 ***
## factor(iid)242   -5.13386    0.79744  -6.438 1.21e-10 ***
## factor(iid)243   -4.94807    0.81254  -6.090 1.13e-09 ***
## factor(iid)244   -6.69053    1.13080  -5.917 3.29e-09 ***
## factor(iid)245   -6.25435    1.19828  -5.219 1.79e-07 ***
## factor(iid)246  -23.16466 3533.55011  -0.007 0.994769
## factor(iid)247  -23.20888 3419.57049  -0.007 0.994585
## factor(iid)248   -5.44004    0.79946  -6.805 1.01e-11 ***
## factor(iid)249  -23.10297 3585.42701  -0.006 0.994859
## factor(iid)250   -4.62803    0.78277  -5.912 3.37e-09 ***
## factor(iid)251   -5.80620    1.11463  -5.209 1.90e-07 ***
## factor(iid)252   -6.93915    0.82881  -8.372  < 2e-16 ***
## factor(iid)253   -5.33375    0.83422  -6.394 1.62e-10 ***
## factor(iid)254  -24.34404 2295.97719  -0.011 0.991540
## factor(iid)255  -23.75640 2288.38579  -0.010 0.991717
## factor(iid)256   -6.21533    0.71868  -8.648  < 2e-16 ***
## factor(iid)257  -24.50493 2428.30256  -0.010 0.991948
## factor(iid)258   -5.53859    0.62451  -8.869  < 2e-16 ***
## factor(iid)259  -24.52437 2355.34990  -0.010 0.991692
## factor(iid)260   -5.14114    0.63109  -8.146 3.75e-16 ***
## factor(iid)261   -5.22645    0.68396  -7.641 2.15e-14 ***
## factor(iid)262  -24.15146 2281.66844  -0.011 0.991555
## factor(iid)263   -5.51093    0.59684  -9.234  < 2e-16 ***
## factor(iid)264   -5.96568    0.69162  -8.626  < 2e-16 ***
## factor(iid)265   -7.01630    1.21152  -5.791 6.98e-09 ***
## factor(iid)266   -6.39135    0.67895  -9.414  < 2e-16 ***
## factor(iid)267  -24.04058 2363.58553  -0.010 0.991885
## factor(iid)268   -5.38541    0.59976  -8.979  < 2e-16 ***
## factor(iid)269   -5.77403    0.61255  -9.426  < 2e-16 ***
## factor(iid)270   -5.36772    0.62480  -8.591  < 2e-16 ***
## factor(iid)271   -6.86757    0.84974  -8.082 6.37e-16 ***
## factor(iid)272  -24.63573 2418.46721  -0.010 0.991872
## factor(iid)273  -23.67182 2254.50101  -0.010 0.991623
## factor(iid)274   -4.73937    0.58329  -8.125 4.47e-16 ***
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## factor(iid)275   -6.98977    1.08007  -6.472 9.70e-11 ***
## factor(iid)276   -5.59527    0.61448  -9.106  < 2e-16 ***
## factor(iid)277   -6.00654    0.65334  -9.194  < 2e-16 ***
## factor(iid)278  -22.98319 2399.16468  -0.010 0.992357
## factor(iid)279   -5.63192    0.60638  -9.288  < 2e-16 ***
## factor(iid)280   -5.33244    0.64444  -8.275  < 2e-16 ***
## factor(iid)281   -6.19586    0.81252  -7.626 2.43e-14 ***
## factor(iid)282   -5.30752    0.57773  -9.187  < 2e-16 ***
## factor(iid)283   -5.88372    0.81726  -7.199 6.05e-13 ***
## factor(iid)284   -7.12366    1.07482  -6.628 3.41e-11 ***
## factor(iid)285   -5.82467    0.80521  -7.234 4.70e-13 ***
## factor(iid)286  -23.07061 2324.64016  -0.010 0.992082
## factor(iid)287  -22.57719 2392.82820  -0.009 0.992472
## factor(iid)288   -6.36032    0.80878  -7.864 3.72e-15 ***
## factor(iid)289   -5.64285    0.64571  -8.739  < 2e-16 ***
## factor(iid)290   -6.59046    0.81819  -8.055 7.95e-16 ***
## factor(iid)291   -5.10256    0.60009  -8.503  < 2e-16 ***
## factor(iid)292   -6.22656    0.71281  -8.735  < 2e-16 ***
## factor(iid)293   -5.99236    0.79631  -7.525 5.26e-14 ***
## factor(iid)294   -7.62577    1.10719  -6.888 5.68e-12 ***
## factor(iid)295  -23.60923 3042.13414  -0.008 0.993808
## factor(iid)296   -5.27509    0.74007  -7.128 1.02e-12 ***
## factor(iid)297   -5.28997    0.69221  -7.642 2.14e-14 ***
## factor(iid)298  -23.89947 2760.76117  -0.009 0.993093
## factor(iid)299   -6.96649    1.13052  -6.162 7.17e-10 ***
## factor(iid)300   -5.54183    0.86044  -6.441 1.19e-10 ***
## factor(iid)301   -6.30770    0.83332  -7.569 3.75e-14 ***
## factor(iid)302  -23.78956 2728.60264  -0.009 0.993044
## factor(iid)303   -7.64967    1.12053  -6.827 8.68e-12 ***
## factor(iid)304   -7.26439    1.09090  -6.659 2.76e-11 ***
## factor(iid)305   -6.13807    0.86054  -7.133 9.83e-13 ***
## factor(iid)306   -7.15158    1.15638  -6.184 6.23e-10 ***
## factor(iid)307   -6.04841    0.74117  -8.161 3.33e-16 ***
## factor(iid)308   -6.96465    1.16386  -5.984 2.18e-09 ***
## factor(iid)309   -6.83897    0.88046  -7.768 8.00e-15 ***
## factor(iid)310   -6.48548    1.12535  -5.763 8.26e-09 ***
## factor(iid)311   -7.64112    1.11728  -6.839 7.97e-12 ***
## factor(iid)312   -7.96402    1.12499  -7.079 1.45e-12 ***
## factor(iid)313   -7.82385    1.11020  -7.047 1.83e-12 ***
## factor(iid)314  -23.61526 2852.70078  -0.008 0.993395
## factor(iid)315   -6.22886    0.86747  -7.181 6.95e-13 ***
## factor(iid)316   -5.56021    0.74257  -7.488 7.00e-14 ***
## factor(iid)317   -6.70988    0.75555  -8.881  < 2e-16 ***
## factor(iid)318  -23.60465 2851.50848  -0.008 0.993395
## factor(iid)319   -5.49614    0.79716  -6.895 5.40e-12 ***
## factor(iid)320  -23.79659 2914.96430  -0.008 0.993486
## factor(iid)321  -24.07737 2745.99860  -0.009 0.993004
## factor(iid)322   -5.52420    0.88987  -6.208 5.37e-10 ***
## factor(iid)323   -5.35899    0.81481  -6.577 4.80e-11 ***
## factor(iid)324   -5.88079    0.88157  -6.671 2.54e-11 ***
## factor(iid)325   -5.70325    0.83973  -6.792 1.11e-11 ***
## factor(iid)326   -5.60737    0.89453  -6.268 3.65e-10 ***
## factor(iid)327  -25.35540 3435.19585  -0.007 0.994111
## factor(iid)328   -7.40756    1.15572  -6.409 1.46e-10 ***
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## factor(iid)329  -23.42044 3502.70117  -0.007 0.994665
## factor(iid)330   -7.33891    1.12110  -6.546 5.90e-11 ***
## factor(iid)331  -23.23301 3537.38391  -0.007 0.994760
## factor(iid)332   -4.71571    0.98272  -4.799 1.60e-06 ***
## factor(iid)333   -5.68844    0.95075  -5.983 2.19e-09 ***
## factor(iid)334  -24.67254 3681.08872  -0.007 0.994652
## factor(iid)335   -6.51708    1.15392  -5.648 1.63e-08 ***
## factor(iid)336   -6.42532    1.14033  -5.635 1.75e-08 ***
## factor(iid)337   -5.89403    0.91357  -6.452 1.11e-10 ***
## factor(iid)338   -5.23120    1.14884  -4.553 5.28e-06 ***
## factor(iid)339   -5.42179    0.80550  -6.731 1.69e-11 ***
## factor(iid)340   -6.72452    1.15359  -5.829 5.57e-09 ***
## factor(iid)341   -6.54949    0.72677  -9.012  < 2e-16 ***
## factor(iid)342   -7.02325    1.08268  -6.487 8.76e-11 ***
## factor(iid)343   -6.15452    0.85953  -7.160 8.05e-13 ***
## factor(iid)344   -6.40365    0.67245  -9.523  < 2e-16 ***
## factor(iid)345   -5.77828    0.67741  -8.530  < 2e-16 ***
## factor(iid)346   -6.27073    0.60334 -10.393  < 2e-16 ***
## factor(iid)347  -22.12836 2677.36100  -0.008 0.993406
## factor(iid)348   -7.18173    1.10926  -6.474 9.52e-11 ***
## factor(iid)349   -5.53289    0.63213  -8.753  < 2e-16 ***
## factor(iid)350   -6.80700    0.71803  -9.480  < 2e-16 ***
## factor(iid)351   -5.18368    0.63800  -8.125 4.48e-16 ***
## factor(iid)352   -5.73397    0.72012  -7.963 1.69e-15 ***
## factor(iid)353   -6.20660    1.08645  -5.713 1.11e-08 ***
## factor(iid)354   -5.50444    0.58915  -9.343  < 2e-16 ***
## factor(iid)355   -7.89540    1.08269  -7.292 3.05e-13 ***
## factor(iid)356   -7.05350    1.15004  -6.133 8.61e-10 ***
## factor(iid)357   -6.53860    0.72848  -8.976  < 2e-16 ***
## factor(iid)358   -7.00573    0.86411  -8.107 5.17e-16 ***
## factor(iid)359   -6.01097    0.78141  -7.692 1.44e-14 ***
## factor(iid)360   -5.34525    0.69108  -7.735 1.04e-14 ***
## factor(iid)361   -6.14984    1.11055  -5.538 3.07e-08 ***
## factor(iid)362   -5.62873    0.66236  -8.498  < 2e-16 ***
## factor(iid)363   -6.00559    0.84068  -7.144 9.09e-13 ***
## factor(iid)364   -5.85159    0.62627  -9.344  < 2e-16 ***
## factor(iid)365   -7.87993    1.11224  -7.085 1.39e-12 ***
## factor(iid)366   -5.17772    0.63403  -8.166 3.18e-16 ***
## factor(iid)367   -7.01104    0.82050  -8.545  < 2e-16 ***
## factor(iid)368   -5.68008    0.64704  -8.779  < 2e-16 ***
## factor(iid)369   -4.85874    0.70814  -6.861 6.83e-12 ***
## factor(iid)370   -8.12671    1.08866  -7.465 8.34e-14 ***
## factor(iid)371   -6.55615    0.72811  -9.004  < 2e-16 ***
## factor(iid)372   -6.63617    0.84769  -7.829 4.93e-15 ***
## factor(iid)373   -6.83673    0.74475  -9.180  < 2e-16 ***
## factor(iid)374   -6.65683    1.09746  -6.066 1.31e-09 ***
## factor(iid)375   -7.04098    0.83565  -8.426  < 2e-16 ***
## factor(iid)376   -6.84032    0.86282  -7.928 2.23e-15 ***
## factor(iid)377   -6.74797    1.09174  -6.181 6.37e-10 ***
## factor(iid)378   -7.59510    1.11763  -6.796 1.08e-11 ***
## factor(iid)379   -6.50000    0.82866  -7.844 4.36e-15 ***
## factor(iid)380   -6.68122    0.83472  -8.004 1.20e-15 ***
## factor(iid)381   -6.86774    0.88537  -7.757 8.70e-15 ***
## factor(iid)382   -5.62346    0.70986  -7.922 2.34e-15 ***
```

```
## factor(iid)383   -6.28154    0.85563  -7.341 2.11e-13 ***
## factor(iid)384   -5.99093    0.63643  -9.413  < 2e-16 ***
## factor(iid)385   -6.13677    0.70077  -8.757  < 2e-16 ***
## factor(iid)386   -6.69167    0.84732  -7.897 2.85e-15 ***
## factor(iid)387   -5.71911    0.67053  -8.529  < 2e-16 ***
## factor(iid)388  -24.49543 2883.89368  -0.008 0.993223
## factor(iid)389   -5.30293    0.60087  -8.825  < 2e-16 ***
## factor(iid)390   -4.40215    0.77727  -5.664 1.48e-08 ***
## factor(iid)391   -7.65783    1.09494  -6.994 2.67e-12 ***
## factor(iid)392  -24.44562 2623.17388  -0.009 0.992565
## factor(iid)393   -5.74964    0.74779  -7.689 1.48e-14 ***
## factor(iid)394  -24.06595 2659.59256  -0.009 0.992780
## factor(iid)395   -6.29232    0.83561  -7.530 5.07e-14 ***
## factor(iid)396   -5.89616    0.82923  -7.110 1.16e-12 ***
## factor(iid)397   -6.70685    1.10878  -6.049 1.46e-09 ***
## factor(iid)398   -5.79205    0.66085  -8.765  < 2e-16 ***
## factor(iid)399   -6.91845    1.16335  -5.947 2.73e-09 ***
## factor(iid)400   -6.06883    0.77359  -7.845 4.33e-15 ***
## factor(iid)401   -7.02645    1.09901  -6.393 1.62e-10 ***
## factor(iid)402   -6.76560    1.09051  -6.204 5.50e-10 ***
## factor(iid)403   -6.34762    1.09318  -5.807 6.38e-09 ***
## factor(iid)404   -5.36006    0.66707  -8.035 9.34e-16 ***
## factor(iid)405  -24.03647 2779.32411  -0.009 0.993100
## factor(iid)406   -5.79408    0.67922  -8.530  < 2e-16 ***
## factor(iid)407   -7.32668    0.87213  -8.401  < 2e-16 ***
## factor(iid)408   -5.56346    0.70989  -7.837 4.61e-15 ***
## factor(iid)409   -5.91203    0.69368  -8.523  < 2e-16 ***
## factor(iid)410   -7.12674    0.87890  -8.109 5.12e-16 ***
## factor(iid)411   -7.69579    1.10440  -6.968 3.21e-12 ***
## factor(iid)412   -6.04669    0.86491  -6.991 2.73e-12 ***
## factor(iid)413  -24.26250 2925.73952  -0.008 0.993383
## factor(iid)414   -4.16435    0.66644  -6.249 4.14e-10 ***
## factor(iid)415  -22.54546 2657.09861  -0.008 0.993230
## factor(iid)416   -4.06381    0.93012  -4.369 1.25e-05 ***
## factor(iid)417   -6.14896    1.18900  -5.172 2.32e-07 ***
## factor(iid)418  -24.55983 3917.98580  -0.006 0.994999
## factor(iid)419   -6.43012    1.13945  -5.643 1.67e-08 ***
## factor(iid)420   -6.40170    0.89631  -7.142 9.18e-13 ***
## factor(iid)421   -5.40426    0.93434  -5.784 7.29e-09 ***
## factor(iid)422   -3.61150    0.95745  -3.772 0.000162 ***
## factor(iid)423   -5.22170    1.19648  -4.364 1.28e-05 ***
## factor(iid)424   -4.57545    1.04368  -4.384 1.17e-05 ***
## factor(iid)425  -22.34300 4575.41474  -0.005 0.996104
## factor(iid)426   -5.05336    1.29815  -3.893 9.91e-05 ***
## factor(iid)427  -23.31659 4170.13603  -0.006 0.995539
## factor(iid)428   -5.13929    0.93918  -5.472 4.45e-08 ***
## factor(iid)429   -5.29037    1.21229  -4.364 1.28e-05 ***
## factor(iid)430  -24.00093 2787.52412  -0.009 0.993130
## factor(iid)431   -5.39355    0.88127  -6.120 9.34e-10 ***
## factor(iid)432   -4.68850    0.65105  -7.201 5.96e-13 ***
## factor(iid)433   -6.09861    0.83871  -7.271 3.56e-13 ***
## factor(iid)434   -7.29713    1.09669  -6.654 2.86e-11 ***
## factor(iid)435   -5.49018    0.84517  -6.496 8.25e-11 ***
## factor(iid)436   -7.01283    0.92866  -7.552 4.30e-14 ***
```

```
## factor(iid)437   -5.59382    0.73703  -7.590 3.21e-14 ***
## factor(iid)438   -6.35011    0.87961  -7.219 5.23e-13 ***
## factor(iid)439   -5.78746    0.72342  -8.000 1.24e-15 ***
## factor(iid)440  -25.56473 3342.75089  -0.008 0.993898
## factor(iid)441   -6.38867    0.96064  -6.650 2.92e-11 ***
## factor(iid)442   -4.42712    0.80732  -5.484 4.16e-08 ***
## factor(iid)443  -24.43060 3548.50280  -0.007 0.994507
## factor(iid)444  -24.91774 3083.03738  -0.008 0.993551
## factor(iid)445   -6.98283    1.14474  -6.100 1.06e-09 ***
## factor(iid)446   -4.85524    0.82000  -5.921 3.20e-09 ***
## factor(iid)447   -7.22099    1.18605  -6.088 1.14e-09 ***
## factor(iid)448   -5.33377    0.90487  -5.895 3.76e-09 ***
## factor(iid)449   -6.60598    0.92791  -7.119 1.09e-12 ***
## factor(iid)450   -5.89437    0.88234  -6.680 2.38e-11 ***
## factor(iid)451  -23.04104 3124.75703  -0.007 0.994117
## factor(iid)452   -6.35988    1.17890  -5.395 6.86e-08 ***
## factor(iid)453   -5.52987    1.17788  -4.695 2.67e-06 ***
## factor(iid)454  -23.84212 4679.23908  -0.005 0.995935
## factor(iid)455  -24.38573 4529.55034  -0.005 0.995704
## factor(iid)456   -6.36320    1.18087  -5.389 7.10e-08 ***
## factor(iid)457  -24.71748 4347.82082  -0.006 0.995464
## factor(iid)458   -4.77429    0.98600  -4.842 1.28e-06 ***
## factor(iid)459  -24.05244 4772.63121  -0.005 0.995979
## factor(iid)460   -6.84023    1.19138  -5.741 9.39e-09 ***
## factor(iid)461  -23.29908 3964.88132  -0.006 0.995311
## factor(iid)462   -4.66100    1.16900  -3.987 6.69e-05 ***
## factor(iid)463  -23.64043 4224.78151  -0.006 0.995535
## factor(iid)464   -6.54795    1.15682  -5.660 1.51e-08 ***
## factor(iid)465  -23.78443 4321.21501  -0.006 0.995608
## factor(iid)466  -24.74007 2772.03247  -0.009 0.992879
## factor(iid)467   -4.84392    0.66321  -7.304 2.80e-13 ***
## factor(iid)468   -6.88238    0.76251  -9.026  < 2e-16 ***
## factor(iid)469   -5.87929    0.69951  -8.405  < 2e-16 ***
## factor(iid)470   -6.13834    0.70029  -8.765  < 2e-16 ***
## factor(iid)471   -7.30466    1.12578  -6.489 8.67e-11 ***
## factor(iid)472   -6.76683    0.86432  -7.829 4.91e-15 ***
## factor(iid)473   -6.96489    1.12111  -6.212 5.22e-10 ***
## factor(iid)474   -6.44829    0.70893  -9.096  < 2e-16 ***
## factor(iid)475   -7.15926    0.88359  -8.102 5.39e-16 ***
## factor(iid)476   -7.36547    1.14915  -6.409 1.46e-10 ***
## factor(iid)477  -24.48101 2746.03602  -0.009 0.992887
## factor(iid)478   -7.58145    0.87535  -8.661  < 2e-16 ***
## factor(iid)479  -24.83323 2612.56355  -0.010 0.992416
## factor(iid)480   -7.06430    0.78010  -9.056  < 2e-16 ***
## factor(iid)481   -7.08568    1.12504  -6.298 3.01e-10 ***
## factor(iid)482   -5.41841    0.73333  -7.389 1.48e-13 ***
## factor(iid)483  -24.51300 2555.83734  -0.010 0.992348
## factor(iid)484   -5.14706    0.91084  -5.651 1.60e-08 ***
## factor(iid)485   -5.76023    0.87615  -6.574 4.88e-11 ***
## factor(iid)486  -23.82310 2848.47038  -0.008 0.993327
## factor(iid)487  -23.18437 2587.67017  -0.009 0.992851
## factor(iid)488   -5.74121    0.85160  -6.742 1.57e-11 ***
## factor(iid)489   -4.77705    0.67142  -7.115 1.12e-12 ***
## factor(iid)490   -6.89986    1.22627  -5.627 1.84e-08 ***
```

```
## factor(iid)491   -6.14864    0.69581  -8.837  < 2e-16 ***
## factor(iid)492   -5.14958    0.70411  -7.314 2.60e-13 ***
## factor(iid)493   -7.68151    1.11826  -6.869 6.46e-12 ***
## factor(iid)494   -6.79353    0.91361  -7.436 1.04e-13 ***
## factor(iid)495   -5.94596    0.91066  -6.529 6.61e-11 ***
## factor(iid)496   -6.31420    1.20685  -5.232 1.68e-07 ***
## factor(iid)497  -24.28311 3873.65068  -0.006 0.994998
## factor(iid)498  -24.08533 4322.76791  -0.006 0.995554
## factor(iid)499   -5.00935    0.91772  -5.458 4.80e-08 ***
## factor(iid)500   -5.88345    1.13575  -5.180 2.22e-07 ***
## factor(iid)501   -6.32599    1.15033  -5.499 3.81e-08 ***
## factor(iid)502  -23.23553 4253.22055  -0.005 0.995641
## factor(iid)503  -22.62999 4008.12670  -0.006 0.995495
## factor(iid)504   -4.96674    0.97924  -5.072 3.94e-07 ***
## factor(iid)505  -23.56209 4330.58009  -0.005 0.995659
## factor(iid)506  -22.31727 4434.81825  -0.005 0.995985
## factor(iid)507   -6.47741    1.15374  -5.614 1.97e-08 ***
## factor(iid)508   -5.51885    1.17837  -4.683 2.82e-06 ***
## factor(iid)509   -4.93260    0.60773  -8.116 4.80e-16 ***
## factor(iid)510   -5.92174    0.59595  -9.937  < 2e-16 ***
## factor(iid)511   -5.37605    0.64082  -8.389  < 2e-16 ***
## factor(iid)512   -7.33542    1.08525  -6.759 1.39e-11 ***
## factor(iid)513   -5.73312    0.58138  -9.861  < 2e-16 ***
## factor(iid)514  -23.63851 2210.34979  -0.011 0.991467
## factor(iid)515   -7.32672    1.08631  -6.745 1.53e-11 ***
## factor(iid)516   -5.52979    0.60169  -9.190  < 2e-16 ***
## factor(iid)517  -23.77459 2403.69471  -0.010 0.992108
## factor(iid)518   -5.62721    0.57075  -9.859  < 2e-16 ***
## factor(iid)519  -23.76008 2388.69088  -0.010 0.992064
## factor(iid)520  -23.73518 2335.39061  -0.010 0.991891
## factor(iid)521   -4.96036    0.55191  -8.988  < 2e-16 ***
## factor(iid)522   -5.75798    0.61130  -9.419  < 2e-16 ***
## factor(iid)523   -6.34888    0.87925  -7.221 5.17e-13 ***
## factor(iid)524   -4.05884    0.57851  -7.016 2.28e-12 ***
## factor(iid)525  -23.23430 2382.94736  -0.010 0.992221
## factor(iid)526   -6.10794    0.80987  -7.542 4.63e-14 ***
## factor(iid)527  -23.20897 2309.07831  -0.010 0.991980
## factor(iid)528  -23.96174 2420.62662  -0.010 0.992102
## factor(iid)529   -5.92564    0.82303  -7.200 6.03e-13 ***
## factor(iid)530   -5.77253    0.83366  -6.924 4.38e-12 ***
## factor(iid)531   -6.86167    1.07731  -6.369 1.90e-10 ***
## factor(iid)532   -4.42107    0.69291  -6.380 1.77e-10 ***
## factor(iid)533   -6.29911    0.83985  -7.500 6.37e-14 ***
## factor(iid)534   -4.90042    0.61401  -7.981 1.45e-15 ***
## factor(iid)535   -5.09577    0.59993  -8.494  < 2e-16 ***
## factor(iid)536   -6.10770    0.74068  -8.246  < 2e-16 ***
## factor(iid)537   -6.66262    0.82645  -8.062 7.52e-16 ***
## factor(iid)538   -5.91306    0.76764  -7.703 1.33e-14 ***
## factor(iid)539   -6.45810    0.84028  -7.686 1.52e-14 ***
## factor(iid)540   -6.95491    1.15796  -6.006 1.90e-09 ***
## factor(iid)541   -6.55588    1.07188  -6.116 9.58e-10 ***
## factor(iid)542   -5.01643    0.72377  -6.931 4.18e-12 ***
## factor(iid)543  -24.01657 2284.41562  -0.011 0.991612
## factor(iid)544   -5.81790    0.81112  -7.173 7.35e-13 ***
```

```
## factor(iid)545   -6.01893    0.86561  -6.953 3.57e-12 ***
## factor(iid)546   -5.80105    0.64783  -8.955  < 2e-16 ***
## factor(iid)547   -6.69800    0.81319  -8.237  < 2e-16 ***
## factor(iid)548   -5.52161    0.63481  -8.698  < 2e-16 ***
## factor(iid)549   -6.03757    0.73224  -8.245  < 2e-16 ***
## factor(iid)550   -5.20047    0.64318  -8.086 6.19e-16 ***
## factor(iid)551   -6.67956    0.83792  -7.972 1.57e-15 ***
## factor(iid)552   -5.73625    0.65836  -8.713  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 9747.0  on 7031  degrees of freedom
## Residual deviance: 4636.5  on 6474  degrees of freedom
##   (1347 observations deleted due to missingness)
## AIC: 5750.5
##
## Number of Fisher Scoring iterations: 18
```

5. Fit a multilevel model, allowing the intercept and the coefficients for the 6 ratings to vary by the rater i.

```
# Vary both intercept and slope
dating_reg5 <- glmer(match~attr_o +sinc_o +intel_o +fun_o +amb_o +shar_o+(1+attr_o|iid)+ (1+sinc_o|iid)+
```

```
## Warning in optwrap(optimizer, devfun, start, rho$lower, control =
## control, : convergence code 1 from bobyqa: bobyqa -- maximum number of
## function evaluations exceeded
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : unable to evaluate scaled gradient
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge: degenerate Hessian with 3
## negative eigenvalues
```

```
summary(dating_reg5)
```

```
## Generalized linear mixed model fit by maximum likelihood (Laplace
##   Approximation) [glmerMod]
##  Family: binomial  ( logit )
## Formula: match ~ attr_o + sinc_o + intel_o + fun_o + amb_o + shar_o +
##     (1 + attr_o | iid) + (1 + sinc_o | iid) + (1 + intel_o |
##     iid) + (1 + fun_o | iid) + (1 + amb_o | iid)
##    Data: dating
##
##      AIC      BIC   logLik deviance df.resid
##   5566.4   5717.3  -2761.2   5522.4     7009
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -1.9128 -0.4395 -0.2914 -0.1508 11.9350
##
## Random effects:
##  Groups Name        Variance  Std.Dev. Corr
##  iid    (Intercept) 2.953e-03 0.054339
##         attr_o      7.076e-03 0.084117 -1.00
```

```
##  iid.1  (Intercept) 4.200e-03 0.064811
##         sinc_o      2.454e-03 0.049541 -0.99
##  iid.2  (Intercept) 9.179e-03 0.095809
##         intel_o     5.177e-05 0.007195 -0.76
##  iid.3  (Intercept) 2.246e-01 0.473919
##         fun_o       5.861e-03 0.076556 -1.00
##  iid.4  (Intercept) 1.000e-03 0.031623
##         amb_o       8.229e-04 0.028687 -0.99
## Number of obs: 7031, groups:  iid, 551
##
## Fixed effects:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept) -5.82920    0.28999 -20.101  < 2e-16 ***
## attr_o       0.21776    0.02908   7.488 7.00e-14 ***
## sinc_o      -0.02652    0.03651  -0.726 0.467608
## intel_o      0.07597    0.03978   1.910 0.056150 .
## fun_o        0.26491    0.04076   6.499 8.06e-11 ***
## amb_o       -0.12815    0.03420  -3.747 0.000179 ***
## shar_o       0.22853    0.02368   9.652  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##         (Intr) attr_o sinc_o intel_ fun_o  amb_o
## attr_o  -0.181
## sinc_o  -0.148 -0.147
## intel_o -0.224 -0.049 -0.418
## fun_o   -0.385 -0.200 -0.142 -0.126
## amb_o   -0.011 -0.085 -0.091 -0.322 -0.219
## shar_o  -0.075 -0.077 -0.069 -0.008 -0.151 -0.205
## convergence code: 0
## unable to evaluate scaled gradient
## Model failed to converge: degenerate  Hessian with 3 negative eigenvalues
```

6. Compare the inferences from the multilevel model in (5) to the no-pooling model in (4) and the complete-pooling model from part (1) of the previous exercise.

```
anova(dating_reg5,dating_reg4,dating_reg1)
```

```
## Data: dating
## Models:
## dating_reg1: match ~ attr_o + sinc_o + intel_o + fun_o + amb_o + shar_o
## dating_reg5: match ~ attr_o + sinc_o + intel_o + fun_o + amb_o + shar_o +
## dating_reg5:     (1 + attr_o | iid) + (1 + sinc_o | iid) + (1 + intel_o |
## dating_reg5:     iid) + (1 + fun_o | iid) + (1 + amb_o | iid)
## dating_reg4: match ~ attr_o + sinc_o + intel_o + fun_o + amb_o + shar_o +
## dating_reg4:     factor(iid) - 1
##              Df    AIC    BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## dating_reg1   7 5625.0 5673.0 -2805.5   5611.0
## dating_reg5  22 5566.4 5717.3 -2761.2   5522.4  88.57     15  1.832e-12
## dating_reg4 557 5750.5 9570.4 -2318.2   4636.5 885.92    535  < 2.2e-16
##
## dating_reg1
## dating_reg5 ***
## dating_reg4 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The last model which is the vary intercept and slope model has the lowest AIC. However, the no-pooling model has significantly lower deviance.