5.1

7. (a) The problem should have geometric distribution.

$$P(T=n) = (\tfrac{5}{6})^{n-1}\tfrac{1}{6}$$

(b) $P(T>3) = 1-P(T\le 3) = 1-(\tfrac{1}{6}+\tfrac{5}{6}\times\tfrac{1}{6}+(\tfrac{5}{6})^2\times\tfrac{1}{6})$

$$= \frac{125}{216}$$

(c) $P(T>6 \mid T>3) = \dfrac{P(T>3\mid T>6)\,P(T>6)}{P(T>3)} = \dfrac{1\times(\frac{5}{6})^6}{(\frac{5}{6})^3} = (\tfrac{5}{6})^3 = \dfrac{125}{216}$

10. (a) The problem should follow Hypergeometric Distribution

$$P(x=k) = \frac{\binom{n_1}{k}\binom{N-n_1}{n_2-k}}{\binom{N}{n_2}}$$

(b) $X=k=n_{12}$

To maximize the expression in (a), we could use the ratio of $\underset{h(N+1,n_1,n_2,n_{12})}{\underset{\shortparallel}{N+1}}$ and $\underset{h(N,n_1,n_2,n_{12})}{\underset{\shortparallel}{N}}$, if the ratio changes from positive to negative, then we have our ~~maximum~~ value of N that maximize the expression in (a)

16. $P(X=k) = \dfrac{\lambda^k}{k!}e^{-\lambda}$ $\quad \lambda = np$

$n = 5\times 60 = 300$ $\quad P = 0.01$ $\quad \lambda = 3$

$P(X\le 1) = P(X=0)+P(X=1)$

$= \dfrac{\lambda^0}{0!}e^{-\lambda} + \dfrac{\lambda^1}{1!}e^{-\lambda}$

$= \cancel{3e^{-3}}\, e^{-\lambda} + 3e^{-\lambda}$

$= 4e^{-\lambda}$

$= 4e^{-3}$

18. (a) $P(X=0)$ $\quad P=\frac{1}{500}$ $\quad n=600$ $\quad \lambda=\frac{600}{500}=\frac{6}{5}$

$P(X=0)=e^{-\lambda}=e^{-\frac{6}{5}}=0.3011$

(b) $P(X=2)=\frac{\lambda^2}{2!}e^{-\lambda}$ $\quad P=\frac{1}{500}$ $\quad n=\cancel{600}\,400$ $\quad \lambda=\cancel{\frac{6}{5}}\,\frac{4}{5}$

$\cancel{=\frac{36}{25}\cdot\frac{1}{2}\cdot e^{-\frac{6}{5}}}$

$\cancel{=\frac{18}{25}e^{-\frac{6}{5}}\;\frac{9}{5}e^{-\frac{6}{5}}}$

$=\frac{16}{25}\times\frac{1}{2}\times e^{-\frac{4}{5}}$

$=0.1438$

(c) $P=\frac{1}{500}$ $\quad n=1000$ $\quad \lambda=2$

$P(X\geq 2)=1-P(X<2)=1-(P(X=0)+P(X=1))$

$=1-(e^{-\lambda}+\frac{\lambda}{1!}e^{-\lambda})$

$=1-(e^{-2}+2e^{-2})$

$=0.5939$

25. $n=100$ $\quad p=0.05$ $\quad \lambda=5$

$P(X=k)=\frac{\lambda^k}{k!}e^{-\lambda}$ $\quad$ k is number of times he got caught

The ~~expected~~ fines he expected to pay:

when $k=1$ amount $=0$ $\quad k=2$ amount $=2\times P(X=2)=2\times\frac{5^2}{2!}e^{-5}=25e^{-5}$

$k\geq 3$ $\quad$ amount $=\sum_{i=3}^{100}(2+5\times(i-2))\frac{\lambda^i}{i!}e^{-\lambda}=16.9865$

if he pays everytime $\quad$ amount: $100\times 0.1=10$

if he doesn't pay $\quad$ amount $=\underbrace{0}_{\text{first time}}+\underbrace{0.1684}_{\text{Second time}}+\underbrace{16.9865}_{\text{Third time to }100^{th}\text{ time}}=17.1549$

27. $n=100$ $\quad P=0.001$ $\quad \lambda=0.1$

$P(X=0)=e^{-0.1}$ $\quad \cancel{P(X=1)=\frac{0.1}{1}e^{-0.1}}$

$P(X\geq 1)=1-P(X=0)=1-e^{-0.1}=0.095$

28. $P=0.04 \quad n=100 \quad \lambda=4$

if all passengers show up or only one passenger doesn't show up, there would be passenger(s) don't have a seat

X is the number of passengers don't show up

$P(X \geq 2)$ is the probability every passenger gets seat.

$$P(X\geq 2) = 1 - P(X=0) - P(X=1)$$
$$= 1 - e^{-4} - 4e^{-4}$$
$$= 0.9084$$

38. (a) $\frac{1}{4} \times (\frac{3}{4})^4 \times C_5^1 = 0.3955$

(b) $\frac{C_5^1 C_{15}^4}{C_{20}^5} = 0.4402$

5.2

1. CDF : $\frac{x-a}{b-a}$ $x \in [a,b]$ PDF $\frac{1}{b-a}$ $x \in [a,b]$

(a) $Y = U + 2 \in [2,3]$

CDF : $F(x) = \frac{x-2}{3-2} = x-2 \quad x \in [2,3]$

PDF : $f(x) = \frac{1}{3-2} = 1 \quad x \in [\cancel{a}2, 3]$

(b) $Y = U^3 \in [0,1] \implies U = Y^{\frac{1}{3}}$

CDF : $F(x) = \frac{x^{\frac{1}{3}} - 0}{1-0} = x^{\frac{1}{3}} \quad x \in [0,1]$

PDF : $f(x) = F'(x) = \frac{1}{3} x^{-\frac{2}{3}} \quad x \in [0,1]$

17. (a) $f(x) = F'(x) = \frac{\pi}{2} \sin(\pi x) \quad x \in [0,1]$

(We know that $\frac{d}{dx}(\sin^2(x)) = \sin(2x)$)

(b) $P(X < \frac{1}{4}) = F(\frac{1}{4}) = \sin^2(\pi \cdot \frac{1}{4} \cdot \frac{1}{2}) = 0.1464$

21. From Theorem 5.1 on page 216, we have

$$F_Y(y) = P(Y \leq y) = P(F(X) \leq y) = P(X \leq F^{-1}(y)) = F_X(F^{-1}(y)) = y$$

37.