Permutation Test

1.

| $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | $X_7$ | $X_8$ | | |
|---|---|---|---|---|---|---|---|---|---|
| 0.225 | 0.262 | 0.217 | 0.240 | 0.230 | 0.229 | 0.235 | 0.217 | | |

| $Y_1$ | $Y_2$ | $Y_3$ | $Y_4$ | $Y_5$ | $Y_6$ | $Y_7$ | $Y_8$ | $Y_9$ | $Y_{10}$ |
|---|---|---|---|---|---|---|---|---|---|
| 0.209 | 0.205 | 0.196 | 0.210 | 0.202 | 0.207 | 0.224 | 0.223 | 0.220 | 0.201 |

$\bar{X} = \frac{1.855}{8} = 0.2319 \qquad \bar{Y} = \frac{2.097}{10} = 0.2097$

Wald statistics is $\dfrac{\bar{X} - \bar{Y}}{\sqrt{\frac{\sigma_X^2}{8} + \frac{\sigma_Y^2}{10}}} = 3.7 = Z$

$P(|Z| > 3.7) = 2P(Z > 3.7) = 0.0002$ P-value

Therefore we reject the $H_0$: no difference in means.

Confidence interval $\bar{X} - \bar{Y} \pm 2\sqrt{\frac{\sigma_X^2}{8} + \frac{\sigma_Y^2}{10}} = [0.01, 0.03]$

Conclusion: These are two different authors.

Simulation

Run a permutation test on the absolute difference of means gave me a p-value of [redacted] 0.004

Same conclusion as the previous test.

2. Hot Dog problem

$X_1, \dots, X_n \sim N(\mu, \sigma^2)$

$\mu_X = \frac{\sum_{i=1}^{n} X_i}{n} = \frac{\sum_{i=1}^{20} X_i}{20} = 156.66$

$\sigma_X^2 = \frac{\sum_{i=1}^{20}(X_i - \bar{X})^2}{n-1} = 512.66 \qquad \sigma_X = \sqrt{\sigma_X^2} = 22.64 \qquad \frac{\sigma_X}{\sqrt{n}} = 5.06 = sd$

$\alpha = 0.05 \qquad \phi^{-1}(1-\alpha) = \phi^{-1}(0.95) = 1.729$ (with $df = 19$)

Confidence Interval $[\mu_X - 1.729 \times 5.06, \mu_X + 1.729 \times 5.06]$

$= [148.1, 167.6]$

3. Reading Score problem

$$X_1, \dots, X_m \sim N(\mu_1, \sigma^2)$$

$$Y_1, \dots, Y_n \sim N(\mu_2, \sigma^2)$$

$H_0: \mu_1 \geq \mu_2 \qquad H_1: \mu_1 < \mu_2$

The test statistic is

$$U = \frac{(m+n-2)^{\frac{1}{2}} (\bar{X}_m - \bar{Y}_n)}{\left(\frac{1}{m} + \frac{1}{n}\right)^{\frac{1}{2}} (S_X^2 + S_Y^2)^{\frac{1}{2}}}$$

$m = 8 \quad n = 6$

$\bar{X} = \frac{1}{8} \sum_1^8 X_i = 1.5125$

$\bar{Y} = \frac{1}{6} \sum_1^6 Y_i = 1.6683$

$S_X^2 = \sum_1^8 (X_i - \bar{X}) = 0.18075$

$S_Y^2 = \sum_1^6 (Y_i - \bar{Y}) = 0.16768$

$$\therefore U = \frac{\sqrt{12} \times (1.5125 - 1.6683)}{\left(\frac{1}{8} + \frac{1}{6}\right)^{\frac{1}{2}} \times (0.18075 + 0.16768)^{\frac{1}{2}}}$$

$$= -1.6429$$

$df = m+n-2 = 12 \qquad \alpha = 0.1$

$U \sim t_{12}$

$t_{12}(1-\alpha) = t_{12}(0.9) = 1.365$

Reject $H_0$ when $U \leq -T^{-1}_{m+n-2}(1-\alpha)$

In this question $-T^{-1}_{m+n-2}(1-\alpha) = -1.365$

$\therefore U \leq -T^{-1}_{m+n-2}(1-\alpha)$

So we reject $H_0$

Mean of group 1 is not large than or equal to group