C4

Hypothesis Test

1. $X \sim Exp(\lambda)$ $f(x) = \lambda e^{-\lambda x}$ $F(x) = 1 - e^{-\lambda x}$

$H_0: \beta \geq 1$ $H_1: \beta < 1$ Rej. if $X \geq 1$

(a) $B(\beta) = P(X \geq 1|\beta) = 1 - P(X < 1)$

$\therefore B(\lambda) = 1 - (1 - e^{-\lambda}) = e^{-\lambda}$

(b)

1. $X \sim Exp(\lambda)$ $f(x) = \lambda e^{-\lambda x}$ $F(x) = 1 - e^{-\lambda x}$

$H_0: \lambda \geq 1$ $H_1: \lambda < 1$ Rej. if $X \geq 1$

(a) $B(\lambda) = P(X \geq 1 | \lambda) = 1 - P(X < 1)$

$\therefore B(\lambda) = 1 - (1 - e^{-\lambda}) = e^{-\lambda}$

(b) $\alpha = \sup_{\lambda \geq 1} B(\lambda) = e^{-1} = 0.3678$

2. It is a binomial distribution, two-tail problem

$f(Y) = \binom{n}{Y} p^Y (1-p)^{n-Y}$

$n = 20$

$\therefore f(Y) = \binom{20}{Y} p^Y (1-p)^{20-Y}$

$H_0: p = 0.2$ $H_1: p \neq 0.2$

Rej.: $Y \geq 7$ or $Y \leq 1$

(a) $B(P) = P(Y \geq 7 | P) + P(Y \leq 1 | P)$

$= 1 - P(Y < 7 | P) + P(Y \leq 1 | P)$

$= 1 - \sum_{0}^{6} \binom{20}{Y} p^Y (1-p)^{20-Y} + \sum_{0}^{1} \binom{20}{Y} p^Y (1-p)^{20-Y}$

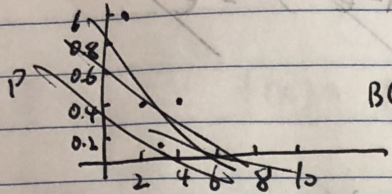
$= \sum_{0,1,7 \cdots 20} \binom{20}{Y} p^Y (1-p)^{20-Y}$

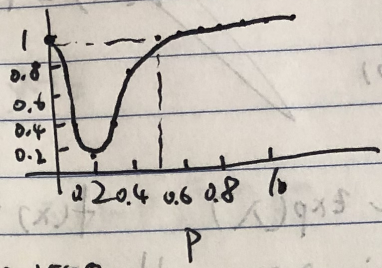
$B(P=0)=1 \quad B(P=0.1)=0.394 \quad B(P=0.2)=0.1558$

$B(P=0.3)=0.399 \quad B(P=0.4)=0.7505 \quad B(P=0.5)=0.9423$

$B(P=0.6)=0.993 \quad B(P=0.7)=0.999 \quad B(P=0.8)=0.999$

$B(P=0.9)=1 \quad B(P=1)=1$





(b) $\alpha = \sup_{p=0.2} B(P) = 0.1558$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$$

$H_0: \mu=\mu_0 \quad H_1: \mu \neq \mu_0$

Rej: $T(x) > c \quad |\bar{X}-\mu_0| > c \qquad \alpha = 0.05$

$$B(\mu) = P(T(x)>c) = P(|\bar{x}-\mu_0|>c) = P\left(\frac{\sqrt{n}|\bar{x}-\mu_0|}{\sigma} > \frac{\sqrt{n}c}{\sigma}\right)$$

$$= P\left(|z| > \frac{\sqrt{n}c}{\sigma}\right)$$

$\alpha = B(\mu) = 0.05$

$$= 1 - \phi\left(\frac{\sqrt{n}c}{\sigma}\right)$$

$$C = \frac{\phi^{-1}(0.975)}{\sqrt{n}} = \frac{1.96}{5} = 0.392$$

$f(P, X) = \binom{9}{x} P^x (1-P)^{9-x}$

$H_0: p=0.4 \qquad H_1: P \neq 0.4$

(a) $P(Y \le C_1 | p=0.4) + P(Y \ge C_2 | P=0.4)$

$= P(Y \le C_1 | P=0.4) + 1 - P(Y < C_2 | P=0.4) < 0.1$

$\therefore \; 0.9 < P(Y < C_2 | P=0.4) - P(Y \le C_1 | P=0.4)$

When $c_1 \geq 2$ $\quad P(Y \leq c_1 \mid p=0.4) \geq 0.23$

$c_2 \leq 5$ $\quad P(Y \geq c_2 \mid p=0.4) \geq 0.26$

$\therefore$ $c_1 \leq 1$ $\quad c_2 \geq 6$

c_1	c_2	P
1	6	0.169
1	7	0.0956
0	6	0.169
-1	6	0.993

$\therefore$ We choose $c_1 = 1$ $\quad c_2 = 7$

(b) ~~The size is 0.0956~~

$$B(p) = P(Y \geq 7 \mid p) + P(Y \leq 1 \mid p)$$

$$= \sum_{0,1,7..9} \binom{9}{Y} p^{Y} (1-p)^{9-Y}$$

$$\alpha = \sup_{p=0.4} B(p) = 0.0956$$