1. Use ~~Binomial~~ Bernoulli Distribution $f(x,p) = p^x(1-p)^{1-x}$

$$L(x,p) = f(x_1)f(x_2)\cdots f(x_{70})$$
$$= p^{\Sigma x_i}(1-p)^{70-\Sigma x_i}$$

$$\ell(x,p) = \Sigma x_i \log p + (70-\Sigma x_i)\log(1-p)$$

$$\frac{\partial \ell}{\partial p} = \frac{\Sigma x_i}{p} - \frac{70-\Sigma x_i}{1-p} = 0$$

$$\Sigma x_i - p\Sigma x_i - 70p + p\Sigma x_i = 0$$
$$\hat{p} = \frac{\Sigma x_i}{70}$$

$$\Sigma x_i = 12$$
$$\therefore \hat{p} = \frac{12}{70} = \frac{6}{35}$$

2. ~~From the previous / above problem, we know $\hat{\theta} = \frac{\Sigma x_i}{n}$~~

~~when all x_i are 0 $\hat{\theta} = \frac{0}{n} = 0$~~

~~x_i are 1 $\hat{\theta} = \frac{n}{n} = 1$~~

From the above problem, we have

$$L(x,\theta) = \theta^{\Sigma x_i}(1-\theta)^{n-\Sigma x_i}$$

when x are all 0

$L(x,\theta) = (1-\theta)^n$ $\quad \ell(x,\theta) = n\log(1-\theta)$

$\frac{\partial \ell}{\partial \theta} = \frac{n}{1-\theta} = 0$ can't solve for $\hat{\theta}$

when x are all 1

$L(x,\theta) = \theta^n$ $\quad \ell(x,\theta) = n\log\theta$

$\frac{\partial \ell}{\partial \theta} = \frac{n}{\theta} = 0$ can't solve for $\hat{\theta}$

3. $X_1, \ldots, X_n \sim Po(\lambda)$ $f(x) = \frac{\lambda^x e^{-\lambda}}{x!}$

$$L(x,\lambda) = f(x_1)\, f(x_2)\, f(x_3) \cdots f(x_n)$$
$$= \frac{\lambda^{x_1} e^{-\lambda}}{x_1!} \cdot \frac{\lambda^{x_2} e^{-\lambda}}{x_2!} \cdots \frac{\lambda^{x_n} e^{-\lambda}}{x_n!}$$
$$= \frac{\lambda^{\Sigma x_i} e^{-\lambda n}}{\sum_{i=1}^{n} x_i!}$$

$$\ell(x,\lambda) = \Sigma x_i \log\lambda + (-n\lambda) - \log \Sigma x_i!$$
$$= \Sigma x_i \log\lambda - n\lambda - \Sigma \log x_i!$$

$$\frac{\partial \ell(x,\lambda)}{\partial \lambda} = \frac{\Sigma x_i}{\lambda} - n = 0$$
$$\Rightarrow \hat{\lambda} = \frac{\Sigma x_i}{n}$$

when all observations are 0

$$L(x,\lambda) = \frac{e^{-\lambda n}}{n}$$
$$\ell(x,\lambda) = -\lambda n - \log n$$
$$\frac{\partial \ell(x,\lambda)}{\partial \lambda} = -n = 0$$

no value for λ

4. $X_1, \ldots, X_n \sim N(\mu, \sigma^2)$ $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2\sigma^2}(x-\mu)^2\right\}$

$$L(x,\mu,\sigma^2) = \prod \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2\sigma^2}(X_i-\mu)^2\right\}$$
$$= \frac{(2\pi)^{-\frac{n}{2}}}{\sigma^n} \exp\left\{-\frac{\Sigma(X_i-\mu)^2}{2\sigma^2}\right\}$$

$$\ell(x,\mu,\sigma^2) = -\frac{n}{2}\log(2\pi) - n\log\sigma + \left(-\frac{\Sigma(X_i-\mu)^2}{2\sigma^2}\right)$$

$$\frac{\partial \ell(x,\mu,\sigma^2)}{\partial \sigma} = -\frac{n}{\sigma} + \frac{\Sigma(X_i-\mu)^2}{\sigma^3} = 0 \Rightarrow \hat{\sigma} = \sqrt{\frac{\Sigma(X_i-\mu)^2}{n}}$$

$$\hat{\sigma}^2 = \frac{\Sigma(X_i-\mu)^2}{n}$$