Deep Learning: Homework 1

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Problem 1

1. Multilayer perceptron: Solar radiation map

2. Convolutional Neural Network: Drug discovery

3. Recurrent Neural Network: Digital asset management in marketing

4. Autoencoder: One-class classification

5. Generative Adversarial Network: Create clothing images and styles from an image

6. Deep reinforcement learning: Personalized news recommendations

Problem 2

1. Let A, B be positive definite matrixs, then $\forall x \in \mathbf{R}^n \setminus 0$, $x^T A x > 0$ and $x^T B x > 0$. Therefore, $x^T A x + x^T B x > 0 \Rightarrow x^T (A + B) x > 0$. Hence, $x^T (A + B) x > 0 \, for all x \in \mathbf{R}^n \setminus 0$ which implies that the sum of two PD matrices is also PD.

2. Let A be a PD matrice and B be a PSD matrix, then $\forall x \in \mathbf{R}^n \setminus 0$, $x^T A x > 0$ and $x^T B x \geq 0$. Therefore, A+B $= x^T A x + x^T B x > 0 \Rightarrow x^T (A + B) x > 0$. Hence, the sum of a PD and PSD is PD.

3. (a) A is not a square matrix, so neither PD and PSD.

   (b) $A^T A$ is PD, since it is a symmetric matrix and its eigenvalues [22.911286 68.088714] are all greater than zero.

   (c) AA^T is PSD, since its eigenvalues[6.80887140e+01 -4.88498131e-15 2.29112860e+01] are all greater or very close to zero.

   (d) B is PD, since it is a symmetric matrix and its eigenvalues [1 1 1] are all greater than zero.

(e) $-B$ is neither PD and PSD, since its eigenvalues [-1 -1 -1] are all less than zero.

(f) C is PD, since it is a symmetric matrix and its eigenvalues [5.82842712 1. 0.17157288] are all greater than zero.

(g) $C - 0.1 \times B$ is PD, since it is a symmetric matrix and its eigenvalues [5.72842712 0.9 0.07157288] are all greater than zero.

(h) $C - 0.01 \times AA^T$ is neither PD and PSD, since its eigenvalues [4.46822067 -1.37698649 -5.19123418] are less than zero.

Problem 3

1.

$$\nabla f_1 = \begin{bmatrix} 2v_1 + 3e^{v_2} \\ 3v_1 e^{v_2} \end{bmatrix}$$

$$\nabla^2 f_1 = \begin{bmatrix} 2 & 3e^{v_2} \\ 3e^{v_2} & 3v_1 e^{v_2} \end{bmatrix}$$

2.

$$\nabla f_2 = \begin{bmatrix} 12v_1^2 v_2 - v_2 \log v_2 \\ 4v_1^3 - v_1 \log v_2 - v_1 \end{bmatrix}$$

$$\nabla^2 f_2 = \begin{bmatrix} 24v_1 v_2 & 12v_1^2 - \log v_2 - 1 \\ 12v_1^2 - \log v_2 - 1 & -\frac{v_1}{v_2} \end{bmatrix}$$

3.

$$J = \begin{bmatrix} 2v_1 + 3e^{v_2} & 3v_1 e^{v_2} \\ 12v_1^2 v_2 - v_2 \log v_2 & 4v_1^3 - v_1 \log v_2 - v_1 \end{bmatrix}$$

Problem 4

1. (a) $Pr(X = x, Y = y) \leq Pr(X = x)$
   The probability for x happen should be equal or grater than the probability that x and y are both happen, since we need to consider that the probability for y happen should be less or equal to one.

   (b) It depends on whether this two random variables are dependent or independent. If they are independent random variables, $Pr(X = x|Y = y) = Pr(X = x)$.

   (c) $Pr(X = x|Y = y) \geq Pr(Y = y|X = x)Pr(X = x)$
   Since $Pr(X = x|Y = y) = \frac{Pr(X=x,Y=y)}{Pr(Y=y)}$,
   $Pr(Y = y|X = x)Pr(X = x) = \frac{Pr(X=x,Y=y)}{Pr(X=x)} Pr(X = x)$,
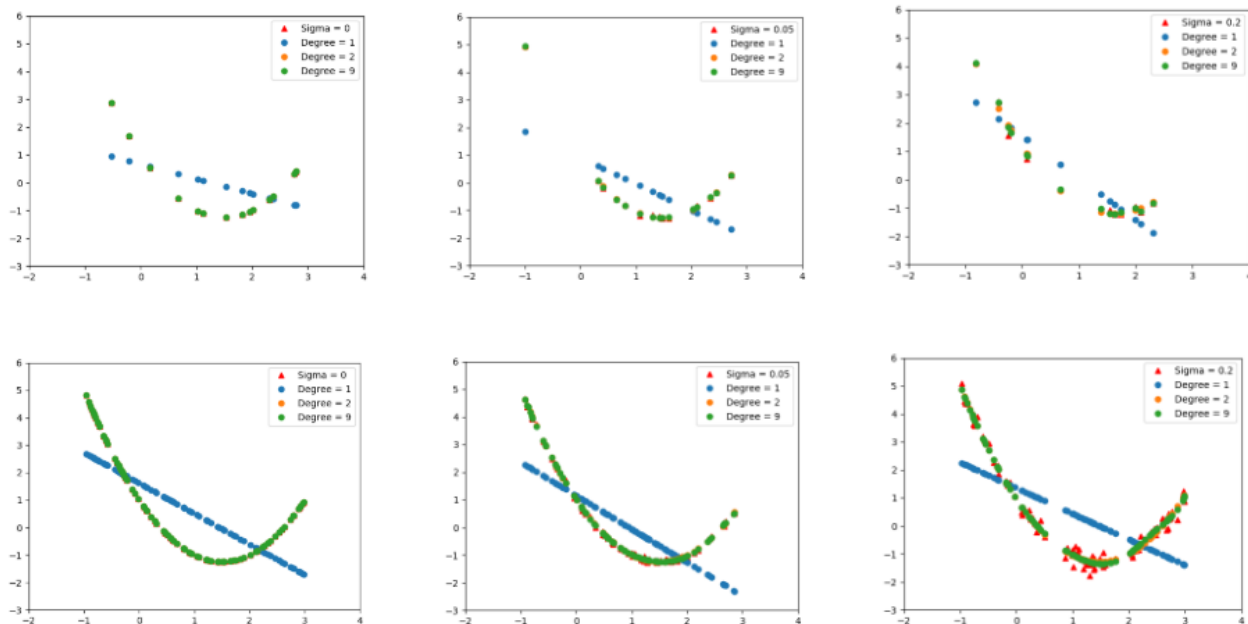   and $Pr(Y = y) \leq 1$.

Problem 5

1. $mse = \frac{1}{n} \sum_{i=1}^{n} (w^T x_i + w_0 - y_i)^2$

   $= \frac{1}{n} [wx - y]^T [wx - y]$

   $= \frac{1}{n} [(wx)^T (wx) + (wx)^T (-y) - y^T (wx) - y^T (-y)]$

   $= \frac{1}{n} [(wx)^T (wx) + y^T y]$

   $\frac{\partial mse}{\partial w} = \frac{1}{n} [2x^T wx - 2x^T y] = 0$

   $w* = (x^T x)^{-1} x^T y$

2. According to the plots, when degree $= 9$ and sigma $=0.2$ is looks like overfit the data.

N	Sigma	Degree	MSE	Weights									
15	0	1	14.54316837	0.93098078	-0.65713384								
		2	5.55E-28	1	-3	1							
		9	2.80E-15	1.00E+00	-3.00E+00	1.00E+00	1.60E-06	-1.73E-07	-3.05E-06	3.60E-06	-1.79E-06	4.19E-07	-3.81E-08
	0.05	1	28.76266461	0.8606674	-0.71379981								
		2	0.02112102453	1.00381929	-3.04773459	1.01979942							
		9	0.009026564303	1.11E+00	-3.06E+00	1.51E-01	6.76E-01	1.16E+00	-2.02E+00	1.20E+00	-3.15E-01	2.81E-02	8.02E-04
	0.2	1	14.22869484	1.17603072	-0.68335129								
		2	0.3750097712	1.0469403	-2.96487379	0.97058011							
		9	0.2102336807	1.17952488	-3.14060687	-0.21991351	1.63241144	-0.25149625	-1.39742186	1.99768538	-1.23214554	0.3520455	-0.03783919
100	0	1	154.1210067	1.69180011	-1.27390864								
		2	5.50E-27	1	-3	1							
		9	1.22E-16	1	-3	1.00E+00	-3.81E-09	4.29E-08	-1.92E-08	-2.15E-08	2.16E-08	-6.78E-09	7.35E-10
	0.05	1	133.231257	0.99957878	-0.77560223								
		2	0.2513029899	1.0062637	-3.000359	0.99806238							
		9	0.226077497	1.03E+00	-3.02E+00	7.94E-01	1.62E-01	1.81E-01	-1.94E-01	2.29E-02	2.53E-02	-8.82E-03	8.09E-04
	0.2	1	146.0396365	1.495408	-1.16811145								
		2	3.781836257	0.97709135	-2.9829376	0.99767143							
		9	3.251049268	0.96897598	-3.36094628	1.29685125	0.80669999	-1.14002863	0.06391825	0.79551026	-0.62026314	0.19046528	-0.02135225