Julia SVM Helper Functions

```
## bv Oin Yu. Feb 2019
## using Julia 1.0.3
gaussian_kernel_matrix(X, Y) = # X's rows are input tensors.
    exp.(- y * pairwise(SqEuclidean(), Matrix(X), dims=1))
gaussian_kernel_matrix(X, X_test, y) = # X's rows are input tensors
    exp.(- y * pairwise(SqEuclidean(), Matrix(X_test), Matrix(X), dims=1))
{\bf function} \ {\bf extract\_each\_classes}({\it X\_ALL}, \ {\it y\_ALL})
    CLASSES = sort(unique(v ALL))
    NO_OF_CLASSES = length(CLASSES)
    L = [sum(y_ALL .== class) for class in CLASSES]
    X_{\text{EACH\_train}} = [X_{\text{ALL}}[:,:,y_{\text{ALL}}] = \text{class}] for class in CLASSES]
    Xs = [hcat(transpose(reshape(X_EACH_train[i], 28*28, L[i])), ones(L[i])) for i = 1:NO_OF_CLASSES]
    ys = ones.(Int, L) .* CLASSES
    return L, Xs, ys
end
function extract_all_test(X_ALL, y_ALL)
    Xs = hcat(transpose(reshape(X_ALL, 28*28, :)), ones(size(X_ALL, 3)))
    return Xs, y_ALL
function prepare_one_vs_rest(X_ALL, y_ALL)
    X = hcat(transpose(reshape(X_ALL, 28*28, :)), ones(size(X_ALL, 3)))
    ys = [(y -> y == pos\_label ? 1 : -1).(y\_ALL) for pos\_label = 0:9]
    return X, ys
function prepare_MNIST_data(class_pos, class_neg, y)
    X_{ALL_{train}}, y_{ALL_{train}} = MNIST.traindata(Float32)
    NO_OF_NEGATIVE = sum(y_ALL_train .== class_neg)
    \label{eq:no_of_positive} \mbox{NO\_OF\_POSITIVE = } \mbox{sum}(\mbox{$y$\_ALL\_train .== class\_pos})
    1 = NO OF NEGATIVE + NO OF POSITIVE
    X_neg_train = X_ALL_train[:,:,y_ALL_train .== class_neg]
    X_pos_train = X_ALL_train[:,:,y_ALL_train .== class_pos]
    y_neg_train = - ones(Int, NO_OF_NEGATIVE)
y_pos_train = ones(Int, NO_OF_POSITIVE)
    randomisation = randperm(1)
    X = hcat(transpose(reshape(cat(X_neg_train, X_pos_train; dims=3), 28*28, 1)), ones(1))[randomisation,:]
    y = vcat(y_neg_train, y_pos_train)[randomisation,:]
    gaussian_kernel_matrix(X, y) = exp.(-y * pairwise(SqEuclidean(), Matrix(X), dims=1)) # X's rows are input tensors.
    K = gaussian_kernel_matrix(X, y)
     # Load test data
    X_{ALL\_test}, y_{ALL\_test} = MNIST.testdata(Float32)
    NO_OF_NEGATIVE_test = sum(y_ALL_test .== class_neg)
    NO_OF_POSITIVE_test = sum(y_ALL_test .== class_pos)
    1 test = NO OF NEGATIVE test + NO OF POSITIVE test
    X_neg_test = X_ALL_test[:,:,y_ALL_test .== class_neg]
    X_pos_test = X_ALL_test[:,:,y_ALL_test .== class_pos]
    y_{neg_{test}} = - ones(Int, NO_OF_NEGATIVE_test)
    y_pos_test = ones(Int, NO_OF_POSITIVE_test)
    X_{\text{test}} = \text{hcat}(\text{transpose}(\text{reshape}(\text{cat}(X_{\text{neg\_test}}, X_{\text{pos\_test}}; \text{dims=3}), 28*28, 1_{\text{test}})), ones(1_{\text{test}}))
    y_{\text{test}} = \text{vcat}(y_{\text{neg_test}}, y_{\text{pos_test}})
    gaussian\_kernel\_matrix(X, X\_test, \ \gamma) = exp.(-\ y * pairwise(SqEuclidean(), Matrix(X\_test), Matrix(X), dims=1))
    K_{\text{test}} = \text{gaussian\_kernel\_matrix}(X, X_{\text{test}}, \gamma)
    return (X, y, K, 1), (X_{test}, y_{test}, K_{test}, 1_{test})
end
# Only as termination criterion for the whole process.
function monitor_kkt_condition(\alpha, \sigma, y; accuracy=1e-2, C::Int32=Int32(1))
    for (idx, \alpha) in enumerate(\alpha)
         if \alpha == 0 \&\& yfx[idx] < 1
             return false
         elseif \alpha == C \&\& yfx[idx] > 1
             return false
         elseif 0 < \alpha < C \&\& !isapprox(y/x[idx], 1; atol=accuracy)
             return false
    return true
function select_new_points(\alpha, \sigma, y, M, support_vector_idx; accuracy=1e-2, C::Int32=Int32(1))
    yfx = y \cdot * \sigma
    list_of_new_ids = Int[]
    list of new min = Float64[]
    for (idx, \alpha) in enumerate(\alpha)
     if (\alpha == 0 && y/x[idx] < 1) || (\alpha == C && y/x[idx] > 1) || (0 < \alpha < C && !isapprox(y/x[idx], 1; atol=accuracy))
```

```
push!(list_of_new_ids, idx)
    push!(list_of_new_min, yfr[idx])
    end
end
list_of_new_ids = list_of_new_ids[sortperm(list_of_new_min)]
setdiff!(list_of_new_ids, support_vector_idx)
return list_of_new_ids
```

select_new_points (generic function with 1 method)

Julia SVM

```
#####################################
## by Qin Yu, Apr 2019
## using Julia 1.1.0
******************************
using Revise, BenchmarkTools
using JLD2, FileIO, MLDatasets
                                                                # Development
                                                                # Data & IO
using LinearAlgebra, Distances, Random, Distributions # Maths
using CUDAdrv, CUDAnative, CuArrays
MAX_MINI_BATCH_ID = 50
@inline sync_threads_and(predicate::Int32) = ccall("llvm.nvvm.barrier0.and", llvmcall, Int32, (Int32,), predicate)
@inline sync_threads_and(predicate::Bool) = ifelse(sync_threads_and(Int32(predicate)) !== Int32(0), true, false)
function kernel_soft_SGD_SVM(\alpha, \sigma, K, y, 1::Int32, C::Int32)
    j = (blockIdx().x-1) * blockDim().x + threadIdx().x
     can_stop = false
     \delta_i = \text{@cuStaticSharedMem}(\text{Float32, 1})
     while !sync_threads_and(can_stop)
        last_\alpha_j = \alpha[j]
         # Adatron:
         for i = 1:1
             if j == i # Online
                  last \alpha = \alpha[i]
                  \mu_{i} = 1 / K[i,i]
                  \delta_{i}[1] = \mu_{i} * (1 - y[i] * \sigma[i])
                  \alpha[i] = \alpha[i] + \delta_i[1]
                  \alpha[i] < 0 && (\alpha[i] = 0; \delta_i[1] = 0 - last_\alpha)
                  \alpha[i] > C \&\& (\alpha[i] = C; \delta_i[1] = C - last_\alpha)
              end
             # Stopping criterion:
         can_stop = false
         isapprox(\alpha[j], last_\alpha_j; atol=1e-4) \&\& (can_stop = true)
     return nothing
end
function optimise_working_set(\alpha, \sigma, K, y; C::Int32=Int32(1))
    1 = Int32(length(\alpha))
     cu_{\alpha} = CuArray{Float32}(\alpha)
    cu_{\sigma} = CuArray\{Float32\}(\sigma)
    cu_K = CuArray{Float32}(K)
    cu_y = CuArray{Float32}(y)
    @cuda threads=1 kernel_soft_SGD_SVM(cu_\alpha, cu_\sigma, cu_K, cu_y, 1, C)
    \alpha = Array\{Float32\}(cu_\alpha)
    return \alpha
function optimise_working_set_CPU(\alpha, \sigma, K, y; C=1)
     1 = length(\alpha)
     while true
         last_{\alpha} = copy(\alpha)
         for i = 1:1
             \mu_{i} = 1 / K[i,i]
              \delta_i = \mu_i * (1 - y[i] * \sigma[i])
              \alpha[i] = \alpha[i] + \delta_i
              \alpha[i] < 0 \&\& (\alpha[i] = 0; \delta_i = 0 - last_\alpha[i])
             \alpha[i] > C \&\& (\alpha[i] = C; \delta_i = C - last\_\alpha[i])
\sigma .+= \delta_i * y[i] * K[i,:]
         end
         all(isapprox.(last_{\alpha}, _{\alpha}; atol=1e-4)) && break
     end
     return \alpha
function \ stochastic\_decomposition\_test(\textit{C, M, ACCURACY, 1, y, K, 1\_test, y\_test, K\_test; usecpu=} false)
     M_{\text{safe}} = \operatorname{div}(M, 4) * 3
     M_{rand} = M - M_{safe}
    \alpha = zeros(Float32, 1)
    \sigma = zeros(Float32, 1)
     selected new idx = []
     support_vector_idx, alpha0_vector_idx = collect(1:M), collect(M+1:1)
```

```
working_set_counter = zeros(Int32, 1)
    batch id = 0
    while !monitor_kkt_condition(\alpha, \sigma, y; C=C) && batch_id < MAX_MINI_BATCH_ID
         current_working_set_counter = working_set_counter[support_vector_idx]
         sorted_support_vector_idx = support_vector_idx[sortperm(current_working_set_counter)]
         if length(support_vector_idx) >= M_safe
             no_of_new_idx = min(length(selected_new_idx), M_rand)
             no of new idx = min(length(selected new idx), M-length(support vector idx))
         no_of_old_idx = length(support_vector_idx)
         no_of_exceeds = no_of_new_idx + no_of_old_idx - M
         if no of exceeds > 0
             abandoned_support_vector_idx = sorted_support_vector_idx[end+1-no_of_exceeds:end]
              setdiff!(support vector idx, abandoned support vector idx)
             union!(alpha0 vector idx, abandoned support vector idx)
         selected_new_idx = selected_new_idx[1:no_of_new_idx]
         union!(support_vector_idx, selected_new_idx)
         setdiff!(alpha0_vector_idx, selected_new_idx)
         working\_set\_counter[support\_vector\_idx] \ .+= \ 1
         \alpha\_{subset}, \ \sigma\_{subset} = \alpha[support\_{vector\_idx}], \ \sigma[support\_{vector\_idx}]
         y_subset, K_subset = y[support_vector_idx], K[support_vector_idx, support_vector_idx]
         if usecpu
             \alpha[\text{support\_vector\_idx}] = \text{optimise\_working\_set\_CPU}(\alpha_{\text{subset}}, \sigma_{\text{subset}}, K_{\text{subset}}, y_{\text{subset}}; C=C)
             \alpha[\text{support\_vector\_idx}] = \underset{\text{optimise\_working\_set}}{\text{optimise\_working\_set}}(\alpha\_\text{subset}, \ \sigma\_\text{subset}, \ K\_\text{subset}, \ y\_\text{subset}; \ C=C)
         end
         \sigma = K * (\alpha .* y)
         for (i index,i) in enumerate(support vector idx)
             if \alpha[i] == 0
                  deleteat!(support_vector_idx, i_index)
                  push!(alpha0_vector_idx, i)
             end
         end
         \texttt{selected\_new\_idx} = \texttt{select\_new\_points}(\alpha, \ \sigma, \ y, \ M, \ \texttt{support\_vector\_idx}; \ \texttt{accuracy=ACCURACY}, \ \textit{C=C})
            = sign.(\sigma)
    \hat{y}_test = sign.(K_test * (\alpha .* y))
                     = count(\hat{y} .!= y) / 1 * 100
    error_rate_test = count(\hat{y}_test .!= y_test) / l_test * 100
    return \alpha, error_rate, error_rate_test, monitor_kkt_condition(\alpha, \sigma, y; C=C)
function stochastic_decomposition(C, M, ACCURACY, 1, y, K; usecpu=false)
    M_{\text{safe}} = \operatorname{div}(M, 4)
    M_{\rm rand} = M - M_{\rm safe}
    \alpha = zeros(Float32, 1)
    \sigma = zeros(Float32, 1)
    selected_new_idx = []
    {\tt support\_vector\_idx,\ alpha0\_vector\_idx = collect(1:M),\ collect(M+1:l)}
    working set counter = zeros(Int32, 1)
    while !monitor_kkt\_condition(\alpha, \sigma, y); accuracy=ACCURACY, C=C) && batch_id < MAX_MINI_BATCH_ID
         batch_id += 1
         current_working_set_counter = working_set_counter[support_vector_idx]
         sorted_support_vector_idx = support_vector_idx[sortperm(current_working_set_counter)]
         if length(support vector idx) >= M safe
             no of new idx = min(length(selected new idx), M rand)
             \verb"no_of_new_idx = \verb"min(length(selected_new_idx), M-length(support_vector_idx))"
         no_of_old_idx = length(support_vector_idx)
         no\_of\_exceeds = no\_of\_new\_idx + no\_of\_old\_idx - M
         if no of exceeds > 0
             abandoned support vector idx = sorted support vector idx[end+1-no of exceeds:end]
              setdiff!(support_vector_idx, abandoned_support_vector_idx)
             union!(alpha0_vector_idx, abandoned_support_vector_idx)
         selected_new_idx = selected_new_idx[1:no_of_new_idx]
         union!(support_vector_idx, selected_new_idx)
         setdiff!(alpha0_vector_idx, selected_new_idx)
         working set counter[support vector idx] .+= 1
         \alpha_subset, \sigma_subset = \alpha_s[support_vector_idx], \sigma_s[support_vector_idx] \gamma_s[subset, K_subset = \gamma_s[support_vector_idx], K[support_vector_idx, support_vector_idx]
              println("using CPU")
              \alpha[\text{support\_vector\_idx}] = \text{optimise\_working\_set\_CPU}(\alpha\_\text{subset}, \ \sigma\_\text{subset}, \ K\_\text{subset}, \ y\_\text{subset}; \ C=C)
         else
```

stochastic_decomposition (generic function with 1 method)

MNIST Training and Testing

```
## by Qin Yu, Apr 2019
## using Julia 1.1.0
time_total_start = time()
    randomisation_list = Array{Int32,1}[]
    \alpha_list = Array{Float32,1}[]
    error_list = Float64[]
    error list test = Float64[]
    finished list = Bool[]
    time ivi list = Float64[]
    for ci = 1:10, cj = 1:10
        ci >= cj && continue
        print("$(ci-1)-vs-$(cj-1) - ")
        time_K_start = time()
        1 = L[ci] + L[cj]
        randomisation = randperm(1)
        y = vcat(ones(Int32, size(ys[ci])), -ones(Int32, size(ys[cj])))[randomisation,:]
        X = vcat(Xs[ci], Xs[cj])[randomisation,:]
        K = gaussian_kernel_matrix(X, \gamma)
        time_K_spent += time() - time_K_start
        \# 1\_test = L\_test[ci] + L\_test[cj]
        # randomisation_test = randperm(l_test)
        # y_test = vcat(ones(Int32, size(ys_test[ci])), -ones(Int32, size(ys_test[cj])))[randomisation_test,:]
# X_test = vcat(Xs_test[ci], Xs_test[cj])[randomisation_test,:]
        # K_test = gaussian_kernel_matrix(X, X_test, y)
        time_GPU_start = time()
        \alpha, error_rate, finished = stochastic_decomposition(C, M, ACCURACY, 1, y, K; usecpu=usecpu)
        # \alpha, error_rate, error_rate_test, finished =
              stochastic\_decomposition(\textit{C}, \textit{M}, \textit{1}, \textit{y}, \textit{K}, \textit{1\_test}, \textit{y\_test}, \textit{K\_test})
        time_GPU_spent_this = time() - time_GPU_start
        push!(time_ivj_list, time_GPU_spent_this)
        time_GPU_spent += time_GPU_spent_this
        println("error rate = $error_rate% ; finished = $finished ; time = $time_GPU_spent_this")
        # println("error rate = $error_rate% ; test error rate = $error_rate_test% ;
              finished = $finished ; time = $time_GPU_spent_this")
        push!(randomisation_list, randomisation)
        push!(\alpha list, \alpha)
        push!(error_list, error_rate)
        # push!(error list test, error rate test)
        push!(finished_list, finished)
    time_total_spent = time() - time_total_start
    return ([time_K_spent, time_GPU_spent, time_total_spent],
            randomisation\_list, \ \alpha\_list, \ error\_list, \ error\_list\_test, \ finished\_list, \ time\_ivj\_list)
end
function MNIST_test_slave(X_ALL, y_ALL, Xs, ys, \alpha_list, randomisation_list, \gamma)
    \hat{y}_{t} = Array{Int}[]
    for ci = 1:10, cj = 1:10
       ci >= cj && continue
        print("$(ci-1)-vs-$(cj-1) - ")
        i += 1
        X_test, y_test = extract_all_test(X_ALL, y_ALL)
        K\_test = gaussian\_kernel\_matrix(vcat(Xs[ci], Xs[cj])[randomisation\_list[i],:], X\_test, y)
        y_{\text{true}} = \text{vcat}(\text{ones}(\text{Int32}, \text{size}(ys[\text{ci}])), -\text{ones}(\text{Int32}, \text{size}(ys[\text{cj}])))[\text{randomisation\_list[i],:}]
        \sigma_{\text{test}} = K_{\text{test}} * (\alpha_{\text{list[i]}} .* y_{\text{true}})
```

```
\hat{v} test = (v \to v > 0 ? \text{ci-1} : \text{ci-1}).(\sigma \text{ test})
        push!(ŷ_test_list, ŷ_test)
    end
    return \hat{y}_{t}test_list
function MNIST_test(y_test, \hat{y}_test_list)
   \hat{y}_{\text{test_matrix}} = \text{hcat}(\hat{y}_{\text{test_list...}})
    \hat{y}_{\text{test}} = \text{mapslices}(\text{mode}, \hat{y}_{\text{test}} = \text{matrix}; \text{dims} = 2)
    \texttt{error\_rate\_test} = \texttt{count}(\hat{y}\_\texttt{test} \ .!= y\_\texttt{test}) \ / \ \texttt{length}(y\_\texttt{test}) \ * \ \texttt{100}
    correct_rate_test = 100 - error_rate_test # First run: 99.1%
    return ŷ test, error rate test, correct rate test
function run(C::Int32=Int32(1), # Penalty
                M::Int=512,
                                        # Max size of minibatch
                ACCURACY=1e-2,
                                        # GPU accuracy = 0.01 * ACCURACY
                v=0.015:
                                        # Gaussian kernel parameter
                usecpu=false)
    X ALL, v ALL = MNIST.traindata(Float32)
    X_ALL_test, y_ALL_test = MNIST.testdata(Float32)
    L, Xs, ys = extract_each_classes(X_ALL, y_ALL)
    L_test, Xs_test, ys_test = extract_each_classes(X_ALL_test, y_ALL_test)
    @time time_list, randomisation_list, \alpha_list, error_list, error_list_test, finished_list, time_ivj_list =
    MNIST_train(L, L_test, Xs, ys, C, M, ACCURACY, \gamma; usecpu=usecpu)
@save "1v1.jld2" time_list, randomisation_list, \alpha_list, error_list, error_list_test, finished_list, time_ivj_list
    Otime \hat{y}_test, error_rate_test = MNIST_test(y_ALL, \hat{y}_test_list)
    println(error_rate_test)
```

run (generic function with 5 methods)

Run SVM

```
run()
                 # 1256.897736s ≈ 20.95min
0-vs-1 - error rate = 0.007895775759968417% : finished = true : time = 11.731913089752197
0-vs-2 - error rate = 0.042083999663328% : finished = true : time = 12.422368049621582
0-vs-3 - error rate = 0.024888003982080638%; finished = true; time = 10.846972942352295
0-vs-4 - error rate = 0.06799830004249893%; finished = true; time = 7.246622085571289
0-vs-5 - error rate = 0.07052186177715092\%; finished = true; time = 18.025906801223755
0-vs-6 - error rate = 0.10978802466007939%; finished = true; time = 10.650660037994385
0-vs-7 - error rate = 0.016409583196586804\%; finished = true; time = 7.11625599861145
0-vs-8 - error rate = 0.08493290300662477\% ; finished = true ; time = 11.790964126586914
0-vs-9 - error rate = 0.07580862533692723\%; finished = true ; time = 7.729828834533691
1-vs-2 - error rate = 0.13385826771653542\%; finished = true; time = 16.983326196670532
1-vs-3 - error rate = 0.06991377301328362\%; finished = true; time = 11.7693190574646
1-vs-4 - error rate = 0.14303877940241577%; finished = true; time = 8.491616010665894
1-vs-5 - error rate = 0.04932993504891885%; finished = true; time = 11.43572211265564
1-vs-6 - error rate = 0.023696682464454978\%; finished = true; time = 22.854277849197388
1-vs-7 - error rate = 0.261397708925963% ; finished = true ; time = 12.761433839797974
1-vs-8 - error rate = 0.1508774716112126% ; finished = true ; time = 25.24915885925293
1-vs-9 - error rate = 0.11031439602868175\%; finished = true ; time = 7.722901821136475
2-vs-3 - error rate = 0.2233435354454463\%; finished = true; time = 51.78821110725403
2\text{-vs-4} - error rate = 0.11864406779661016\% ; finished = true ; time = 24.21775197982788
2-vs-5 - error rate = 0.043940592319184464%; finished = true; time = 24.209162950515747
2-vs-6 - error rate = 0.0421017177500842% ; finished = true ; time = 26.605338096618652
2-vs-7 - error rate = 0.29452671193651314%; finished = true; time = 31.75123906135559
2-vs-8 - error rate = 0.20323482089931408\%; finished = true ; time = 57.46496510505676
2-vs-9 - error rate = 0.07558578987150416\%; finished = true ; time = 15.855940103530884
3-vs-4 - error rate = 0.05011275369581559%; finished = true; time = 15.090398073196411
3-vs-5 - error rate = 0.3808864265927978\%; finished = true; time = 66.38876104354858
3-vs-6 - error rate = 0.008299443937256203\%; finished = true; time = 14.185173988342285
3-vs-7 - error rate = 0.18554372378186512\% ; finished = true ; time = 28.159248113632202
3-vs-8 - error rate = 0.35052578868302453\%; finished = true; time = 55.56105613708496
3-vs-9 - error rate = 0.25662251655629137%; finished = true; time = 22.970326900482178
4-vs-5 - error rate = 0.03551451655864335%; finished = true; time = 19.871274948120117
4-vs-6 - error rate = 0.13605442176870747%; finished = true; time = 15.57249402999878
4-vs-7 - error rate = 0.21475179648137438% ; finished = true ; time = 42.69997715950012
4-vs-8 - error rate = 0.09407337723424271%; finished = true; time = 19.575289964675903
4-vs-9 - error rate = 0.5003816470189127\%; finished = true; time = 73.25518202781677
5\text{-vs-}6 - error rate = 0.17638239703677575\% ; finished = true ; time = 26.81020998954773
5-vs-7 - error rate = 0.06845798391237377%; finished = true; time = 20.46792197227478
5-vs-8 - error rate = 0.17743080198722497%; finished = true; time = 53.68216514587402
5-vs-9 - error rate = 0.12313104661389623%; finished = true; time = 23.383502960205078
6\text{-vs-7} - error rate = 0.008208158909956496\%; finished = true; time = 6.258177995681763
6-vs-8 - error rate = 0.05098139179199593\%; finished = true; time = 16.659406185150146
6-vs-9 - error rate = 0.02528018875874273%; finished = true; time = 7.46078896522522
 7-vs-8 - error rate = 0.115549686365137% ; finished = true ; time = 18.93709897994995
7\text{-vs-9} - error rate = 0.4830522351400033\%; finished = true; time = 76.98206496238708
8-vs-9 - error rate = 0.1440677966101695\%; finished = true; time = 44.56295299530029
Total training time 1278.997176 seconds (19.93 M allocations: 159.586 GiB, 1.11% gc time)
Final error rate 0.895%
```