Adiabatic three-waveguide coupler

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We propose an adiabatic method for transfer of light in a three-waveguide directional coupler in a single mode. In our scheme the propagation coefficients of the two outer waveguides are identical, but may vary simultaneously along the propagation direction. In contrast to earlier work [E. Paspalakis, Opt. Commun. **258**, 30 (2006)], where the coupling coefficients are designed to act in the counterintuitive order for an input in one of the outer waveguides, here we may have initially any light input in each of the waveguides and any order of coupling coefficients. Consequently, all three adiabatic eigenstates of the system are used in the dynamics. These solutions allow us to design some interesting waveguide devices.

I. INTRODUCTION

Stimulated Raman adiabatic passage (STIRAP) [1–4] is a simple and powerful technique for complete and robust population transfer in three-state quantum systems. In this technique, the population is transferred adiabatically from an initially populated state 1 to a target state 3, which are coupled via an intermediate state 2 by two pulsed fields, pump and Stokes. A unique and very useful feature of STIRAP is that in the counterintuitive order of pulses—the Stokes before the pump pulse—the intermediate state 2 never gets populated, even transiently. Therefore, the properties of the intermediate state, including possible population decay, are largely irrelevant. The reason for this unique feature is that the evolution of STIRAP proceeds via an adiabatic dark state, which does not involve state 2. The dark state is associated with state 1 initially and state 3 in the end, thereby providing an adiabatic route from 1 to 3.

In the past decade the concept of STIRAP has been extended beyond the realm of quantum physics. An important field, where this concept is now successfully applied, is waveguide optics [5–12]. In this field the STIRAP mechanism leads to an effective technique for light transfer between the waveguides in an array, even when they are not directly coupled. Examples includes analogies of STIRAP in a three-waveguide directional coupler [5–7], fractional STIRAP [8], and extensions of STIRAP to multiple states [9–12]. An important feature of the above adiabatic light transfer processes is their robustness with respect to the waveguide design parameters, which is related to the fact that the light waves are associated with the same spatially evolving eigenstate of the system. This robustness leads, for instance, to an increased spectral bandwidth. It is important to notice that, in contrast to the atomic systems, where the middle state may decay, in waveguides the middle waveguide is not lossy. This is a conceptional difference between STIRAP in waveguides and atomic systems, which allows us to extend its use beyond the dark state, by including the other two adiabatic states of the system. This leeway provides additional control over the distribution of light in the waveguides.

In this paper we exploit all three adiabatic eigenstates of STIRAP by using the adiabatic evolution matrix for a three-waveguide structure. We examine the spatial evolution of the light distribution in the single-mode waveguides for different light inputs in all three waveguides. Contrary to the standard STIRAP approach, here we also use the shape of detuning as an additional control parameter.

II. ADIABATIC EVOLUTION

The propagation of the electric-field amplitudes $c_1(z)$, $c_2(z)$, and $c_3(z)$ of light waves traveling in three evanescently coupled optical waveguides, shown schematically in Fig. 1, is described by a system of three coupled differential equations written in a matrix form as [5]

$$i\frac{d}{dz}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 & \Omega_P & 0 \\ \Omega_P & \Delta & \Omega_S \\ 0 & \Omega_S & 0 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}, \quad (1)$$

where Ω_P and Ω_S are the coupling coefficients between waveguides $1 \leftrightarrow 2$ and $2 \leftrightarrow 3$, respectively, and are functions of z. This can be achieved with changing the distance between the waveguides along the propagation direction. The $1 \leftrightarrow 3$ coupling is neglected because the structure is planar. Note that if the waveguides differ slightly, the coupling to the left may slightly differ from the coupling to the right and Ω_P (or Ω_S) represents the geometrical average of the two values. Here Δ is the difference between the propagation constants of the outer waveguides and the middle one and it changes also in the propagations direction. The absolute squares of the amplitudes $c_k(z)$ are the dimensionless light intensities in the waveguides, normalized to the total input light intensity: $I_k(z) = |c_k(z)|^2$ ($k = 1,2,3$). Obviously, $I_1(z) + I_2(z) + I_3(z) = 1$ in the lossless case.

As Ω_P, Ω_S, and Δ are proposed to be functions of the coordinate z, we can write Eq. (1) in the so-called adiabatic basis [3,4,13–15] (for the three-state atom this is the basis of the instantaneous eigenstates of the Hamiltonian)

$$i\frac{d}{dz}\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} \Omega\cot\phi & i\dot\vartheta\sin\phi & i\dot\phi \\ -i\dot\vartheta\sin\phi & 0 & -i\dot\vartheta\cos\phi \\ -i\dot\phi & i\dot\vartheta\cos\phi & -\Omega\tan\phi \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix},$$
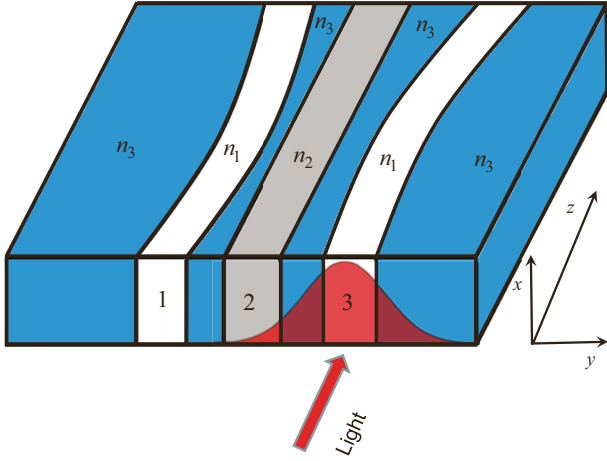
$$(2)$$

FIG. 1. Three evanescently coupled waveguides labeled 1, 2, and 3 are made of slabs with refractive index n_1 or n_2 embedded in a medium with a refractive index n_3. The refractive index n_2 or the width of the waveguide 2 may change along the coordinate z. Here a Gaussian-shaped light beam is injected initially in waveguide 3.

where the overdot denotes d/dz and ϕ, ϑ, and Ω are z-dependent variables defined as

$$\tan(\vartheta) = \frac{\Omega_P(z)}{\Omega_S(z)}, \tag{3a}$$

$$\tan(2\phi) = \frac{2\Omega(z)}{\Delta(z)}, \tag{3b}$$

$$\Omega(z) = \sqrt{\Omega_P^2(z) + \Omega_S^2(z)}. \tag{3c}$$

The connection between the original amplitudes $\mathbf{c}(z) = [c_1(z), c_2(z), c_3(z)]^T$ and the adiabatic amplitudes $\mathbf{a}(z) = [a_1(z), a_2(z), a_3(z)]^T$ is given by $\mathbf{c}(z) = R(z)\mathbf{a}(z)$, where $R(z)$ is the orthogonal transformation matrix

$$R(z) = \begin{bmatrix} \sin\phi\sin\vartheta & \cos\vartheta & \cos\phi\sin\vartheta \\ \cos\phi & 0 & -\sin\phi \\ \sin\phi\cos\vartheta & -\sin\vartheta & \cos\phi\cos\vartheta \end{bmatrix}. \tag{4}$$

When the system evolves adiabatically there are no transitions between the adiabatic states, hence $|a_1(z)|$, $|a_2(z)|$, and $|a_3(z)|$ remain constant [3,4,13–15]. Mathematically, adiabatic evolution means that in Eq. (2) the nondiagonal terms can be neglected compared to the diagonal ones. This occurs when

$$|\sin\phi\,\dot{\vartheta}| \ll |\Omega\cot\phi|, \tag{5a}$$

$$|\cos\phi\,\dot{\vartheta}| \ll |\Omega\tan\phi|, \tag{5b}$$

$$|\dot{\phi}| \ll \Omega|\tan\phi + \cot\phi|. \tag{5c}$$

Hence adiabatic evolution requires a smooth z dependence of Ω_P, Ω_S, and Δ.

The evolution matrix $U^A(z_f, z_i)$ in the adiabatic basis connects the initial and final states: $\mathbf{a}(z_f) = U^A(z_f, z_i)\mathbf{a}(z_i)$, where z_i and z_f are the z coordinates at the input and output, respectively. For adiabatic evolution, this matrix is diagonal

and contains phase factors

$$U^A(z_f, z_i) = \begin{bmatrix} \exp(i\eta_1) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \exp(i\eta_2) \end{bmatrix}, \tag{6}$$

with $\eta_1 = \int_{z_i}^{z_f} \Omega \tan\phi\,dz$ and $\eta_2 = \int_{z_i}^{z_f} \Omega \cot\phi\,dz$. The evolution matrix in the original basis is

$$U(z_f, z_i) = R(z_f)U^A(z_f, z_i)R^{-1}(z_i), \tag{7}$$

or explicitly

$$U_{11}(z_f, z_i) = \sin\vartheta_i \sin\vartheta_f(e^{i\eta_2}\cos\phi_i\cos\phi_f \\ + e^{i\eta_1}\sin\phi_i\sin\phi_f) + \cos\vartheta_i\cos\vartheta_f, \tag{8a}$$

$$U_{12}(z_f, z_i) = \sin\vartheta_f(e^{i\eta_1}\cos\phi_i\sin\phi_f \\ - e^{i\eta_2}\cos\phi_f\sin\phi_i), \tag{8b}$$

$$U_{13}(z_f, z_i) = \cos\vartheta_i \sin\vartheta_f(e^{i\eta_2}\cos\phi_i\cos\phi_f \\ + e^{i\eta_1}\sin\phi_i\sin\phi_f) - \cos\vartheta_f\sin\vartheta_i, \tag{8c}$$

$$U_{21}(z_f, z_i) = \sin\vartheta_i(e^{i\eta_1}\cos\phi_f\sin\phi_i \\ - e^{i\eta_2}\cos\phi_i\sin\phi_f), \tag{8d}$$

$$U_{22}(z_f, z_i) = e^{i\eta_1}\cos\phi_i\cos\phi_f + e^{i\eta_2}\sin\phi_i\sin\phi_f, \tag{8e}$$

$$U_{23}(z_f, z_i) = \cos\vartheta_i(e^{i\eta_1}\cos\phi_f\sin\phi_i \\ - e^{i\eta_2}\cos\phi_i\sin\phi_f), \tag{8f}$$

$$U_{31}(z_f, z_i) = \cos\vartheta_f \sin\vartheta_i(e^{i\eta_2}\cos\phi_i\cos\phi_f \\ + e^{i\eta_1}\sin\phi_i\sin\phi_f) - \cos\vartheta_i\sin\vartheta_f, \tag{8g}$$

$$U_{32}(z_f, z_i) = \cos\vartheta_f(e^{i\eta_1}\cos\phi_i\sin\phi_f \\ - e^{i\eta_2}\cos\phi_f\sin\phi_i), \tag{8h}$$

$$U_{33}(z_f, z_i) = \cos\vartheta_i \cos\vartheta_f(e^{i\eta_2}\cos\phi_i\cos\phi_f \\ + e^{i\eta_1}\sin\phi_i\sin\phi_f) + \sin\vartheta_i\sin\vartheta_f, \tag{8i}$$

with $\phi_{i,f} = \phi(z_{i,f})$ and $\vartheta_{i,f} = \vartheta(z_{i,f})$. Equations (8) for the elements of $U(z_f, z_i)$ give the general adiabatic solution for arbitrary input intensities in the three waveguides. Below we analyze several possible light-switching cases with Gaussian-shaped couplings $\Omega_P(z)$ and $\Omega_S(z)$,

$$\Omega_P(z) = \Omega_0 \exp[-\alpha(z - z_0)^2/L^2], \tag{9a}$$

$$\Omega_S(z) = \Omega_0 \exp[-\alpha(z + z_0)^2/L^2]. \tag{9b}$$

A. Case 1: Light swapping

For our first example we consider that the light is initially in the outer waveguides only $\mathbf{c}(z_i) = [c_1(z_i), 0, c_3(z_i)]^T$. We assume the following initial and final conditions:

$$\phi_i = \phi_f = \vartheta_i = 0, \quad \vartheta_f = \pi/2, \tag{10}$$

which lead to the following final light field distributions among waveguides:

$$\mathbf{c}(z_f) = [e^{i\eta_2}c_3(z_i), 0, -c_1(z_i)]^T \tag{11}$$
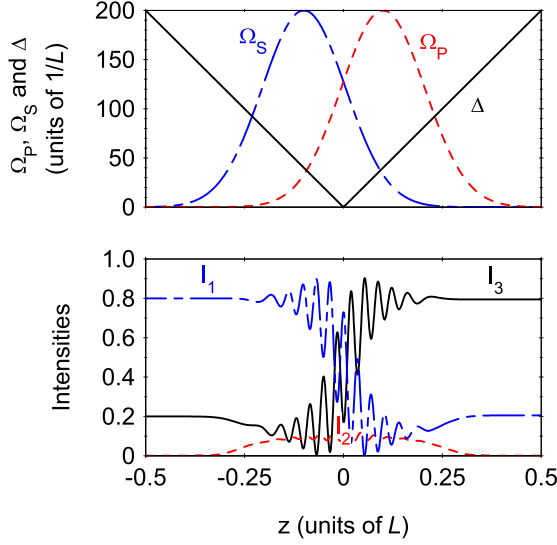
FIG. 2. Adiabatic light transfer between waveguides. Shown on top is the phase mismatch Δ and couplings Ω_P and Ω_S with shapes as in Eqs. (9) and (12). The parameters are $\Omega_0 = 200/L$, $\Delta_0 = 400/L$, $\alpha = 45$, and $z_0 = 0.1L$. The bottom shows intensities $I_n = |c_n(z)|^2$ vs z when initially $\mathbf{c}(z_i) = \frac{1}{\sqrt{5}}[2,0,1]^T$.
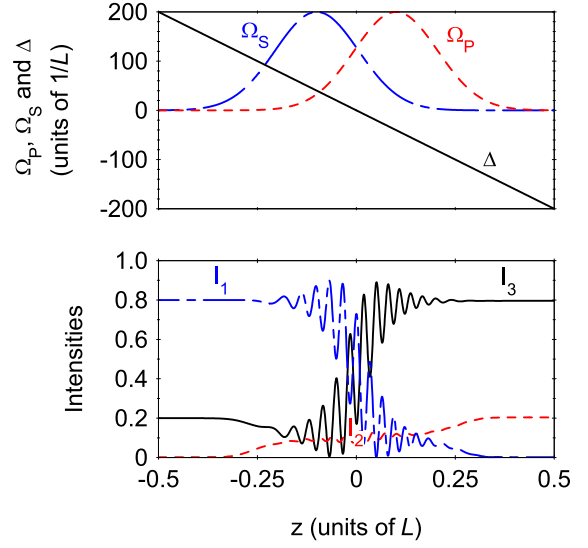

FIG. 3. Adiabatic light transfer between waveguides. Shown on top is the phase mismatch Δ and couplings Ω_P and Ω_S with shapes as in Eqs. (9) and (15). The parameters are $\Omega_0 = 200/L$, $\Delta_0 = -400/L$, $\alpha = 45$, and $z_0 = 0.1L$. The bottom shows intensities $I_n = |c_n(z)|^2$ vs z when initially $\mathbf{c}(z_i) = \frac{1}{\sqrt{5}}[2,0,1]^T$.

representing essentially a swapping of the fields in waveguides 1 and 3. This case can be achieved if Ω_S is before Ω_P and Δ is the modulus of a linear function

$$\Delta(z) = \Delta_0 |z|. \tag{12}$$

Figure 2 shows a simulation of such light swapping.

B. Case 2: Light permutation

Again we consider light initially in the outer waveguides $\mathbf{c}(z_i) = [c_1(z_i),0,c_3(z_i)]^T$, but for the initial and final conditions

$$\phi_i = \vartheta_i = 0, \quad \phi_f = \vartheta_f = \pi/2. \tag{13}$$

We find the following final light distribution:

$$\mathbf{c}(z_f) = [0, -e^{i\eta_2}c_3(z_i), -c_1(z_i)]^T, \tag{14}$$

thus essentially light field permutation. This case can be achieved if Ω_S is before Ω_P and

$$\Delta(z) = \Delta_0 z. \tag{15}$$

Figure 3 shows the results.

C. Case 3: Light swapping and splitting

Once more we consider the input as $\mathbf{c}(z_i) = [c_1(z_i),0,c_3(z_i)]^T$, but now for initial and final conditions

$$\phi_i = \vartheta_i = 0, \quad \phi_f = \pi/4, \quad \vartheta_f = \pi/2, \tag{16}$$

which lead to

$$\mathbf{c}(z_f) = [e^{i\eta_2}c_3(z_i)/\sqrt{2}, -e^{i\eta_2}c_3(z_i)/\sqrt{2}, -c_1(z_i)]^T. \tag{17}$$

Thus the light entering waveguide 3 is equally split among waveguides 1 and 2 while the light of waveguide 1 is moved

to waveguide 3. This case can be achieved if Ω_S is before Ω_P and

$$\Delta(z) = \Delta_0 z \theta(-z), \tag{18}$$

where $\theta(z)$ is the Heaviside step function. Figure 4 shows the results. Note that in this specific case the final light distribution is independent of the sign of Δ_0.
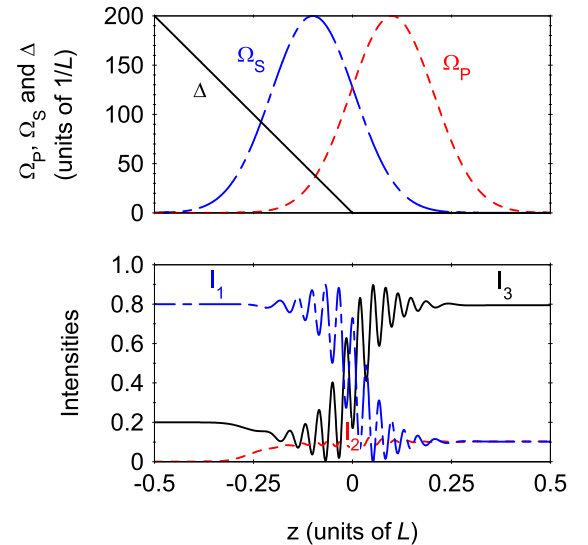

FIG. 4. Adiabatic light transfer between waveguides. Shown on top is the phase mismatch Δ and couplings Ω_P and Ω_S with shapes as in Eqs. (9) and (18). The parameters are $\Omega_0 = 200/L$, $\Delta_0 = -400/L$, $\alpha = 45$, and $z_0 = 0.1L$. The bottom shows intensities $I_n = |c_n(z)|^2$ vs z when initially $\mathbf{c}(z_i) = \frac{1}{\sqrt{5}}[2,0,1]^T$.
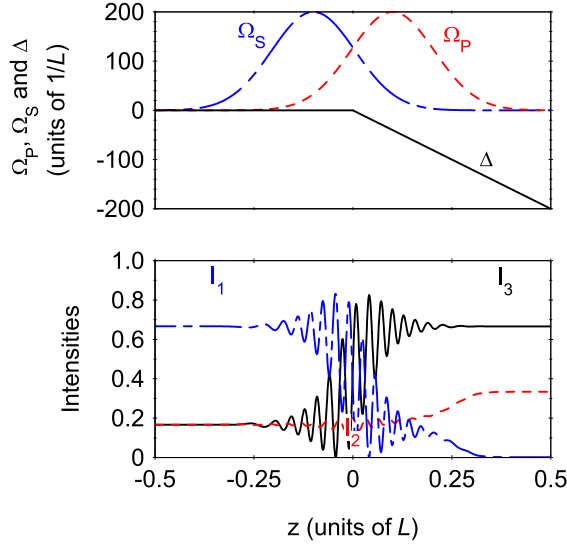
FIG. 5. Adiabatic light transfer between waveguides. Shown on top is the phase mismatch Δ and couplings Ω_P and Ω_S with shapes as in Eqs. (9) and (21). The parameters are $\Omega_0 = 200/L$, $\Delta_0 = -400/L$, $\alpha = 45$, and $z_0 = 0.1L$. The bottom shows intensities $I_n = |c_n(z)|^2$ vs z when initially $\mathbf{c}(z_i) = \frac{\sqrt{2}}{\sqrt{3}}[1, -\frac{1}{2}, \frac{1}{2}]^T$.

D. Case 4: Light mixing

In all three previous examples we have considered optical injection in the outer waveguides only. Let us now assume that light is injected in all three waveguides $\mathbf{c}(z_i) = [c_1(z_i), c_2(z_i), c_3(z_i)]^T$. If we now ensure the initial and final conditions

$$\phi_i = \pi/4, \quad \vartheta_i = 0, \quad \phi_f = \vartheta_f = \pi/2, \tag{19}$$

then the final light distribution becomes

$$\mathbf{c}(z_f) = \left[\frac{e^{i\eta_1}[c_2(z_i) + c_3(z_i)]}{\sqrt{2}}, \frac{e^{i\eta_2}[c_3(z_i) - c_2(z_i)]}{\sqrt{2}}, \right.$$
$$\left. -c_1(z_i) \right]^T \tag{20}$$

so that waveguides 1 and 2 carry a superposition of the fields injected in waveguides 2 and 3. This case can be achieved if Ω_S is before Ω_P and

$$\Delta(z) = \Delta_0 z \theta(z). \tag{21}$$

Figure 5 illustrates this case.

E. Case 5: Bright STIRAP

Here let us assume that initially the light is in only one of the outer waveguides $\mathbf{c}(z_i) = [c_1(z_i), 0, 0]^T$. If we now ensure the initial and final conditions

$$\phi_i = \phi_f = \pi/2, \quad \vartheta_i = \pi/2, \quad \vartheta_f = 0, \tag{22}$$

the light will move to the other outer waveguide

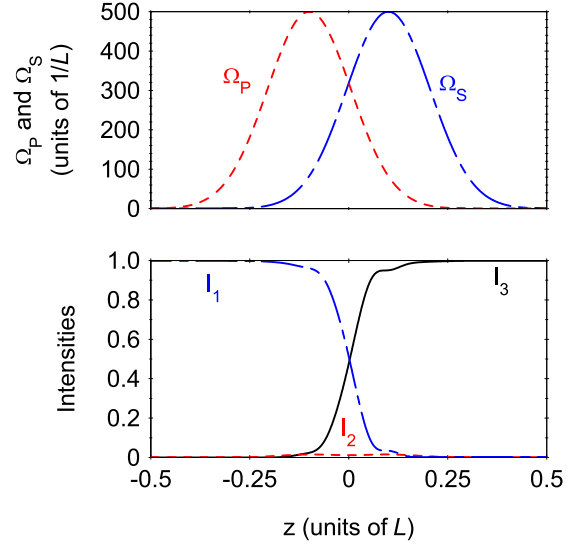$$\mathbf{c}(z_f) = [0, 0, -c_1(z_i)]^T. \tag{23}$$

FIG. 6. Adiabatic light transfer between waveguides. Shown on top are couplings Ω_P and Ω_S with shapes as in Eq. (9). The parameters are $\Omega_0 = 500/L$, $\Delta = 4000/L$, $\alpha = 45$, and $z_0 = -0.1L$. The bottom shows intensities $I_n = |c_n(z)|^2$ vs z when initially $\mathbf{c}(z_i) = [1, 0, 0]^T$.

This case can be achieved if Ω_P is before Ω_S and $\Delta \gg \sqrt{\Omega_P^2 + \Omega_S^2}$ and is known in the field of atomic physics as bright-state STIRAP [16]. Figure 6 shows the results.

F. Case 6: STIRAP

One can realize many other cases using different initial conditions for the light input and the two angles, but we will show just one more case, when the adiabatic technique proposed here duplicates the famous STIRAP technique. For this specific case we consider the initial distribution $\mathbf{c}(z_i) = [c_1(z_i), 0, 0]^T$. For

$$\phi_i = \phi_f = \pi/4, \quad \vartheta_i = 0, \quad \vartheta_f = \pi/2, \tag{24}$$

all light will be transferred to the other outer waveguide

$$\mathbf{c}(z_f) = [0, 0, -c_1(z_i)]^T, \tag{25}$$

as in case 5.

G. Robustness

From the definition of the angles ϕ and ϑ [Eq. (3)], we see that the needed values of $0, \pi/4$ and $\pi/2$ are obtained for limits

$$\phi \xrightarrow[\Omega/\Delta \to 0+]{} 0, \quad \phi \xrightarrow[\Omega/\Delta \to \infty]{} \pi/4, \quad \phi \xrightarrow[\Omega/\Delta \to 0-]{} \pi/2; \tag{26a}$$

$$\vartheta \xrightarrow[\Omega_P/\Omega_S \to 0]{} 0 \quad \vartheta \xrightarrow[\Omega_P/\Omega_S \to 1]{} \pi/4, \quad \vartheta \xrightarrow[\Omega_P/\Omega_S \to \infty]{} \pi/2. \tag{26b}$$

The proposed adiabatic switching between waveguides has the useful property that light transfer depends only on the initial and final values of the angles ϕ and ϑ. Therefore, the light redistribution is frequency independent and is robust against variations of the propagation length, temperature, etc. The only restriction that we have to follow during the light evolution is

the adiabatic condition [see Eq. (5)], thus a slow change of the angles ϕ and ϑ is needed.

III. CONCLUSION

We have used the analogy between the equations that describe the light propagating in an array of three coupled waveguides and the time-dependent Schrödinger equation of three-state atoms in order to describe the adiabatic light switching. In contrast to the famous STIRAP-based technique in waveguide couplers, where only the dark state is used, we use all three adiabatic states for light transfer. In this sense the approach proposed here is more general and a variety of light-switching waveguide devices can be designed.

Waveguide structures based on the present approach have great potential for various integrated optical devices. For instance, they can implement the basis for integrated homodyne or heterodyne detectors or permit the preparation of specific quantum photonic states on an integrated basis.

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