CS 325 - Homework 5

1. *(7 pts)* Let X and Y be two decision problems. Suppose we know that X reduces to Y in polynomial time. State whether the following statements are true or false and give a brief explanation.

a. If Y is NP-complete then so is X.
b. If X is NP-complete then so is Y.
c. If Y is NP-complete and X is in NP then X is NP-complete.
d. If X is NP-complete and Y is in NP then Y is NP-complete.
e. If X is in P, then Y is in P.
f. If Y is in P, then X is in P.
g. X and Y can't both be in NP.

2. *(3 pts)* Two well-known NP-complete problems are 3-SAT and TSP, the Traveling Salesman Problem. The 2-SAT problem is a SAT variant in which each clause contains at most two literals. 2-SAT is known to have a polynomial-time algorithm. State whether the following statements are true or false and give a brief explanation.

a. 3-SAT $\leq_p$ TSP.

b. If P $\neq$ NP, then 3-SAT $\leq_p$ 2-SAT.

c. If TSP $\leq_p$ 2-SAT, then P = NP.

3. *(10 pts)* A Hamiltonian path in a graph is a simple path that visits every vertex exactly once. Show that HAM-PATH = { (G, u, v): there is a Hamiltonian path from u to v in G } is NP-complete. You may use the fact that HAM-CYCLE is NP-complete.

4. *(10 pts)* K-COLOR. Given a graph G = (V,E), a k-coloring is a function c: V -> {1, 2, … , k} such that c(u) $\neq$ c(v) for every edge (u,v) $\in$ E. In other words the number 1, 2, .., k represent the k colors and adjacent vertices must have different colors. The decision problem K-COLOR asks if a graph can be colored with at most K colors.

The 3-COLOR decision problem is NP-complete by using a reduction from SAT. Use the fact that 3-COLOR is NP-complete to prove that 4-COLOR is NP-complete.