Lab06-Linear Programming

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1. **Controlling Air Pollution.** The three main types of pollutants in an airshed are particulate matter, sulfur oxides, and hydrocarbons. The new standards require that the steelworks reduce its annual emission of these pollutants by the amounts shown in the following table:

Pollutant	**Required Reduction in Annual Emission Rate (Million Pounds)**
Particulates	60
Sulfur oxides	150
Hydrocarbons	125

The steelworks has two primary sources of pollution, namely, the blast furnaces for making pig iron and the open-hearth furnaces for changing iron into steel. In both cases the engineers have decided that the most effective types of abatement methods are (1) increasing the height of the smokestacks, (2) using filter devices (including gas traps) in the smokestacks, and (3) including cleaner, high-grade materials among the fuels for the furnaces. Note that each of these methods has a technological limit on how heavily it can be used (e.g., a maximum feasible increase in the height of the smokestacks), but there also is considerable flexibility for using the method at a fraction of its technological limit.

The following table shows how much emission (in millions of pounds per year) can be eliminated from each type of furnace by fully using any abatement method to its technological limit. For purposes of analysis, it is assumed that each method also can be used less fully to achieve any fraction of the emission-rate reductions shown in this table. Furthermore, the fractions can be different for blast furnaces and for open-hearth furnaces. For either type of furnace, the emission reduction achieved by each method is not substantially affected by whether the other methods also are used.

	Taller Smokestacks		**Filters**		**Better Fuels**	
Pollutant	**Blast Furnaces**	**Open-Hearth Furnaces**	**Blast Furnaces**	**Open-Hearth Furnaces**	**Blast Furnaces**	**Open-Hearth Furnaces**
Particulates	12	9	25	20	17	13
Sulfur oxides	35	42	18	31	56	49
Hydrocarbons	37	53	28	24	29	20

The total annual cost from the maximum feasible use of an abatement method (in millions of dollars) was shown in the following table. The board of directors wants to figure out how to achieve these reductions with minimum annual cost. Please design a scheme for them.

Abatement Method	**Blast Furnaces**	**Open-Health Furnaces**
Taller smokestacks	8	10
Filters	7	6
Better fuels	11	9

(a) Formulate a linear programming with necessary explanations.

(b) Transform your LP into its standard form.

(c) Transform your LP into its dual form.

(d) Assume that the clean air standards have been relaxed. The steelworks only needs to meet any two of the three pollutants emission standards. Please update your LP in (a) to satisfy the relaxed clean air standards. (Hint: You can refer to Reference14-ModelFormulation.pdf)

Solution.

(a) First, set some variables with certain meaning:

x_1: Usage-rate of taller smokestacks used in blast furnaces.

x_2: Usage-rate of taller smokestacks used in open-hearth furnaces.

x_3: Usage-rate of filters used in blast furnaces.

x_4: Usage-rate of filters used in open-hearth furnaces.

x_5: Usage-rate of better fuels used in blast furnaces.

x_6: Usage-rate of better fuels used in open-hearth furnaces.

Then, the total cost can be calculated from table:

$$f_{cost} = 8x_1 + 10x_2 + 7x_3 + 6x_4 + 11x_5 + 9x_6$$

Then, we should meet the requirement of reduction presented in the first table.

$$12x_1 + 9x_2 + 25x_3 + 20x_4 + 17x_5 + 13x_6 \geq 60 \qquad (1)$$

$$35x_1 + 42x_2 + 18x_3 + 31x_4 + 56x_5 + 49x_6 \geq 150 \qquad (2)$$

$$37x_1 + 53x_2 + 28x_3 + 24x_4 + 29x_5 + 20x_6 \geq 125 \qquad (3)$$

After that, there must be some constraints on "rate":

$$x_1, x_2, x_3, x_4, x_5, x_6 \leq 1 \qquad (4)$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0 \qquad (5)$$

To sum up, the LP can be expressed as:

$$\min\ f_{cost}, \quad s.t.\ (1), (2), (3), (4), (5)$$

(b) To transform the LP into standard form, we should transfer the f_{cost} function to f'_{cost}.

$$f'_{cost} = -8x_1 - 10x_2 - 7x_3 - 6x_4 - 11x_5 - 9x_6$$

Change the inequality (1)(2)(3) which have "$\geq$" to (6)(7)(8) .

$$-12x_1 - 9x_2 - 25x_3 - 20x_4 - 17x_5 - 13x_6 \leq -60 \qquad (6)$$

$$-35x_1 - 42x_2 - 18x_3 - 31x_4 - 56x_5 - 49x_6 \leq -150 \qquad (7)$$

$$-37x_1 - 53x_2 - 28x_3 - 24x_4 - 29x_5 - 20x_6 \leq -125 \qquad (8)$$

Then, the standard form can be expressed as:

$$\max\ f'_{cost}, \quad s.t.\ (6), (7), (8), (4), (5)$$

(c) We should set some multipliers to deal with the dual form:

$$y_1 \ \text{for} \ (6), \qquad y_2 \ \text{for} \ (7), \qquad y_3 \ \text{for} \ (8)$$

As for the inequality (4):

$$y_1^{'} \ \text{for} \ x_1 \leq 1, \quad y'_2 \ \text{for} \ x_2 \leq 1, \quad ..... \quad y_6^{'} \ \text{for} \ x_6 \leq 1$$

Then we can get the new form:

$$g_{cost} = -60y_1 - 150y_2 - 125y_3 + y_1^{'} + y_2^{'} + y_3^{'} + y_4^{'} + y_5^{'} + y_6^{'}$$

$$-12y_1 - 35y_2 - 37y_3 + y_1^{'} \geq -8 \qquad (9)$$
$$-9y_1 - 42y_2 - 53y_3 + y_2^{'} \geq -10 \qquad (10)$$
$$-25y_1 - 18y_2 - 28y_3 + y_3^{'} \geq -7 \qquad (11)$$
$$-20y_1 - 31y_2 - 24y_3 + y_1^{'} \geq -6 \qquad (12)$$
$$-17y_1 - 56y_2 - 29y_3 + y_1^{'} \geq -11 \qquad (13)$$
$$-13y_1 - 49y_2 - 20y_3 + y_1^{'} \geq -9 \qquad (14)$$
$$y_1, y_2, y_3, y_1^{'}, y_2^{'}, y_3^{'}, y_4^{'}, y_5^{'}, y_6^{'} \geq 0 \qquad (15)$$

Therefore, the dual form can be expressed as:

$$\min g_{cost}, \quad s.t. \ (9), (10), (11), (12), (13), (14), (15)$$

(d) We can get the answer from (a).Then f_{cost}, and inequality (4) (5) don't need to be changed, so we just reconsider inequality(1)(2)(3):
We can import some binary variables, x_7, x_8, x_9, which must be 0 or 1. Then, the (1)(2)(3) can be written as:

$$12x_1 + 9x_2 + 25x_3 + 20x_4 + 17x_5 + 13x_6 \geq 60 - 60x_7 \qquad (16)$$
$$35x_1 + 42x_2 + 18x_3 + 31x_4 + 56x_5 + 49x_6 \geq 150 - 150x_8 \qquad (17)$$
$$37x_1 + 53x_2 + 28x_3 + 24x_4 + 29x_5 + 20x_6 \geq 125 - 125x_9 \qquad (18)$$
$$x_7 + x_8 + x_9 \leq 1 \qquad (19)$$
$$x_7, x_8, x_9 \quad \text{can only be 0 or 1} \qquad (20)$$

Therefore, the LP can be expressed as:

$$\min f_{cost}, \quad s.t. \ (16), (17), (18), (19), (20), (4), (5)$$

2. **Factory Production.** An engineering factory makes seven products (PROD 1 to PROD 7) on the following machines: four grinders, two vertical drills, three horizontal drills, one borer and two planer. Each product yields a certain contribution to profit (in £/unit). These quantities (in £/unit) together with the unit production times (hours) required on each process are given below. A dash indicates that a product does not require a process.

There are marketing limitations on each product in each month, given in the following table:

It is possible to store up to 100 of each product at a time at a cost of £0.5 per unit per month (charged at the end of each month according to the amount held at that time). There are no stocks at present, but it is desired to have a stock of exactly 50 of each type of product at the end of June. The factory works six days a week with two shifts of 8h each day. It may be assumed that each month consists of only 24 working days. Each machine must be down for maintenance in one month of the six. No sequencing problems need to be considered.

When and what should the factory make in order to maximize the total net profit?

	PROD 1	**PROD 2**	**PROD 3**	**PROD 4**	**PROD 5**	**PROD 6**	**PROD 7**
Contribution to profit	10	6	8	4	11	9	3
Grinding	0.5	0.7	0.2	-	0.3	0.2	0.5
Vertical drilling	0.1	0.2	0	0.3	-	0.6	-
Horizontal drilling	0.2	-	0.8	-	-	-	0.6
Boring	0.05	0.03	-	0.07	0.1	-	0.08
Planing	-	-	0.01	-	0.05	0.02	0.04

	PROD 1	**PROD 2**	**PROD 3**	**PROD 4**	**PROD 5**	**PROD 6**	**PROD 7**
January	500	1000	300	300	800	200	100
February	600	500	200	0	400	300	150
March	300	600	0	0	500	400	100
April	200	300	400	500	200	0	100
May	0	100	500	100	1000	300	0
June	500	500	100	300	1100	500	60

(a) Use *CPLEX Optimization Studio* to solve this problem. Describe your model in *Optimization Programming Language* (OPL). Remember to use a separate data file (.dat) rather than embedding the data into the model file (.mod).

(b) Solve your model and give the following results.

  i. For each machine:

    A. the month for maintenance.

  ii. For each product:

    A. The amount to make in each month.

    B. The amount to sell in each month.

    C. The amount to hold at the end of each month.

  iii. The total selling profit.

  iv. The total holding cost.

  v. The total net profit (selling profit minus holding cost).

Solution.

(a) The related files are attached in **.zip** file, which is tested available.

(b) We can use Cplex to get optimal solution as follows:

  – The following chart represents the months for maintenance:

	Grinding	**V-drilling**	**H-drilling**	**Boring**	**Planing**
January	0	0	0	0	0
February	0	0	0	0	0
March	0	0	0	0	0
April	0	0	0	1	0
May	2	1	1	0	1
June	2	1	2	0	1

Figure 1: Maintenance Table

  – The following chart (fig.2) represents the amount to make in each month:

  – The following chart (fig.3) represents the amount to sell in each month:

  – The following chart (fig.4) represents the amount to hold in each month:

- Then, we can easily get Three variables:
  The total selling profit: 111730 £
  The total holding cost: 425 £
  The total net profit: 111305 £

Products (大小 7)	Months (大小 6)					
	1	2	3	4	5	6
"PROD1"	500	600	400	0	0	550
"PROD2"	1000	500	700	0	100	550
"PROD3"	300	200	0	400	500	150
"PROD4"	300	0	100	0	100	350
"PROD5"	800	400	600	0	1000	1150
"PROD6"	200	300	400	0	300	550
"PROD7"	100	150	200	0	0	110

Figure 2: Making Table

Products (大小 7)	Months (大小 6)					
	1	2	3	4	5	6
"PROD1"	500	600	300	100	0	500
"PROD2"	1000	500	600	100	100	500
"PROD3"	300	200	0	400	500	100
"PROD4"	300	0	0	100	100	300
"PROD5"	800	400	500	100	1000	1100
"PROD6"	200	300	400	0	300	500
"PROD7"	100	150	100	100	0	60

Figure 3: Selling Table

Products (大小 7)	Months (大小 6)					
	1	2	3	4	5	6
"PROD1"	0	0	100	0	0	50
"PROD2"	0	0	100	0	0	50
"PROD3"	0	0	0	0	0	50
"PROD4"	0	0	100	0	0	50
"PROD5"	0	0	100	0	0	50
"PROD6"	0	0	0	0	0	50
"PROD7"	0	0	100	0	0	50

Figure 4: Holdinging Table

Remark: Include your .pdf, .tex, .oplproject, .project, .mod and .dat files for uploading.