Minimizing the Number of States in a DFA

Smaller is better!

Minimal DFA

- Given any DFA, there is an equivalent DFA containing the minimum number of states
- The minimal DFA is unique
- It is possible to directly obtain the minimal DFA from any DFA
- The algorithm presented here is adapted from Aho, Sethi, and Ullman.

Minimal DFA Algorithm 1

- The algorithm starts by partitioning the states in the DFA into sets of states that will ultimately be combined into single states.
- The first partitioning creates 2 sets:
  - One set contains all the accepting states
  - The other set contains all the nonaccepting states
- The process now goes through one or more iterations where it considers the transitions on each character of the alphabet

Minimal DFA Algorithm 2

- Iterate until no further partitioning is possible:
  - For each set G of states in partition Π, consider the transitions for each input symbol **a** from any state in G.
  - Two states **s** and **t** belong in the same subgroup iff for all input symbols **a**, states **s** and **t** have transitions into states in the same subgroup of Π.
  - Replace G in Π by the set of subgroups formed.

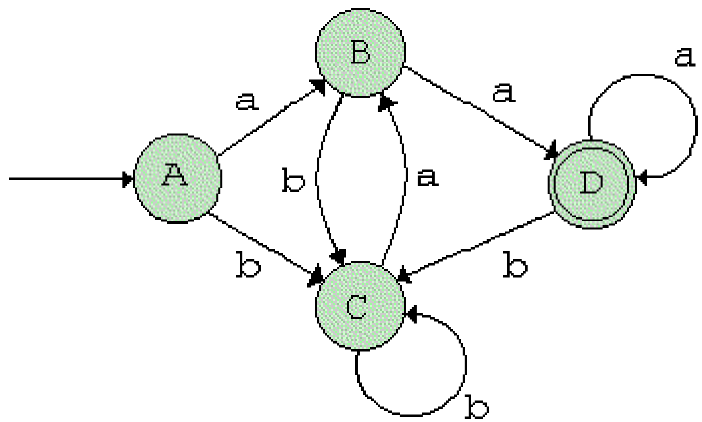
Minimal DFA Algorithm 3

- Choose one state in each group of the partition Π as the representative for that group. The representatives will be the states of the reduced DFA **M’**.
- The start state of **M’** will be the group that contains the start state of the original DFA.
- Any group that contains an accepting state from the original DFA will be an accepting state of the minimal DFA **M’.**

Minimal DFA Algorithm 4

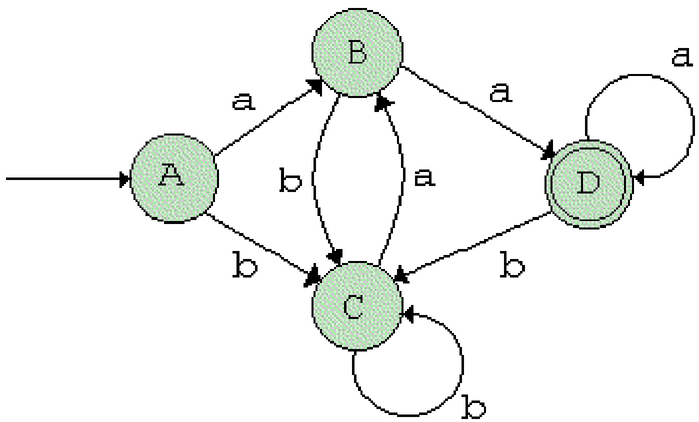
- Remove any dead state **d** from **M’.**
  - a dead state is one that has transitions to itself on all input symbols.
  - Any transitions to **d** from other states become undefined.
- Remove any states unreachable from the start state from **M’.**

Example 1: Minimize the DFA



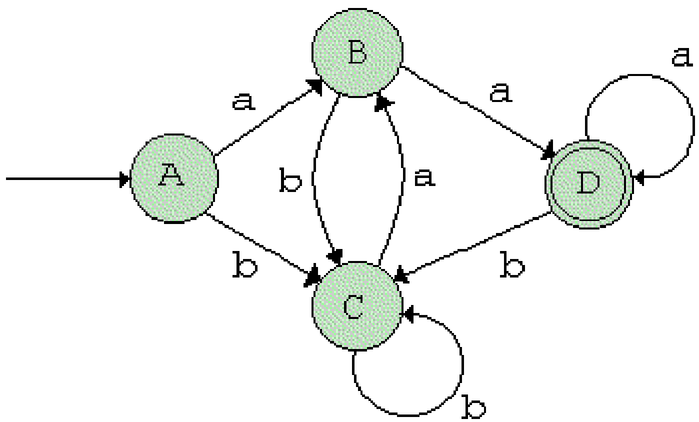
- We start with two groups
  - Accepting states: { D }
  - Nonaccepting states: { A, B, C }
- Since the singleton set { D } cannot be partitioned any further, we concentrate on { A, B, C }

Example 1, continued



- For input a and states in group { A, B, C }
  - T(A, a) = B
  - T(B, a) = D   (maps into a different subgroup)
  - T(C, a) = B
- We must split the group { A, B, C } into two subgroups, { A, C } and { B}

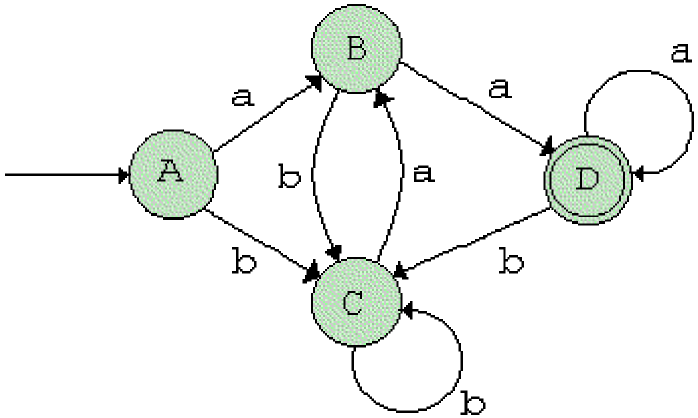
Example 1, continued



- At this point, Π = { A, C }, { B }, { D }
- Consider transitions from { A, C } on **a** and **b**
  - T(A, a) = B T(C, a) = B (same group)
  - T(A, b) = C T(C, b) = C (same group)
- No further partitioning is possible.

Example 1, continued

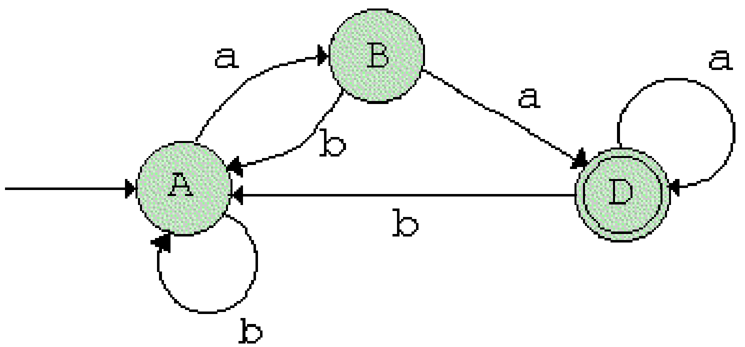
- Π = {A,C}, {B}, {D}
- Choose A as representative from {A,C}:
  - Remove row C from table
  - Replace all occurrences of C with A
- Resulting minimal DFA is shown on next slide



	a	b	
A	B	C	start
B	D	C	
C	B	C	
D	D	C	accept

Example 1, conclusion

- Minimal DFA



	a	b	
A	B	A	start
B	D	A	
D	D	A	accept

Example 2

- Minimize the DFA given in tabular form at right
- Π = {A,B,C,D}, {E}
- Consider a-transitions:
  - T(A, a) = B
  - T(B, a) = B
  - T(C, a) = B
  - T(D, a) = B
- So far, no repartitioning is needed.

	a	b	
A	B	C	start
B	B	D	
C	B	C	
D	B	E	
E	B	C	accept

Example 2, continued

- Consider b-transitions:
  - T(A, b) = C
  - T(B, b) = D
  - T(C, b) = C
  - T(D, b) = E
- Since A, B, and C have transitions into the same group, they stay grouped together
- D has a transition into a different group, so it becomes a new singleton group.
- Π = {A,B,C}, {D}, {E}

	a	b	
A	B	C	start
B	B	D	
C	B	C	
D	B	E	
E	B	C	accept

Example 2, continued

- Π = {A,B,C}, {D}, {E}
- It should be obvious that a-transitions will not cause any repartitioning, so let's consider the b-transitions from {A,B,C}
  - T(A, b) = C
  - T(B, b) = D
  - T(C, b) = C
- {A,B,C} splits into {A,C}, {B}

	a	b	
A	B	C	start
B	B	D	
C	B	C	
D	B	E	
E	B	C	accept

Example 2, continued

- Π = {A,C}, {B}, {D}, {E}
- Since rows A and C are identical, no further partitioning can be done.
- We choose A as the representative from the set {A,C}
- The minimal DFA is shown on the next page.

	a	b	
A	B	C	start
B	B	D	
C	B	C	
D	B	E	
E	B	C	accept

Example 2, conclusion

- The minimal DFA is shown at right.

	a	b	
A	B	A	start
B	B	D	
D	B	E	
E	B	A	accept

Checkpoint: Minimize the DFA

	a	b	
A	G	F	start
B	C	G	
C	B	D	
D	G	E	
E	B	H	accept
F	A	D	
G	B	D	
H	A	E	accept

Checkpoint Solution

	a	b	
A	G	F	start
B	C	G	
C	B	D	
D	G	E	
E	B	H	accept
F	A	D	
G	B	D	
H	A	E	accept

- Π={A,B,C,D,F,G},{E,H}
- T({A,B,C,D,F,G}, a):
  - T(A, a) = G
  - T(B, a) = C
  - T(C, a) = B
  - T(D, a) = G
  - T(F, a) = A
  - T(G, a) = B
  - all map to same group—no repartitioning (yet)

Checkpoint Solution

	a	b	
A	G	F	start
B	C	G	
C	B	D	
D	G	E	
E	B	H	accept
F	A	D	
G	B	D	
H	A	E	accept

- Π={A,B,C,D,F,G},{E,H}
- T({A,B,C,D,F,G},b):
  - T(A, b) = F
  - T(B, b) = G
  - T(C, b) = D
  - T(D, b) = E (different group)
  - T(F, b) = D
  - T(G, b) = D
  - Partition {A,B,C,D,F,G} into {A,B,C,F,G}, {D}

Checkpoint Solution

	a	b	
A	G	F	start
B	C	G	
C	B	D	
D	G	E	
E	B	H	accept
F	A	D	
G	B	D	
H	A	E	accept

- Π={A,B,C,F,G},{D},{E,H}
- T({E,H}, a):
  - T(E, a) = B
  - T(H, a) = A
  - map into same group
- T({E,H}, b):
  - T(E, b) = H
  - T(H, b) = E
  - map into same group
- No repartitioning necessary (at least not yet)

Checkpoint Solution

	a	b	
A	G	F	start
B	C	G	
C	B	D	
D	G	E	
E	B	H	accept
F	A	D	
G	B	D	
H	A	E	accept

- Π={A,B,C,F,G},{D},{E,H}
- T({A,B,C,F,G}, a):
  - T(A, a) = G
  - T(B, a) = C
  - T(C, a) = B
  - T(F, a) = A
  - T(G, a) = B
  - all map to same group, so no repartitioning results from this

Checkpoint Solution

	a	b	
A	G	F	start
B	C	G	
C	B	D	
D	G	E	
E	B	H	accept
F	A	D	
G	B	D	
H	A	E	accept

- Π={A,B,C,F,G},{D},{E,H}
- T({A,B,C,F,G}, b):
  - T(A, b) = F
  - T(B, b) = G
  - T(C, b) = D
  - T(F, b) = D
  - T(G, b) = D
  - {A,B} and {C,F,G} map to different groups, so we repartition {A,B,C,F,G} into the groups {A,B} and {C,F,G}

Checkpoint Solution

	a	b	
A	G	F	start
B	C	G	
C	B	D	
D	G	E	
E	B	H	accept
F	A	D	
G	B	D	
H	A	E	accept

- Π={A,B},{C,F,G},{D},{E,H}
- T({A,B}, a) $\rightarrow$ {C,F,G}
- T({A,B}, b) $\rightarrow$ {C,F,G}
- T({C,F,G}, a) $\rightarrow$ {A,B}
- T({C,F,G}, b) $\rightarrow$ {D}
- T({D}, a) $\rightarrow$ {C,F,G}
- T({D}, b) $\rightarrow$ {E, H}
- T({E,H}, a) $\rightarrow$ {A, B}
- T({E,H}, b) $\rightarrow$ {E, H}
- No further partitioning is possible

Checkpoint Solution

	a	b	
A	G	F	start
B	C	G	
C	B	D	
D	G	E	
E	B	H	accept
F	A	D	
G	B	D	
H	A	E	accept

- Π={A,B},{C,F,G},{D},{E,H}
- Representatives:
  - A = {A,B}
  - C = {C,F,G}
  - D = {D}
  - E = {E,H}
- The minimal DFA is shown on the next slide.

Checkpoint Solution

	a	b	
A	C	C	start
C	A	D	
D	C	E	
E	A	E	accept