Underdetermined Low-Complexity Wideband DOA Estimation with Uniform Linear Arrays

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1. Introduction

- Direction-of-arrival (DOA) estimation is an important area in array signal processing.
- In recent years, underdetermined DOA estimation with sparse arrays has attracted wide attention due to the capability of offering higher number of DOFs with fewer sensors.
- Among most of the existing methods, sparse array configurations are adopted, but the frequency information are not involved for DOFs improvement.
- In this paper, an underdetermined low-complexity DOA estimation method based on difference co-arrays in the spatio-spectral domain has been proposed employing a widebnad ULA.

1. Introduction

- For a uniform linear array (ULA) with specially designed spacing and system settings, the received signals are first decomposed into different frequency bins via discrete Fourier transform (DFT).
- By grouping the frequencies of interest into several pairs, a generalized complexity reduction method is proposed to merge the redundant entries in both the autocorrelation matrices at each frequency and the cross-correlation matrices across different frequencies.
- A group sparsity based low-complexity method has been proposed to find the directions of arrival (DOAs) of the impinging signals with better performance and lower complexity achieved compared with existing methods.

2. System Model Design

- In this paper, the LFMCW signal is employed as an example, and the LFMCW signal with a bandwidth B is

$$s(t) = Ae^{j(2\pi f_c t + \pi\alpha \cdot \mathrm{mod}(t,T)^2 + \varphi)} \ . \tag{1}$$

- We simply consider an M-sensor ULA with the interelement spacing d, and its sensor position set is given as

$$\mathbb{S} = \{md,\ 0 \leq m \leq M-1, m \in \mathbb{Z}\} \ . \tag{2}$$

$$\begin{array}{cccccc} \bullet & \bullet & \bullet & \bullet & \cdots\cdots & \bullet \\ 0 & d & 2d & 3d & & (M-1)d \end{array}$$

2. System Model Design

- The echo signal observed at the m-th sensor is

$$x_m(t) = \sum_{k=1}^{K} \gamma_k(t) \cdot s\left[t - \tau_m(\theta_k)\right] + \bar{n}_m(t) \ , \tag{3}$$

  where $\gamma_k(t)$ is the time-varying reflection coefficient, it is in general frequency-dependent. $\tau_m(\theta_k)$ represents the time delay.

- As the picture shows that, each received signal is divided into non-overlapping groups with length L.

$$\begin{array}{cccc} 1th & 2th & \cdots\cdots & Pth \\ \bullet\ \bullet\ \cdots\ \bullet & \bullet\ \bullet\ \bullet\ \cdots\ \bullet & \cdots\cdots & \bullet\ \bullet\ \cdots\ \bullet \\ \leftarrow\ L\ \rightarrow & \leftarrow\ L\ \rightarrow & \cdots\cdots & \leftarrow\ L\ \rightarrow \end{array}$$

2. System Model Design

- Then, the output array signal model at the l-th frequency bin and the p-th DFT group after an L-point DFT can be expressed as

$$\mathbf{X}[l,p] = \mathbf{A}(l,\boldsymbol{\theta})\mathbf{S}[l,p] + \overline{\mathbf{N}}[l,p] \; , \tag{4}$$

where $\mathbf{S}[l,p] = \left[S_1[l,p], \ldots, S_K[l,p]\right]^T$ with $S_k[l,p]$ being the DFT of the received echo signals $\gamma_k[i]s[i]$.

- The steering vector in the steering matrix $\mathbf{A}(l,\boldsymbol{\theta})$ is

$$\mathbf{a}(l,\theta_k) = \left[1, e^{-j\frac{2\pi d}{\lambda_l}\sin(\theta_k)}, \ldots, e^{-j\frac{2\pi(M-1)d}{\lambda_l}\sin(\theta_k)}\right]^T \; .$$

2. System Model Design

- For this L-point DFT, the frequency interval $f_\triangle$ is $\frac{f_s}{L}$, we design the frequency interval $f_\triangle$ to satisfy

$$f_\triangle = \frac{f_s}{L} = \frac{c}{2d} \, , \tag{5}$$

Then, the wavelength of the l-th frequency bin can be expressed as

$$\lambda_l = \frac{c}{\frac{f_s l}{L}} = \frac{2d}{l} \, ,$$

- Accordingly, $\mathbf{a}(l, \theta_k)$ can be updated to

$$\mathbf{a}(l, \theta_k) = \left[1, e^{-j\pi l \sin(\theta_k)}, \ldots, e^{-j\pi(M-1)l \sin(\theta_k)}\right]^T . \tag{6}$$

3. Underdetermined Low-Complexity DOA Estimation Based on Difference Co-Arrays

- These frequency bins of interest are divided into N pairs, with l_{n_1} and l_{n_2} being the frequency components of the n-th pair ($l_{n_1} \in \Phi_l$, $l_{n_2} \in \Phi_l$, and $l_{n_1} \neq l_{n_2}$).

- Denote $\alpha, \beta \in \{l_{n_1}, l_{n_2}\}$, The correlation matrices (including both auto-correlation matrices and cross-correlation matrices) of the two frequencies can be obtained by

$$\begin{aligned} &\mathbf{R}_{\mathbf{x}}[\alpha, \beta] = \mathrm{E}\left\{\mathbf{X}[\alpha, p] \cdot \mathbf{X}^H[\beta, p]\right\} \\ =&\sum_{k=1}^{K} \sigma_k^2[\alpha, \beta]\mathbf{a}(\alpha, \theta_k)\mathbf{a}^H(\beta, \theta_k) + \sigma_{\bar{n}}^2[\alpha, \beta]\mathbf{I}_M \cdot \delta(\alpha - \beta) \ . \end{aligned}$$

3. Underdetermined Low-Complexity DOA Estimation Based on Difference Co-Arrays

- For $\alpha = \beta$, the parameters $\sigma_k^2[\alpha, \alpha]$, representing both $\sigma_k^2[l_{n_1}, l_{n_1}]$ and $\sigma_k^2[l_{n_2}, l_{n_2}]$, denotes the power of the k-th signal at the α-th frequency bin, whereas $\sigma_{\bar{n}}^2[\alpha, \alpha]$ is the corresponding noise power.

  It is worth nothing that both $\sigma_k^2[\alpha, \alpha]$ and $\sigma_{\bar{n}}^2[\alpha, \alpha]$ are real and positive.

- For $\alpha \neq \beta$, $\sigma_k^2[\alpha, \beta]$ is not zero due to correlation among different frequencies offered by the LFMCW waveforms, and $\sigma_{\bar{n}}^2[\alpha, \beta] = 0$. $\delta(\alpha - \beta)$ is the Kronecker delta function.

  $\sigma_k^2[l_{n_1}, l_{n_2}]$ and $\sigma_k^2[l_{n_2}, l_{n_1}]$ are in general complex values owing to the phase shift between different frequency bins.

- Since $\mathbf{R}_{\mathbf{x}}[l_{n_1}, l_{n_2}] = \mathbf{R}_{\mathbf{x}}^H[l_{n_2}, l_{n_1}]$, we only use the former in the estimation process for complexity reduction.

3. Underdetermined Low-Complexity DOA Estimation Based on Difference Co-Arrays

- Vectorizing $\mathbf{R}_{\mathbf{x}}[\alpha, \beta]$ yields a virtual array model

$$\begin{aligned} \mathbf{z}[\alpha, \beta] &= \mathrm{vec}\left\{\mathbf{R}_{\mathbf{x}}[\alpha, \beta]\right\} \\ &= \widetilde{\mathbf{A}}[\alpha, \beta]\widetilde{\mathbf{s}}[\alpha, \beta] + \sigma_{\bar{n}}^2[\alpha, \beta]\widetilde{\mathbf{I}}_{M^2} \cdot \delta(\alpha - \beta) \ , \end{aligned} \tag{7}$$

with the equivalent steering matrix

$$\widetilde{\mathbf{A}}[\alpha, \beta] = [\widetilde{\mathbf{a}}(\alpha, \beta, \theta_1), \ldots, \widetilde{\mathbf{a}}(\alpha, \beta, \theta_K)] \ , \tag{8}$$

where $\widetilde{\mathbf{a}}(\alpha, \beta, \theta_k) = \mathbf{a}^*(\beta, \theta_k) \otimes \mathbf{a}(\alpha, \theta_k)$ with $\otimes$ as the Kronecker product, and $\widetilde{\mathbf{s}}[\alpha, \beta] = \left[\sigma_1^2[\alpha, \beta], \ldots, \sigma_K^2[\alpha, \beta]\right]^T$. $\widetilde{\mathbf{I}}_{M^2} = \mathrm{vec}\{\mathbf{I}_M\}$ is an $M^2 \times 1$ column vector.

3. Underdetermined Low-Complexity DOA Estimation Based on Difference Co-Arrays

- For $\alpha = \beta$, the self-difference co-array lags is generated as

$$\{(\alpha m_1 - \alpha m_2), 0 \leq m_1, m_2 \leq M-1\} \ . \tag{9}$$

- For $\alpha \neq \beta$, the cross-difference co-array lags in the spatio-spectral domain is generated as

$$\{\pm(\alpha m_1 - \beta m_2), 0 \leq m_1, m_2 \leq M-1\} \ . \tag{10}$$

- Specifically, l_{n_1} and l_{n_2} can be chosen to be co-prime, and then the decomposed signals at two frequency bins are equivalent to the received signals of two uniform linear sub-arrays in a co-prime array with $2M - 1 - \mathrm{floor}\{\frac{M-1}{\max(l_{n_1}, l_{n_2})}\}$ sensors.

3. Underdetermined Low-Complexity DOA Estimation Based on Difference Co-Arrays

- Note that the auto-correlation matrix $\mathbf{R_x}[\alpha, \alpha]$ is both Hermitian and Toeplitz, i.e.,

$$\begin{bmatrix} r_0 & r_1 & r_2 & \cdots & r_{M-1} \\ r_1 & r_0 & r_1 & \cdots & r_{M-2} \\ r_2 & r_1 & r_0 & \cdots & r_{M-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r_{M-1} & r_{M-2} & r_{M-2} & \cdots & r_0 \end{bmatrix}$$

- After removing the redundant lags in $\mathbf{R_x}[\alpha, \alpha]$, we obtain a column vector $\mathbf{z_c}[\alpha, \alpha]$ with its merged m-th entry given by

$$z_c^m[\alpha, \alpha] = \frac{1}{M-m} \sum_{\hat{m}=m}^{M-1} R_x^{\hat{m}, \hat{m}-m}[\alpha, \alpha] \ , \tag{11}$$

where $m = 0, 1, \ldots, M-1$.

3. Underdetermined Low-Complexity DOA Estimation Based on Difference Co-Arrays

- The m_1-th row and m_2-th column of the cross-correlation matrix $R_x^{m_1,m_2}[l_{n_1}, l_{n_2}]$ is given by

$$R_x^{m_1,m_2}[l_{n_1}, l_{n_2}] = \sum_{k=1}^{K} \sigma_k^2[l_{n_1}, l_{n_2}] e^{-j\pi(l_{n_1}m_1 - l_{n_2}m_2)\sin\theta_k} \ .$$

- $R_x^{m_1,m_2}[l_{n_1}, l_{n_2}]$ and $R_x^{\hat{m}_1,\hat{m}_2}[l_{n_1}, l_{n_2}]$ are equal if and only if $l_{n_1}m_1 - l_{n_2}m_2 = l_{n_1}\hat{m}_1 - l_{n_2}\hat{m}_2$, which can be modified into

$$l_{n_1}(m_1 - \hat{m}_1) = l_{n_2}(m_2 - \hat{m}_2) \ , \qquad (12)$$

where $0 \le m_1, m_2, \hat{m}_1, \hat{m}_2 \le M - 1$.

3. Underdetermined Low-Complexity DOA Estimation Based on Difference Co-Arrays

- Assume that η_n is the greatest common divisor between l_{n_1} and l_{n_2}, and then (12) can be updated to

$$\begin{aligned} &\frac{l_{n_1}}{\eta_n}(m_1 - \hat{m}_1) = \frac{l_{n_2}}{\eta_n}(m_2 - \hat{m}_2) \ , \\ &m_1 - \hat{m}_1 = \bar{k}\frac{l_{n_2}}{\eta_n} \bigcap m_2 - \hat{m}_2 = \bar{k}\frac{l_{n_1}}{\eta_n} \ , \ \bar{k} \in \mathbb{Z} \ . \end{aligned} \tag{13}$$

where $\frac{l_{n_1}}{\eta_n}$ and $\frac{l_{n_2}}{\eta_n}$ are co-prime.

Then, we have

$$\begin{aligned} R_x^{\hat{m}_1,\hat{m}_2}[l_{n_1}, l_{n_2}] &= R_x^{m_1,m_2}[l_{n_1}, l_{n_2}] \\ &= R_x^{\bar{k}\frac{l_{n_2}}{\eta_n}+\hat{m}_1, \bar{k}\frac{l_{n_1}}{\eta_n}+\hat{m}_2}[l_{n_1}, l_{n_2}] \ . \end{aligned} \tag{14}$$

3. Underdetermined Low-Complexity DOA Estimation Based on Difference Co-Arrays

- Thus, we can obtain a new smoothed cross-correlation matrix $\mathbf{R_c}[l_{n_1}, l_{n_2}]$ by combining the equal entries together, with its m_1-th row and m_2-th column given by

$$R_c^{m_1,m_2}[l_{n_1}, l_{n_2}] = \frac{\sum_{\bar{k}=\hat{k}_{\min}}^{\hat{k}_{\max}} R_x^{\bar{k}\frac{l_{n_2}}{\eta_n}+m_1, \bar{k}\frac{l_{n_1}}{\eta_n}+m_2}[l_{n_1}, l_{n_2}]}{\hat{k}_{\text{len}}}, \tag{15}$$

with

$$\begin{aligned} \hat{k}_{\text{len}} &= \hat{k}_{\max} - \hat{k}_{\min} + 1 \ , \\ \hat{k}_{\min} &= \text{ceil}\left\{\max\left\{\frac{-m_1\eta_n}{l_{n_2}}, \frac{-m_2\eta_n}{l_{n_1}}\right\}\right\} \ , \\ \hat{k}_{\max} &= \text{floor}\left\{\min\left\{\frac{\eta_n(M-1-m_1)}{l_{n_2}}, \frac{\eta_n(M-1-m_2)}{l_{n_1}}\right\}\right\} \ , \end{aligned} \tag{16}$$

- Define two sets of indexes (m_1, m_2), $m_1, m_2 \in \mathbb{Z}$, given by

$$\Phi_M = \{(m_1, m_2), 0 \leq m_1, m_2 < M\} \ ,$$
$$\widetilde{\Phi}_n = \left\{(m_1, m_2), \frac{l_{n_2}}{\eta_n} \leq m_1 < M \bigcap \frac{l_{n_1}}{\eta_n} \leq m_2 < M\right\} \ .$$

Note that $\widetilde{\Phi}_n = \emptyset$ is an empty set when $\frac{l_{n_2}}{\eta_n} \geq M \bigcup \frac{l_{n_1}}{\eta_n} \geq M$.

- The set $\Phi_n = \Phi_M - \widetilde{\Phi}_n$ represents the unique entries without redundancy in the smoothed cross-correlation matrix $\mathbf{R_c}[l_{n_1}, l_{n_2}]$.

- For the n-th frequency pair, the number of unique entries without redundancy in $\mathbf{R_c}[l_{n_1}, l_{n_2}]$ is

$$M_n = \begin{cases} \frac{M(l_{n_1}+l_{n_2})}{\eta_n} - \frac{l_{n_1} l_{n_2}}{\eta_n^2}, & \widetilde{\Phi}_n \neq \emptyset, \\ M^2, & \widetilde{\Phi}_n = \emptyset. \end{cases} \tag{17}$$

3. Underdetermined Low-Complexity DOA Estimation Based on Difference Co-Arrays

- $\mathbf{R_x}[\alpha,\alpha]$ is simplified into a vector, rewritten as

$$\mathbf{z_c}[\alpha,\alpha] = \mathbf{A_c}[\alpha,\alpha]\widetilde{\mathbf{s}}[\alpha,\alpha] + \sigma_{\tilde{n}}^2[\alpha,\alpha]\mathbf{v}_M, \tag{18}$$

where $\mathbf{A_c}[\alpha,\alpha] = [\mathbf{a}(\alpha,\theta_1),\ldots,\mathbf{a}(\alpha,\theta_K)]$, and $\mathbf{v}_M$ has a size of $M\times 1$, being all zeroes except for a 1 at the zeroth entry.

- By vectorizing $\mathbf{R_c}[l_{n_1},l_{n_2}]$, we obtain

$$\begin{aligned}\mathbf{z_c}[l_{n_1},l_{n_2}] &= \mathrm{vec}\left\{\mathbf{R_c}[l_{n_1},l_{n_2}]\right\} \\ &= \widetilde{\mathbf{A}}[l_{n_1},l_{n_2}]\widetilde{\mathbf{s}}[l_{n_1},l_{n_2}]\ .\end{aligned} \tag{19}$$

3. Underdetermined Low-Complexity DOA Estimation Based on Difference Co-Arrays

- For the n-th frequency pair, with the same search grid of K_g potential incident angles $\theta_{g,0}, \cdots, \theta_{g,K_g-1}$, we construct

$$\widetilde{\mathbf{A}}_{\mathbf{cg}}[\alpha, \alpha] = \left[\mathbf{a}(\alpha, \theta_{g,0}), \ldots, \mathbf{a}(\alpha, \theta_{g,K_g-1})\right] \ . \tag{20}$$

- We use $z_{c,\bar{m}}[l_{n_1}, l_{n_2}]$, $0 \leq \bar{m} \leq M^2 - 1$, to denote the $\bar{m}$-th entry in the column vector $\mathbf{z_c}[l_{n_1}, l_{n_2}]$, and row vectors $\widetilde{\mathbf{a}}_{\mathbf{r},\bar{m}}[l_{n_1}, l_{n_2}]$ and $\widetilde{\mathbf{a}}_{\mathbf{g},\bar{m}}[l_{n_1}, l_{n_2}]$ are the $\bar{m}$-th row of the matrices $\widetilde{\mathbf{A}}[l_{n_1}, l_{n_2}]$ and $\widetilde{\mathbf{A}}_{\mathbf{g}}[l_{n_1}, l_{n_2}] = \left[\widetilde{\mathbf{a}}(l_{n_1}, l_{n_2}, \theta_{g,0}), \ldots, \widetilde{\mathbf{a}}(l_{n_1}, l_{n_2}, \theta_{g,K_g-1})\right]$, respectively.

- Denote $\bar{m}_{m_0} \in \phi_n$, $0 \leq m_0 \leq M_n - 1$ as the row indexes corresponding to the unique co-array lags without redundancy, where $\phi_n = \{m_1 + m_2 M, (m_1, m_2) \in \Phi_n\}$ with M_n elements.

3. Underdetermined Low-Complexity DOA Estimation Based on Difference Co-Arrays

- By keeping all the row indexes $\bar{m}_{m_0}$ associated with unique entries, the following matrices can be generated

$$\begin{aligned}
\bar{\mathbf{z}}_{\mathbf{c}}[l_{n_1}, l_{n_2}] &= \left[z_{c,\bar{m}_0}[l_{n_1}, l_{n_2}], \ldots, z_{c,\bar{m}_{M_n-1}}[l_{n_1}, l_{n_2}]\right]^T, \\
\widetilde{\mathbf{A}}_{\mathbf{c}}[l_{n_1}, l_{n_2}] &= \left[\widetilde{\mathbf{a}}^T_{\mathbf{r},\bar{m}_0}[l_{n_1}, l_{n_2}], \ldots, \widetilde{\mathbf{a}}^T_{\mathbf{r},\bar{m}_{M_n-1}}[l_{n_1}, l_{n_2}]\right]^T, \\
\widetilde{\mathbf{A}}_{\mathbf{cg}}[l_{n_1}, l_{n_2}] &= \left[\widetilde{\mathbf{a}}^T_{\mathbf{g},\bar{m}_0}[l_{n_1}, l_{n_2}], \ldots, \widetilde{\mathbf{a}}^T_{\mathbf{g},\bar{m}_{M_n-1}}[l_{n_1}, l_{n_2}]\right]^T.
\end{aligned}$$

- Therefore, the model in (19) can be simplified into

$$\bar{\mathbf{z}}_{\mathbf{c}}[l_{n_1}, l_{n_2}] = \widetilde{\mathbf{A}}_{\mathbf{c}}[l_{n_1}, l_{n_2}]\widetilde{\mathbf{s}}[l_{n_1}, l_{n_2}] \ . \qquad (21)$$

3. Underdetermined Low-Complexity DOA Estimation Based on Difference Co-Arrays

- We obtain the low-complexity virtual array model under the CS framework, given by

$$\mathbf{z_c}[n] = \widetilde{\mathbf{A}}_{\mathbf{cg}}[n]\widetilde{\mathbf{s}}_{\mathbf{g}}[n] + \mathbf{V}\mathbf{w}[n] = \widetilde{\mathbf{A}}^{\circ}_{\mathbf{cg}}[n]\widetilde{\mathbf{s}}^{\circ}_{\mathbf{g}}[n] \ , \tag{22}$$

where $\mathbf{z_c}[n] = \left[\mathbf{z}_{\mathbf{c}}^{T}[l_{n_1}, l_{n_1}], \mathbf{z}_{\mathbf{c}}^{T}[l_{n_2}, l_{n_2}], \bar{\mathbf{z}}_{\mathbf{c}}^{T}[l_{n_1}, l_{n_2}]\right]^{T}$, $\widetilde{\mathbf{A}}_{\mathbf{cg}}[n] = \mathrm{blkdiag}\left\{\widetilde{\mathbf{A}}_{\mathbf{cg}}[l_{n_1}, l_{n_1}], \widetilde{\mathbf{A}}_{\mathbf{cg}}[l_{n_2}, l_{n_2}], \widetilde{\mathbf{A}}_{\mathbf{cg}}[l_{n_1}, l_{n_2}]\right\}$. The matrix $\mathbf{V} = [\widetilde{\mathbf{v}}_1, \widetilde{\mathbf{v}}_2]$ has a size of $(2M + M_n) \times 2$ with $\widetilde{\mathbf{v}}_1 = \left[\mathbf{v}_M^T, \mathbf{0}_M^T, \mathbf{0}_{M_n}^T\right]^T$ and $\widetilde{\mathbf{v}}_2 = \left[\mathbf{0}_M^T, \mathbf{v}_M^T, \mathbf{0}_{M_n}^T\right]$, where $\mathbf{0}_M^T$ denotes an $M \times 1$ column vector consisting of all zeros. $\widetilde{\mathbf{A}}^{\circ}_{\mathbf{cg}}[n] = \left[\widetilde{\mathbf{A}}_{\mathbf{cg}}[n], \mathbf{V}\right]$, $\widetilde{\mathbf{s}}_{\mathbf{g}}[n] = \mathrm{vec}\left\{\widetilde{\mathbf{S}}_{\mathbf{g}}[n]\right\}$, $\mathbf{w}[n] = \left[\sigma^2_{\bar{n}}[l_{n_1}, l_{n_1}], \sigma^2_{\bar{n}}[l_{n_2}, l_{n_2}]\right]^T$, and $\widetilde{\mathbf{s}}^{\circ}_{\mathbf{g}}[n] = \left[\widetilde{\mathbf{s}}_{\mathbf{g}}^T[n], \mathbf{w}^T[n]\right]^T$.

3. Underdetermined Low-Complexity DOA Estimation Based on Difference Co-Arrays

- For DOA estimation across the frequency range of interest with N pairs, a $K_g \times 3N$ matrix $\mathbf{R}_\mathbf{g}$, and a $(3K_g+2)N \times 1$ column vector $\mathbf{r}^\circ_\mathbf{g}$, are constructed as

$$\begin{aligned} \mathbf{R}_\mathbf{g} &= \left[\widetilde{\mathbf{S}}_\mathbf{g}[0], \widetilde{\mathbf{S}}_\mathbf{g}[1], \ldots, \widetilde{\mathbf{S}}_\mathbf{g}[N-1]\right] , \\ \mathbf{r}^\circ_\mathbf{g} &= \left[\widetilde{\mathbf{s}}^{\circ T}_\mathbf{g}[0], \widetilde{\mathbf{s}}^{\circ T}_\mathbf{g}[1], \ldots, \widetilde{\mathbf{s}}^{\circ T}_\mathbf{g}[N-1]\right]^T . \end{aligned} \tag{23}$$

Denote $\hat{\mathbf{r}}_\mathbf{g} = \left[\left\|\mathbf{r}_{\mathbf{g},0}\right\|_2, \left\|\mathbf{r}_{\mathbf{g},2}\right\|_2, \ldots, \left\|\mathbf{r}_{\mathbf{g},K_g-1}\right\|_2\right]^T$ with the row vector $\mathbf{r}_{\mathbf{g},k_g}$, $0 \le k_g \le K_g - 1$ representing the k_g-th row of the matrix $\mathbf{R}_\mathbf{g}$.

3. Underdetermined Low-Complexity DOA Estimation Based on Difference Co-Arrays

- Finally, the group sparsity based low-complexity wideband DOA estimation method for multiple frequency pairs can be expressed as

$$\min_{\mathbf{r}_{\mathbf{g}}^{\circ}} \quad \|\hat{\mathbf{r}}_{\mathbf{g}}\|_1, \quad \text{subject to} \quad \left\|\mathbf{z}_{\mathbf{cg}} - \widetilde{\mathbf{B}}_{\mathbf{cg}}^{\circ}\mathbf{r}_{\mathbf{g}}^{\circ}\right\|_2 \leq \varepsilon \ , \tag{24}$$

where $\mathbf{z}_{\mathbf{cg}} = \left[\mathbf{z}_{\mathbf{c}}^T[0], \mathbf{z}_{\mathbf{c}}^T[1], \ldots, \mathbf{z}_{\mathbf{c}}^T[N-1]\right]^T$. $\|\cdot\|_1$ is the ℓ_1 norm, $\|\cdot\|_2$ is the ℓ_2 norm, and the $(2MN + \sum_{n=0}^{N-1} M_n) \times (3K_g + 2)N$ block diagonal matrix $\widetilde{\mathbf{B}}_{\mathbf{cg}}^{\circ}$ is generated by

$$\widetilde{\mathbf{B}}_{\mathbf{cg}}^{\circ} = \mathrm{blkdiag}\left[\widetilde{\mathbf{A}}_{\mathbf{cg}}^{\circ}[0], \widetilde{\mathbf{A}}_{\mathbf{cg}}^{\circ}[1], \ldots, \widetilde{\mathbf{A}}_{\mathbf{cg}}^{\circ}[N-1]\right] . \tag{25}$$

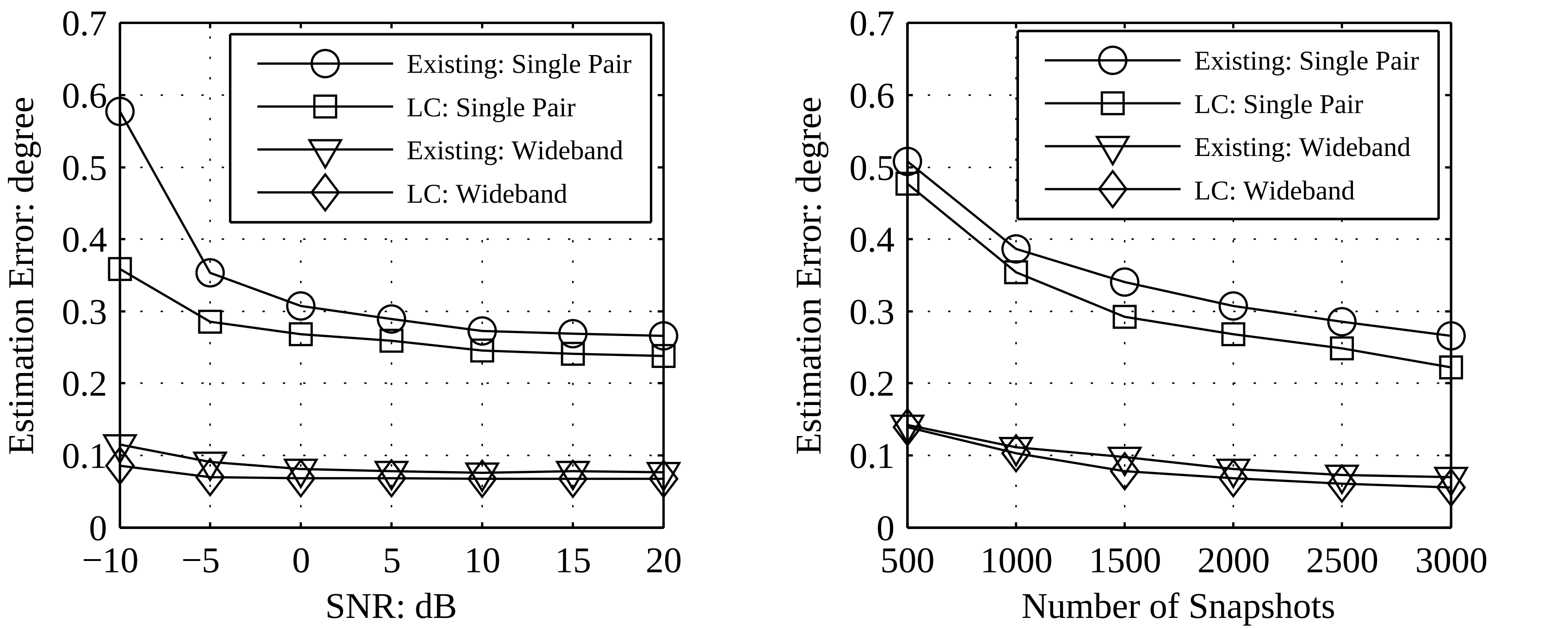
4. Simulation Results

Table 1: Number of Entries in Vectors/Matrices

Vector / Matrix	Theoretical results		Example	
	Existing method (Single Pair)	LC method (Single Pair)	Existing method (Single Pair)	LC method (Single Pair)
$\widetilde{\mathbf{s}}^{\circ}_{\mathbf{g}}[n]$	$3K_g + 2$	$3K_g + 2$	5405	5405
$\mathbf{z_c}[n]$	$3M^2$	$2M + M_n$	147	61
$\widetilde{\mathbf{A}}^{\circ}_{\mathbf{cg}}[n]$	$3M^2(3K_g + 2)$	$(2M + M_n)(3K_g + 2)$	794535	329705
	Existing method (Wideband)	LC method (Wideband)	Existing method (Wideband)	LC method (Wideband)
$\mathbf{r}^{\circ}_{\mathbf{g}}$	$(3K_g + 2)N$	$(3K_g + 2)N$	27025	27025
$\mathbf{z_{cg}}$	$3M^2 \cdot N$	$2MN + \sum_{n=1}^{N} M_n$	735	259
$\widetilde{\mathbf{B}}^{\circ}_{\mathbf{cg}}$	$3(3K_g + 2)(MN)^2$	$(2MN + \sum_{n=1}^{N} M_n)(3K_g + 2)N$	19863375	6999475
Computation Time			116.5474 s	55.2154 s

4. Simulation Results

- RMSE results obtained by different estimation methods (source number $K = 16$):



(a) RMSE results vs input SNR.

(b) RMSE results vs snapshot number

Figure 1: RMSE results obtained by different estimation methods.

5. Conclusion

- An underdetermined low-complexity DOA estimation method based on the group sparsity concept has been proposed employing a widebnad ULA.

- The received signals were first decomposed into different frequencies by DFT, and these frequency bins were divided into several pairs for co-array generation in the spatio-spectral domain.

- A generalized complexity reduction method was then proposed to merge the redundant entries in both the auto-correlation matrices at each frequency and the cross-correlation matrices across frequencies, followed by the group sparsity based method for wideband DOA estimation with reduced complexity achieved.

- As shown by simulations, the proposed low-complexity estimation method outperformed the existing method in terms of both estimation accuracy and computation time.

Thank you!