VICTORIA UNIVERSITY OF WELLINGTON
Te Whare Wananga o te Upoko o te Ika a Maui

School of Engineering and Computer Science

COMP 307 — Lectures 02 and 03

Problem Solving and Search Techniques

Mengjie Zhang (Meng)

mengjie.zhang@ecs.vuw.ac.nz

Outline

- Why Search?
- Search strategies
- Uninformed/Blind search
- Informed search/Heuristic search
- Local search: Hill Climbing
- local search in continuous space
- Genetic beam search
- Advanced discussions
- Game Playing (x)

Why Search (1)

Many puzzle and game playing problems need search

- The Monkey and Bananas Problem
- The Missionaries and Cannibals Problem
- The 8-puzzle
- The Tower of Hanoi
- Wolf, Goat and Cabbage
- Water jug
- Route finding
- Chess, Bridge, Go

Why Search (2)

Many real-world complex and engineering problems need search

- Touring problems
- Traveling Salesperson Problem (TSP)
- VLSI layout (Cell Layout and Channel routing)
- Robot nevigation
- Automatic Assembly sequencing
- University timetabling
- Job shop scheduling

Search will be used in almost all AI techniques such as machine learning and evolutionary computation

State Space Search

Problem solving as State Space Search

- State: a state of the world.
  – State space: Collection of all possible states
  – Initial state: where the search starts
  – Goal state: where the search stops
- Operators: Links between pairs of states, that is, what actions can be taken in any state to get to a new state.
- A path in the state space is a sequence of operators leading from one state to another
- Solve problem by searching for path from initial state to goal state through legal states
- Each operator has an associated cost.
  – Path cost: the sum of the costs of operators along the path.

An Example: 8 Puzzle

7	2	4
5		6
8	3	1

Start State

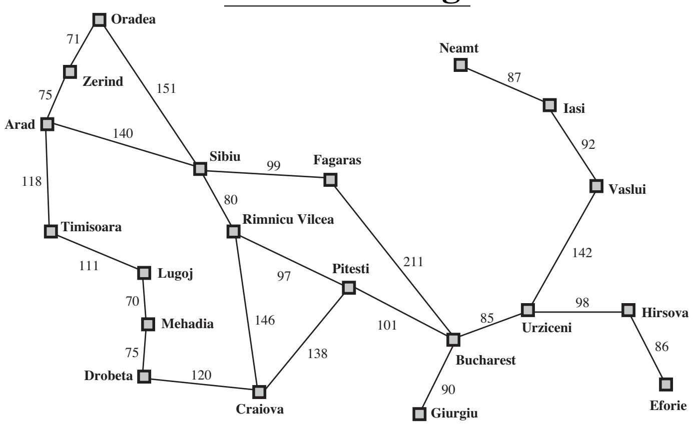
	1	2
3	4	5
6	7	8

Goal State

Formulating problems:

- States: location of the 8 tiles
- Operators: blank moves left, right, up, or down
- Path cost: length of path

Route Finding



- State: current location on map (part of Romania)
  – Initial state: city A
  – Goal state: city B
- Operators: move along a road to another location
- Path cost: sum on lengths of road

General Search Algorithm

For general tree-search:

```
Initialise the frontier using the initial state of the  problem
 loop
   If the frontier/fringe is empty
     then return failure
   choose a leaf node and remove it from the frontier
   If the node represents the goal state
     then return the corresponding solution (as success)
   else expand the node and put the children notes on the frontier
             (and ordering)
 end loop
```

For general graph-search, similar(see text book)

Search Variants and Performance Measure

Three search variants:

- FIFO (queue): pop the oldest element
- FILO (stack): pop the newest element
- Priority queue: pop the element with the highest prority based on some ordering function

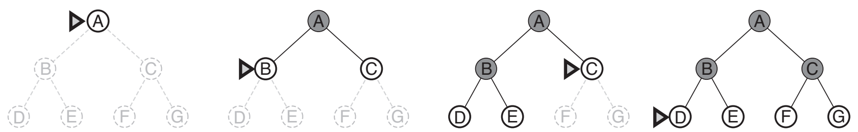
Four measures:

- Completeness: is the strategy guaranteed to find a solution when one exists?
- Optimality: does the strategy find the highest-quality solution (lowest cost) when there are several solutions?
- Time complexity: how long does it take to find a solution?
- Space complexity: how much memory does it need to perform the search?

Search Strategies

- Uninformed (blind) search
  – Breadth first
  – Uniform cost
  – Depth first
  – Depth limited
  – Iterative deepening
  – Bidirectional
- Informed (Heuristic) search
  – Greedy best-first search
  – A* search
- Beyond classic search
  – Hill climbing
  – Gradient descent
  – Simulated Annealing
  – Beam search
  – Bound and bound (x)
  – dynamic programming (x)

Breadth First Search

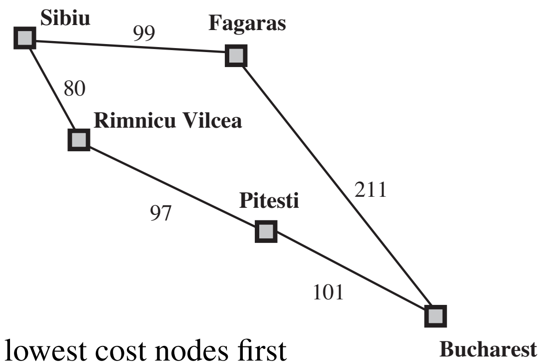


- Start at the initial state;
- Find all states reachable from the initial state;
- Find all states reachable from the states available at the previous step;
- Repeat the previous step until the final state is reached

Issues for Breadth-first Search

- Breadth-first search is guaranteed to find the shortest path
  – Complete
  – Optimal if all operators have same cost (so shallowest solution is cheapest solution)
- In practice, breadth-first search is very expensive
  – Time complexity $O(b^d)$
  – Space complexity $O(b^d)$
- b: the branching factor of nodes (ie, average number of children a node has)
- d: the depth of the desired solution

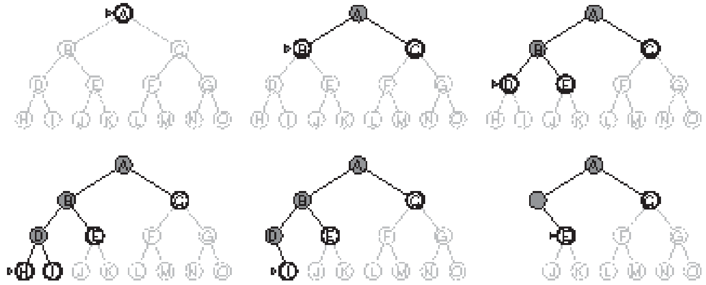
Uniform Cost Search



- Expand lowest cost nodes first
- Same as breadth-first search if all operators have the same cost
- Complete
- Optimal if all operators have positive cost
- Time Complexity $O(b^d)$
- Space Complexity $O(b^d)$

Depth-first Search

- Make the initial state the current state;
- If current state is a final state then succeed
  – else make the current state a state reachable from the current state;
- Repeat previous step until success, backtracking if necessary

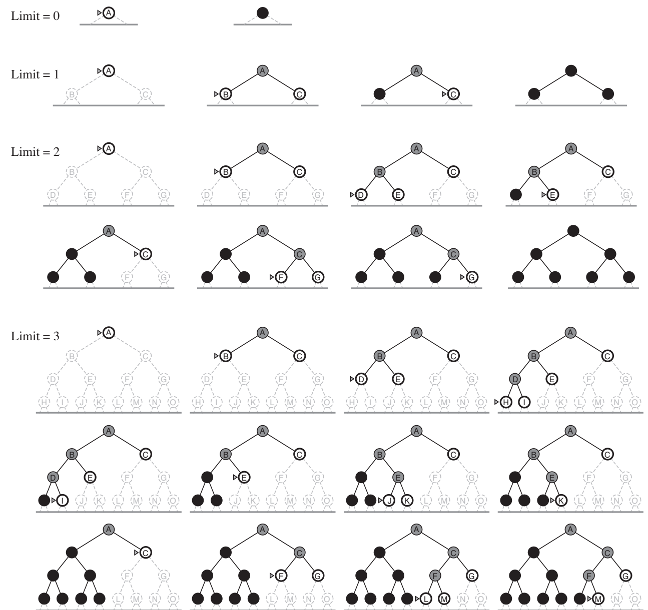


Issues for DFS and Depth Limited Search

- Depth-first search is prone to being lost
  – Complete only if search tree is finite
  – Not optimal
- In practice, depth-first search is (more) efficient
  – Time Complexity $O(b^m)$
  – Space Complexity $O(bm)$ (m=max depth of search tree)
- Good when many solutions in deep (but finite) trees

- **Depth Limited Search:** Like depth-first, but with depth cut off
- Complete only if solution is at depth $<=$ c where c is the depth limit
- Not optimal; Time complexity $O(b^c)$; Space complexity $O(bc)$

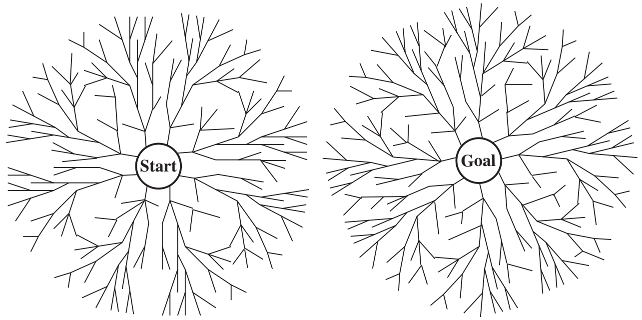
Iterative Deepening (Depth-first) Search



Iterative Deepening Search

- Optimal and Complete (if operators all have same cost)
- Time complexity $O(b^d)$
- Space complexity $O(bd)$
- Expands some nodes multiple times
  – But wasted effort is actually quite small and goes down as the branching factor goes up
- In general, iterative deepening is the preferred search method when there is a large search space and the depth of the solution is not known.

Bidirectional Search



- Two breadth-first search: one forward from the initial state and the other backward from the goal state
- Complete and optimal
- Time complexity $O(b^{d/2})$
- Space complexity $O(b^{d/2})$
- Not applicable for some problems

Heuristic Search

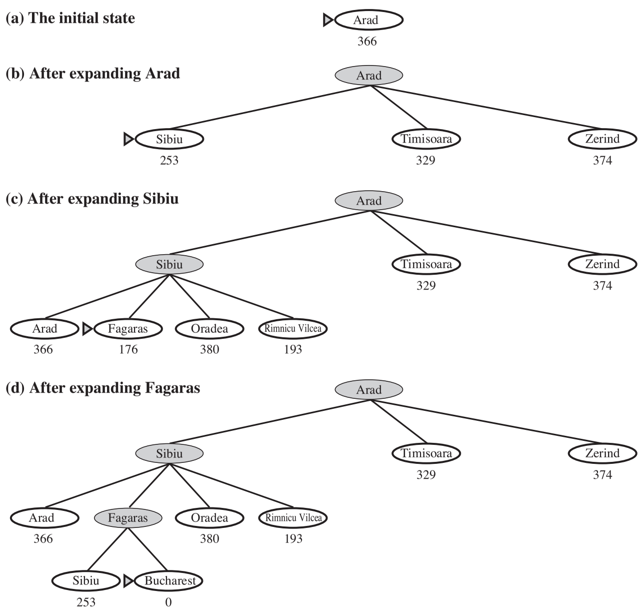
- Looking ahead and estimating cost to goal
  – no information: blind search
  – perfect information: search is easy
  – partial information: look ahead gives fuzzy picture
- A heuristic function estimates the cost from the current state to the goal state
- Why Heuristics?
  – Chess: b=35 d=100
  – Go: b=361, d=?
  – (b: branching factor; d: the depth of the desired solution)
- h(n) = estim. cost of the cheapest path from node n to goal state
- Admissible heuristics: h(n) **never** overestimates the cost to reach the goal
  – Route Finding: straight line distance
  – Find heuristics by relaxing the problem

Greedy (Best First) Search

- Minimize estimated cost to reach the goal, i.e., always expand node whose state looks closest to the goal state (Bucharest)
- f(n) = h(n)
- Route Finding — straight line distance:

Arad	366	**Mehadia**	241
Bucharest	0	**Neamt**	234
Craiova	160	**Oradea**	380
Drobeta	242	**Pitesti**	100
Eforie	161	**Rimnicu Vilcea**	193
Fagaras	176	**Sibiu**	253
Giurgiu	77	**Timisoara**	329
Hirsova	151	**Urziceni**	80
Iasi	226	**Vaslui**	199
Lugoj	244	**Zerind**	374

Greedy (Best First) Search

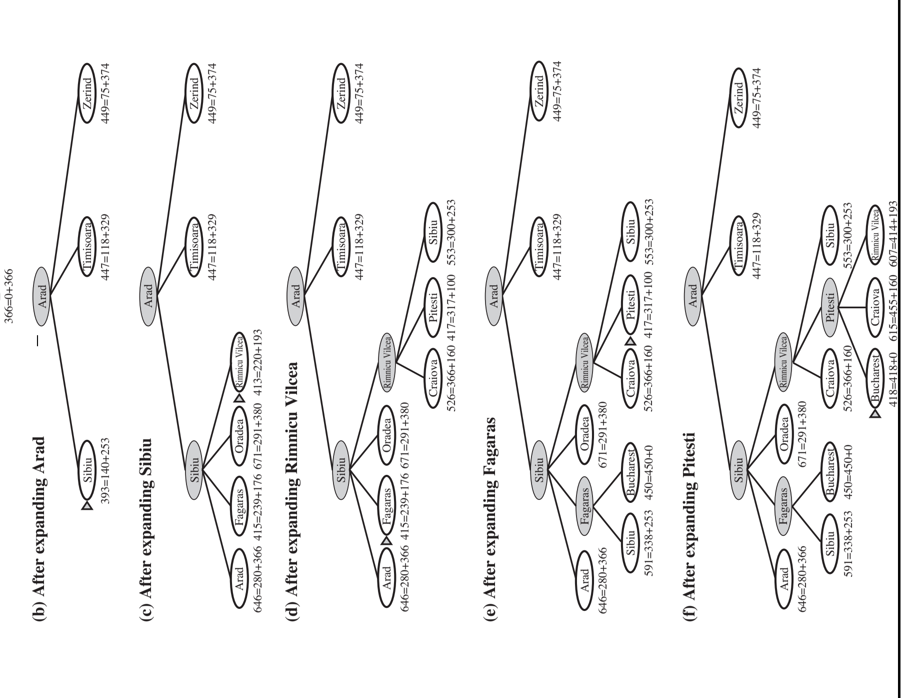


Issues for Greedy search

- Not optimal, Not complete
- Can go down false paths
- Worst case time & space complexity is $O(b^m)$ (m: maximum depth of the search tree)
- In practise, nonetheless, can work well
- Does not consider path cost g(n): cost from the start node to node n

A* search

- A* attempts to minimize total estimated path cost:
- $f(n) = g(n) + h(n)$
- g(n): cost from the start node to node n
- h(n): estimated cost of the cheapest path from node n to the goal
- f(n): estimated cost of the cheapest solution through n



Issues for A* search

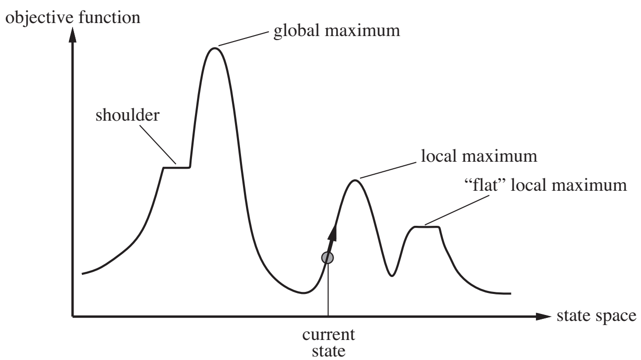
- If f^* is (true) cost of solution path, then
  – A* expands all nodes with $f(n) \leq f^*$
- Optimal and complete
  – Condition: h(n) must be admissible, that is, h(n) never over-estimates the cost to goal
- Both time and space complexity are $O(b^d)$, because A* stores all the nodes it visits.

Local Search

- So far, the search techniques we have discussed assume observable, deterministic, known environments where the solution is a sequence of actions.
- In those algorithms, the *path* is a solution to the problem.
- In many other problems, however, the path is *irrelevant* or does NOT matter
- e.g. integrated circuit design (ECEN), factor-floor layout, job shop scheduling (COMP/OPRE), automatic programming (SWEN/COMP), telecommunication network optimisation (NWEN/COMP), Vehicle routing, or portfolio management (other), classification, many data mining tasks
- Local search algorithms operate using a single current node (in general) to move to neighbours of that node
- Local search algorithms are not systematic:
  – use very little memory
  – can often find reasonable solutions in large or even infinate state space

Local Search — Hill Climbing

- Local search is useful for solving optimisation problems
- Aim to find the best state according to an *objective function*
- Only keep *one* state (node) and its evaluation h(n): the quality
- A loop that continually moves in the direction of increasing h(n). (decreasing h(n): if h(n) is the cost)
- Choose the best successor; if more than one, choose at random.



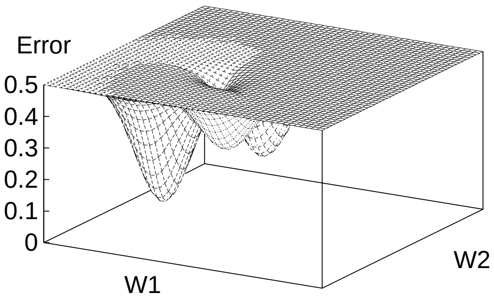
Simulated Annealing

- HC never makes "worse" moves toward states with a higher cost.
- HC can easily get stuck on a local optimum (maximum) and is almost guaranteed to be incomplete.
- Purely random walk is complete in general, but can be extremely inefficient
- Can we combine the advatages of the HC and RW together?
- This is the idea of SA.

- SA also borrows the idea from mental annealing in Physics or Mechanical Engineering that used to temper or harden mentals
  – by heating them to a high temperature then gradually cooling them
  – allowing them to reach a low energy crystalline state.
- Instead of using the best move, SA uses a random move.
- But still uses the HC idea — if the move improves the situation, accept it; otherwise, accept it with some probability less than 1. It uses the probability and "temperature" to control the loop

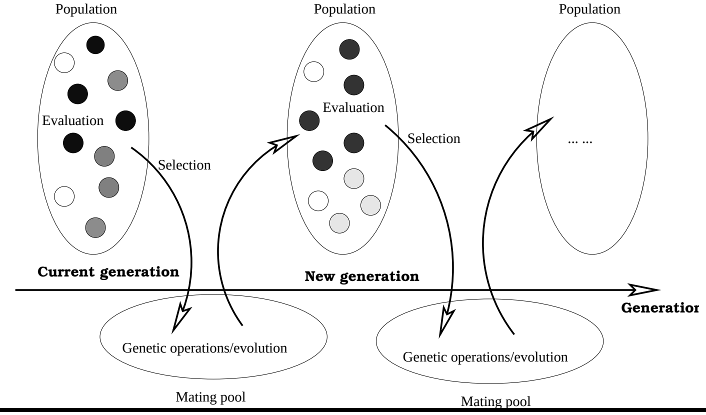
Gradient Descent Search

- HC only considers discrete space
- If the problem (objective function) is continuous, then the idea of HC can still work but the mechanism will need to change
- Gradient Descent (or Ascent) search
- E.g. neural network training using back propagation



(Genetic) Beam Search

- Like HC, BS also cocerns solution space rather than the path.
- Rather considering 1 neighbour only, BS considers one or more neighbours
- Process a beam with some randomly generated multiple *candidate solutions* and modify the candidate solutions



Building Solutions vs Searching Solutions

- Building solutions step by step
  – search path/space is partial solutions
  – Classic search
  – all uninformed/blind search: BFS, UC, DFS, DLS, IDS, BiDS
  – all classic/heuristic search: greedy BFS, A*, etc.
- Searching solution space
  – path does NOT matter or irrelevant
  – modifying solutions step by step
  – HC, GDS, SA, BS
- # Search solutions
  – A single solution per run: classic search, HC, GD, SA
  – Multiple solutions per run: Beam search
- Partial solutions vs candidate solutions
  – Partial: HC, GD, SA
  – Candidate: (genetic) beam search

More Discussions

- How many states would be checked (as the current) for neighbours during search?
  – 1 states/nodes: HC
  – 1 or more states: BS — mutation (1), crossover (>=2), ...
  – in gradient direction: GD
  – random? SA, ...
- Fringe/frontier
  – pruning and ordering
  – genetic operators?
  – Pareto front?
- When to stop?
  – goal? classical search
  – local optima? HC, GD
  – random temperature? SA
  – convergince? BS

Further Discussions

- Paths and solutions: Explicit graphs (including trees)?
- Paths and solutions: Implicit graphs (construct as you go)?
- Local search vs Global search
- Online search vs offline search
- Satisficing vs. Optimising?
- Dynamic environments?

Game Playing

- States are board positions
- Operators are moves of the game
- Initial state is initial board position
- Final state is winning (or drawn) position
- COMP348

Summary

- Applications of Search
- Search strategies and techniques
  - Uninformed
  - Informed
  - Local search
- Other search techniques
  - Bound and Bound
  - Dynamic programming
  - ...
- Characteristics
- Suggested readings: Chapters 3 and 4
- Next Topic: Machine Learning