A Survey on Multiobjective Evolutionary Algorithms for the Solution of the Portfolio Optimization Problem and Other Finance and Economics Applications

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***Abstract*—The coinciding development of multiobjective evolutionary algorithms (MOEAs) and the emergence of complex problem formulation in the finance and economics areas has led to a mutual interest from both research communities. Since the 1990s, an increasing number of works have thus proposed the application of MOEAs to solve complex financial and economic problems, involving multiple objectives. This paper provides a survey on the state-of-the-art of research, reported in the specialized literature to date, related to this framework. The taxonomy chosen here makes a distinction between the (widely covered) portfolio optimization problem and the other applications in the field. In addition, potential paths for future research within this area are identified.**

***Index Terms*—Economics, evolutionary algorithms (EAs), finance, multiobjective evolutionary algorithms (MOEAs), multiobjective optimization.**

I. Introduction

MANY PROBLEMS in all sorts of domains can be formulated as optimization problems, which need the application of specialized methods for their solution. If great importance was traditionally placed on mathematical programming methods, the complexity of the models would have led researchers to concentrate their efforts on the development of solution heuristics analogous with biological, social, or physical phenomena observed in nature. In this framework, great attention has been dedicated to evolutionary algorithms (EAs). This class of metaheuristics relies on emulation of the Darwinian theory (i.e., the "survival of the fittest" mechanism), in order to evolve a population of solutions toward a good adaptation to their environment (i.e., to produce solutions that are a good approximation of the global optimum that we wish to achieve). Since their very inception, EAs have been successfully adopted to solve problems in many different application areas, ranging from engineering to ecology to the social sciences [1]–[4].

EAs also require little domain information to operate, which makes them less susceptible to the specific mathematical features of a given problem (e.g., they can deal with nonconvexities and/or discontinuities in objective function space). Therefore, due to the complexity of the involved problems, finance and economics applications have been of great interest for researchers from the evolutionary computation community [5], [6]. EAs have been, therefore, widely applied these types of problems since the 1980s, as indicated in [7].

However, the majority of the financial problems that have been tackled with EAs only deal with a single-objective function. Nevertheless, many authors have indeed pointed out the fact that many problems in finance and economics involve multiple conflicting objectives. In such problems, the aim is no longer to identify one optimal solution, but rather a set of solutions representing the best possible tradeoffs among the objectives of the problem.

Within this context, a major motivation for using multiobjective evolutionary algorithms (MOEAs) instead of single-objective evolutionary algorithms (SOEAs) or other techniques is their ability to simultaneously handle a set of solutions, called a *population*, since the aim is to identify a set of efficient (Pareto-optimal) solutions. This can be attained in a single MOEA run while several runs should be necessary when applying techniques such as, e.g., ϵ-constraint or goal programming. Besides, single-objective-based strategies might be unable to deal with complex shapes of the Pareto front (for instance, the aggregation function technique cannot determine the nonconvex parts of the front), while MOEAs' performance is not affected by this issue. Furthermore, an essential feature desired when addressing multiobjective problems is the achievement of a uniform distribution of the efficient solutions over the Pareto front. Thus, most MOEAs implement specific procedures that enforce diversity preservation techniques (such as niching or use of density metrics) as a secondary criterion to evaluate the solution fitness. Conversely, single-objective optimization methods might focus on a particular region of the front and neglect others. So, the coinciding emergence

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of multiobjective financial applications and of MOEA-related developments led to a mutual interest from both research communities. In fact, the mathematical nature of the financial models currently available and the high complexity of the search spaces produced by such models, make MOEAs particularly suitable to deal with such applications. However, it is somewhat surprising to find that, in spite of the existence of an increasing trend, the number of financial problems solved with MOEAs until now, remains relatively scarce, when compared to the use of SOEAs [7].

The aim of this paper is thus twofold: we aim to attract the attention of evolutionary multiobjective optimization (EMOO) researchers toward this application domain and we seek to encourage researchers involved with financial applications to consider the use of MOEAs.

This paper constitutes an updated extension of the surveys published in [8] and [9]. It is worth noting, however, that, in contrast with the survey done by Schlottmann and Seese [9], our objective is to provide a broader coverage of applications, while sacrificing, to a certain extent, a more in-depth analysis of them.

The taxonomy adopted for the purposes of this paper is based on the topics covered by the works that were revised and is divided in two main parts. The first one is devoted to studies dealing with the portfolio optimization problem (which is, by far, the most popular application of MOEAs reported in the specialized literature), while the second one presents all other types of applications.

Note, however, that the discussion provided next will be unbalanced because of the high number of publications devoted to solving portfolio optimization problems using MOEAs as compared to the application of MOEAs in other problems arising in economics and finance. Due to the wide variety of papers that focus on portfolio optimization, we present a more detailed study of this area. This paper is divided in two main research lines: the first one is related to the realism of the models adopted (e.g., integration of additional constraints and objectives currently used in funds management) and the second topic deals with the adaptation of MOEAs proposed for the solution of this problem.

The other applications covered in this paper are divided into five main classes:

1) financial time series;
2) stock ranking;
3) risk-return analysis;
4) financial and trading decision-support tools;
5) economic modeling.

Applications of MOEAs in each of these problem areas will be discussed next.

The remainder of this paper is organized as follows. Section II provides a short introduction to multiobjective optimization that is required to make this paper self-contained. In Section III, we review the use of MOEAs in investment portfolio optimization while other kinds of applications are described in Section IV. Finally, Section V provides our conclusions and some guidelines for future investigation.

II. Basic Concepts on EMOO

In this section, a basic overview on multiobjective optimization and evolutionary solution techniques is provided. This will involve, first, a short presentation of the formulation and aims of multiobjective problems. Second, the classical MOEAs mentioned in this paper will be briefly defined.

A. Formulation of Multiobjective Problems

We are interested in solving constrained optimization problems, whose recognized common formulation can be written as follows[1]:

$$\text{minimize } \vec{f}(\vec{x}) := \left[f_1(\vec{x}), f_2(\vec{x}), \ldots, f_k(\vec{x})\right] \quad (1)$$

subject to

$$g_i(\vec{x}) \le 0 \qquad i = 1, 2, \ldots, m \quad (2)$$

$$h_i(\vec{x}) = 0 \qquad i = 1, 2, \ldots, p \quad (3)$$

where $\vec{x} = [x_1, x_2, \ldots, x_n]^T$ is the vector of decision variables, $f_i : \mathbb{R}^n \to \mathbb{R}$, $i = 1, \ldots, k$ are the objective functions, and $g_i, h_j : \mathbb{R}^n \to \mathbb{R}$, $i = 1, \ldots, m$, and $j = 1, \ldots, p$ are the constraint functions of the problem.

To describe the concept of optimality in which we are interested, we will introduce next a few definitions.

Definition 1: Given two vectors $\vec{x}, \vec{y} \in \mathbb{R}^k$, we say that $\vec{f}(\vec{x}) \le \vec{f}(\vec{y})$ if $f_i(\vec{x}) \le f_i(\vec{y})$ for $i = 1, \ldots, k$, and that $\vec{x}$ **dominates** $\vec{y}$ (denoted by $\vec{x} \prec \vec{y}$) if $\vec{f}(\vec{x}) \le \vec{f}(\vec{y})$ and $\vec{f}(\vec{x}) \ne \vec{f}(\vec{y})$.

Definition 2: We say that a vector of decision variables $\vec{x} \in \mathcal{X} \subset \mathbb{R}^n$ is *nondominated* with respect to $\mathcal{X}$, if there does not exist another $\vec{x}' \in \mathcal{X}$ such that $\vec{f}(\vec{x}') \prec \vec{f}(\vec{x})$.

Definition 3: We say that a vector of decision variables $\vec{x}^* \in \mathcal{F} \subset \mathbb{R}^n$ ($\mathcal{F}$ is the feasible region) is *Pareto-optimal* if it is nondominated with respect to $\mathcal{F}$.

Definition 4: The *Pareto-optimal set* $\mathcal{P}^*$ is defined by
$$\mathcal{P}^* = \{\vec{x} \in \mathcal{F} | \vec{x} \text{ is Pareto-optimal}\}.$$

Definition 5: The *Pareto front* $\mathcal{PF}^*$ is defined by
$$\mathcal{PF}^* = \{\vec{f}(\vec{x}) \in \mathbb{R}^k | \vec{x} \in \mathcal{P}^*\}.$$

We thus wish to determine the Pareto-optimal set from the set $\mathcal{F}$ of all the decision variable vectors that satisfy (2) and (3). Note, however, that in practice, not all the Pareto-optimal set is usually desirable (e.g., it may not be desirable to have different solutions that map to the same values in objective function space) or achievable.

B. State-of-the-Art MOEAs in this Survey

The first attempts to adapt EAs to the solution of multiobjective optimization problems date back to the mid-1980s, when Schaffer [10] proposed the vector-evaluated genetic algorithm (VEGA), which is considered the first MOEA ever proposed. Over the years, many other MOEAs have been proposed, and they have been used in a wide variety of application domains [11], [12].

In general, an MOEA consists of two main components: 1) a selection mechanism that aims to select the solutions

[1] Without loss of generality, we will assume only minimization problems.

representing the best possible tradeoffs among all the objectives, and 2) a diversity maintenance (also called "density estimator") mechanism that avoids convergence of the population to a single solution. This allows an MOEA to generate several different solutions in a single run. Another important component of modern MOEAs is elitism, which refers to maintaining the best solutions that have been generated so far. Elitism is normally implemented through the use of an external archive in which the best solutions generated at each iteration are stored.

The aim of this section is to provide neither a general description of the historical developments nor particular processes implemented in MOEAs. We rather propose here a short presentation of the MOEAs mentioned in the remainder of this survey, in order to make it a self-contained work. The MOEAs are presented in alphabetical order.

1) *AbYSS* [13]*:* This approach is based on scatter search with a small population, whose members are combined and subsequently improved by a (1+1)-evolution strategy (ES) to construct new individuals (applying SPEA2's density estimator, which is based on a clustering algorithm [14]). It also uses an external archive to store the nondominated solutions obtained during the search, using the crowding distance of the nondominated sorting genetic algorithm (NSGA)-II [15] as a density estimator.

2) *FastPGA* [16]*:* FastPGA computes the fitness of each solution according to a crowding distance for the nondominated solutions and to the number of dominating or dominated individuals for the others. Additionally, a population regulation operator dynamically adapts the population size to the number of existing nondominated solutions.

3) *Indicator-Based Evolutionary Algorithm (IBEA)* [17]*:* IBEA is an MOEA that defines the optimization goal in terms of an arbitrary binary performance measure (indicator). Then, IBEA directly uses this measure in the selection process, through a replacement strategy that tries (in a greedy way) to optimize the value of the indicator for the current population. In contrast to other existing MOEAs, IBEA can be adapted to the preferences of the user and, moreover, does not require any additional diversity preservation mechanism, such as fitness sharing, clustering, and so on.

4) *MOCeLL* [18]*:* This approach uses an external archive to store the nondominated solutions found during the search. For each generation, a number of individuals in the current population is replaced by randomly chosen solutions from the external archive, in order to enhance the diversity preservation. The genetic operations are performed only for two individuals belonging to the same neighborhood. When the external archive is full, a crowding distance-based density estimator is adopted to determine if a new nondominated solution should be included or not. This process allows production of an evenly distributed set of solutions.

5) *MOEA Based on Decomposition (MOEA/D)* [19]*:* This algorithm uses an aggregation method to decompose a multiobjective optimization problem into N single-objective optimization subproblems, with each one related to the others through a neighborhood structure. N is a parameter set by the user in order to control the spacing between a set of uniformly distributed points approximating the Pareto front. Then, MOEA/D solves these subproblems simultaneously by evolving a population of solutions and taking advantage of the solutions obtained for neighboring subproblems. One of the key components in MOEA/D are its decomposition methods, two of them being, in the initial MOEA/D version, the weighted Chebyshev approach and the weighted sum approach.

6) *Multiobjective Genetic Algorithm (MOGA)* [20]*:* This algorithm is a nonelitist MOEA that was very popular in the early days of EMOO [12]. MOGA implements a variant of the Pareto ranking selection originally proposed by Goldberg [2]. Nondominated individuals have a rank equal to 1, while the dominated ones are penalized according to the population density in the corresponding region of the surface of the tradeoff solutions. A fitness is then assigned to each individual, by interpolating from the best (rank 1) to the worst. MOGA also implements fitness sharing and mating restrictions.

7) *Nondominated Sorting Genetic Algorithm (NSGA)* [21]*:* NSGA is a nonelitist MOEA based on several layers of classifications of the individuals as suggested by Goldberg [2]. Before selection is performed, the population is ranked on the basis of nondomination; all nondominated individuals are classified into one category (with a dummy fitness value, which is proportional to the population size, to provide an equal reproductive potential for these individuals). To maintain the diversity of the population, these classified individuals are shared with their dummy fitness values. Then this group of classified individuals is ignored and another layer of nondominated individuals is considered. The process continues until all individuals in the population are classified.

8) *NSGA-II* [15]*:* In NSGA-II, for each solution one has to determine how many solutions dominate it and the set of solutions to which it dominates. The NSGA-II estimates the density of solutions surrounding a particular solution in the population by computing the average distance of two points on either side of this reference solution along each of the objectives of the problem. This value is called the *crowding distance*. During selection, the NSGA-II uses a crowded-comparison operator that takes into consideration both the nondomination rank of an individual in the population and its crowding distance (i.e., nondominated solutions are preferred over dominated solutions, but between two solutions with the same nondomination rank, the one that resides in the less crowded region is preferred).

9) *Pareto Archived Evolution Strategy (PAES)* [22]*:* PAES is an elitist MOEA consisting of a (1+1)-ES, whose main highlight is the use of an adaptive grid in the external archive that is used to store the nondominated solutions that are generated during the search. This adaptive grid is used like a coordinate system to locate the nondominated solutions inside the external archive. Once the archive has reached a certain (predefined) limit, then new solutions are allowed only if they will occupy the less densely populated grids. Thus, this mechanism allows the proper distribution of the nondominated solutions along the Pareto front.

10) *Pareto Envelope-Based Selection Algorithm (PESA)* [23]*:* PESA uses, like PAES [22], a small internal population

and a larger external population (which is an archive that stores the nondominated solutions generated during the search). Additionally, the external population has an adaptive grid that is used to preserve diversity in both populations: for selection purposes (based on a crowding metric) and to filter out the solutions entering the external archive.

11) *Strength Pareto Evolutionary Algorithm* 2 *(SPEA2)* [14]*:* SPEA2 maintains an external archive of the nondominated solutions found during the search and updates it at each generation. This update is performed through a specific technique for estimating the neighborhood density for each solution. For each individual in the external archive, a "strength value" is computed, according to the number of solutions that it dominates and to the number of solutions that dominate it.

12) *VEGA* [10]*:* VEGA is commonly regarded as the first attempt for modifying traditional genetic algorithms (GAs) in order to solve multiobjective optimization problems. It works by decomposing the population into a number of subpopulations equal to the number of objective functions of the problem. In each subpopulation, the solutions are evaluated according to only one objective and the survivors are selected by a roulette-wheel mechanism. This simple working mode is very similar to a classical GA, but it has several limitations. The main one is that it is unable to identify appropriate tradeoffs regions since it is designed to favor solutions that are the best only with respect to one objective. The fuzzy versions of VEGA are designed to overcome this drawback and make use of fuzzy decision rules, based on verbal statements on the quality of each objective.

III. Portfolio Optimization Problem

Selecting investment portfolios is a very common practice, covering a wide range of applications going from relatively simple portfolios held by individuals (containing a few stocks, bank investments, real estate holdings, etc.) to huge portfolios managed by professional investors for companies or pension funds (containing many stocks, treasure bonds, etc.). The solution of this problem lies on the intersection of finance, mathematics, and computer science, and, because of its importance, it has attracted a considerable amount of research within each of these three areas.

The pioneering methodology for solving portfolio optimization problems was formulated by Markowitz [24] and is based on the duality of the two features desired by any investor (or decision-maker): assuring a certain return (i.e., money earned) and, at the same time, avoiding the risks of possible losses due to market fluctuations. Formulating this problem in optimization terms, Markowitz states that, ideally, the investor searches for the optimal portfolio, i.e., the portfolio that minimizes the risk (within a defined tolerance) while maximizing the return.

Within the Markowitz mean-variance formalism, a portfolio is defined by a vector of real numbers that contains the weight (i.e., ratio of the total invested capital) corresponding to each available asset. Then, one wishes to maximize the weighted sum of the assets' expected rate of return and, simultaneously, to minimize the risk expressed as the variance of the portfolio's rate of return (since this defines the level of uncertainty about the future payoff at a certain time). This model can be solved using quadratic programming (QP) [25].

However, the comparison between the model's simplistic assumptions and the real-world conditions that decision makers are faced with (additional requirements on the number or the relative ratio of selected assets, probabilistic distributions of the returns, etc.) has led to the necessity of improving and consequently, to an increasing complexity of the models addressing the portfolio selection problem. Also, additional constraints, as well as new risk representation schemes, make impossible the solution of the problem through the use of classical, exact techniques. This higher complexity, coinciding with increased interest in research on MOEAs, has encouraged the development and application of this more robust class of optimization techniques. As such, the first use of MOEAs for optimizing investment portfolios was apparently proposed in 1993 [26].

This section focuses on two features: 1) the various proposals made to improve and extend the formulation of the portfolio selection problem toward more realistic models, and 2) the advances in the adaptation and implementation of evolutionary computation techniques used for solving this problem. Rather than a chronological description of the existing studies, the taxonomy chosen here groups together the specialized works according to their research line(s): addition of additional constraints to the initial Markowitz formulation, consideration of new objectives, design of new problem-devoted optimization methods (which include, in the evolutionary framework, recent trends in the portfolio encoding techniques), and studies comparing the performance of state-of-the-art MOEAs when applied to the portfolio selection problem. Please note that this thematic organization results in an unavoidable repetition of the quoted references, since each work typically investigates several of the previously mentioned features.

A. *Recent Trends in the Formulation of Portfolio Optimization Problems*

The classical Markowitz mean-variance model addresses the simultaneous minimization of risk and maximization of return, according to the following mathematical formulation for an N-assets problem

$$\text{minimize Risk, maximize Return} \tag{4}$$

subjected to

$$\text{Return} = \sum_{i=1}^{N} w_i r_i \tag{5}$$

$$\text{Risk} = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i \sigma_{ij} w_j \tag{6}$$

$$\sum_{i=1}^{N} w_i = 1 \tag{7}$$

$$0 \le w_i \le 1 \tag{8}$$

where w_i is the weight assigned to asset i, r_i is the associated expected return, and σ_{ij} denotes the elements of the covariance matrix of all the investment alternatives. The two last constraints of the model impose, respectively, the sum of asset weights to be equal to one (budget constraint) and the weights to be positive (no short-sell is allowed).

As mentioned earlier, the simple initial Markowitz model was modified and adapted to real-world requirements and different operating modes. With respect to the improvements brought, researchers have mainly emphasized: 1) the integration of realistic constraints; 2) the addition of new objectives (mostly for the formulation of risk indicators); and 3) the use of Sharpe's ratio (to a moderate extent). It is worth noting that, in the framework of this paper, we only focus on the model developments that are combined with the utilization of MOEAs (and not on works using a single-objective approach either based on EAs or on any other technique).

1) *Consideration of Real-World Constraints:* The Markowitz model initially considered only one strong constraint, setting the sum of asset weights to 1 (which means that the sum of the invested amounts must exactly meet the available capital). However, many examples of other constraints are often used in real-world fund management, which researchers tend to include into their models. Although the most common ones are the typical floor-ceiling constraints or the cardinality constraints, more advanced constraints have been proposed by some authors and are detailed in the remainder of this section.

Note that the linear budget constraint does not involve computational difficulty for a QP solution technique [25]. Conversely, in most cases, the introduction of new constraints leads to a nonconvex search space and QP cannot be used anymore. This clearly motivates the use of metaheuristics.

The real-world constraints that have been more frequently added to the portfolio optimization problem are the following.

a) *Floor-ceiling constraints:* These constraints impose lower and/or upper bounds on the values of each asset weight, instead of the 0 (minimal) to 1 (maximal) bounds. This means that an asset cannot represent less or more than some proportion of the total invested capital. This requirement is explained by the following: 1) regarding the lower bound, not devoting very small percentages of the capital to many securities (since this would result in high transaction costs); and 2) concerning the upper bound, not assigning a too large ratio of the total invested capital to one asset, in order to minimize risk (by sharing it among several assets). The mathematical expression of the floor-ceiling constraint is

$$\forall i = 1, \dots, N, l_i \leq w_i \leq u_i. \tag{9}$$

To the best of our knowledge, this kind of constraint was first introduced in [27] as buy-in thresholds, and was subsequently taken up in [28]–[35]. Note that [36] proposed a similar version of the floor-ceiling constraints, which is called *5-10-40 constraint*, in reference to the German investment law [37] in which an upper bound is defined for each individual asset and for the sum of all "heavyweight" assets in the portfolio.

In most cases, the floor-ceiling constraints are handled either through an appropriate encoding technique (see Section III-B1) or by a specific repair procedure (which may slightly differ from one author to another, even though the global methodology is basically always the same one).

b) *Total weight assigned to asset classes:* This constraint is very similar to the floor-ceiling constraint and lays down bounds on the total capital assigned to a class or sector of assets (for instance, securities from the steel industry may represent one sector). So, in the same way bounds are assigned to each asset weight for the floor-ceiling constraint, the sum of the weights assigned to securities belonging to a same sector might also be restricted within some determined limits. This approach was adopted in [29] and [35]. Similarly, in [38], sectors are considered and sorted (in decreasing order) regarding market capitalization. Then, the total weight of securities in one given sector should be greater than the total weight of securities in any following sector (i.e., a sector with lower capitalization). The mixed-integer model thus involves an additional vector of binary variables, indicating, for each sector, if an asset belonging to it has been selected or not.

c) *Cardinality constraints:* The cardinality constraint forces the number of assets selected in a portfolio to respect some restrictions. This constraint has two versions. The first (or exact) version imposes the number of selected securities to be equal to a given value K and the second (or soft) version only provides lower and/or upper bounds (Z_L, Z_U) on this number. The mathematical formulation typically involves new binary variables z_i, denoting the presence or absence of asset i in the considered portfolio: $z_i = 1$ if $w_i > 0$, and $z_i = 0$, otherwise. Then, the (soft) cardinality constraint is formulated as

$$Z_L \leq \sum_{i=1}^{N} z_i \leq Z_U. \tag{10}$$

As mentioned earlier, it is important to note that when no cardinality constraints are imposed on the problem, QP can be used to solve it in an exact manner. However, when cardinality constraints are imposed, no exact method exists for solving it. In the multiobjective statement of this problem, the Pareto front may be discontinuous and/or nonconvex, which also causes difficulties when using approaches based on linear aggregating functions.

Reference [39] appears to be the first work to introduce the cardinality constraint. The exact version was then adopted in [30], [33], [35], [38], and [40], while the soft version was used in [27], [28], [31], [32], [36], and [41].

The most commonly implemented strategies to handle cardinality constraints lie on classical techniques integrated into an EA (i.e., penalty functions, domination-based rules, and others). In [27], the authors adopted a repair mechanism that first removes all surplus assets (those with the smallest weights w_i) from the portfolio to meet the cardinality constraints (similar methods are used for the other constraints). Additionally, in order to examine the effect of this repair mechanism, the authors use two strategies, adopting or not adopting Lamarckism (when using Lamarckism, the repaired solution is kept; otherwise, only its objective function is used). The comparative results are further discussed in Section III-B1, which focuses on encoding techniques.

More specific constraint handling methods are proposed in other papers. In [40], the authors indicated that if only the cardinality constraint is imposed on the problem and the others are ignored, then cardinality can be considered a third objective (in addition to the traditional risk and return objectives). Then, the 2-D cardinality constrained frontier can be extracted for any particular cardinality k. However, if additional constraints need to be considered, this approach is no longer viable and the authors thus propose to search for each cardinality constrained front in parallel, and constructively use information from these fronts to improve the search process of the others.

In [35], a k-means clustering method was adopted; considering that K assets must be exactly chosen for the portfolio, K clusters are defined such that the variability (in terms of mean and variance of returns) inside a cluster is minimized and the variability between different clusters is maximized. Selecting exactly one asset from each cluster thus allows removal of the cardinality constraint, which consequently simplifies the model. This strategy is validated by comparing the efficient frontier obtained from the initial problem (with all the assets) with the modified one (clustered investable universe). The study shows that, regardless of the cluster selection, the efficient frontiers are quite close to those produced with a Markowitz mean-variance model (solved using QP).

d) *Roundlot or minimum lots constraint:* In many real-world applications, the amount invested in a security must be a multiple of the minimum transaction lot, which represents the smallest volume of this security that can be purchased. Thus, the weight w_i of any asset i is not directly a decision variable anymore, but, instead, it has to be computed through a lot purchasing price (c_i) and an integer number of lots that should be purchased (x_i) as follows:

$$\forall i = 1, ..., N, \ w_i = \frac{x_i c_i}{\sum_{i=1}^{N} x_i c_i}. \tag{11}$$

This type of constraint has been adopted in [27], [33], [38], [41]–[44].

e) *Turnover constraints:* Finally, some authors consider further restrictions on the change in the assets weight, with respect to a previous weight allocation. This formalism might be especially useful when considering a multiperiod investment horizon. In [29], the difference between the current assignment w and the previous one w' must be larger than a certain threshold (if there is any change in the asset weight) as follows:

$$\forall i = 1, ..., N, \left|w_i - w'_i\right| \geq \Delta_i \text{ or } \left|w_i - w'_i\right| = 0. \tag{12}$$

Additionally, the sum of the absolute change from the previous allocation must be smaller than the maximum turnover ratio

$$\sum_{i=1}^{N} \left|w_i - w'_i\right| \leq \text{TR}. \tag{13}$$

Similarly, purchase or sale constraints are proposed in [34]

$$\forall i = 1, ..., N, \max\left(w_i - w'_i, 0\right) \leq \overline{B_i} \text{ and } \max\left(w'_i - w_i, 0\right) \leq \overline{S_i} \tag{14}$$

as well as trading constraints

$$\forall i = 1, ..., N, w_i = w'_i \text{ or } w_i \geq w'_i + \underline{B_i} \text{ or } w_i \leq w'_i - \underline{S_i} \tag{15}$$

where $\underline{B_i}$ and $\overline{B_i}$ represent minimal and maximal purchasing thresholds, respectively (i.e., when $w_i > w'_i$) and $\underline{S_i}$ and $\overline{S_i}$ represent minimal and maximal sale thresholds, respectively (i.e., when $w'_i > w_i$).

2) *New Objectives:* If the "maximizing return–minimizing risk" strategy (the investor is risk-averse and wants to maximize his or her profit) is generally not questioned, other issues (and mainly the computation of risk as the variance of the security returns) have generated many criticisms. These criticisms include: 1) the assumption of a multivariate normal distribution of an asset rate of return does not hold in practice (the distributions are typically asymmetric); 2) the common approaches disregard the individual investor's preferences, who sometimes prefers portfolios that lie behind the nondominated frontier; 3) the fact that variance equally accounts for upward and downward deviation, unlike business executives who rather view risk as the probability of not meeting a fixed target rate of return (thus, upward deviations are not penalizing and should not be included in the risk representation scheme); 4) the consideration, apart from the fluctuations of a portfolio value around its mean due to market volatility, of the possibility that a portfolio may lose a significant amount of its value because of catastrophic, nonpredictable, and low-probability events; and 5) the issue of the accuracy of the expected return rates forecast in the small sample situation (since the computation of the variance–covariance matrix is based on the large sample theory), etc. These reasons encouraged researchers to define new ways of representing the risk of a given portfolio. Even Markowitz proposed a variation of the mean-variance model, introducing the semivariance of a portfolio value [25], which represents the probability of possible losses.

Accordingly, Arnone *et al.* [26] adopted lower partial moments, which refer to the downside part of the distribution of returns (appropriately, this measure is called *downside risk*). The use of downside risk makes the problem more difficult, because the shape of the objective surface is generally nonconvex, therefore forbidding the use of QP to find exact solutions. The same measure for risk was also adopted in [45].

Similarly, Yan *et al.* [46] computed risk as the semivariance (equivalent to the downside risk) of portfolio returns. Moreover, they do not consider a single investment period, but rather T periods; the decision-maker must determine, at the beginning of each period, how to set the weights of the N investment assets. Aiming at a target value d representing the growth rate of the initially invested capital over the T periods, the semivariance risk is equal to 0 if the target is reached; otherwise, it is equal to the quadratic distance to the target. Consequently, the authors address the bi-objective problem of maximizing growth rate d and minimizing semivariance (through an ϵ-constraint strategy).

Another risk measure used in several works is value-at-risk (VaR), which is defined as the α-quantile (typically $\alpha = 5\%$) of the distribution of a portfolio's losses. In other words, VaR represents the maximum loss on a portfolio that can be expected with a certain confidence level $100(1-\alpha)$, over a certain time interval [47]. In fact, VaR has become a popular risk measure since it was recommended and adopted by the Bank for International Settlements and by U.S. regulatory agencies in 1988. Like downside risk, VaR is a nonconvex measure. Subbu *et al.* [48] defined a model that maximized the expected return while minimizing both the portfolio VaR and surplus variance. Bradshaw *et al.* [49] adopted VaR for the design of the risk criterion. In [29], the authors considered three bi-objective models, which differ one from another in the risk representation. The first one addresses the Markowitz mean-variance optimization, while the other two are based on VaR and expected shortfall (i.e., the conditional mean value of the losses given that the losses have exceeded VaR) instead of variance.

Additionally, many other risk indicators have been proposed in the multiobjective portfolio selection framework. Ehrgott *et al.* [50] extended the Markowitz mean-variance model with the design of five new objectives to be maximized [derived from a cooperation with Standard and Poor's (S&P)]: 1) 12-month performance of an asset; 2) three-year performance of an asset; 3) annual dividend of a portfolio; 4) S&P star ranking; and 5) volatility.

In [51], the objective was to compare several MOEAs on two portfolio optimization formulations. While the first one involves the two classical objectives (mean-variance of returns), the second one includes a third criterion, i.e., maximization of the annual dividend yield.

Chang *et al.* [30] proposed various risk measures which included semivariance, mean absolute deviation, and variance with skewness (as a third objective). Skewness represents the fact that the security price distribution might not be symmetrical. Maximizing skewness thus biases the return distribution toward profits instead of losses.

In [31] and [32], a return-risk model is optimized but, unlike classical approaches, a loss distribution is computed by generating scenarios for the stochastic return variable. Thus, the risk and return over a given period can be computed according to the generated set of scenarios, allowing a wide-open range of choices for the risk representation measure. According to this strategy, three measures (standard deviation, VaR, and conditional VaR) are adopted in the first work. The second one extends the study to two additional risk measures, i.e., mean-absolute downside semideviation and expected shortfall.

In [52], the authors were specifically interested in the critical step of computing an accurate forecast for the return rate and variance–covariance matrix, according to the classical Markowitz model. They pointed out that the methods typically used for this computation lie on the large samples theory, which may not be reliable when dealing with small problems. Thus, they proposed the use of Gray and possibilistic regression models in order to design three objectives: the expected return, the uncertainty risk, and the relation risk.

All the above-mentioned papers discuss portfolio selection in a stochastic environment. In [53], however, the author underlined the fact that in some situations in which it is difficult to use probability theory, investors can make use of fuzzy set theory to reflect the vagueness and ambiguity of security returns. Within this perspective, some authors adopted a framework that represents uncertainty on security returns through fuzzy variables. Huang [53] computed the risk as the semivariance of the expected returns, while Li *et al.* [54] proposed skewness as a third objective (in addition to expected return and variance). In these two former cases, the objective is computed as a credibility index, which is obtained by fuzzy simulation.

Anagnostopoulos and Mamanis [55] argued that in multiobjective optimization, a constraint must be considered an objective if it is not easy to fix a right-hand side value for the constraint without knowing the levels of the other objectives. Thus, the authors decide to add the number of assets included in the portfolio as an objective. This way, the formulation of the portfolio selection problem comprises three objectives: maximize the return, minimize the risk, and minimize the number of assets in a portfolio. The results indicated that the algorithms provided a good approximation of the return-risk Pareto front for different values of the number of assets. Thus, it can be said that the tri-objective formulation of the portfolio selection problem generalizes the mean-variance approach; this strategy provides the investor with portfolios that are not mean-variance efficient (i.e., when projected on the two-objectives plane) but, since these solutions have fewer assets, they represent nondominated solutions when considering the three objectives.

Finally, apart from the risk measures, other criteria can be considered, such as transaction costs [42], [43], [56] or, as mentioned in the previous section, the cardinality constraint can also be considered as an additional objective [40].

3) *Use of Sharpe's Ratio:* Apart from the determination of the efficient frontier representing the optimal risk-return portfolios, some authors considered the use of another selection criterion, the Sharpe's ratio. This index measures how much excess profit per risk unit delivers a certain portfolio and is parameterized by a value R_f that indicates the observed (or desired) return of a risk-free portfolio. As mentioned in [57], an important consequence of the use of this selection criterion is the fact that specific regions of the Pareto front turn out to be more relevant. The inclusion of this selection criterion within the optimization process thus allows focusing on this region where the Sharpe's index attains its maximum value. This operating mode can be seen as a way of incorporating preferences into the evolutionary search.

This concept is illustrated in Fig. 1 (which is reproduced from [28]). The point R_f represents the risk-free return available in the market. Line CC represents the capital market line, which is a straight line that passes through R_f and is tangential to the efficient frontier F. The intersection of F and CC defines the efficient portfolio in point a; any combination of it and the risk-free asset, attainable by either lending or borrowing at rate R_f, will allow operating at any point on the capital market line, above the efficient frontier, resulting in a higher return for any given amount of risk than any optimal portfolio on F.

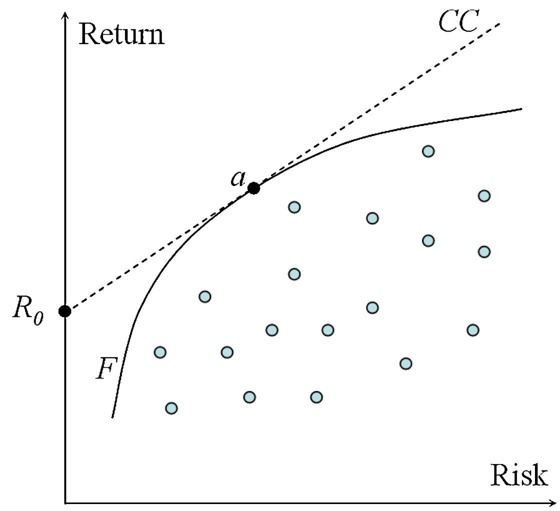

Fig. 1. Illustration of Sharpe's index in the risk-return space.

Mathematically, the efficient portfolio is the point on the Pareto front that maximizes Sharpe's ratio as follows:

$$S_r = \frac{R_P - Rf}{\sigma_P} \tag{16}$$

where R_P and σ_P are the expected return and the risk expressed as the variance of a portfolio P, respectively. Thus, maximizing S_r comes down to increasing the slope of the capital market line CC, until reaching a tangential position with the feasible region in the objective space (last feasible intersection between CC and the feasible region).

Sharpe's index can be used either as a single objective in the optimization process or as an additional objective in a multiobjective setting. For instance, Aranha and Iba [58], [59] solved the portfolio selection problem adopting Sharpe's ratio as the only optimization criterion.

On the other hand, in [28], the mean-variance formulation was solved with an MOEA in which the selection is carried out through a Pareto-ranking procedure. To break ties between solutions from the same front, the authors propose to use Sharpe's ratio instead of the classical density estimators (such as niche count or crowding distance) that are generally adopted in state-of-the-art MOEAs. This allows the focus of the search to take place around the efficient portfolio point.

The resulting preference-based MOEA (PMOEA) is compared against a classical MOEA (using a niche count as tie breaker within the selection step) and against an SOEA that optimizes Sharpe's ratio, for five problems drawn from the OR-Library [60] and for three locations of the risk-free point R_f. Surprisingly, in almost all cases, the number of fitness function evaluations required to reach solutions within 5% of the optimal fitness was better for the PMOEA. The authors also noted that the benefit of integrating preferences is higher when the gap between the efficacy of the SOEA and the MOEA is larger. Therefore, the PMOEA strategy allows location of an efficient portfolio with a higher accuracy than the SOEA. Additionally, it requires fewer fitness function evaluations to find such a solution and also provides a set of alternative solutions close to this target point (which may be useful for the decision-maker to choose a final portfolio).

In [57], the efficacy of several MOEAs (see Section III-B3) is compared on the mean-variance formulation. Subsequently, the point closest to the efficient portfolio is identified in the efficient frontier produced by each MOEA and compared to the solution of an SOEA using Sharpe's index as an objective function (for an equal number of objective function evaluations). Quite obviously, the results show that the SOEA consistently returns a solution very close to the efficient portfolio and outperforms the MOEAs. This statement remains true for higher R_f values, with the MOEA solution quality decreasing drastically in this case.

Considering this preliminary result, the authors then propose comparing the Pareto front produced by the MOEAs with the approximated set obtained by running several times the SOEA when varying the R_f parameter. In each of the 30 performed executions of the SOEA, the efficient portfolio obtained with a specific value of the risk-free return rate constitutes an approximated point of the Pareto front. The SOEA, in this configuration, cannot achieve the MOEAs' efficacy because of its discretization, even though it does not perform so badly.

Table I summarizes the various developments brought to the Markowitz mean-variance model mentioned in this section, according to the constraint and objective research lines. The classical case (Markowitz model) is characterized by the "basic constraint" (i.e., capital budget constraint) and mean-variance objectives. Note that all models include the basic constraint, so the references appearing in the corresponding column are those that only have this constraint.

This section presented the addition and refinements of new and more realistic features in the formulation of the portfolio selection problem. As mentioned before, the resulting complexity of the model makes the solution step very difficult, or even impossible, for classical methods such as QP (used within the original Markowitz formalism). Due to the multiobjective nature of the problem, MOEAs represent a viable alternative, but their application requires necessary adaptation efforts. Therefore, the next section is devoted to the presentation of research that proposes or improves the use of EAs for solving portfolio optimization problems.

B. Focusing on the Solution Techniques

The aim of the portfolio optimization problem is to provide a set of portfolios belonging to the Pareto front (i.e., the best possible tradeoffs among the objectives), among which the investor is able to choose the most appropriate option. A clear advantage of MOEAs in this framework is their ability to produce, in one single run, a complete approximation of the Pareto front.

However, when tackling the portfolio optimization problem with MOEAs, various issues arise. First, the representation (or encoding) of a portfolio is not straightforward, mainly because of the basic (budget) constraint (of course, including real-world constraints has similar implications). Various studies thus particularly focus on this point, aiming to propose encoding techniques that always produce feasible portfolios. Additionally, since reaching the optimal Pareto front is the crucial goal of an MOEA, many works deal with the development of new techniques that improve the quality of the final

TABLE I
OVERVIEW ON MODEL DEVELOPMENTS (NEW OBJECTIVES OR CONSTRAINTS)

Objectives		Constraints				
	Basic	Floor-Ceiling	Asset Classes	Cardinality	Min. Lots	Turnover
Mean-variance	[50], [51]	[27]–[29], [31], [33]–[36], [55]	[35], [38], [55]	[27], [28], [31], [33], [35], [36], [38], [39], [61]	[27], [33], [38], [42], [44]	[29], [34]
Downside risk/semivariance	[26], [45], [46]	[30]		[30]		
VaR	[47]–[49]	[29], [31]		[31]		[29]
Surplus variance	[48]					
Expected shortfall		[29], [32]		[32]		[29]
12-month performance	[50]					
3-year performance	[50]					
Annual dividend yield	[50], [51]					
S&P star ranking	[50]					
Mean abs. dev.		[30], [32]		[30], [32], [41]	[41]	
Skewness		[30]		[30]		
Uncertainty/relation risk	[52]					
Fuzzy approach: Skewness	[54]					
Fuzzy approach: Semivariance	[53]					
Transaction cost	[56]				[42]	
Sharpe's index	[57]–[59]	[28]		[28]		

solutions attained. Finally, and as the use of MOEAs became more popular, some researchers started to present comparative studies in which several MOEAs are applied to some instances of portfolio optimization problems. The following sections provide an analysis of the most representative work along these three research lines.

1) *Encoding Techniques:* The encoding is an issue of great importance when dealing with the portfolio selection problem, which is really an allocation problem. Thus, a direct representation (i.e., using decision variables as usually done with GAs for representing the weights of each stock) does not work well. The reason is that this type of representations will frequently produce infeasible solutions in which the values allocated do not add up to one, which is the basic constraint imposed on the problem.

a) *Shoaf and Foster* [62], [63]*:* These authors adopted a representation that has a single field of k+1 bits for each asset. The first bit indicates whether the position on that holding will be long (one) or short (zero). The remaining k bits are an unsigned index onto an "allocation wheel," representing the resources to be allocated. The wheel is divided into 2k equal sections, each indexed by a k-bit binary value. For any asset represented as a long position, the wheel proportion between its index and the index of the next long position, plus the proportion of any enclosed short-position wedges, is the total proportion of resource allocation for that holding. The idea is that, e.g., the resources from a short sale of a stock are used to purchase additional shares of the long position stock whose index most immediately precedes its index. The greatest benefit of this encoding is that the total investment represented by a chromosome is always 100% of the available resources (i.e., the solutions are always feasible). Conversely, its main drawback is a higher sensitivity to the mutation and crossover rates, since the encoding is epistatic (i.e., a change in the index of one holding generally affects one or two other holding allocations). The authors compared their approach with respect to QP (the most common approach used to solve this problem) and highlighted that their GA could determine portfolio allocations with similar risk and higher rates of return than QP.

b) *Vedarajan et al.* [56]*:* Two encoding schemes, both based on real-value genes, were proposed in this case. The first one has no specific features but, since the sum of weights might not add up to one, the budget constraint is handled through a static penalty function, aggregated in the fitness computation. The second encoding scheme is based on a variation of random keys (initially proposed in [64]); the assets are included in the portfolio in the decreasing order of their weights, until the sum of the included asset weights is higher than one; the weight of the last included asset is then modified to respect the budget constraint.

c) *Streichert et al.* [27]*:* The authors experimented with both a binary encoding (with and without Gray codes) and a real-numbers encoding. They adopted the NSGA [21] with tournament selection, fitness sharing, one-point mutation, and discrete three-points crossover. Since the authors had determined from preliminary experiments that Pareto-optimal solutions were normally composed of a limited selection of the available assets, they noted the similarities of the problem with the 1-D binary knapsack problem. Since the knapsack problem has been solved using EAs, the authors adopted this encoding in addition to the vector of decision variables (the weights). So, each bit from the knapsack determined if an asset would be used or not. The genetic operators were applied separately to each of the two segments of the chromosome.

As mentioned in Section III-A1, different constraints are additionally considered in this paper and the effect of the use (or not) of Lamarckism after the repair process constitutes a parameter of the study. The S-metric was employed to compute the hypervolume (the region of the objective space dominated by the approximation obtained) of the Pareto front [65], and this value was used as a performance assessment measure.

The results indicated that when no Lamarckism is adopted and no additional constraints are imposed on the problem, the use of the knapsack encoding clearly outperforms the standard representation. From the different encodings adopted, the traditional binary encoding is the best one and the real-numbers encoding is the worst.

Without cardinality constraints, the standard GA is slightly better than the GA with knapsack encoding, but the effect is hardly noticeable because all the approaches perform very well. But the standard GA is still much better than without Lamarckism. However, when cardinality constraints are imposed on the problem and without using Lamarckism, one cannot make clear distinctions in the results anymore. Conversely, when using Lamarckism, the standard GA outperforms the GA with the knapsack encoding regarding convergence rate and reliability, for the case in which cardinality constraints are considered.

When additional constraints (floor-ceiling, minimum round-lot) are considered, for the case in which no Lamarckism is adopted, the standard GA presents premature convergence. If Lamarckism is adopted, then the negative effect of the neutral search space is apparently removed, which significantly increases the efficiency of the standard GA. Real-numbers encoding exhibits a slightly better performance than binary encoding in this case. Using the knapsack encoding, the GA does not present premature convergence, but its performance is poor. The use of Lamarckism causes, again, a very significant performance improvement. However, binary encoding is better than real-numbers encoding in this case.

d) *Chiam et al.* [28]*:* The main point of this paper consisted of the design of a hybrid representation scheme and associated genetic operators. The chromosome was divided into two parts: the first one was a permutation vector containing the identity tags of the available securities, while the second one represented the weights corresponding to each security. The securities were integrated into the portfolio in the order they appeared in the permutation vector, until the sum of the associated weight exceeded one. Then, the weights of the included assets were normalized. This process was slightly modified through two simple repair processes. The first one was to handle the floor-ceiling constraints (considered as hard constraints) and the second one was to handle the cardinality constraints (which were considered as soft constraints and infeasible solutions might be included in the population).

Additionally, the first population is generated according to three possible initialization techniques, resulting in three different MOEAs: 1) without any constraint on the asset weights; 2) with a maximum asset weight equal to 0.1; and 3) with a varying maximum asset weight. The purpose of such a strategy is to change the diversity (in terms of asset number in each portfolio) of the initial population. Besides, an external archive maintains the nondominated solutions found during the search; the selection step is performed on the current population plus this external archive, with a binary tournament based on feasibility, Pareto-dominance, and niche count (in the normalized objective space).

The three resulting MOEAs (with one of the above-mentioned initialization methods each) are compared against a classical real-number-encoded MOEA. The comparison criteria account for the distance of the approximated Pareto set to the unconstrained true Pareto front (this is called "generational distance" [66]) and for the approximated Pareto set diversity (using the maximum spread and the spacing performance measures [65], [67]). The first computational experiments (driven on five problems drawn from the OR-Library [60]) were carried out for the unconstrained problem, and showed the clear superiority of the proposed MOEA with the varying maximum asset weight as the initialization method. Then, when applied to the constrained problem, this latter MOEA version could reach solutions close to the unconstrained true front, albeit depending on the harshness of the imposed floor-ceiling and cardinality bounds. The authors also highlighted the influence of the constraints on the size of the associated portfolios, as well as the size of the portfolios according to their location in the Pareto front. As expected, portfolios located in the low risk–low returns region were those having many assets, since diversifying inversion on many items has the effect of distributing the risk; conversely, portfolios with high returns (and high risks) involved few assets.

e) *Aranha and Iba* [58]*:* Here, the authors developed a specific representation scheme, quite similar to genetic programming (GP) [68], which adapts a tree-based portfolio construction. Terminal nodes (leaves) must identify a security tag while the other nodes (trunk) determine the weight of each leftside subtree. This allows producing purely feasible portfolios (the asset weights are implicitly normalized). Moreover, each subtree can act as an independent feasible tree, so that a fitness can also be assigned to it.

This encoding technique (which imposes a limit on the tree depth) involves the development of adapted genetic operators that randomly modify selected subtrees; in the mutation step, the chosen subtree is replaced by a new randomly generated one, while in the crossover step, the subtrees of two parents are exchanged according to their associated fitness. This latter mechanism produces an offspring combining the best subtrees of each parent.

Before numerical experimentation, the authors show that, in most cases, the tree-based representation technique should speed up convergence. Then, the new encoding scheme is compared to a classical array-based representation. The results obtained on two problems (with 100 and 225 possible assets), in terms of the best solution found, are quite similar. However, a deeper analysis of the selected portfolios shows that the number of selected assets is much lower with the tree-based encoding, which highlights the fact that the array-based method is not able to discard assets that weakly contribute to the global portfolio.

f) *Anagnostopoulos and Mamanis* [55]*:* As described in Section III-A2, the authors formulated the portfolio selection problem adopting three objectives: return, risk, and number of assets in the portfolio. The formulation of the problem also included class constraints. In order to handle these constraints, the authors utilized a problem-specific chromosome representation, in which a real-valued vector was used. In order to avoid the construction of infeasible solutions, they also used a repairing technique: when a new chromosome did not contain an asset from a specific class, a randomly selected asset was added to the chromosome.

2) *Original Multiobjective Optimization Techniques:* A typical approach when dealing with multiobjective problems

is to reduce the number of criteria in order to come down to a single-objective problem. To do so, the most commonly used methods are the aggregation of the weighted objectives in a linear function and the ϵ-constraint strategy.

For the former technique, the weights associated with each objective direct the search toward a specific region of the Pareto front, so that multiple executions are required to produce a complete approximation of the Pareto set. Additionally, the aggregation function technique does not allow construction of the whole Pareto front when it is not convex. Regarding the ϵ-constraint strategy, all objectives but one are considered as constraints of the model. The right-hand-side term of the new constraints represents an extreme acceptable value; several executions (varying the value of the RHS term) are also needed to determine the complete Pareto front. Works using these two strategies are reviewed in the next sections.

The above-mentioned remarks have encouraged and promoted the development of algorithms adapted to the multiobjective nature of the problem and which do not face the above-mentioned problem; with MOEAs, the Pareto front might be approximated in a single run, and regardless of its characteristics (convexity, continuity, etc.). Because of the diversity of the techniques proposed in the devoted literature, the second part of this section presents each associated work in a paper-by-paper fashion.

a) *Weighted linear aggregation function:* In [26], a GA was adopted and the population was divided into different subpopulations that encoded different weight combinations and produced, in consequence, different portions of the Pareto front. In a further paper, Loraschi *et al.* [45] used a distributed GA for the same problem formulation, and showed that the distributed version offered a significantly better return for a given risk level than its sequential counterpart.

Chang *et al.* [39] proposed a steady-state GA with binary tournament selection, uniform crossover, and a boundary mutation operator. The issue of the encoding was dealt with by a simple repair procedure that transforms infeasible solutions into feasible ones. Besides using a GA, the authors also experimented with tabu search [69] and simulated annealing (SA) [70] (all of them using the same linear aggregating function). For their comparative study, the authors constructed five test data sets considering the stocks involved in five different capital market indices from around the world. First, an unconstrained version of the problem was solved and results were compared with respect to those generated by an exact method. The GA was the best overall performer in this case, followed by SA. Tabu search produced a very large mean percentage error with respect to the two other approaches. In a second experiment, the authors considered the cardinality constrained version of the problem. This time, there was no clear winner, since some approaches produced smaller mean percentage errors in some problems, but greater values in others. However, regarding the contribution of each algorithm to the Pareto-optimal set (constructed as the union of results produced), the GA was the approach that contributed the most, followed by tabu search and then by SA.

A GA was proposed in [62] to solve the classical Markowitz formulation. A specific encoding scheme (detailed in Section III-B1) was implemented and compared with respect to QP using end-of-week closing data accumulated over an 11-month period beginning on October 3, 1994. The GA adopted two-point crossover, roulette-wheel selection, and bit-flip mutation. Results indicated that the GA could find portfolio allocations with similar risk and higher rates of return than QP.

In a further paper, Shoaf and Foster [63] analyzed the computational complexity of their approach and proved that it is dominated by the sorting required by their special encoding. Assuming that quicksort is adopted, the complexity is $O(n \log n)$. Moreover, their results indicated a good scalability of the GA up to 100 stocks (i.e., the algorithm's complexity remained $O(n \log n)$, as expected). However, aiming to be able to explore faster and in a more effective manner the potentially large and highly multimodal search space of this problem, the authors also proposed a parallel model based on islands.

In [30], 500 combinations of the return-risk weights were used in order to construct the Pareto front. A simple GA (uniform crossover, problem-specific mutation) was employed to solve the resulting single-objective problem. The approach was validated on a benchmark consisting of three problems with a number of securities varying between 33 and 99. The Pareto front, drawn for different values for the cardinality constraint (K), showed that for larger K values, the region with high returns (and high risks) could not be reached. At the same time, the CPU time increased with K.

Hochreiter [31], [32] proposed a GA optimizing a single objective, which is built as a linear aggregation of both risk–return criteria. The GA adopted a real-numbers encoding, followed by a necessary normalization repair procedure to respect the total budget constraint. The genetic operators were N-point and intermediate crossovers and a mutation step in which asset weights were multiplied by a randomly generated factor. A 15-assets problem from Dow Jones historical data was solved. The authors highlighted the differences observed, according to the used risk measure (e.g., standard deviation, VaR, conditional VaR, mean-absolute downside semideviation, and expected shortfall; see Section III-A2), between the best obtained portfolios as well as the associated loss distributions.

In [41], the portfolio selection problem was modeled by a fuzzy mathematical programming approach. The fuzzy model is based on the mean-absolute deviation variant of the Markowitz model. In the proposed model, the expected return and the risk objectives were fuzzified using a logistic membership function. Thus, both the objective (risk) and the constraints (expected return and other usual constraints) were considered to be fuzzy. Among the constraints considered we can find the expected return, a lower bound for the number of assets in the portfolio, lower and uppers bounds for the budget, and the minimum transaction lots. The authors adopted a GA combined with a local search algorithm in order to improve the population before applying evolutionary operators.

b) *ϵ-constraint strategy:* In [52], the multiobjective problem was reduced to a single-objective problem (i.e., minimize risk), by including the expected return (greater than a minimum acceptable value) in the constraints. The solution phase was undertaken with a GA using binary encoding, uniform crossover, roulette-wheel selection, and fitness evaluation through crowding distance (no further details are provided). A numerical example with six stock assets was solved to illustrate the proposed methodology. A 55-solutions Pareto front was obtained.

Soleimani *et al.* [38] considered the return objective as a constraint while the variance was minimized. The solution method was a single-objective GA, whose main features were a completely deterministic selection technique (the fittest half of the population survives) and a random assorting recombination crossover method [71] that was used to generate the other half of the population. Computational experiments were first driven on a small nine-stock problem, in order to compare it with the results obtained by a mathematical programming method (LINGO package); the GA's solutions were between 0.45% and 7.77% far away from the optimum found by LINGO. However, LINGO was unable to solve a 30-stock problem to optimality within 24 h execution, so only the GA was tested on larger (randomly generated) instances, consisting of 500 and 2000 securities. The authors concluded on the good quality of the achieved results in reasonable computational times (lower than 7 min), despite the lack of comparison with any other solution technique.

c) *Ehrgott et al.* [50]*:* Five different objectives were considered in this paper (refer to Section III-A2) and the authors also allowed the incorporation of the user's preferences through the construction of decision-maker specific utility functions and an additive global utility function. Using this global utility function as the objective function to be optimized, the authors performed a study in which they compared four approaches: 1) a two-phase local search algorithm; 2) SA [70]; 3) tabu search [69]; and 4) a GA. The two-phase local algorithm, SA, and tabu search, shared the same neighborhood structure. Results on a funds database indicated that the GA was the best performer, followed by SA. In randomly generated instances, however, the two-phase local search algorithm had a better performance, followed by the GA.

d) *Lin et al.* [42], [43]*:* These authors adopted the NSGA-II [15] to solve the investment portfolio optimization problem with fixed transaction costs and minimum lots. In this paper, integers encoding, simulated binary crossover [72] and parameter-based mutation [15] were implemented within the NSGA-II.

An interesting aspect of this paper is that the authors adopted genetic algorithm for numerical optimization of constrained problems (GENOCOP) [73] to handle the constraints of the problem. However, since GENOCOP requires that the initial population is feasible in order to handle linear constraints, the authors adopted the same NSGA-II to find feasible solutions. The problem solved in this case is really single objective, but it is considered as a special case of the multiobjective problem.

When all the individuals in the population are feasible, the NSGA-II is stopped and the solutions are fed into GENOCOP, which handles the original constraints of the problem. The authors validated their approach using data from the OR-Library [60]. The results indicated that by investing in more stocks, the maximum risk was significantly decreased. The authors also experimented with fitness scaling, which they found to be useful to make their MOEA more efficient. However, the results were not compared with respect to any other approach.

e) *Fieldsend et al.* [40]*:* The MOEA adopted in this case is a (1+1)-ES previously used in [61]. The algorithm maintains a set of the different cardinality constrained Pareto fronts, each of which is initialized with a random portfolio. The algorithm proceeds at each iteration by first randomly selecting an archive with cardinality k and copying a portfolio from it. Such a copied portfolio is then adjusted (either only weight adjustment or also dimensionality change). The resulting portfolio is evaluated in terms of its return and risk and compared to the others previously stored portfolios to see if it is nondominated. Evidently, any dominated portfolios are removed. The approach was validated using stock data from the U.S. S&P 100 index and emerging markets stock. Results are compared with respect to the unconstrained problem, which is solved using QP. Preliminary results showed that it was possible to replicate closely the mean and variance of an efficient portfolio using a relatively low number of stocks.

f) *Subbu et al.* [48]*:* The solution technique adopted in this case is a hybrid multiobjective optimization algorithm that combines EAs with linear programming for investment portfolio optimization. The adopted EA, proposed in this paper, is the Pareto-sorting evolutionary algorithm (PSEA), which uses a small population size and an archive that retains the nondominated solutions found along the search. PSEA is initialized with a randomized linear programming algorithm, which stochastically identifies a sample of the boundaries of the search space by solving thousands of randomized linear programs. Additionally, a fast dominance filter is implemented to differentiate between dominated and nondominated solutions. Following a "divide-and-conquer" strategy, a set of solutions is decomposed in order to work on smaller chunks of such set.

The actual search for optimal portfolios was performed by another approach called target-objective genetic algorithm (TOGA), which is based on both goal programming and the VEGA [10]. TOGA attempts to find solutions that are as close as possible to a predefined target for one or more objectives. These approaches are all part of a more complex system developed at General Electric and currently used in real-world problems with hundreds to thousands of assets. The system also allows the incorporation of progressive preferences and provides 2-D projections of the obtained Pareto fronts.

g) *Branke et al.* [36]*:* A hybrid algorithm was proposed in this paper, in order to obtain a continuous Pareto front, by combining an MOEA (NSGA-II [15]) and the critical line algorithm [25]. NSGA-II handled the permutation variables that specify if the corresponding assets are included or not in the portfolio and was used to define convex subsets of the original search space; subsequently, the critical line algorithm was applied on every subset to form the complete Pareto front.

Thus, each solution in the MOEA is no longer represented by a single point but rather by a partial front in the mean-variance space, determined by the critical line algorithm, which is referred to by the authors as an "envelope."

Several envelopes may be aggregated to determine the entire front. Using this feature, the nondominated sorting procedure and crowding distance computation are modified in order to determine, for the former, which is the best aggregated front a solution participates to and, for the latter, which is the proportion of this front the solution is contributing to.

Although a simple binary encoding would be sufficient (since the asset weights are handled by the critical line algorithm), a permutation encoding is adopted in order to provide some feedback from the critical line algorithm to the MOEA; an asset receiving a high weight (averaged on the number of efficient solutions found by the critical line algorithm) will appear sooner in the permutation. Uniform order-based crossover and swap mutation are used as genetic operators.

Two additional features are also added to the proposed strategy. First, the duplicate elimination technique removes all individuals showing a frontier completely covered by (or included in) the frontier of another individual (this solution, therefore, does not really contribute individually to the construction of the aggregated front). Besides, a variable population size is used in order to increment the number of individuals when the first aggregated front (i.e., the Pareto-front approximation) is constituted by a number of solutions higher than the current population size.

The approach is tested on four problems drawn from the OR-Library [60] and compared with NSGA-II without the critical line algorithm (and which consequently produces point-based solutions). The authors underlined that, because of its envelope-based working mode, the hybrid algorithm requires a much smaller (initial) population size than the classical NSGA-II. The comparison was performed according to metrics that basically represent the area between the Pareto front obtained with the considered algorithms and an ideal Pareto front (i.e., the Pareto front for the problem without nonconvex constraints and determined by the critical line algorithm). Experimental results highlighted the superiority of the envelope-based algorithm, which provided a better approximation of the ideal front (especially in the extreme regions) and converged faster than the simple MOEA. Finally, its clear advantage over other existing approaches was its ability to produce a continuous Pareto front.

h) *Yan et al.* [46]*:* Considering expected return and semivariance on T periods as optimization objectives, the authors of this paper included the risk minimization in the constraints and performed several executions (varying the right handside term of the maximum allowable risk) with a single-objective GA.

The specificity of the proposed GA lies on the replacement of the mutation step by a particle swarm optimization (PSO)-based operator [74]; the considered individuals are modified via a vector accounting for the historical best personal position and the current best global position. Note that the velocity from the previous iteration does not appear in the PSO-based mutation process (as it does in a classical PSO strategy). No further details are provided about the handling of the budget constraint or about the adopted encoding scheme. Two numerical experiments were carried out, using two nonreferenced examples that considered, respectively, 3 and 6 periods and 4 and 12 securities. The resulting Pareto fronts were compared against the solutions obtained by a direct search method (no details were provided about this technique) and the EA outperformed the latter one.

i) *Bradshaw et al.* [49]*:* The portfolio selection problem with VaR as the risk objective was solved by these authors using a classical MOEA (the authors argue that it is loosely similar to SPEA2 [14]). The main features of the algorithm consist of the design of genetic operators that maintain the solutions' feasibility and a varying size population. Regarding the crossover and mutation operators, when some portfolio exceeds or is below the total budget, the weight of a randomly selected asset (or several assets, if necessary) is modified so that the sum of all weights equals one. Concerning the varying size population, when the number of nondominated solutions is higher than the current population size, the latter is increased in order to keep all those solutions.

The resulting algorithm was tested on a 20 assets instance and a sensitivity analysis was carried out on the number of generations and on the initial population size. Results showed that a small initial population size allowed to reduce the computational time without deteriorating the quality of the resulting Pareto front. The results were not matched against any other solution method.

j) *Krink and Paterlini* [29]*:* An original solution technique was proposed by these authors for three models that differ in the objectives that were considered in each case (refer to Section III-A2); the expected return criterion was combined with either variance (model MV), VaR (model MVaR), or expected shortfall (model MES). The solution technique was inspired on several algorithms. First, the operators were taken from differential evolution (DE) [75] (in its rand/1/exp version, meaning that the individuals involved in the mutation step are randomly selected and combined in only one differentiation, and that the crossover step is based on an exponential distribution). For the selection step, they adopted the nondominated sorting from NSGA-II, together with its multiobjective-based rules and crowding distance to choose from the union of the parents and the mutated offspring. Finally, Michalewicz's GENOCOP approach [73] was used to handle the linear constraints of the problem. Unlike the classical DE technique, all the mutated individuals were compared in this case to all the parents, in order to replace the worst ones in the following population.

The resulting algorithm [differential evolution for multiobjective portfolio optimization (DEMPO)] is compared against NSGA-II [15] using four metrics that measure, respectively, the distance between the approximated front and the true Pareto front (which is generated by a QP method for the unconstrained problem), the uniformity of the approximated front, the hypervolume [76] covered by the approximated front and Knowles and Corne's MOSTATS metric [22] (which statistically computes the attainment surfaces [77] for a given number of runs for each considered algorithm).

Previously tested on six multiobjective mathematical problems, this comparative study shows that NSGA-II beats DEMPO for the simplest instances, while the reverse is observed for the more complex problems. Concerning the computational times, DEMPO is almost three times faster than NSGA-II for an equal number of fitness function evaluations.

Then, the three portfolio optimization models were solved on a real-world instance; data from the Italian stock exchange were used. For the MV model, DEMPO outperformed NSGA-II and provided a Pareto front very close to the true Pareto front generated by QP. Moreover, apart from the uniformity metric, DEMPO obtained the best values of the performance indicators. The authors claim that the reasons for DEMPO's superiority are the (already reported) particularly good efficacy of DE on continuous optimization problems and the GENOCOP-like constraint-handling method adopted for the equality constraints of the problem. NSGA-II's results were not provided for the two remaining models (MVaR and MES), so it is difficult to assess DEMPO's behavior on those formulations.

k) *Aranha and Iba* [59], [78]*:* As mentioned in the previous section on encoding techniques, Aranha and Iba [58], [78] proposed a tree-based representation scheme. The authors subsequently improved their solution technique by implementing a local search procedure in order to modify the trunk nodes of a tree, starting by the deepest nonterminal nodes. The resulting memetic algorithm, when compared to more classical portfolio representations, was found to work quite well on three real-world data examples.

l) *Gomez et al.* [34]*:* The authors of this paper proposed a hybrid approach based on SA [70] and MOEAs for dealing with a model involving cardinality, floor-ceiling, trading, and turnover constraints. A population of solutions was modified according to a neighborhood (not detailed in the paper), and then it was Pareto ranked and the offspring were selected if they improved the risk criterion. No details were provided about the selection process, the constraint-handling method nor the cooling schedule adopted for the SA algorithm.

m) *Vijayalakshmi and Michel* [35]*:* As mentioned in Section III-A1, the most relevant aspect of the approach proposed in [35] lies on the use of k-means clustering to deal with the cardinality constraints. Then, a $(\lambda+\mu)$-ES is adopted to optimize a linear function that aggregates return and risk objectives. Moreover, a "plus" selection scheme (involving more offspring than parents), uniform crossover and mutation steps, and repair procedures (to handle the floor-ceiling and class constraints) were adopted within the ES algorithm.

The strategy was numerically validated on two real-world problems (Indian and Japanese stock exchanges, with 200 and 225 assets, respectively). The Pareto fronts produced by multiple executions of the ES (modifying the return-risk weights in the objective function) and by QP were compared only in terms of their visualization. From this analysis, the authors concluded that the solutions generated by their ES were very close to those generated by QP. Additionally, a data envelopment analysis technique was carried out to compare both approaches, concluding that they were similar in terms of efficiency and robustness.

n) *Drezewski and Siwik* [79]*:* These authors emphasized the necessity of procedures, inside MOEAs, that preserve the diversity of the population (in order to avoid converging to a limited region of the Pareto front or to avoid getting trapped in false Pareto fronts). As such, they present the use of a co-ES as an alternative for dealing with this issue. Coevolution is defined as the prolonged mutual interaction between two (or more) species. The nature of these interactions might be positive or negative, according to the considered species. For instance, *mutualism* means that all involved species benefit from the relationship, *competition* means that the species have a negative effect on the others (they compete for the same resources), and *predation* involves a benefit for the predator and, conversely, a negative impact for the prey.

This last scheme (predator–prey) is the one selected by the authors in this paper. They underline that in addition to the diversity preservation aspect, coevolution provides a useful analogy with market-oriented economic systems. In this sense, agents can be seen as virtual entities interacting with their environment and with other agents, making them an ideal tool for modeling social or financial structures. Coevolutionary multiagent systems (CoEMASs) therefore consists in evolving a population of agents, divided into several species.

The implementation of the CoEMAS requires the definition of an environment (with its associated topology), of the species evolving in this environment, and of the nature of the existing information and resources. The selection mechanism is based on the closed circulation of resources within the system, this transfer being possible between preys and from preys to predators. To completely determine a species, sets of sexes, actions, and relationships (with other species) must be defined for each of them. The genotype of each agent consists of two vectors: the first one represents the decision variables of the tackled problem, while the second one is used to compute the standard deviation implemented in the self-adaptive mutation step. Additionally, each agent is assigned a profile, a goal corresponding to this profile, and a strategy to attain the goal.

This modeling and solution system was implemented for the solution of the Markowitz mean-variance formulation. For this problem, elitism had to be integrated into the evolutionary process in order to preserve nondominated agents that, as simulation time passed, only met a majority of the other nondominated agents and, thus, could not increase their resources' level. The computational experiments were carried out on data considering three and 17 stocks from the Warsaw stock exchange, on a three-year period. The results were compared against those generated by Niched-Pareto genetic algorithm (NPGA) [80] and a "classical" predator–prey evolution strategy (PPES). They showed that if the number of nondominated solutions found was similar for all three solution techniques, CoEMAS was unable to provide a good diversity of solutions over the whole Pareto front (which was one of the main justifications for the use of coevolution); CoEMAS only identified a small, restricted subset of the Pareto front, located on the low risk, low return region. This trend became even more pronounced as the number of iterations increased. The authors however argued that when applied to test problems (proposed by Laumanns, Kursawe, and Zitzler), CoEMAS had presented the best results.

o) *Zhang et al.* [81]*:* In this paper, the objective, rather than improving state-of-the-art solutions on a specific portfolio optimization formulation, was to present an improvement for the original version of an algorithm introduced a few years earlier, namely, MOEA/D (its main features are described in Section II). The authors propose the normal boundary intersection–Chebyshev style decomposition to derive single-objective subproblems from the initial multiobjective problem. The portfolio optimization problem is only used to provide test instances to evaluate the performance of the novel decomposition approach proposed. The model used is the classical mean-variance of return model, which includes cardinality constraints (an upper bound for the number of selected assets). The authors tested their algorithm on eight instances with a number of assets ranging from 30 to 150. They conclude that their approach finds the best solutions in almost all cases when comparing it against the NSGA-II [15]. However, they highlight that the implementation of their method is nontrivial, since it involves extra constraints and, therefore, the need for an efficient constraint-handling technique.

3) *Studies Comparing Different Solution Strategies:*

a) *Vedarajan et al.* [56]*:* Considering the Markowitz mean-variance model, the authors first adopted a GA with a linear aggregating function combining the two objectives into a single scalar value. The weights were varied in order to generate different nondominated solutions. Binary tournament selection, one-point crossover, and bit-flip mutation were implemented within the GA. The latter was compared against the NSGA [21].

As mentioned in Section III-B1, both GAs use random keys encoding, which requires that the solutions are sorted. The results from both solution techniques were compared only in a graphical form (no real numerical comparison, using performance measures, was reported in this paper) and seemed similar in quality, although the NSGA provided a much more diverse set of solutions.

According to the authors, there are, however, other advantages of using the NSGA; when adopting QP techniques, one has to work with a covariance matrix and such matrix needs to be positive definite at all times. It turns out that when working with real-world problems, as the number of portfolio holdings increases, it becomes difficult to maintain this matrix as positive definite because of numerical imprecisions associated with the use of floating point arithmetic. This is not an issue with MOEAs, since they do not use this matrix. The authors noted that, in practice, portfolio management involves other costs as well, such as transaction costs, broker fees, and others. So, the authors added another objective: minimizing the transaction costs. The new problem has one additional constraint related to the maximum allowable transaction cost, which is difficult to handle with QP because of the way in which transactions normally take place in practice. Thus, in this case, the use of an MOEA brings even more advantages, since the NSGA was able to produce the 3-D Pareto front in a single run.

b) *Radziukyniene and Zilinskas* [51]*:* Four algorithms were compared by these authors on two portfolio selection problem formulations: the first one considers the two classical objectives, while the second one includes the annual dividend yield as a third criterion. The algorithms, whose main features have been described in Section II, are FastPGA[16], MOCeLL [18], AbYSS [13], and NSGA-II [15].

These four algorithms were compared according to five criteria (merging the Pareto-optimal solutions generated by the four algorithms, in order to form the reference true Pareto front): generational distance (GD), inverted generational distance (IGD), hypervolume (HV), spread (S), and computational time. GD and IGD [66] represent, respectively, how far the approximation is from the true Pareto front and how far are the elements in the Pareto-optimal set from those in the approximated set. Each algorithm had its own set of parameters, but all of them were run with an equal number of fitness function evaluations. For the two-objective problem, MOCeLL (for GD, IGD, and HV) and AbySS (for S and computational time) were found to be the most competitive solution techniques. For the three-objective problem, MOCeLL outperformed the other algorithms for all the performance measures adopted, except for the computational time (NSGA-II was the fastest technique).

c) *Duran et al.* [57]*:* The authors of this paper compared the performance of three MOEAs and an SOEA, according to two criteria: the efficiency of the algorithms in achieving a good-quality Pareto front for the Markowitz model and the performance in locating the efficient portfolio, determined with Sharpe's ratio (see Section III-A3). The SOEA used Sharpe's index as its only objective function and approximated the Pareto front by performing several runs for different values of Sharpe's index.

The MOEAs adopted were NSGA-II [15], SPEA2 [14], and IBEA [17], whose main features were defined in Section II.

The Pareto fronts generated by each of these approaches were compared according to the hypervolume and to the R_2 performance measure [82], which estimates the extent to which a certain front approximates another one (the reference set in this case, which was formed with the union of the nondominated sets generated by the four approaches being compared). The solution sets found by the three MOEAs were quite similar, but IBEA seemed to be the best one with respect to the two above-mentioned performance measures. Because of its discretized working mode, the SOEA could not reach a good coverage of the Pareto front, but it produced an approximation that was not too far from the "true" Pareto front. Focusing on Sharpe's ratio, the SOEA obviously identified with more accuracy the best portfolio with respect to it. The differences with respect to the values obtained by the MOEAs became larger as the risk-free return rate parameter increased.

d) *Skolpadungket et al.* [33]*:* In this paper, six MOEAs were compared on the classical mean-variance model, including exact cardinality constraints, round-lot constraints, and fixed asset weights bounds: the canonical version of VEGA [10], two fuzzy versions of VEGA, MOGA [20], NSGA-II [15], and SPEA2 [14]. The fuzzy versions of VEGA are designed to overcome this drawback and make use of fuzzy decision rules, based on verbal statements on the quality of each objective.

For all the considered algorithms, a mixed binary-real encoding was adopted, while a repair procedure was implemented in order to produce feasible offspring after the (three-point) crossover and (uniform perturbation) mutation steps.

The efficient frontiers produced by the six MOEAs were compared with respect to the distance to the true (unconstrained) Pareto front (i.e., generational distance) and in terms of the diversity of the solutions obtained (this latter feature was accounted for by visual inspection of the Pareto fronts). The computational results indicated that SPEA2 was the best solution technique, with respect to both generational distance and solutions dispersion. MOGA and NSGA-II solutions lie quite close, while the three VEGA versions are strongly outperformed by the Pareto-ranking-based techniques.

e) *Diosan* [83]*:* Within the mean-variance formulation framework, this author compared the performance of three MOEAs: PESA [23], NSGA-II [15], and SPEA2 [14]. For the three algorithms, a real-numbers encoding scheme was adopted, followed by a normalization repair procedure in order to satisfy the capital constraint.

The algorithms' performances, evaluated on a real-world data set, were compared with respect to two unary metrics: S (portion of the objective space dominated by the approximation of the Pareto front generated by the MOEA) and δ (uniformity of the distribution of the points along the Pareto front) [15]. Additionally, binary set coverage [65] was also adopted; this metric, comparing the Pareto sets produced by any two algorithms A_1 and A_2, is computed as the ratio of non-dominated solutions produced by algorithm A_1 that dominate the solutions produced by algorithm A_2, and conversely. The author concluded that PESA outperformed the other two algorithms regarding the S and coverage performance measures, while NSGA-II was the best with respect to the δ measure.

f) *Anagnostopoulos and Mamanis* [55]*:* The authors of this paper solved a variation of the Markowitz model with three objectives: return value, risk, and number of assets in the portfolio. Three different MOEAs were compared in this paper: NSGA-II [15], PESA [23], and SPEA2 [65]. The performance of the algorithms was evaluated using the hypervolume and the ϵ-indicator. The experiments showed that SPEA2 was the best for both the constrained and the unconstrained multiobjective portfolio optimization problem.

IV. Other Financial Applications

In this section, several financial problems, for which a solution technique based on MOEAs has been proposed, are briefly reviewed. Unlike the previously treated portfolio selection problem, in this case, the diversity of works is not big. Thus, the topics addressed in this section do not share specific similarities and, therefore, they are presented in an independent manner.

A. Financial Time Series

The aim of this research topic is to find patterns in financial time series, such that predictions can be made regarding the behavior of a certain stock. Typically, neural networks (NNs) have been applied to this problem, but the use of different types of EAs has also been reported, independently or in combination with the use of NNs.

The use of an evolutionary multiobjective approach in this area has not attracted much interest so far and the references devoted to this topic are, therefore, scarce. There are, however, some references in which MOEAs have been used for time series prediction, although not in a finance-related domain (e.g., [84]). Additionally, there is also some work on predicting customers patterns [85], which is also related to this topic.

1) *Ruspini and Zwir:* Ruspini and Zwir [86] and Zwir and Ruspini [87] used the NPGA [80] for automatic derivation of qualitative descriptions of complex objects. The NPGA is a nonelitist MOEA that uses a tournament selection scheme based on Pareto dominance. The authors applied their methodology to the identification of significant technical-analysis patterns in financial time series. Two objectives were considered: quality of fit (which measures the extent to which the time-series values correspond to a financial uptrend, downtrend, or head-and-shoulders interval) and extent (which measures, through a linear function, the length of the interval being explained). The NPGA was actually adopted to determine crisp intervals corresponding to downtrends, uptrends, and head-and-shoulders intervals. Niching and tournament selection were used in this application. The authors of this paper concluded, from a graphical analysis of the obtained results, that their approach allows a satisfactory description of complex qualitative problems such as financial time series.

2) *Abraham et al.* [88]*:* These authors proposed the unification of five types of NNs (Levenberg–Marquardt algorithm, support vector machine, difference boosting NN, Takagi–Sugeno fuzzy inference system using a NN algorithm, and multiexpression programming) to model the trends of two stock market indexes (NASDAQ and NIFTY). The quality of the approximation was evaluated according to four criteria, which basically represent different ways of computing the error between the predicted and the actual index values. The data set, consisting of a large number of historical values for both indexes, was divided into two parts, for training and testing, respectively.

The unification of these five NNs algorithms was carried out through a linear combination of the outputs from each algorithm. The weights in this linear function represented the decision variables of a multiobjective optimization problem, which considered the above-mentioned error metrics as the optimization criteria. Two solution schemes were adopted, NSGA-II [15] and PAES [22].

The computational experiments conducted by the authors showed that the output combinations obtained by any of the MOEAs adopted in this paper reached better error measures than any NN used in an independent manner. However, even though the weights obtained by each optimization method were different, no superiority between them could be established.

B. Stock Ranking

The aim of this problem is to classify stocks as strong or weak performers based on technical indicators (TIs) and then use this information to select stocks for investment and for making recommendations to customers. Next, we report the use of MOEAs in this application area.

1) *Mullei and Beling* [89]*:* The authors of this paper used a GA with a linear combination of weights to select rules for a classifier system adapted to rank stocks based on profitability. Up to nine objectives were considered in this case, related to conjunctive attribute rule tests. This problem was solved using a classifier system from the so-called Pittsburgh approach [2]. The authors used binary encoding, roulette-wheel selection, one-point crossover, and uniform mutation. The approach was validated using five large historical (U.S.) stock data sets covering approximately three years (1995–1998) of weekly data on a universe of 16 stocks. Results were compared against those produced by a technique related to the synthesis of polynomial networks called STATNET. The results were inconclusive since no technique was able to outperform the other in all cases.

2) *Hassan and Clack* [90]*:* Stock picking refers to a low-frequency investment practice, which applies models that guide the choice for buying or selling stocks in an extremely unstable market environment. The main objective is not to accurately predict stock prices, but rather to determine a consistent stock ranking. GP is a popular technique for evolving stock-picking models, but the time series used for model training are highly subject to fluctuations that might affect the model effectiveness; the environment is not the same as that used for training, and the model is not appropriate anymore. When considering the optimization of several objectives (typically risk and return), this loss of effectiveness results in a Pareto front that could be far from optimal in the new environment; in addition, the relative position of the solutions on the Pareto front might switch. The authors of this paper, noting that retraining is not an adequate parry, proposed to look for robust models that would still perform reasonably well when changes in the environment occur. In this view, they put forward two hypotheses and suggested appropriate responses. The first was that the search process should focus on particular niches in phenotypic space, rather than trying to fit to specific data. Thus, it might be useful to cluster the solutions and applying mating restrictions in order to only recombine solutions belonging to the same cluster. The second assumption was that diversity would favor smaller trees, avoid overfitting, and promote more robust solutions; increasing the mutation rate might help to obtain a better robustness. Note that these two techniques (mating restriction and diversity preservation) are antagonistic since they are methods devoted to intensification and exploration, respectively.

Then, the aim of this paper was to produce solutions that worked reasonably well when applied to another data set (i.e., that the model could properly adapt to new environments). To quantify robustness in this dynamic environment, three aspects must be accounted for.

a) The nearness of the solutions with respect to the (new) Pareto front.
b) The change in the relative position of the solutions. The k-means clustering algorithm was used to group together solutions into three clusters (high risk–high returns, low risk–low returns, and intermediate location) and the change in the cluster membership of any solution was recorded. Additionally, any change in the rank order (computed in objective function space) was also taken into account.
c) The good spread and distribution of the new Pareto front formed during the validation stage.

The first and last aspects could be evaluated through classical performance indexes used for classical MOEAs (the spacing and hole-relative-size [91] performance measures were adopted here), and new performance measures were developed by the authors to account for the cluster and rank-order preservation.

An MOEA similar to SPEA2 [14] was adapted by the authors to operate in a GP working mode [i.e., a multiobjective genetic programming (MOGP) approach]. The only important change introduced was that mating was allowed only for parents belonging to the same cluster. The resulting algorithm was tested on an investment strategy problem with 25 stocks from the U.K. market. The training set used the data from 48 months and the validation set data from 20 months. At the start of each month, the stocks were ranked according to the model produced by the MOGP implementation. The stocks from the bottom quartile were sold and, if the number of stocks in the portfolio was insufficient, then stocks from the top quartile were bought. The experiments investigated and compared the results obtained by four algorithms: standard SPEA2 (without mating restrictions nor mutation), SPEA2 without mating restrictions but with mutation, SPEA2 without mutation but with mating restrictions, and SPEA2 with mating restrictions and mutation. Finally, the computational results highlighted the benefits brought by both mating restrictions and diversity preservation strategies, since for all the performance measures adopted, except for one (the hole-relative-size metric), SPEA2 (with its new mechanisms) obtained the best solutions.

C. *Risk-Return Analysis*

Credit portfolios handled by banks are also investment portfolios, but they operate under different rules and, therefore, they are not modeled using the original Markowitz approach. Next, we will describe applications of MOEAs to this area.

1) *Schlottmann and Seese* [92], [93]*:* An approach similar to the NSGA-II [15] was used in this case for solving portfolio selection problems relevant to real-world banking. In the problem studied by the authors, a bank has a fixed supervisory capital budget. This is an upper limit for investments into a portfolio consisting of a subset of assets (e.g., loans to be given to different customers of the bank), each of which is subject to the risk of the default (capital risk). So, in this case, besides having an expected rate of return (as in the original Markowitz problem), each asset has also an expected default probability (which is set *a priori*) and a net exposure, within a fixed risk horizon. The authors adopted binary decision variables to indicate whether or not a certain net exposure would be held in the portfolio or not. Only if an asset was held in the portfolio, the bank had to allocate a supervisory capital amount from

its available (but scarce) resources. Thus, the return objective function had to be adjusted for default risk (i.e., expected loss). The resulting problem had a discrete constrained search space with many local optima and two conflicting objective functions. Unlike the original NSGA-II, the authors adopted an external archive containing the nondominated solutions found during the search. They also incorporated a gradient-based local search operator which was, however, rather heuristic. For validation purposes, the authors designed their own test cases with a structure similar to real-world data from German banks. They compared their hybrid MOEA with respect to the same MOEA without the local search mechanism. Results indicated that the use of local search significantly improved performance (the average improvement was found to be between 17% and 95% for the set coverage metric [65]).

2) *Mukerjee et al.* [94]*:* The authors used the NSGA-II [15] to determine risk-return tradeoffs for a bank loan portfolio manager. The idea is the same as before: the bank manager aims to maximize shareholder wealth. This implies maximizing the net worth of the bank, which, in turn, involves maximizing the net interest margin of the bank. However, there are a number of regulatory constraints imposed on the bank, such as the maintenance of adequate capital, interest-rate risk exposure, and so on. The authors adopted a portfolio credit risk model based on the standard deviation of the return over the entire portfolio. The objectives were to maximize mean return on the portfolio and minimize the variance on the return. For validating their approach, the authors adopted data from the CreditMetrics Technical Document. The authors studied an elastic loan demand model in which they assumed that the amount of loan applications received in a given loan category was a function of the interest rate charged. The authors used the NSGA-II for this model, adopting the interest rates as their decision variables.

An interesting aspect of this paper is that the authors compared the performance of the NSGA-II with respect to that of the ϵ-constraint method (using a simple GA for the individual single-objective optimizations performed by this method). Only graphical comparisons were presented, since the aim was to show that the NSGA-II could achieve the same convergence as the ϵ-constraint method, while providing a much wider distribution of nondominated solutions.

3) *Teive et al.* [95]*:* Similarly, in stock market portfolios, risk management plays an important role in the electricity market to guide investors under both contract uncertainties and energy prices. The above authors proposed solving the portfolio optimization problem in the electricity market by using an MOEA associated with a multicriteria decision strategy to select the most appropriate alternative. One particular aspect of this paper is the consideration of flexible contracts, such as option contracts (call or put options). The return value of the contracts portfolio was computed taking into account sport market, forward contracts, and option contracts (call or put options). The authors developed an MOEA called Pareto GA. The optimization problems considered four objectives: minimize risk, minimize VaR, minimize CVaR, and maximize the return value. The proposed algorithm was applied to a problem taken from the Brazilian electricity market in which it was assumed that the trader agent would buy and sell 1000 MWh at the liberalized market in the same region. After obtaining an approximation of the Pareto front, a multicriteria analysis was performed to reduce the set of alternatives.

D. Financial and Trading Decision-Support Tools

Many decision-making problems in financial forecasting are based on the analysis of time series, typically indicating the market price movements, in order to decide what behavior should be adopted (for instance, sell, buy, or hold). However, the large number of interactive factors and multiple conflicting objectives involved in a financial forecasting problem generally leads to a huge and complex search space.

In this framework, objective technical analysis is widely used by professional traders, because its accuracy can be indisputably quantified. The following two studies adopt MOEAs in order to derive trading rules or strategies, which constitute efficient decision-support tools within the financial forecasting problems.

1) *Li and Taiwo* [96]*:* These authors proposed an extension of a single-objective strategy referred to as financial GP (FGP) [97] using GP to define rules that might help for financial decision making. Some (single-objective) improvements of FGP have been subsequently proposed in [98]–[100]. These rules are built through decision trees constituted by elemental blocks such as operators, thresholds, and financial indicators. The terminal leaves of the tree were binary recommendations (positive or negative) about buying or selling, according to the prediction of an increase of closing prices over a defined period. The rules were derived from a training data set and were subsequently applied on a test set in order to validate the rules' responses and their extrapolating efficacy.

The multiobjective FGP (MOFGP) approach adopted by the authors was based on SPEA2 [14] (i.e., they used an external archive and a strength-based selection step applying nondomination ranking and diversity) in order to produce near-optimal, well-spread, and evenly distributed Pareto fronts. The objectives were: 1) the rate of failure (proportion of positions, i.e., decisions, that were wrongly predicted positive over the number of actual positive positions), and 2) the rate of missing chances (number of wrongly predicted negative positions over the number of actual positive positions).

Numerical experiments were carried out on a 6.5 years training data set and on a 3.5 years testing data set. The MOFGP was compared with respect to the outcome generated from multiple runs of FGP and the decomposed results showed that FGP performed slightly better than its multiobjective counterpart. However, the authors emphasized the fact that the solutions generated by MOFGP were obtained with a single run of the algorithm. Additionally, they noted a significant difference between the Pareto fronts produced on the training set and those produced on the testing set, which means that the generalization ability of MOFGP needs to be improved. Finally, the authors highlighted the good spread of solutions along the Pareto front that they obtained, which covered almost the whole range of variation of the objectives. They also emphasized the fact that there was no need of any extra parameter tuning (unlike the FGP strategy that uses an aggregation function and, therefore, requires that the user sets the weight associated to each criterion).

2) *Chiam et al.* [101]*:* Technical trading strategies (TTSs) are trading rules that analyze the forecasting of future market movements, based on the past history of market actions. "Technician" traders usually make decisions (basically, sell, buy, or hold for short or long positions) relying on several TIs, which account for the past market information including weighted average of the closing prices on a certain period, magnitude of market oscillations, etc. Then, these TIs are aggregated in a generally weighted linear function and, at each instant, a decision is taken if the function value exceeds (upward or downward) a certain threshold.

The aim of this paper is, therefore, to evolve a population of trading agents, each of which consists of several TIs that are taken into account in the design of a TTS (i.e., the aggregation function defining the trading decisions to be taken) and of specific thresholds for the decision-making process. Each TI considered in an agent has its associated weight (for the aggregation function) and parameters. The agents may consider a variable number of TIs, so the associated chromosomes have a variable size. In addition to the maximization of the total return used in single-objective approaches, the authors proposed the simultaneous minimization of risk exposure, computed as the ratio of days with holding assets (i.e., for each trade, the difference between sell and buy dates) with respect to the considered period length.

In the MOEA proposed for solving this problem, a binary tournament was carried out for the selection step, evaluating the individuals through a Pareto-ranking procedure that integrated fitness sharing in the presence of ties (this is similar to NPGA [80]). Additionally, problem-oriented genetic operators were designed: a crossover operator that mixes the parent TIs between two offspring chromosomes (without modifying their associated weights and parameters), while, for mutation, a TI can be added to or removed from an individual, or some random noise can be introduced to the TI parameters. Also, weights or noise can be added to the decision-making thresholds. Furthermore, an insertion procedure introducing some new randomly created individuals was performed at each generation, in order to enhance the population diversity.

The computational experiments first aimed at evaluating the accuracy and distribution of the obtained Pareto front when varying the number and nature of the TIs to be considered by the trading agents. The solutions resulting from the different TIs combination were compared according to the coverage and spread measures [67]. The results showed that some TIs biased the search, focusing it on particular regions of the nondominated set (for instance, preferentially providing high-return, high-risk solutions). In a second set of experiments, the authors tried to evaluate the ability of the strategies generated by the MOEA on a training data set, to generalize to another testing data set. Their results highlighted the low correlation between the return obtained on the training set and on the testing set, which invalidated this assumption, which is traditionally made in single-objective approaches. However, a low-positive correlation was established with respect to risk, between the training set and the testing set.

3) *Lohpetch and Corne* [102]*:* Recently, Lohpetch and Corne [102] used GP for discovering technical trading rules. This kind of approach attempts to forecast the future direction of equity prices based on the patterns that are revealed from historical equity price data. GP uses a tree-based encoding to express trading rules, in which nodes represent a function set composed by arithmetic, Boolean, and relational operators, while the terminal set comprises a collection of basic financial indicators (e.g., opening and closing price, trend lines indicators). The rules can be interpreted, for instance, as a recommendation to buy. A simple but effective trading approach is the buy-and-hold (B&H) strategy that states that for a given period, one should buy the stock at the beginning of the period and sell it at the end. In this paper, the performance of the proposed multiobjective GP algorithm was compared against that of the B&H strategy. The authors evaluated several multiobjective configurations using a combination of two or three of the following objectives:

a) market return (MR), which represents the excess of return over that of the B&H strategy;
b) performance consistency, which is the number of times for which MR outperformed the B&H strategy in several successive periods;
c) complexity penalizing factor, which is the depth of the tree.

For the multiobjective approach, the authors adopted NSGA-II [15]. This way, instead of evaluating a single rule, a set of rules were evaluated. The main finding of this paper was that multiobjective approaches were able to outperform single-objective approaches, as well as the B&H strategy.

4) *Butler and Daniyal* [103]*:* In this paper, the authors proposed a robust trading model by predicting the movements of a stock market index (Dow Jones Industrial Average). The model was created using an evolutionary artificial neural network (EANN) trained for multiobjective optimization. Many approaches for classifying investment returns only take into account the direction of prices changes. For this reason, the authors adopted a multiobjective approach that trains the NN on movement and magnitude. The proposed approach was based on the neuroevolution of augmenting topologies method [104], which starts with a population of simple perceptrons and gradually evolves more complex network structures. The crossover operator took the genomes of two candidate artificial neural networks (ANNs) and, for each innovation number, in either of the genomes, a copy was created in the new genome. An innovation number is an historical marker that identifies the original ancestor of each genome. Mutation consisted of four operations: add a node, add a connection, change connection weights, and backpropagation mutation. The main finding of this paper was that an EANN trained to recognize direction and magnitude in stock market was better equipped to create superior investment returns than one trained only to recognize direction changes.

5) *Larkin and Ryan* [105]*:* These authors proposed a hybrid forecasting system for markets whose goal is to predict events based on historical events and finding the optimal way

to capitalize on those predictions. The hybrid system consisted of a master GA and a slave GP. The GA was responsible for finding the best way to teach the slave GP how to spot rising and falling markets as they occurred. The goal of GP was to build classifiers for forecasting rises or falls from historical trading exchanges. Classifiers took the form of a signal generation tree, and both were found with separate GP runs. The signal generators took the form of a standard GP tree comprised of primitive mathematical operations and functions regularly found in the field of technical analysis (e.g., moving average of the price difference over a given range, and highest price difference observed in given range). This hybrid forecasting system was evaluated using ten well-known equity stocks from the NYSE and NASDAQ trading exchange.

6) *Li and Krause* [106]*:* In this paper, the optimization of financial markets is addressed through a strategy based on near-zero intelligence trading agents. These agents apply specific trading rules, such as tick size (i.e., minimum differences between prices at which orders can be submitted), priority rules (determining the rationing of orders in the case of an imbalance between buy and sell orders at the transaction price), multiple prices (determining the transaction prices), or market transparency (defined as the ability of market participants to observe the information in the trading process). The impact on the financial market is evaluated in terms of the trading volume and bid-ask spread.

The optimization technique is a multiobjective extension of the population-based incremental learning algorithm, which attempts to create a probability vector, measuring the probability of each bit position having a "1" in a binary solution string. Then, the probability vector is moved toward the vector that shows the best performance in a similar manner to a competitive learning process. The probabilities are subject to mutation controlled by a parameter determining the degree of randomness involved in the new probability.

The numerical experiments confirmed the fact that objectives are conflicting (low spread are associated with a low trading volume, and conversely). The authors also highlight that their major conclusion is that larger tick sizes are required for higher trading volumes (and associated higher bid-ask spreads).

E. Economic Modeling

1) *Mardle et al.* [107], [108]*:* In this paper, the authors used a GA with a weighted goal programming approach to optimize a fishery bioeconomic model. Bioeconomic models have been developed for a number of fisheries as a means of estimating the optimal level of exploitation of the resource and for assessing the effectiveness of the different management plans available at achieving the desirable objectives. The foundations of fisheries bioeconomic modeling comes from the economic theory of the open-access or common-property fishery, which is based on a logistic population growth model. In this case, the authors developed a model for the North Sea fishery. Four objectives were considered: 1) maximize profit; 2) maintain historic relative quota shares among countries; 3) maintain employment in the industry; and 4) minimize discards. GENOCOP III [73] was used for the evolutionary optimization process. Real-numbers encoding and arithmetic crossover were adopted. This evolutionary approach was compared to the application of traditional goal programming (as implemented within GAMS [109] and solved with CONOPT) in a model of the North Sea Demersal fishery. The GA was considered to be competitive but not necessarily better than goal programming in this application.

2) *Huang* [110]*:* In this paper, the capital budgeting problem was presented through an example in which resources (machines) were bought in order to maximize profits. The constraints accounted for the available capital for buying as well as for the production capacities that had to meet a fixed demand. The author considered an uncertain environment for the unitary net profit, resource unit price, and future demands; these parameters were represented as trapezoidal fuzzy numbers. Thus, the profit maximization subjected to capital and demand constraints was reformulated in two different ways, by means of a fuzzy simulator that computed the credibility indexes required to evaluate the "possibility" of the fuzzy events, as follows:

a) a single-objective model, which maximized the profit, given the assumption of predefined levels of credibility (for the objective and for the constraints);
b) a multiobjective model, which simultaneously minimized the deviation between cost of bought resources and available capital and the distance between real profit and target profit (also given the assumption of predefined levels of credibility).

The multiobjective nature of the second model was addressed through a lexicographic-goal programming (since targets were fixed in the problem definition) approach. The solution technique was a simple GA implementing two-point crossover and a uniform perturbation-based mutation operator. The selection was carried out by a roulette wheel, while a rank-based fitness evaluation was performed in order to determine the best individuals in the current population. For the multiobjective case, the lexicographic approach (which imposes priority objectives) was implemented in the individuals' rank computation. Feasibility was maintained during the search by forbidding the genetic operators to produce infeasible offspring.

The proposed methodology was validated on a five-resources problem, successively treated with both the single-objective and the multiobjective strategies. The solutions were compared neither against the optimal values nor against those provided by any other optimization technique.

3) *Abido and Elazouni* [111]*:* In the construction industry, the management of financial resources of ongoing projects within a contractor's portfolio guarantees to run profitable and sustainable business. Typically, contractors manage financial resources to timely procure cash for all projects in their portfolios. Finance-based scheduling ensures that the expenditures of individuals projects within the portfolio do not exceed the credit limit. In this context, the authors of this paper proposed designing finance-based schedules for simultaneous construction projects. The resulting multiobjective optimization problem involved G objectives representing the profit

values of individual projects to be maximized subject to financial constraints. The problem was solved using SPEA2 [14] with problem-specific crossover and mutation operators. The experimental results showed that SPEA2 was able to generate results similar to those reported in the literature using a single run.

4) *Gaspar-Cunha et al.* [112]*:* The authors of this paper proposed an application, which could be regarded as a financial-related subject, of an MOEA to perform bankruptcy prediction. In the bankruptcy prediction problem, the aim is to infer the probability that a company becomes distressed, over a specified period, given a set of financial statements. The problem consisted of classifying data into two groups, healthy and bankrupted companies, and then training a binary classifier to learn the pattern to distinguish between the two cases. In this paper, a support vector machine (SVM) was adopted as a classifier. A classifier can be evaluated considering the proportion of misclassified cases. Two different types of errors can occur. The proportion of positive cases incorrectly classified (e_I) and the proportion of negative cases incorrectly classified (e_{II}). Another way of evaluating a classifier is by means of the sensitivity and the precision. The authors also defined the harmonic mean of the sensitivity and precision (F_m) as a measure to evaluate a classifier. In this paper, the reduced Pareto set genetic algorithm was adopted [113]. The variables of the problems were the parameters for the SVM (i.e., regularization and kernel parameters). Eight different formulations of the problem were evaluated combining two or three of the following objectives: minimization of the number of cases incorrectly classified, maximization of F_m, and minimization of either e_I or e_{II}.

To the authors' best knowledge, no other application of MOEAs in economics exist, although several of them are certainly possible (e.g., in negotiation strategies [114]).

V. Conclusion

This paper has provided a state-of-the-art survey of applications of MOEAs in economics and finance reported in the specialized literature. We have presented a taxonomy of applications divided into two large groups. The first one dealt with the portfolio optimization problem and represented the vast majority of the work reported in this domain. The main research lines for this research area have been devoted to: 1) the development of models that, in addition to classical formulations, include more realistic features, particularly by proposing new classes of constraints and objectives; and 2) the adaptation of MOEAs' working mode to these new models, taking advantage of their ability to produce a set of nondominated solutions in a single run and to deal with complex formulations of problems arising in this application domain.

It is worth noting, however, that MOEAs do not constitute the only alternative for solving these complex optimization problems; other metaheuristics have also been found able to provide robust and good quality results. For example, Armananzas and Lozano [115] applied a greedy local search algorithm [116], SA [70], and ant colony optimization [117] to portfolio optimization problems, adopting a lexicographic approach by which objectives are optimized separately. Another example is the work of Stummer and Sun [118], in which the results obtained by SA, tabu search, and variable neighborhood search are compared. The three techniques addressed the multiobjective nature of the problem by minimizing the Chebyshev distance to a utopian point.

The second class mentioned in our taxonomy groups together all other financial and economics applications into five classes: 1) financial time series; 2) stock ranking; 3) risk-return analysis; 4) financial and trading decision-support tools; and 5) economic modeling.

It is clear that the use of MOEAs in economics and finance still remains relatively scarce and that a number of areas remain to be explored. Some of them include the following.

1) *Model discovery:* This is an interesting area in econometrics in which nonparametric models are assumed, and one tries to use an EA to derive a model for a certain type of problem (e.g., forecasting nonlinear time series). Normally, ANNs have been used for the model itself, but several researchers have used EAs to find the most appropriate ANN that models the problem of interest.
2) *Data mining:* The use of data mining techniques for learning complex patterns is a very promising research area in economics and finance. For example, the mining of financial time-series for finding patterns that can provide trading decision models is a topic worth exploring.
3) *Forecasting stock prices:* Although long-term forecasting is not possible for the stock market, it is possible to perform short-term forecasting with heuristics. The use of GP in this area has become increasingly popular, since GP can be used for symbolic regression, emulating the tasks traditionally performed by ANNs.
4) *Risk management:* The study of risk and the reaction of an agent to it is a very interesting research area. Some researchers have studied, e.g., the formation process of risk preferences in financial problems.

Many other future research possibilities exist, including the study of consumers' patterns, credit scoring, economic growth, and auction games.

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