Applications of Multi-Objective Evolutionary Algorithms in Economics and Finance: A Survey

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Abstract— **This paper provides a state-of-the-art survey of applications of multi-objective evolutionary algorithms in economics and finance reported in the specialized literature. A taxonomy of applications within this area is proposed, and a brief review of the most representative research reported to date is then provided. In the final part of the paper, some potential paths for future research within this area are identified.**

I. Introduction

The use of evolutionary algorithms (EAs) in all sorts of application domains has become increasingly popular in the last few years, producing a wide variety of interesting applications ranging from engineering and computer science to ecology, sociology and medicine [1]. From these diverse application areas of evolutionary algorithms, economics and finance constitutes a very promising field, given the high complexity that many problems in these areas have.[1] The use of evolutionary algorithms in economics and finance is, by no means, an emerging research area, since it has been around since the late 1980s. Chen and Kuo [4] already reported the existence of about 400 publications in this field by 2001, which is a clear indication of the high interest raised by this area. On the other hand, the use of evolutionary algorithms for solving multi-objective optimization problems (an area frequently called "evolutionary multi-objective optimization" or EMOO) has also raised a lot of interest within the last few years.[2] This paper presents a survey on the use of multi-objective evolutionary algorithms (MOEAs) for solving problems in economics and finance.[3] The use of MOEAs in this research field is, currently, still relatively scarce, mainly when compared with respect to the use of single-objective evolutionary algorithms [5], [2]. Thus, one of the main purposes of this survey is precisely to attract the attention of EMOO researchers towards this field, which is very promising and contains a wide variety of interesting and challenging problems. Rather than competing with the excellent survey written by Schlottmann and Seese [3], we consider this paper as an extension (and upgrading) of such work, although in this case our goal was to have a broader coverage of applications, thus sacrificing a more in-depth analysis of them, such as the one provided by Schlottmann and Seese.

The remainder of this paper is organized as follows. Section II presents a brief introduction to multi-objective optimization. In Section III, we propose a taxonomy of applications. In Section IV, we review the use of MOEAs in investment portfolio optimization. In Section IX, we describe some of the future applications that remain to be done using MOEAs. Finally, Section X contains our conclusions.

II. Basic Concepts

We are interested in solving problems of the type[4]:

$$\text{minimize } \vec{f}(\vec{x}) := [f_1(\vec{x}), f_2(\vec{x}), \dots, f_k(\vec{x})] \qquad (1)$$

subject to:

$$g_i(\vec{x}) \leq 0 \quad i = 1, 2, \dots, m \qquad (2)$$

$$h_i(\vec{x}) = 0 \quad i = 1, 2, \dots, p \qquad (3)$$

where $\vec{x} = [x_1, x_2, \dots, x_n]^T$ is the vector of decision variables, $f_i : \mathbb{R}^n \to \mathbb{R}$, $i = 1, ..., k$ are the objective functions and $g_i, h_j : \mathbb{R}^n \to \mathbb{R}$, $i = 1, ..., m$, $j = 1, ..., p$ are the constraint functions of the problem.

To describe the concept of optimality in which we are interested, we will introduce next a few definitions.

Definition 1. Given two vectors $\vec{x}, \vec{y} \in \mathbb{R}^k$, we say that $\vec{x} \leq \vec{y}$ if $x_i \leq y_i$ for $i = 1, ..., k$, and that $\vec{x}$ **dominates** $\vec{y}$ (denoted by $\vec{x} \prec \vec{y}$) if $\vec{x} \leq \vec{y}$ and $\vec{x} \neq \vec{y}$.

Definition 2. We say that a vector of decision variables $\vec{x} \in \mathcal{X} \subset \mathbb{R}^n$ is **nondominated** with respect to $\mathcal{X}$, if there does not exist another $\vec{x}' \in \mathcal{X}$ such that $\vec{f}(\vec{x}') \prec \vec{f}(\vec{x})$.

Definition 3. We say that a vector of decision variables $\vec{x}^* \in \mathcal{F} \subset \mathbb{R}^n$ ($\mathcal{F}$ is the feasible region) is **Pareto-optimal** if it is nondominated with respect to $\mathcal{F}$.

Definition 4. The **Pareto Optimal Set** $\mathcal{P}^*$ is defined by:

$$\mathcal{P}^* = \{\vec{x} \in \mathcal{F} | \vec{x} \text{ is Pareto-optimal}\}$$

Definition 5. The **Pareto Front** $\mathcal{PF}^*$ is defined by:

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[1]As indicated by Schlottmann and Seese [2], [3], many problems in finance are NP-complete (e.g., constrained portfolio selection).

[2]The second author maintains the EMOO repository, which currently contains over 2,840 bibliographic references on this topic. The EMOO repository is located at: `http://delta.cs.cinvestav.mx/~ccoello/EMOO/`.

[3]Note that genetic algorithms adopting a linear aggregating function are also consider MOEAs in this paper.

[4]Without loss of generality, we will assume only minimization problems.

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$$\mathcal{PF}^* = \{\vec{f}(\vec{x}) \in \mathbb{R}^k | \vec{x} \in \mathcal{P}^*\}$$

In words, the Pareto front is the image of the Pareto optimal set and is normally displayed in graphical form, in order to have a better idea of the trade-offs available among the objectives. Thus, we wish to determine the Pareto optimal set from the set $\mathcal{F}$ of all the decision variable vectors that satisfy (2) and (3). Note however that in practice, and particularly when dealing with real-world problems, not all the Pareto optimal set is normally desirable (e.g., it may not be desirable to have different solutions that map to the same values in objective function space) or achievable.

III. A Taxonomy of Applications

Unlike the taxonomy of applications of EAs in economics and finance provided by Chen [5], we have decided to adopt a more pragmatic view, and classify applications of MOEAs in economics and finance based on the topics covered by the papers revised when elaborating this survey. This led us to a taxonomy containing five groups of applications:

1) Investment portfolio optimization
2) Financial time series
3) Stock ranking
4) Risk-Return analysis
5) Economic modelling

Each of these topics will be reviewed in the following sections. Note, however, that the discussion provided will be unbalanced, since most of the work done so far in this area has focused on investment portfolio optimization.

IV. Investment Portfolio Optimization

Investment portfolios are very common nowadays, and they can vary from simple portfolios held by individuals (containing a few stocks, bank investments, real estate holdings, etc.), to huge portfolios managed by professional investors (containing lots of stocks, bonds, treasury bills, etc.). Investment portfolios are projected to provide a certain return (i.e., money earned), but they also have an associated risk. Normally, a high risk corresponds to a high expected return and viceversa. Ideally, one would like to minimize the risk (within the tolerance allowed by the investor) while maximizing the return. This is the so-called optimal investment portfolio that one wishes to obtain by using optimization techniques. In the specialized literature, these problems are traditionally studied using the Markowitz portfolio selection model [6]. In this model, a portfolio is defined by a vector of real numbers which contains the weight corresponding to each asset available. Each asset has an expected rate of return paid at a certain time. Then, one wishes to maximize the return function (the weighted sum of the assets' expected rate of return), while minimizing the risk (the standard deviation of the portfolio rate of return, since this defines the level of uncertainty about the future payoff at a certain time). There are also different constraints, depending on the type of problem to be solved. For example, the weights normally have lower bounds (i.e., they must normally be greater than zero), upper bounds and possibly other constraints related to how diversified is the portfolio. The use of MOEAs for optimizing investment portfolios is, by far, the most popular topic within this area that has been reported in the specialized literature. Next, we will review the most representative work in this area.

A. Arnone et al.

Apparently, Arnone et al. [7] were the first to use MOEAs for optimizing investment portfolios. Although the authors adopt the Markowitz model and aim to maximize return while minimizing risk, they do not adopt the variance of the distribution of portfolio returns as their measure of risk. Instead, they use lower partial moments, which refer to the down-side part of the distribution of returns (appropriately, this measure is called *downside risk*). The use of downside risk makes the problem more difficult, because the shape of the objective surface is generally non-convex. Therefore, quadratic programming can no longer be used to find exact solutions. The authors adopt a genetic algorithm (GA) with a weighted linear aggregating function to solve this problem. The weights are called trade-off coefficients. So, the authors adopt different populations in order to encode different weight combinations and produce, in consequence, different portions of the Pareto front. In a further paper, Loraschi et al. [8] use a distributed GA in the same problems, and show that the distributed version offers a significantly better return for a given risk level than its sequential counterpart.

B. Shoaf and Foster

Shoaf and Foster [9] use a GA with a linear combination of weights for portfolio selection based on the Markowitz model. They minimize portfolio variance (i.e., risk) and maximize the expected return of the portfolio. The authors raise an issue of great importance when dealing with this problem: the encoding. The portfolio selection problem is really an allocation problem. Thus, a direct representation (i.e., using decision variables as usually done with GAs for representing the weights of each stock) does not work well. The reason is that this type of representation will frequently produce infeasible solutions in which the values allocated do not sum to 1.0, which is a constraint imposed on the problem. The authors adopt a representation which has a single field of $k+1$ bits for each asset. The first bit indicates whether the position on that holding will be long (one) or short (zero). The remaining k bits are an unsigned index onto an "allocation wheel", representing the resources to be allocated. The wheel is divided into 2^k equal sections, each indexed by a k-bit binary value. For any asset represented as a long position, the wheel proportion between its index and the index of the next long position, plus the proportion of any enclosed short position wedges, is the total proportion of resource allocation for that holding. The idea is that, for example, the resources from a short sale of a stock are used to purchase additional shares of the long position stock whose index most immediately precedes its index. The greatest benefit of this encoding is that the total investment represented by a chromosome is always 100% of the available resources (i.e., solutions are always feasible). The main drawback of this representation is its higher sensitivity to the mutation and crossover rates, since the encoding is epistatic (i.e., a change in the index of one holding generally affects one or two other holding allocations). The authors compare their approach with respect to quadratic programming (the most common approach used to solve this problem) using end-of-week closing data accumulated over an eleven month period beginning on October 3, 1994. The GA adopts two-point crossover, roulette-wheel selection and bit-flip mutation. Results indicated that the GA could find portfolio allocations with similar risk and higher rates of return than quadratic programming.

In a further paper, Shoaf and Foster [10] analyze the computational complexity of their approach. They indicate that the complexity of their approach is dominated by the sorting required by their special encoding. Thus the algorithmic complexity of their approach is $O(n \log n)$ (assuming quicksort is adopted). They also study the scalability of their approach. Their results indicate good scalability of the GA up to 100 stocks (i.e., the algorithm complexity remains as $O(n \log n)$, as expected). However, aiming to be able to explore faster and in a more effective manner the potentially large and highly multimodal search space of this problem, the authors also propose a parallel model based on islands.

C. Vedarajan et al.

Vedarajan et al. [11] also adopt the two objectives from Markowitz's model: maximize the expected return of the portfolio and minimize its risk. The authors adopt first a GA with a linear aggregating function that combines the two objectives into a single scalar value, and in which the weights are varied in order to generate different nondominated solutions. The authors also come across the same encoding problem reported by Shoaf and Foster, and propose two approaches to deal with it: (1) a penalty function, and (2) a Random Keys encoding [12], which also requires that the solutions are sorted. The GA adopts binary tournament selection, one-point crossover, and bit-flip mutation. The authors also adopt the Nondominated Sorting Genetic Algorithm (NSGA) [13], which is a non-elitist MOEA based on several layers of classifications of the individuals as suggested by Goldberg [14]. Before selection is performed, the population is ranked on the basis of nondomination: all nondominated individuals are classified into one category (with a dummy fitness value, which is proportional to the population size, to provide an equal reproductive potential for these individuals). To maintain the diversity of the population, these classified individuals are shared with their dummy fitness values. Then this group of classified individuals is ignored and another layer of nondominated individuals is considered. The process continues until all individuals in the population are classified. The NSGA adopts Random Keys encoding to handle the only constraint of the problem.

For testing their algorithms, the authors consider a portfolio consisting of five large capital stocks from different industries (i.e., Boeing, Disney, Exxon, Mc Donald's and Microsoft) and compute their returns, variances, covariances, etc. over a five years period. The authors show their results in graphical form, although no real numerical comparison takes place (i.e., using performance measures). The results from the NSGA and the GA with a linear aggregating function seem similar in quality, although the NSGA provides a much more diverse set of solutions. According to the authors, there are, however, other advantages of using the NSGA. When adopting quadratic programming (the traditional mathematical programming approach) for solving this problem, one has to work with a covariance matrix, and such matrix needs to be positive definite at all times. It turns out that when working with real-world problems, as the number of portfolio holdings increases, it becomes difficult to maintain this matrix positive definite, because of numerical imprecissions associated to the floating point arithmetic. This is not an issue with MOEAs, since they do not use this matrix. Later on, the authors indicate that, in practice, portfolio management involves other costs as well, such as transaction costs, broker fees, etc. So, it is desirable to minimize these costs as well, and the authors add then another objective: minimize transaction costs. The new problem has one additional constraint related to the maximum transaction cost permitted. However, handling this constraint with quadratic programming is difficult, because of the way in which transactions normally take place in practice. Thus, in this case, the use of a MOEA brings even more advantages, since the NSGA was able to produce the three-dimensional Pareto front in a single run.

D. Chang et al.

Chang et al. [15] use a GA with a linear aggregating function that considers the same two objectives as before: minimize the total variance (risk) associated with the portfolio and ensure that the portfolio has a certain expected return. An interesting aspect of this work is that the authors adopt cardinality constraints (i.e., constraints on the exact number of stocks to hold in a portfolio). It is important to note that, when no cardinality constraints are imposed on the problem, quadratic programming can be used to solve it in an exact manner. However, when cardinality constraints are imposed, no exact method exists for solving it. In the multi-objective statement of this problem, the Pareto front may be discontinuous and/or non-convex, which will create difficulties to the use of approaches based on linear aggregating functions. The authors use a steady state GA with binary tournament selection, uniform crossover and a boundary mutation operator. In this case, the issue of the encoding is dealt with a simple repair procedure that transforms infeasible solutions into feasible ones. Besides using a GA, the authors also experiment with tabu search [16] and simulated annealing [17] (all of the them using the same linear aggregating function). For their comparative study, the authors constructed five test data sets considering the stocks involved in five different capital market indices from around the world. First, an unconstrained version of the problem is solved and results are compared with respect to those generated by an exact method. The GA is the best overall performer in this case, followed by simulated annealing. Tabu search produced a very large mean percentage error with respect to the two other approaches. In a second experiment, the authors consider the cardinality constrained version of the problem. This time, there is no clear winner, since some approaches produced smaller mean percentage errors in some problems, but greater values in others. However, regarding the contribution of each algorithm to the Pareto optimal set (constructed from the union of results produced), the GA is the approach that contributed the most, followed by tabu search and by simulated annealing.

E. Lin et al.

Lin et al. [18], [19] consider a variation of the investment portfolio optimization problem in which fixed transaction costs and minumum lots are adopted. The authors adopt the Nondominated Sorting Genetic Algorithm-II (NSGA-II) [20] with integers encoding, simulated binary crossover (SBX) [21] and parameter-based mutation [20]. In the NSGA-II, for each solution one has to determine how many solutions dominate it and the set of solutions to which it dominates. The NSGA-II estimates the density of solutions surrounding a particular solution in the population by computing the average distance of two points on either side of this point along each of the objectives of the problem. This value is the so-called *crowding distance*. During selection, the NSGA-II uses a crowded-comparison operator which takes into consideration both the nondomination rank of an individual in the population and its crowding distance (i.e., nondominated solutions are preferred over dominated solutions, but between two solutions with the same nondomination rank, the one that resides in the less crowded region is preferred). An interesting aspect of this work is that the authors adopt GENOCOP to handle the constraints of the problem [22]. However, since GENOCOP requires that the initial population is feasible in order to handle linear constraints, then the authors adopt the same NSGA-II to find feasible solutions. The problem solved in this case is really single-objective, but it is considered as a special case of the multi-objective problem. When all the individuals in the population are feasible, the NSGA-II is stopped and they are fed into GENOCOP, which handles the original constraints of the problem.

The authors validate their approach using data from the OR-Library [23]. The results indicated that, by investing in more stocks, the maximum risk was significantly decreased. The authors also experiment with fitness scaling, which they find to be useful to make their MOEA more efficient. However, the results are not compared with respect to any other approach.

F. Fieldsend et al.

Fieldsend et al. [24] deal with the cardinality constraint portfolio selection problem studied by Chang et al. [25]. The authors indicate that if only the cardinality constraint is imposed on the problem and the others are ignored, then cardinality can be considered as a third objective (additional to the traditional risk and return objectives). Then, the 2-dimensional cardinality constrained frontier could be extracted for any particular cardinality k. However, if additional constraints need to be considered, this approach is not viable. The authors propose to search for each cardinality constrained front in parallel, and constructively use information from these fronts to improve the search process of the others. The MOEA adopted is a (1+1)-evolution strategy previously used by the authors [26]. The algorithm maintains a set of sets of the different cardinality constrained efficient frontiers, each of which is initialized with a random portfolio. The algorithm proceeds at each iteration by first selecting (in a random manner) an archive with cardinality k and copying a portfolio from it. Such a copied portfolio is then adjusted (either only weight adjustment or also dimensionality change). The resulting portfolio is evaluated in terms of its return and risk and compared to the others previously stored to see if its nondominated. Evidently, any dominated portfolios are removed. The approach is validated using stock data from the US S&P 100 index and emerging markets stock. Results are compared with respect to the unconstrained problem, which is solved using quadratic programming. Preliminary results show that it is possible to replicate closely the mean and variance of an efficient portfolio using a relatively number of stocks.

G. Streichert et al.

Streichert et al. [27] consider the investment portfolio optimization problem with cardinality constraints (which restrict the maximum number of assets used in the portfolio), buy-in thresholds constraints (which give the minimum amount that is to be purchased), and roundlots constraints (which give the smallest volumes that can be purchased for each asset). The authors use the NSGA [28] with tournament selection, fitness sharing, one-point mutation and discrete 3-point crossover. The authors experiment with both a binary encoding (with and without Gray codes) and a real-numbers encoding. Since the authors had determined from preliminary experiments that Pareto optimal solutions are normally composed of a limited selection of the available assets, they note the similarities of the problem with the one-dimensional binary knapsack problem. Since the knapsack problem has been solved using EAs, the authors adopt this encoding in addition to the vector of decision variables (the weights). So, each bit from the knapsack determines if an asset will be used or not. The genetic operators are applied separately to each of the two segments of the chromosome. The authors also adopt a repair mechanism that first removes all surplus assets from the portfolio to meet the cardinality constraints. Then, other similar mechanisms are adopted to satisfy the other constraints. In order to examine the effect of this repair mechanism, the authors adopt Lamarckism (when using Lamarckism, the repaired solution is kept; otherwise, only its objective function values are used). Test data from the OR-Library [23] is used by the authors. They also use the S-metric to compute the hypervolume of the Pareto front

[29] as a performance assessment measure. The results are quite interesting. When no Lamarckism is adopted, and no additional constraints are imposed on the problem, the use of the knapsack encoding clearly outperforms the standard representation. From the different encodings adopted, the traditional binary encoding is the best and the real-numbers encoding is the worst. However, when cardinality constraints are imposed on the problem, one cannot make clear distinctions in the results anymore. When using Lamarckism, the standard GA outperforms the GA with the knapsack encoding regarding convergence rate and reliability, for the case in which cardinality constraints are considered. Without cardinality constraints, the effect is less noticeable because all the approaches perform very well, but the standard GA is still much better than before. When additional constraints are considered, for the case in which no Lamarckism is considered, the standard GA presents premature convergence. If Lamarckism is adopted, then the negative effect of the neutral search space is apparently removed, which significantly increases the efficiency of the standard GA. Real-numbers encoding exhibits a slightly better performance than the binary encoding in this case. Using the knapsack encoding, the GA does not present premature convergence, but its performance is poor. The use of Lamarckism causes, again, a very significant performance improvement. However, binary encoding is better than real-numbers encoding in this case.

H. Ehrgott et al.

Ehrgott et al. [30] propose an extension to the Markowitz mean-variance model. The authors maximize five objectives (derived from a cooperation with Standard and Poor's): (1) 12-month performance of an asset, (2) 3-year performance of an asset, (3) annual dividend of a portfolio, (4) Standard and Poor's star ranking, and (5) volatility. The authors also allow the incorporation of the user's preferences through the construction of decision-maker specific utility functions and an additive global utility function. Using this global utility function as the objective function to be optimized, the authors perform a study in which they compare four approaches: (1) a two-phase local search algorithm, (2) simulated annealing, (3) tabu search, and (4) a genetic algorithm. The two-phase local algorithm, simulated annealing and tabu search, share the same neighborhood structure. Results on a fund database indicated that the genetic algorithm was the best performer, followed by simulated annealing. In randomly generated instances, however, the two-phase local search algorithm had a better performance, followed by the genetic algorithm.

I. Armañanzas and Lozano

Armañanzas and Lozano [31] apply a greedy local search algorithm [32], simulated annealing [17] and ant colony optimization (ACO) [33] to portfolio optimization problems. For the neighborhood exploration adopted by the greedy algorithm and simulated annealing, the authors adopt a scheme by which the algorithm first looks for the two assets that add the highest risk to the base solution. Then, it selects which of them contributes less to the total portfolio profit. Such an asset will be the *pivot* asset from which the neighborhood will be created. The ACO implementation is also interesting, because the authors adopt a lexicographic approach by which the objectives are optimized separately (this is similar to the MOAQ [34]): first, they optimize the risk, then the profit, and finally a trade-off between these two previous objectives. The approaches are compared using test data from the OR-Library [23]. The results indicated that ACO and simulated annealing provided the best performance, but there was no clear winner between them. The greedy algorithm showed the worst performance in the experiments reported by the authors.

J. Subbu et al.

Subbu et al. [35] propose a hybrid multiobjective optimization approach that combines evolutionary algorithms with linear programming for investment portfolio optimization. The authors maximize the portfolio expected return, while minimizing both the surplus variance and the portfolio value at risk. They also consider duration and convexity mismatch constraints, as well as linear portfolio investment constraints. The authors adopt the Pareto Sorting Evolutionary Algorithm (PSEA), which uses a small population size and an archive that retains the nondominated solutions found along the search. PSEA is initialized with a Randomized Linear Programming (RLP) algorithm, which stochastically identifies a sample of the boundaries of the search space by solving thousands of randomized linear programs. They also use a fast dominance filter, which decomposes a set of solutions by working on smaller chunks of such a set of solutions. This process is used to differentiate between dominated and nondominated solutions. The actual search for optimal portfolios is performed by another approach called Target Objective Genetic Algorithm (TOGA), which is based on both goal programming and VEGA [36]. TOGA attempts to find solutions that are as close as possible to a pre-defined target for one or more objectives. These approaches are all part of a more complex system developed at General Electric and currently used in real-world problems with hundreds to thousands of assets. The system also allows the incorporation of progressive preferences and provides 2-D projections of the Pareto fronts obtained.

V. FINANCIAL TIME SERIES

Here, the idea is to find patterns in financial time series, such that predictions can be made regarding the behavior of a certain stock. Typically, neural networks have been applied on this problem, but the use of different types of evolutionary algorithms has also been reported. Next, we will describe the use of MOEAs in this application domain.

A. Ruspini and Zwir

Ruspini and Zwir [37] & Zwir and Ruspini [38] used the Niched-Pareto Genetic Algorithm (NPGA) [39] for automatic derivation of qualitative descriptions of complex objects. The NPGA uses a tournament selection scheme based on

Pareto dominance. The authors apply their methodology to the identification of significant technical-analysis patterns in financial time series. Two objectives are considered: quality of fit (measures the extent to which the time-series values correspond to a financial uptrend, downtrend, or head-and-shoulders interval) and extent (measures, through a linear function, the length of the interval being explained). The NPGA is really used to determine crisp intervals[5] corresponding to downtrends, uptrends and head-and-shoulders intervals. Niching and tournament selection are used in this application.

Not many people have worked in this area using a multi-objective approach, but there are other references in which MOEAs are used for time series prediction, although not in a finance-related domain (see for example [40]). Additionally, there is also some work on predicting customers patterns [41], which is also related to this topic.

VI. Stock Ranking

The aim of this problem is to classify stocks as strong or weak performers based on technical indicators and then use this information to select stocks for investment and for making recommendations to customers. Next, we report the use of MOEAs in this application area.

A. *Mullei and Beling*

Mullei and Beling [42] use a GA with a linear combination of weights to select rules for a classifier system adopted to rank stocks based on profitability. Up to nine objectives are considered by the authors, related to conjunctive attribute rule tests. This problem is solved using a classifier system from the so-called Pitt approach [14]. The authors use binary representation, roulette wheel selection, one-point crossover and uniform mutation. The approach is validated using 5 large historical (U.S.) stock data sets covering approximately 3 years (1995-1998) of weekly data on a universe of 16 stocks. Results are compared against a technique related to the synthesis of polynomial networks called STATNET. Results were inconclusive since no technique was able to outperform the other in all cases.

VII. Risk-Return Analysis

Credit portfolios handled by banks are also investment portfolios, but they operate under different rules and, therefore, they are not modeled using the original Markowitz approach. Next, we will describe applications of MOEAs to this area.

A. *Schlottmann and Seese*

Schlottmann and Seese [43], [44] use an approach similar to the NSGA-II [20] for solving portfolio selection problems relevant to real-world banking. In the problem studied by the authors, a bank has a fixed supervisory capital budget. This is an upper limit for investments into a portfolio consisting of a subset of assets (e.g., loans to be given to different customers of the bank), each of which is subject to the risk of the default (capital risk). So, in this case, besides having an expected rate of return (as in the original Markowitz problem), each asset also has an expected default probability (which is set *a priori*) and a net exposure, within a fixed risk horizon. The authors adopt binary decision variables to indicate whether or not a certain net expossure is to be held in the portfolio or not. Only if an asset is held in the portfolio, the bank has to allocate a supervisory capital amount from its available (but scarce) resources. Thus, the return objective function has to be adjusted for default risk (i.e., expected loss). The resulting problem has a discrete constrained search space with many local optima and two conflicting objective functions. Unlike the original NSGA-II, the authors adopt an external archive containing the nondominated solutions found along the search. They also incorporate a gradient-based local search operator which is, however, rather heuristic. For validation purposes, the authors designed their own test cases with a structure similar to real-world data from German banks. They compared their hybrid MOEA with respect to the same MOEA without the local search mechanism. Results indicated that the use of local search significantly improved performance (the average improvement was computed to be between 17% and 95% for the set coverage metric [29]).

[5]Fuzzy logic is used to describe the model.

B. *Mukerjee et al.*

Mukerjee et al. [45] use the NSGA-II [20] to determine risk-return trade-offs for a bank loan portfolio manager. The idea is the same as before: the bank manager aims to maximize shareholder wealth. This implies maximizing the net worth of the bank, which in turn involves maximizing the net interest margin of the bank. However, there are a number of regulatory constraints imposed on the bank, such as the maintenance of adequate capital, interest-rate risk exposure, etc. The authors adopt a portfolio credit risk model based on the standard deviation of the return over the entire portfolio. Two objectives are considered: (1) maximize mean return on the portfolio, and (2) minimize the variance on the return. For validating their approach, the authors adopt data from the CreditMetrics Technical Document. The authors studied an elastic loan demand model in which they assume that the amount of loan applications received in a given loan category is a function of the interest rate charged. The authors used the NSGA-II for this model, adopting the interest rates as their decision variables.

An interesting aspect of this work is that the authors compare the performance of the NSGA-II with respect to the ε-constraint method (using a simple genetic algorithm for the individual single-objective optimizations performed by this method). Only graphical comparisons are presented, since the aim was to show that the NSGA-II could achieve the same convergence of the ε-constraint method, while providing a much wider distribution of nondominated solutions.

VIII. Economic Modelling

Mardle et al. [46], [47] use a GA with a weighted goal programming approach to optimize a fishery bioeconomic model. Bioeconomic models have been developed for a number of fisheries as a means of estimating the optimal level of exploitation of the resource and for assessing the effectiveness of the different management plans available, at achieving the desirable objectives. The foundations of fisheries bioeconomic modelling comes from the economic theory of the open-access or common-property fishery, which is based on a logistic population growth model. In this case, the authors develop a model for the North Sea fishery. Four objectives are considered: (1) maximize profit, (2) maintain historic relative quota shares among countries, (3) maintain employment in the industry and (4) minimize discards. GENOCOP III [48] is used for the evolutionary optimization process. Real-numbers representation and arithmetic crossover are employed. The evolutionary approach is compared to the application of traditional goal programming (developed in GAMS [49] and solved with CONOPT) in a model of the North Sea demersal fishery. The GA is considered competitive but not necessarily better than goal programming in this application.

We are not aware of any other applications of MOEAs in economics, although several are certainly possible (e.g., in negotiation strategies [50]).

IX. Future Applications

As we could see in this survey, the use of MOEAs in economics and finance is still relatively scarce and has mostly focused on the optimization of investment portfolios. Thus, a lot of areas remain to be explored. Some of them are the following:

- **Model discovery:** This is an interesting area in econometrics in which non-parametric models are assumed, and one tries to use an evolutionary algorithm to derive a model for a certain type of problem (e.g., forecasting nonlinear time series). Normally, artificial neural networks (ANNs) have been used for the model itself, but several researchers have used evolutionary algorithms to find the most appropriate ANN that models the problem of interest.
- **Data mining:** The use of data mining techniques for learning complex patterns is a very promising research area in economics and finance. For example, the mining of financial time-series for finding patterns that can provide trading decision models is a very promising topic worth exploring [5].
- **Forecasting stock prices:** Although long-term forecasting is not possible for the stock market, it is normally possible to perform short-term forecasting with heuristics. The use of genetic programming (GP) in this area has become increasingly popular, since GP can be used for symbolic regression, emulating the tasks traditionally performed by ANNs.
- **Risk management:** The study of risk and the reaction of an agent to it, is a very interesting research area. Some researchers have studied, for example, the formation process of risk preferences in financial problems [5].
- **Coevolution:** The use of co-evolutionary approaches for certain problems in economics and finance (e.g., for studying artificial foreign exchange markets) is a very interesting topic that certainly deserves attention. Coevolutionary MOEAs are still not too common, but their potential use in financial areas may boost the interest of researchers in paying more attention to them.

Many other possible areas exist, including the study of consumers' patterns, credit scoring, economic growth, and auction games, just to mention a few.

X. Conclusions

This paper has presented a state-of-the-art survey on the use of MOEAs for solving problems in economics and finance. We have identified a taxonomy of applications that consists of five large groups: investment portfolio optimization, financial time series, stock ranking, risk-return analysis and economic modelling. From these five groups, the first (investment portfolio optimization) is, by far, the most popular in the current literature. However, as indicated in Section IX many other application areas exist for MOEAs to be applied. We expect that this paper can motivate researchers interested in economics and finance to learn more about MOEAs, and to apply them in more problems within these areas.

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