MATH 304 Numerical Analysis and Optimization Project

# Traffic flow prediction using LSR and SVR

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(xg54)

# Task

# Predict Traffic Flow

| | A | B | C | D | E | F | G | H | I | J | K | L | M |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | TMU ID | Legacy TM | Site Name | | | | | | | | | | |
| 2 | 233C060F! | 30016383 | TMU Site 9956/1 on link A1081 westbound between A505/B653 and M1 J10; GPS Ref: 509008;218723; Westbound | | | | | | | | | | |
| 3 | | | | | | | | | | | | | |
| 4 | Local Date | Local Tim | Day Type | Total Carr | Total Flow | Total Flow | Total Flow | Total Flow | Speed Val | Quality Inc | Network L | NTIS Model Version | |
| 5 | 2016/6/1 | 0:14:00 | 7 | 151 | 145 | 2 | 0 | 4 | 80.71 | 15 | 2E+08 | 4 | |
| 6 | 2016/6/1 | 0:29:00 | 7 | 131 | 116 | 6 | 4 | 5 | 81.01 | 15 | 2E+08 | 4 | |
| 7 | 2016/6/1 | 0:44:00 | 7 | 147 | 140 | 1 | 1 | 5 | 81.6 | 15 | 2E+08 | 4 | |
| 8 | 2016/6/1 | 0:59:00 | 7 | 101 | 94 | 3 | 2 | 2 | 81.07 | 15 | 2E+08 | 4 | |
| 9 | 2016/6/2 | 1:14:00 | 9 | 194 | 184 | 3 | 5 | 2 | 79.08 | 15 | 2E+08 | 4 | |
| 10 | 2016/6/2 | 1:29:00 | 9 | 129 | 114 | 7 | 4 | 4 | 80.51 | 15 | 2E+08 | 4 | |
| 11 | 2016/6/2 | 1:44:00 | 9 | 83 | 74 | 2 | 3 | 4 | 80.4 | 15 | 2E+08 | 4 | |
| 12 | 2016/6/2 | 1:59:00 | 9 | 74 | 60 | 4 | 5 | 5 | 83.23 | 15 | 2E+08 | 4 | |
| 13 | 2016/6/2 | 2:14:00 | 9 | 58 | 39 | 7 | 5 | 7 | 82.68 | 15 | 2E+08 | 4 | |
| 14 | 2016/6/2 | 2:29:00 | 9 | 36 | 28 | 3 | 2 | 3 | 79.13 | 15 | 2E+08 | 4 | |
| 15 | 2016/6/2 | 2:44:00 | 9 | 68 | 61 | 4 | 1 | 2 | 83.22 | 15 | 2E+08 | 4 | |
| 16 | 2016/6/2 | 2:59:00 | 9 | 72 | 59 | 6 | 4 | 3 | 83.48 | 15 | 2E+08 | 4 | |
| 17 | 2016/6/2 | 3:14:00 | 9 | 70 | 51 | 6 | 3 | 10 | 82.95 | 15 | 2E+08 | 4 | |
| 18 | 2016/6/2 | 3:29:00 | 9 | 68 | 62 | 3 | 1 | 2 | 80.85 | 15 | 2E+08 | 4 | |
| 19 | 2016/6/2 | 3:44:00 | 9 | 61 | 52 | 4 | 2 | 3 | 79.16 | 15 | 2E+08 | 4 | |
| 20 | 2016/6/2 | 3:59:00 | 9 | 75 | 64 | 4 | 4 | 3 | 80.47 | 15 | 2E+08 | 4 | |
| 21 | 2016/6/2 | 4:14:00 | 9 | 90 | 73 | 9 | 3 | 5 | 82.18 | 15 | 2E+08 | 4 | |
| 22 | 2016/6/2 | 4:29:00 | 9 | 95 | 79 | 7 | 2 | 7 | 81.16 | 15 | 2E+08 | 4 | |
| 23 | 2016/6/2 | 4:44:00 | 9 | 128 | 109 | 13 | 2 | 4 | 79.87 | 15 | 2E+08 | 4 | |
| 24 | 2016/6/2 | 4:59:00 | 9 | 161 | 137 | 11 | 7 | 6 | 80.75 | 15 | 2E+08 | 4 | |
| 25 | 2016/6/2 | 5:14:00 | 9 | 227 | 200 | 13 | 7 | 7 | 78.43 | 15 | 2E+08 | 4 | |
| 26 | 2016/6/2 | 5:29:00 | 9 | 290 | 261 | 16 | 7 | 6 | 78.35 | 15 | 2E+08 | 4 | |

**X:** Local Time
**Y:** Total Carriage Flow

**Function** *hours()*

Convert X to double

*1.23*

…

…

*23.68*

# Evaluation Metrics

$$mse = \sum_{i=1}^{N} \frac{1}{N} (\hat{y}_i - y_i)^2$$

$$r^2 = \frac{[N \sum_{i=1}^{N} (\hat{y}_i y_i) - (\sum_{i=1}^{N} \hat{y}_i)(\sum_{i=1}^{N} y_i)]^2}{[N \sum_{i=1}^{N} \hat{y}_i{}^2 - (\sum_{i=1}^{N} \hat{y}_i)^2][N \sum_{i=1}^{N} y_i{}^2 - (\sum_{i=1}^{N} y_i)^2]}$$

$$r^2 = \frac{S_t - S_r}{S_t}$$

$\boldsymbol{S_t}$: Total sum of the squares around the mean for the dependent variable.

$\boldsymbol{S_r}$: Sum of the squares of residuals around the regression line.

$\boldsymbol{S_t - S_r}$ : Quantifies the improvement due to describing data in terms of a straight line rather than as an average value.

**Perfect fit**
$S_r = 0$ and $r^2 = 1$
The line explains 100 percent of the variability of the data.

# Algorithms

Least-square Regression (LSR) and Support Vector Regression (SVR) are applied in this project to predict the traffic flow at different times of a day.

## LSR (Polynomial)

$$f(t) = a_n t^n + a_{n-1} t^{n-1} + \ldots + a_2 t^2 + a_1 t^1 + a_0$$
$$where\ n = 1, 2, \ldots, 9\ldots$$

## SVR (Polynomial)

$$k(x, z) = (x^T z)^d$$

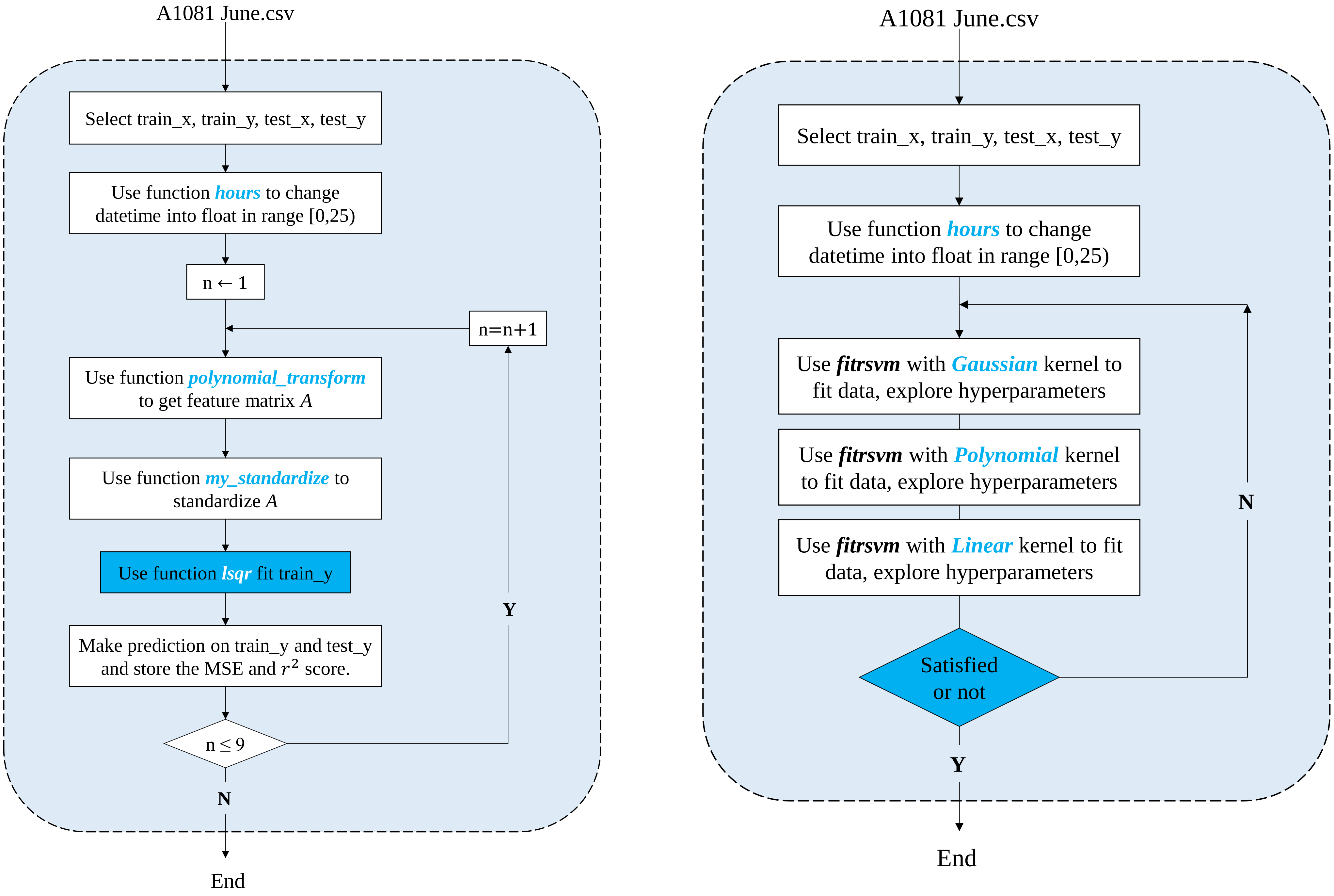
## SVR (Linear)

$$k(x, z) = x^T z$$

## SVR (Gaussian)

$$k(x, z) = exp(-\frac{\|x - z\|_2^2}{2\sigma^2})$$

# Flow chart

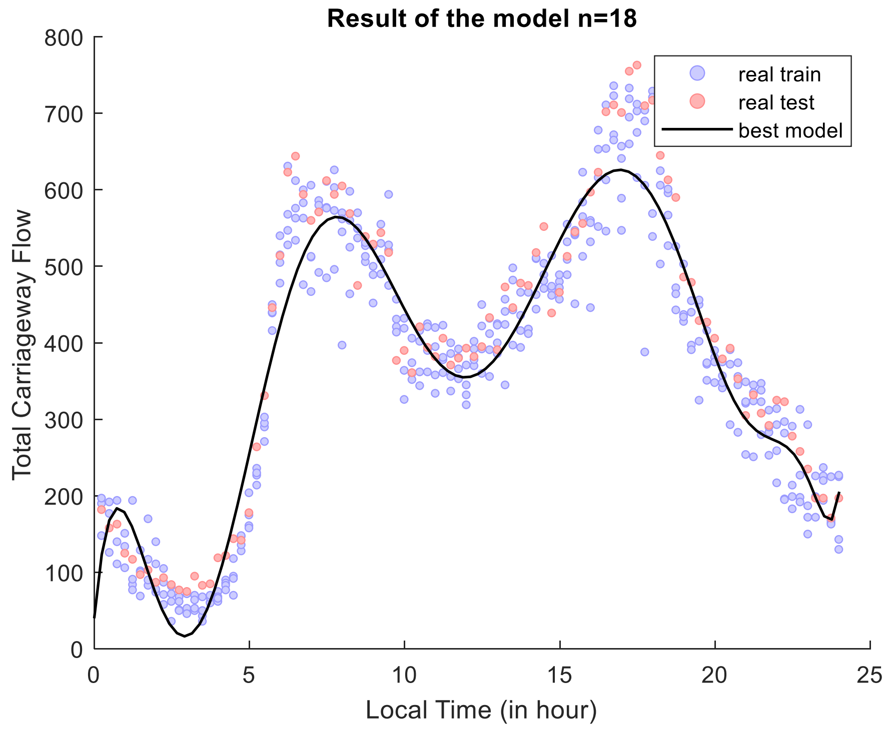


A1081 June.csv
Select train_x, train_y, test_x, test_y
Use function hours to change datetime into float in range [0,25)
n ← 1
n=n+1
Use function polynomial_transform to get feature matrix A
Use function my_standardize to standardize A
Use function lsqr fit train_y
Make prediction on train_y and test_y and store the MSE and $r^2$ score.
n ≤ 9
Y
N
End
A1081 June.csv
Select train_x, train_y, test_x, test_y
Use function hours to change datetime into float in range [0,25)
Use fitrsvm with Gaussian kernel to fit data, explore hyperparameters
Use fitrsvm with Polynomial kernel to fit data, explore hyperparameters
Use fitrsvm with Linear kernel to fit data, explore hyperparameters
Satisfied or not
N
Y
End

# Experimental Results

# Polynomial Regression

| Model | Training error | | Test error | |
|---|---|---|---|---|
| | MSE | $r^2$ | MSE | $r^2$ |
| n = 1 | 29834 | 0.10841 | 32158 | 0.11421 |
| n = 2 | 13194 | 0.60569 | 15084 | 0.58450 |
| n = 3 | 12732 | 0.61952 | 14313 | 0.60576 |
| n = 4 | 12687 | 0.62084 | 14152 | 0.61019 |
| n = 5 | 11969 | 0.64230 | 13453 | 0.62945 |
| n = 6 | 6315.0 | 0.81128 | 6852.6 | 0.81125 |
| n = 7 | 6269.7 | 0.81263 | 6816.2 | 0.81235 |
| n = 8 | 3080.2 | 0.90794 | 3346.1 | 0.90783 |
| n = 9 | 2953.2 | 0.91174 | 3329.7 | 0.90829 |
| n = 10 | 2891.8 | 0.91358 | 3295.7 | 0.90922 |
| n = 11 | 2869.1 | 0.91426 | 3284.6 | 0.90953 |
| n = 12 | 2870.4 | 0.91422 | 3284.6 | 0.90953 |
| n = 13 | 2870.4 | 0.91422 | 3289.6 | 0.90939 |
| n = 14 | 2878.8 | 0.91397 | 3309.6 | 0.90884 |
| n = 15 | 2844.0 | 0.91500 | 3188.3 | 0.91218 |
| n = 16 | 2826.1 | 0.91554 | 3135.0 | 0.91365 |
| n = 17 | 2814.0 | 0.91591 | 3094.0 | 0.91478 |
| n = 18 | 2813.1 | 0.91593 | 3075.3 | 0.91529 |
| n = 19 | 2825.6 | 0.91556 | 3082.2 | 0.91510 |
| n = 20 | 2851.3 | 0.91479 | 3113.6 | 0.91424 |

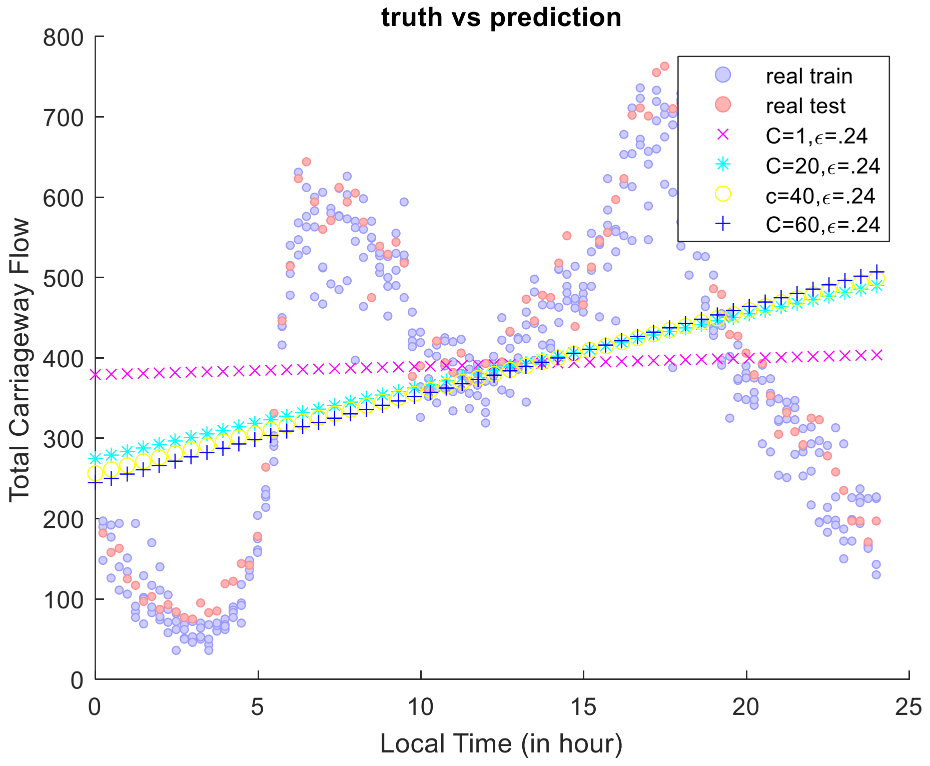



The actual flow and the flow predicted by the polynomial model n=18.

# SVR (Linear)

| Model | SVR (Linear kernel) | | | |
|---|---|---|---|---|
| Setting | Default | Case1 | Case2 | Optimized |
| Train MSE | 30325.68 | 30021.82 | 30019.77 | 30019.77 |
| Train $r^2$ | 0.094 | 0.1028 | 0.1029 | 0.1029 |
| Test MSE | 32012.13 | 31699.00 | 31696.34 | 31696.34 |
| Test $r^2$ | 0.118 | 0.1268 | 0.1269 | 0.1269 |

| | |
|---|---|
| Opt. hp | BoxConstraint=58, KernelScale=13.2, Epsilon=0.236 |
| Case 1 | BoxConstraint=20 , KernelScale=13.2, Epsilon=0.236 |
| Case 2 | BoxConstraint=19, KernelScale=12, Epsilon=0.01 |
| Optimized | BoxConstraint=19, KernelScale=12, Epsilon=0.01 |

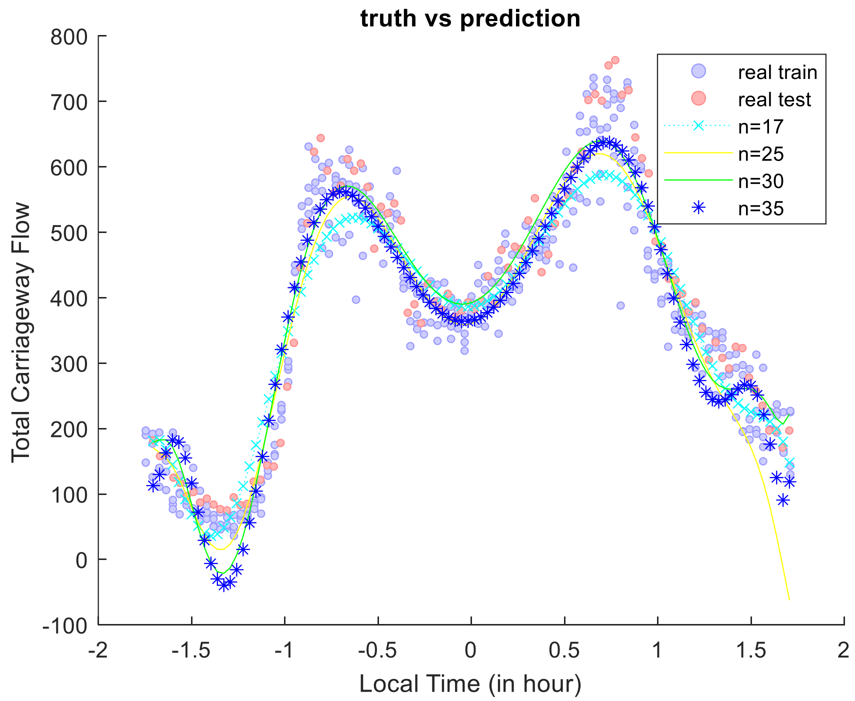



Explore effects of BoxConstraint on SVR(Linear).

# SVR (Polynomial)

| Model | SVR (Polynomial kernel) | | | |
|---|---|---|---|---|
| Setting | Default | Case1 | Case2 | Optimized |
| Train MSE | 13963.17 | 3399.41 | 2701.63 | 2701.63 |
| Train $r^2$ | 0.582713 | 0.8984 | 0.9193 | 0.9193 |
| Test MSE | 16819.28 | 4029.85 | 2974.85 | 2974.85 |
| Test $r^2$ | 0.5367 | 0.8890 | 0.9181 | 0.9181 |

| Opt. hp | BoxConstraint=521.78, KernelScale=2.47, Epsilon=0.22 |
|---|---|
| Case 1 | PolynomialOrder=17, Opt. hp |
| Case 2 | PolynomialOrder=25, Opt. hp |
| Optimized | PolynomialOrder=25, Opt. hp |

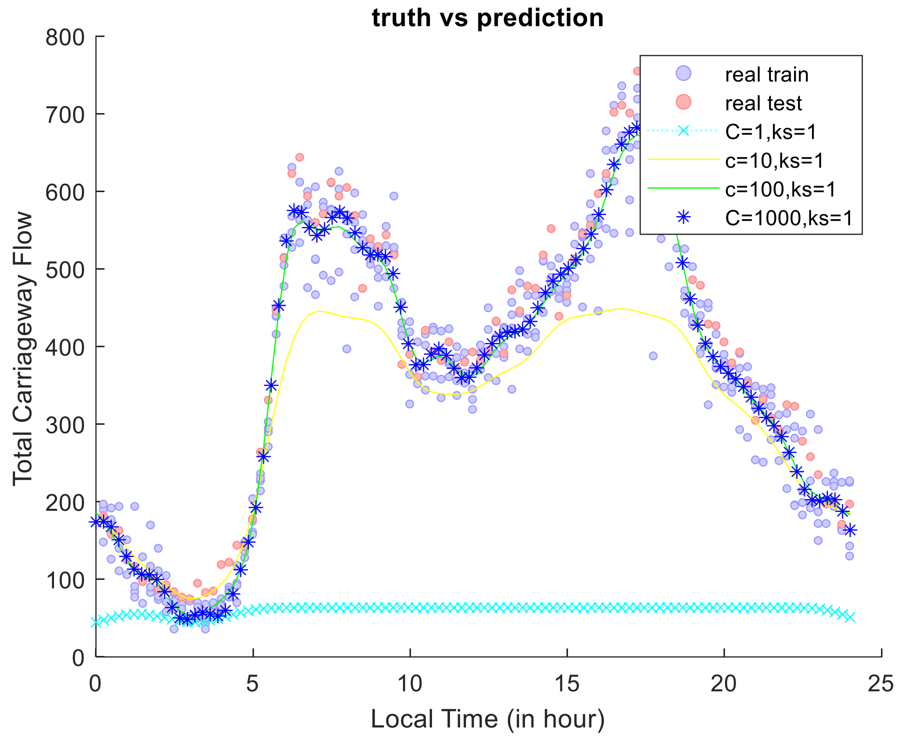



Explore effects of PolynomialOrder on SVR (Polynomial).

# SVR (Gaussian)

| Model | SVR (Gaussian kernel) | | | |
|---|---|---|---|---|
| Setting | Default | Case1 | Case2 | Optimized |
| Train MSE | 1587.85 | 1594.13 | 1752.91 | 1752.91 |
| Train $r^2$ | 0.9525 | 0.9506 | 0.9476 | 0.9476 |
| Test MSE | 1503.10 | 1591.13 | 1470.62 | 1470.62 |
| Test $r^2$ | 0.9586 | 0.9562 | 0.9595 | 0.9595 |

| | |
|---|---|
| Opt. hp | BoxConstraint=137, KernelScale=1, Epsilon=0.45 |
| Case 1 | BoxConstraint=150, KernelScale=1, Epsilon=0.45 |
| Case 2 | BoxConstraint=185, KernelScale=1.3, Epsilon=0.45 |
| Optimized | BoxConstraint=185, KernelScale=1.3, Epsilon=0.45 |




Explore effects of BoxConstraints on SVR (Gaussian).

# SVR (Gaussian)

| Model | SVR (Gaussian kernel) | | | |
|---|---|---|---|---|
| Setting | Default | Case1 | Case2 | Optimized |
| Train MSE | 1587.85 | 1594.13 | 1752.91 | 1752.91 |
| Train $r^2$ | 0.9525 | 0.9506 | 0.9476 | 0.9476 |
| Test MSE | 1503.10 | 1591.13 | 1470.62 | 1470.62 |
| Test $r^2$ | 0.9586 | 0.9562 | 0.9595 | 0.9595 |

| | |
|---|---|
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| Case 1 | BoxConstraint=150, KernelScale=1, Epsilon=0.45 |
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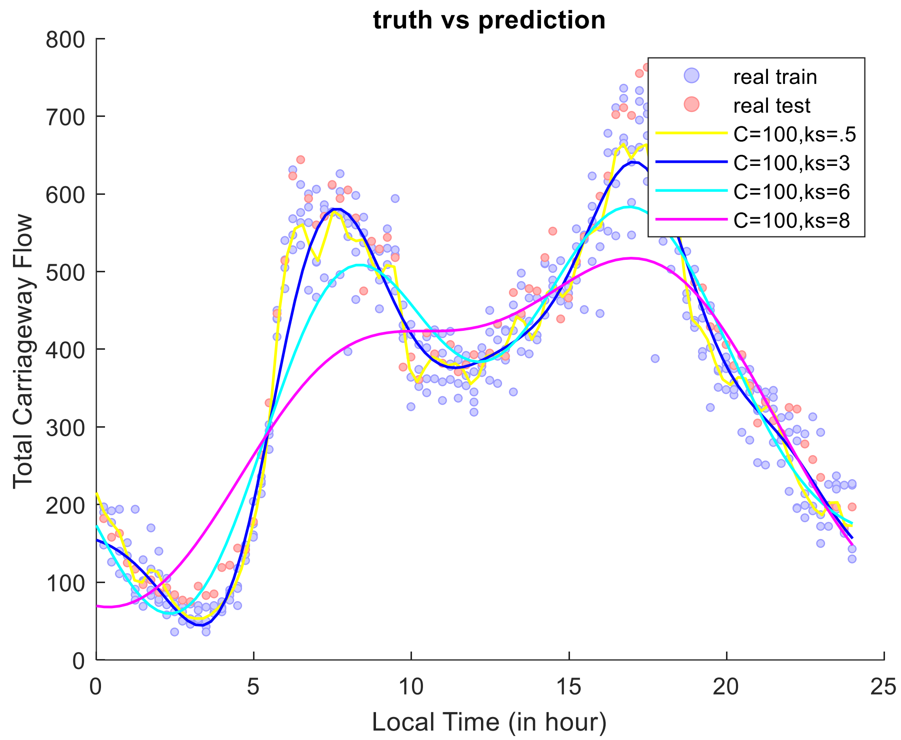
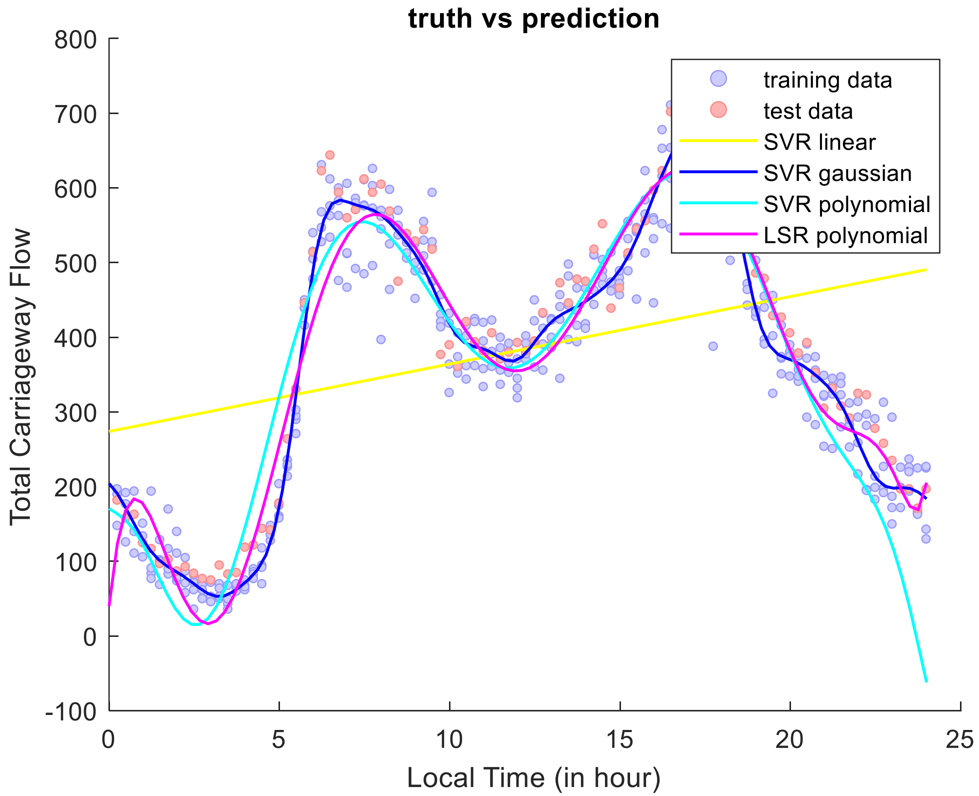



Explore effects of KernelScale on SVR (Gaussian).

# Compare performance




| Model | LSR polynomial | SVR linear | SVR polynomial | SVR gaussian |
|---|---|---|---|---|
| Train MSE | 2813.1 | 30019.77 | 2701.63 | 1752.91 |
| Train $r^2$ | 0.91593 | 0.1029 | 0.9193 | 0.9476 |
| Test MSE | 3075.3 | 31696.34 | 2974.85 | 1470.62 |
| Test $r^2$ | 0.91529 | 0.1269 | 0.9181 | 0.9595 |

Predictions made by the optimized models.

# Discussion

# Factors that impact performance

## PolynomialOrder

| Model | SVR (Polynomial kernel) | | | |
|---|---|---|---|---|
| Setting | Default | Case1 | Case2 | Optimized |
| Train MSE | 13963.17 | 3399.41 | 2701.63 | 2701.63 |
| Train $r^2$ | 0.582713 | 0.8984 | 0.9193 | 0.9193 |
| Test MSE | 16819.28 | 4029.85 | 2974.85 | 2974.85 |
| Test $r^2$ | 0.5367 | 0.8890 | 0.9181 | 0.9181 |

| Model | Training error | | Test error | |
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| | MSE | $r^2$ | MSE | $r^2$ |
| n = 1 | 29834 | 0.10841 | 32158 | 0.11421 |
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| n = 6 | 6315.0 | 0.81128 | 6852.6 | 0.81125 |
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| n = 18 | 2813.1 | 0.91593 | 3075.3 | 0.91529 |
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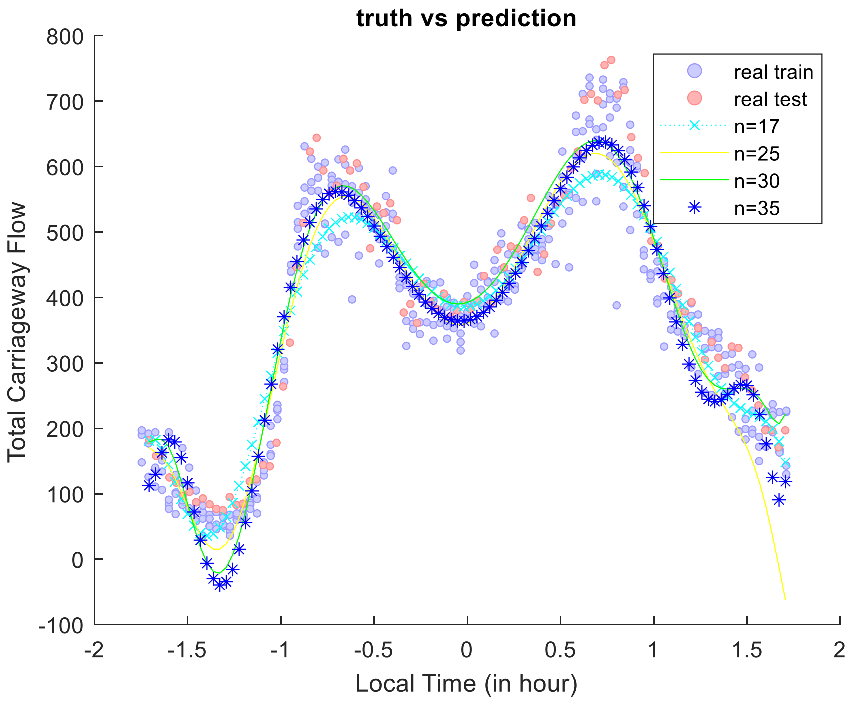
# Factors that impact performance

## PolynomialOrder




When polynomial order $n$ is relatively small, the larger the $n$ is, the better the performance.

While a larger $n$ also means a bigger risk of overfitting.

Explore effects of PolynomialOrder on SVR (Polynomial).

# Factors that impact performance

## BoxConstraint

$$\text{minimize} \quad \frac{1}{2}\|W\|_2^2 + C\sum_{i=1}^{N}(\xi_i - {\xi_i}^*)$$

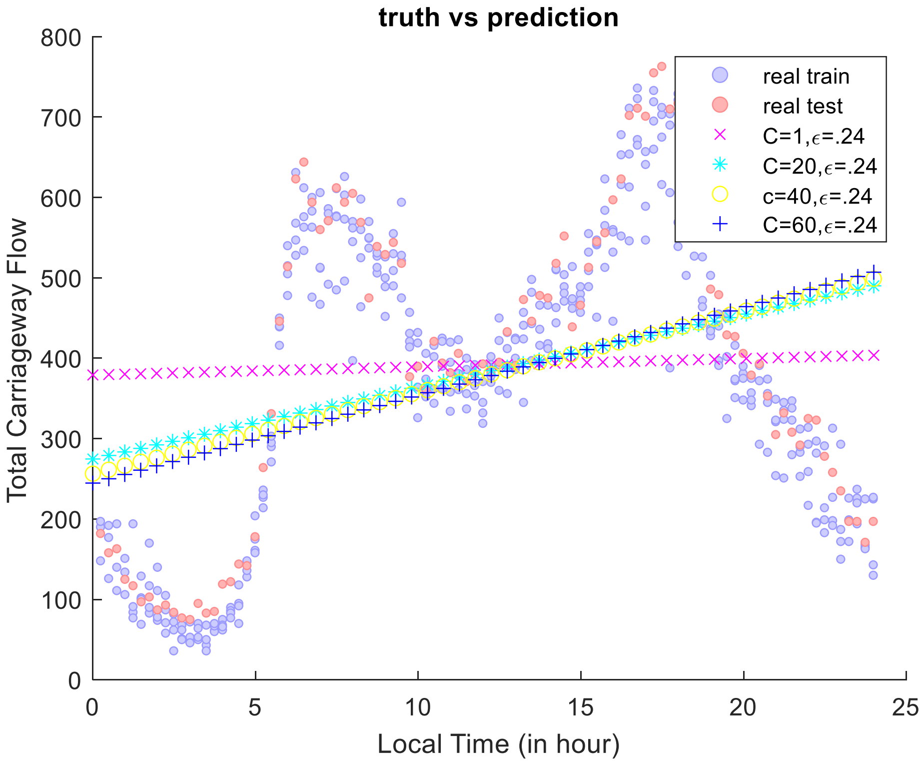
$$\begin{aligned} \text{subject} \quad & y_i - (Wx_i + b) \le \epsilon + {\xi_i}^*, \qquad {\xi_i}^* \ge 0 \\ & Wx_i + b - y_i \le \epsilon + {\xi_i}^*, \qquad {\xi_i}^* \ge 0 \end{aligned}$$

Large C: lower bias, higher variance
Small C: higher bias, lower variance

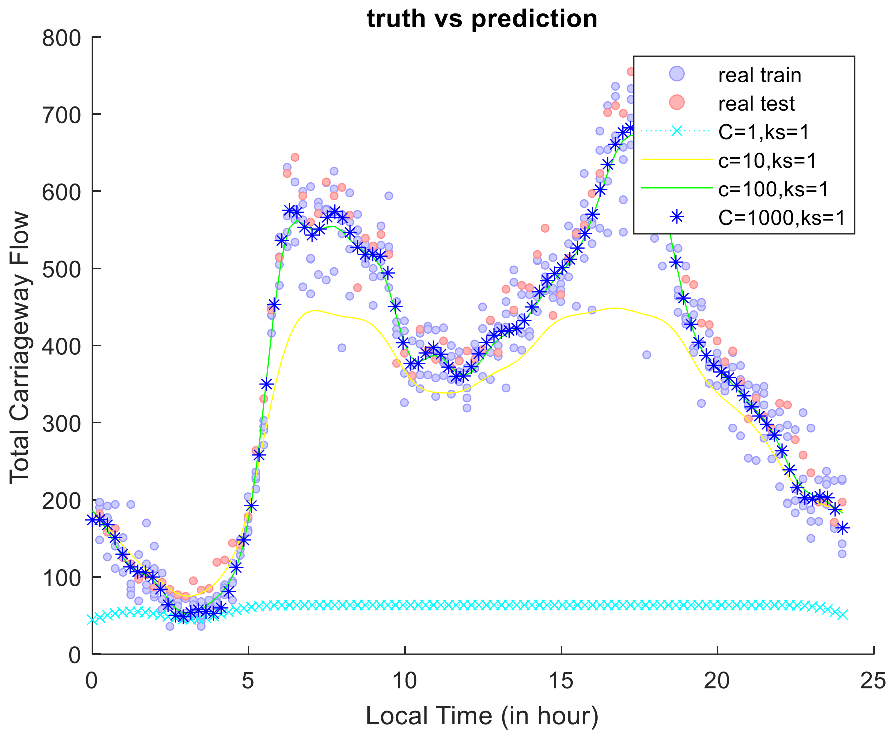
$C$ determines the amount up to which deviations larger than $\epsilon$ are tolerated.
A small $C$ helps prevent overfitting (regularization).

# Factors that impact performance

## BoxConstraint




Explore effects of BoxConstraint on SVR(Linear).



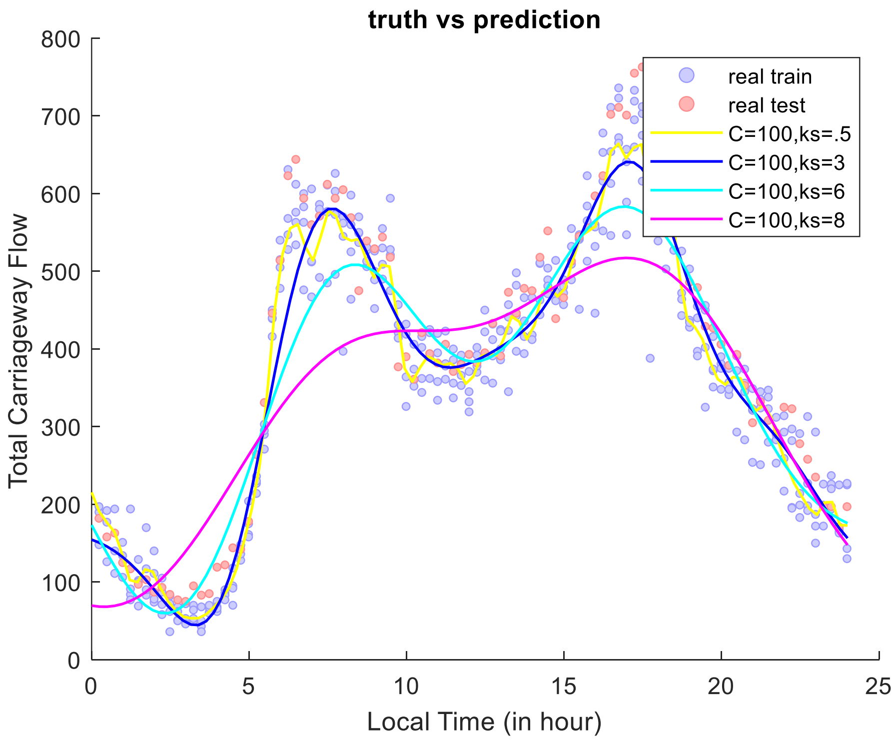

Explore effects of BoxConstraints on SVR (Gaussian).

# Factors that impact performance

## KernelScale

The software divides all elements of the predictor matrix $X$ by the value of KernelScale. KernelScale determines the extent to which the features vary smoothly.

Large KernelScale: higher bias, lower variance
Small KernelScale: lower bias, higher variance




Explore effects of KernelScale on SVR (Gaussian).

# Limits of my approach

1. Other hyperparameters of SVR (Polynomial) also plays an important role and are under-explored in our project currently.

2. For SVR (Polynomial) and SVR (Gaussian), we directly use the suggested value in in our trials instead.

3. Control variable, not a best method.

# Improvements

1. Given more time, we plan to first decide on the bond of each hyperparameter and then do gridsearch.

2. Do random search for a large number of times.

# Conclusion

Least-square Regression (LSR) and Support Vector Regression (SVR) are applied in this project to predict the traffic flow at different times of a day.

**LSR (Polynomial)**

**SVR (Linear)**

**SVR (Polynomial)**

**SVR (Gaussian)**

PolynomialOrder

When polynomial order $n$ is relatively small, the larger the $n$ is, the better the performance.

While a larger $n$ also means a bigger risk of overfitting.

BoxConstraint

Large C: lower bias, higher variance
Small C: higher bias, lower variance

KernelScale

Large KernelScale: higher bias, lower variance
Small KernelScale: lower bias, higher variance

# Thank you