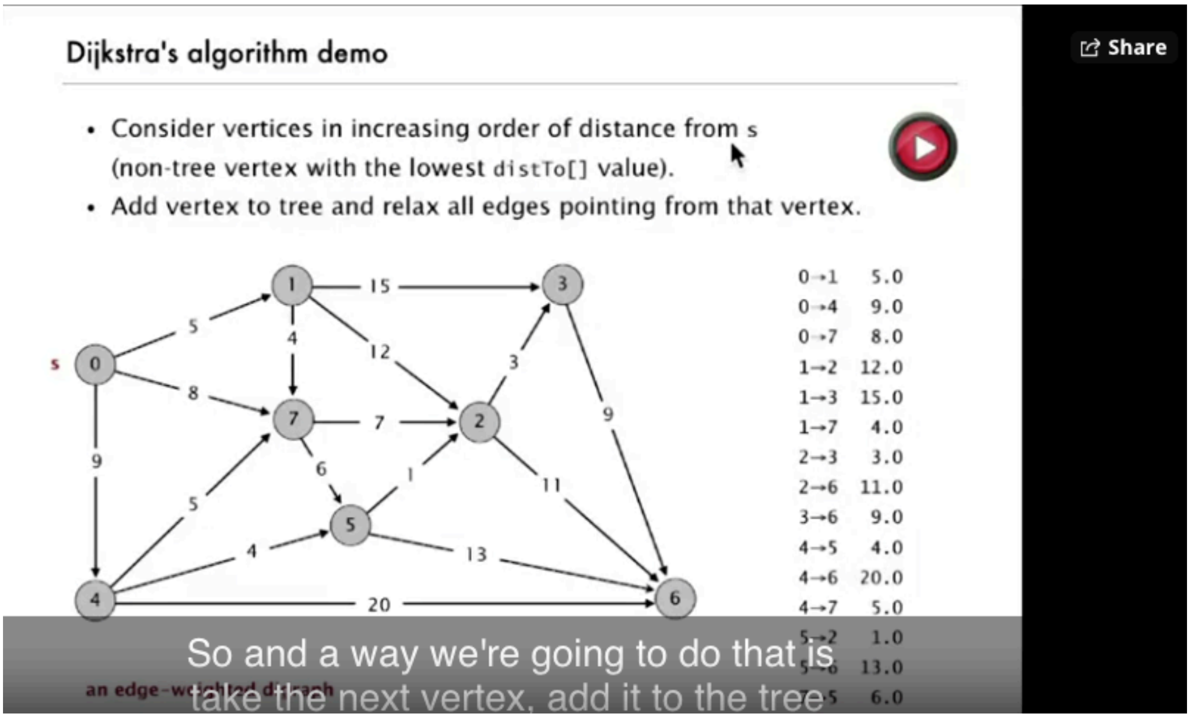
Smallest path

edgeTo[ ]记录最短路径的上一个节点, distTo[ ]记录距离的累加

relax 加入边的操作, 如果初始到v的距离加上v到w的距离小于之前最短到w的距离, 最短的距离等于到v的距离加w的距离, edgeTo的w的父节点改成v - > w边, 这里数组不是用一个节点, 而是用边来追踪路径.

```
private void relax(DirectedEdge e){
    int v = e.from(), w = e.to();
    if(distTo[w] > distTo[v] + e.weight()){
    distTo[w] = distTo[v] + e.weight();
        edgeTo[w] = e;
    }
}
```

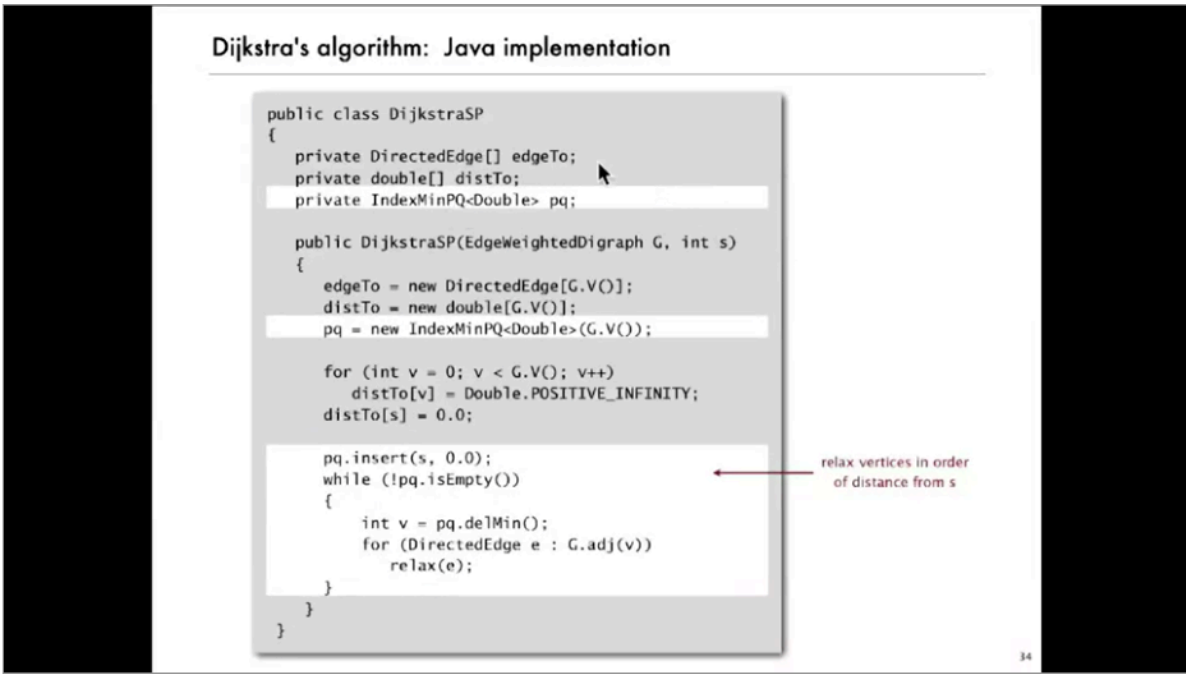
Dijkstra‘s algorithm 有向图

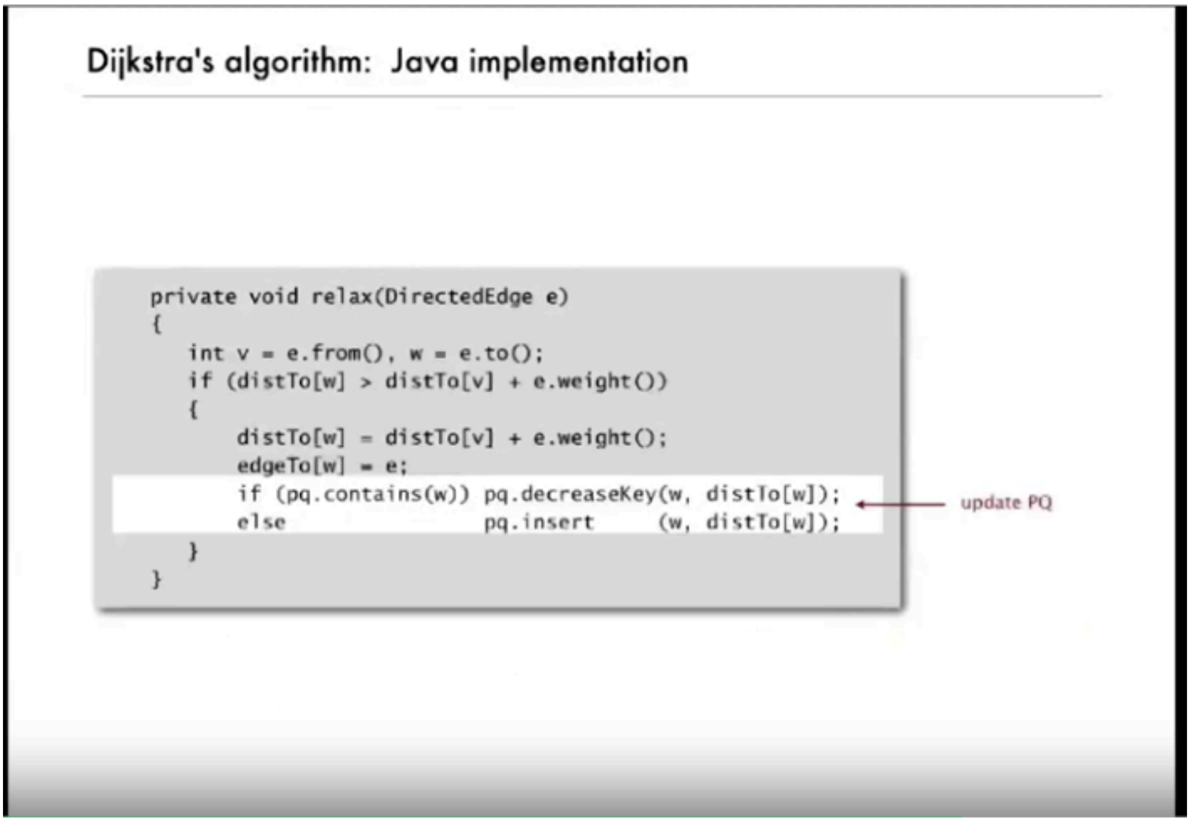


如图, 起点0, 先找最短的edge 0->1 因为他是0出发最短的, 所以不会有比他更短的了, 然后1->7,2,3 如果distTo(1) + edge(1-k)小于distTo(k),就是relax所有的边, 如果小于就更新. 然后0 -1检查过了, 继续下一个最短的, (0-7) 把7所有的边relax.

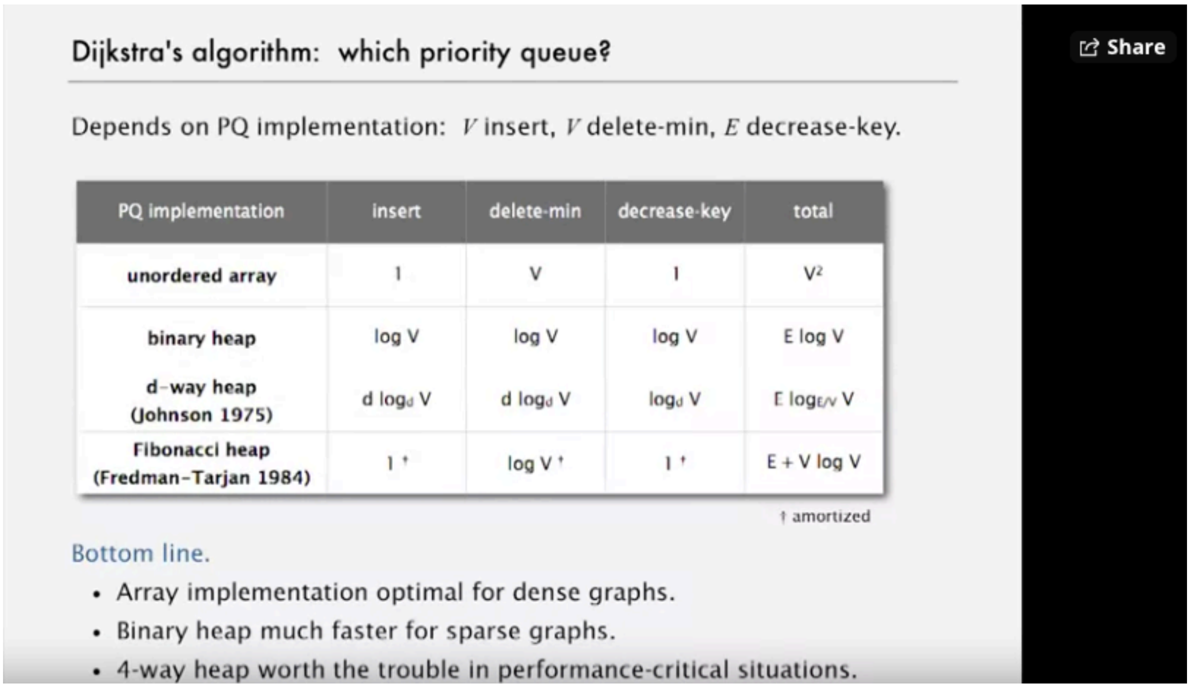
这里有一个decreaseKey的方法, 建议使用PriorityQueue.remove(Obj o)然后再使用PriorityQueue.add(Obj o)的方法 简单一些.

实现




Dijkstra's algorithm: Java implementation
private void relax(DirectedEdge e)
{
int v = e.from(), w = e.to();
if (distTo[w] > distTo[v] + e.weight())
{
distTo[w] = distTo[v] + e.weight();
edgeTo[w] = e;
if (pq.contains(w)) pq.decreaseKey(w, distTo[w]);
else pq.insert (w, distTo[w]);
}
}
update PQ

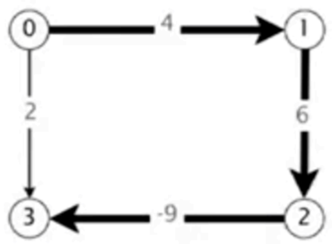
复杂度


Dijkstra's algorithm: which priority queue?
Share
Depends on PQ implementation: V insert, V delete-min, E decrease-key.
PQ implementation | insert | delete-min | decrease-key | total
unordered array | 1 | V | 1 | V²
binary heap | log V | log V | log V | E log V
d-way heap (Johnson 1975) | d log_d V | d log_d V | log_d V | E log_E/V V
Fibonacci heap (Fredman-Tarjan 1984) | 1 † | log V † | 1 † | E + V log V
† amortized
Bottom line.
• Array implementation optimal for dense graphs.
• Binary heap much faster for sparse graphs.
• 4-way heap worth the trouble in performance-critical situations.

拓扑排序, 有向无环图, 无负值 acyclic(无环)

如果有负数的图, 0->3贪心的最短路径不会考虑到2->3是负数

Dijkstra. Doesn't work with negative edge weights.



Dijkstra selects vertex 3 immediately after 0.
But shortest path from 0 to 3 is 0→1→2→3.

复杂度

Single source shortest-paths implementation: cost summary

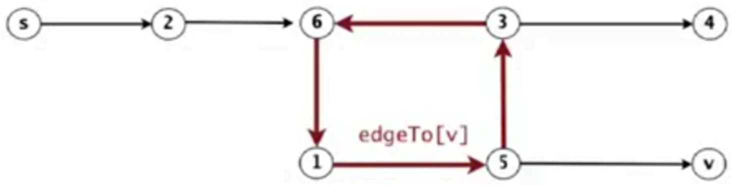
algorithm	restriction	typical case	worst case	extra space
topological sort	no directed cycles	E + V	E + V	V
Dijkstra (binary heap)	no negative weights	E log V	E log V	V
Bellman-Ford	no negative cycles	E V	E V	V
Bellman-Ford (queue-based)		E + V	E V	V

Remark 1. Directed cycles make the problem harder.
Remark 2. Negative weights make the problem harder.
Remark 3. Negative cycles makes the problem intractable.

如果最后一次还有更新,那么他肯定存在着环

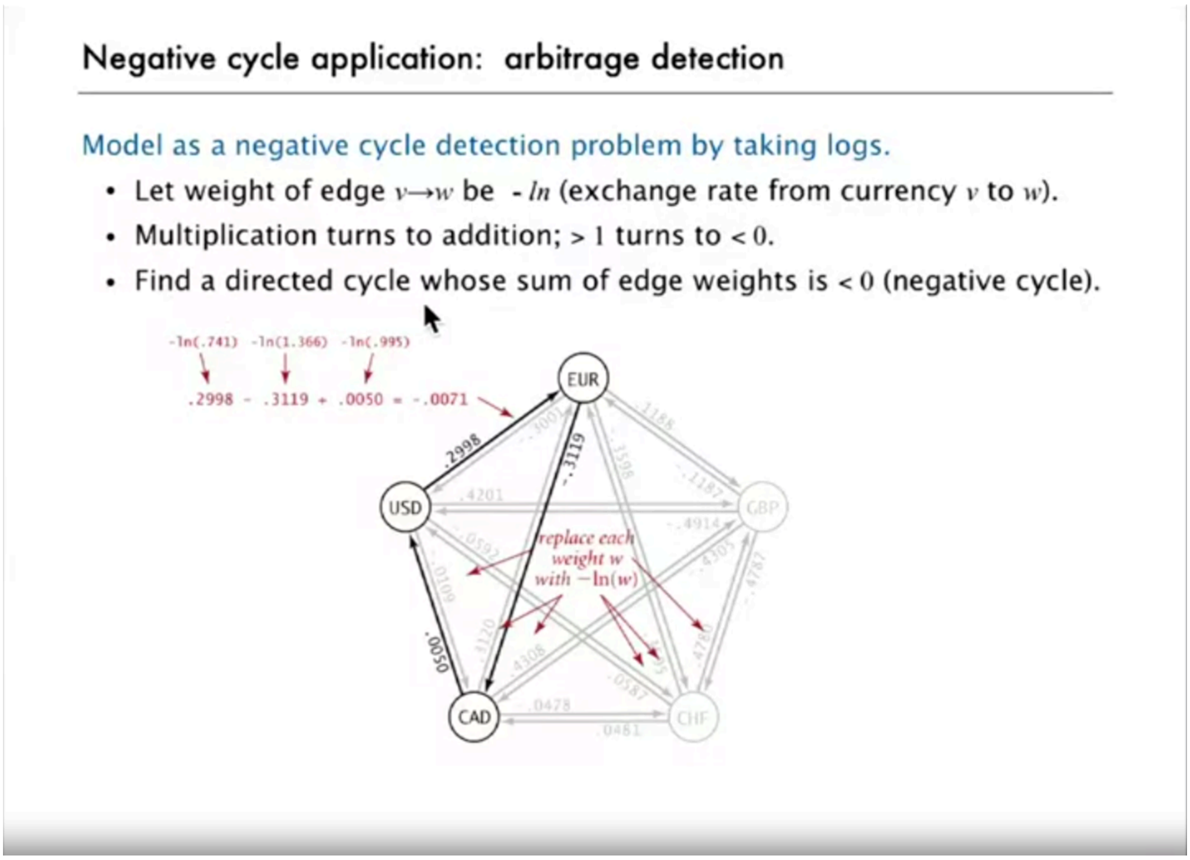
Finding a negative cycle

Observation. If there is a negative cycle, Bellman-Ford gets stuck in loop, updating `distTo[]` and `edgeTo[]` entries of vertices in the cycle.



Proposition. If any vertex v is updated in phase V, there exists a negative cycle (and can trace back `edgeTo[v]` entries to find it).

汇率问题转化成negative cycle



work schedule转换成shortest path

Critical path method

CPM. To solve a parallel job-scheduling problem, create edge-weighted DAG:

- Source and sink vertices.
- Two vertices (begin and end) for each job.
- Three edges for each job.
  - begin to end (weighted by duration)
  - source to begin (0 weight)
  - end to sink (0 weight)
- One edge for each precedence constraint (0 weight).

job	duration	must complete before
0	41.0	1 7 9
1	51.0	2
2	50.0	
3	36.0	
4	38.0	
5	45.0	
6	21.0	3 8
7	32.0	3 8
8	32.0	2
9	29.0	4 6