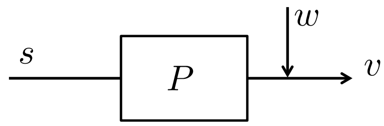
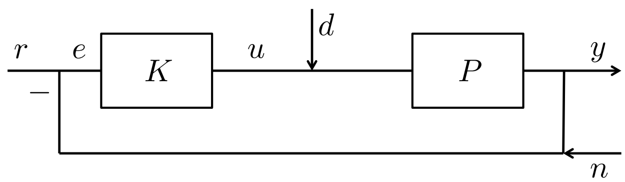
1 Example of tracking

Step 1 Open loop id for unknown plant P. Run experiment to obtain (input, output) sequence (s, v).



Assume the measurement noise w in a bounded set with IQC description $\mathcal{Q}_w$. Convert the observation to IQCs about the plant $\mathcal{Q}_P$.

Step 2 Place the candidate controller K in the closed loop. Run experiment to obtain (reference, output) sequence (r, y).



Given the IQC description of the uncertain plant, the disturbance d and the measurement noise n as $\mathcal{Q}_P$, $\mathcal{Q}_d$, $\mathcal{Q}_n$. The feasibility problem for validation of K is as follows: does there exist (d, n) such that:

$$
\begin{aligned}
&(r, e) \in \mathcal{Q}_T, \quad \text{tracking performance} \\
&d \in \mathcal{Q}_d, \quad n \in \mathcal{Q}_n, \quad \text{and} \quad (K(r - (y + n)) + d, y) \in \mathcal{Q}_P.
\end{aligned}
$$

Naive question: why not replace Step 2 with IQC synthesis? why not replace Step 1 with point estimator plus uncertainty set?

2 Setup

2.1 Basic idea

Given:

- The part of plant that is known a priori, denoted by P.
- Support of the of the unknown part of the plant Δ: an *initial* description in the form of integral quadratic constraints on its input-output pair $(v, s) \in \mathcal{Q}_\Delta = \{Q_1, \ldots, Q_L\}$.
- Support of the controller K: a (discrete) set of candidate controllers $K \in \mathcal{K} = \{K_1, \ldots, K_M\}$.
- Support of the unobserved exogenous noise w: a bounded set in the form of IQCs $w \in \mathcal{Q}_W$.

Goal: learn about the system so that we can find a controller $K \in \mathcal{K}$ which stabilizes the system and optimizes the ℓ_2 gain $\|T_{w\to y}\|_\infty$.

The basic idea of "controller invalidation" is as follows.

1. Consider closed-loop system with a candidate controller $K \in \mathcal{K}$ in place.

   Given $P, \mathcal{Q}_\Delta, \mathcal{Q}_W$, given length T observation $(u_t, y_t, z_t)_{t=1}^T$ generated by the closed loop system, solve the following feasibility problem.

   Does there exist signal $(w_t, v_t, s_t)_{t=1}^T$ such that the following holds:

   (a) (in time domain)

   $$(z, y, v) = P(u, w, s), \quad u = Kz.$$

   Let $F_u(P, K)$ denote the linear fractional transformation, the two time domain equalities are equivalent to

   $$(y, v) = F_u(P, K)(w, s).$$

   (b) IQCs for system uncertainties are satisfied:

   $$w \in \mathcal{Q}_W, \quad (v, s) \in \mathcal{Q}_\Delta$$

   (c) Performance achieved, namely $\|T_{w\to y}\|_\infty \le \gamma$. Note that This closed loop constraint can also be described as a IQC $\mathcal{Q}_{cl}$ for the pair of signal (w, y):

   $$(w, y) \in \mathcal{Q}_{cl}.$$

   These can be turned into an LMI.

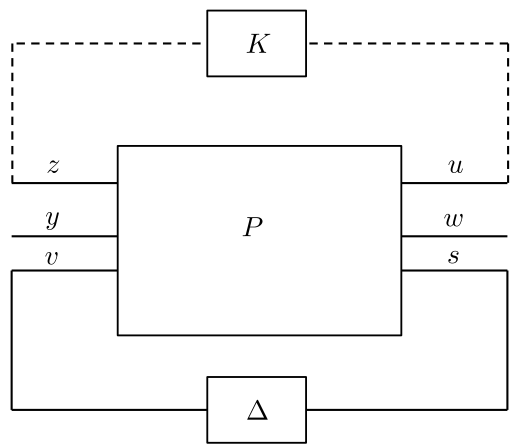
2. If we cannot find such (w, v, s), we should rule out K from $\mathcal{K}$, because there does not exist Δ and w in the uncertainty set such that K is feasible.

3. In the event that we find such (w, v, s), namely we can find some w and Δ in the uncertainty set for which K is feasible, we should keep K in $\mathcal{K}$.

4. Moreover, for each $K_i \in \mathcal{K}$, we can potentially identify a set of Δ that are consistent with the observation for some $w \in \mathcal{Q}_w$. Denote it by Δ_{K_i}.
   Find the tightest IQC description for the set $\cap_i \Delta_{K_i}$ and denote it by $\mathcal{Q}'_\Delta$. Can we confidently rule out the subset of Δ in $\mathcal{Q}_\Delta \backslash \mathcal{Q}'_\Delta$? No system in this subset can generate the observed sequences.

Remark: We need to do this test for every K in the discrete set $\mathcal{K}$ to determine whether to keep it or throw it out. If more than one K is left, we can try:

- Get new observations for each K with the same setup. Run the same invalidation procedure.
- With the same observation, search for better performance so that sub-optimal controllers would be ruled out.
- Impose more IQCs (obtained from observation data) to shrink the uncertainty set of Δ.



3 Discussions

From the Poolla paper (Time domain approach to model validation), only the unstructured uncertainty set (convex ball) can be treated. More complicated IQC, and how to obtain more complicated IQC from data?

Naive questions

1. how does this approach compare to controller synthesis with the IQC for Δ? – we can use the observation sequences without id + control.

After studying the 0-th order question.

1. Start with a compact set for $\mathcal{K}$, we want to test one instance of K and be able to rule out a measurable subset of $\mathcal{K}$.
2. Incorporate the information in the observation sequences to get a finer description of the uncertainty set Δ in terms of more IQCs in $\mathcal{Q}_\Delta$, which would give us more invalidation power.
3. Experiment design. Note that in the closed loop id, choosing a particular controller K determines the u input sequence. We want to select a controller $K \in \mathcal{K}$ so that we can shave of a large portion of $\mathcal{K}$ with the invalidation procedure.