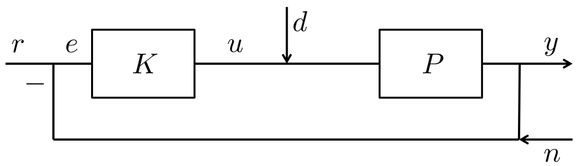
1 PID control

Consider the system Our goal is to test whether a given controller K can cause the plant P



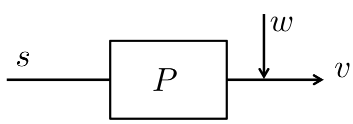
track a known signal $r(t)$, without placing K in the closed loop, over a fixed time horizon T. The challenge is that the plant P is unkown, and lies in some class of plants $\mathcal{P}$. We also assume that our plant is subject a class $\mathcal{D}$ of disturbances, and noises $\mathcal{N}$. We then ask if there exists a plant $P \in \mathcal{P}$ such that, for any admissible disturbance $d \in \mathcal{D}$ and noise $n \in \mathcal{N}$, does there exist a $P \in \mathcal{P}$ such that we achieve some performance objective $Q_{T,y}(r, y)$ of the reference and output. For example, $Q_{T,y} = \sum_{t=1}^{T}(r(t) - y(t))^2 = \|r - y\|_2$. Formally, we compute

$$\begin{aligned} &\inf_{P\in\mathcal{P}} \sup_{n\in\mathcal{N}, d\in\mathcal{D}} Q_{T,y}(r, y) \\ &\text{subject to:} \quad e = r - (y + n), \quad u = Ke, y = P(u + d) \end{aligned} \tag{1.1}$$

For a threshold γ, we say that K is *invalidated* over $(\mathcal{P}, \mathcal{N}, \mathcal{D})$ if the optimal value of Equation 1.1 is $> \gamma$; otherwise, we say that $(\mathcal{P}, \mathcal{N}, \mathcal{D})$ *corroborates* K.

1.1 Shrinking the set $\mathcal{P}$ with open loop ID

If $\mathcal{P}$ is too unrestrictive, then the above optimization problem won't be able to invalidate a K which doesn't meet the performance objective on the true plant, because there may be another plant $P' \in \mathcal{P}$ for which the objective is met. Thus, we use open-loop experiments to constrain P. Namely, we consider the plant



We then assume that take N measurements of the form $v^{(j)} = Ps^{(j)} + w^{(j)}$, where $s^{(j)}$ are probe signals, and $w^{(j)}$ is stochastic noise. In the spirit of compressed sensing, we will for the moment abstract away the stochasticity of $w^{(j)}$ by assumping that the noise vectors $w = (w^{(1)}, \ldots, w^{(N)})$ lie in some stochastic-like set $\mathcal{W}$. For example, $\mathcal{W}$ can include spectral norm constraints on the matrix whose columns are formed by $w^{(j)}$ which would be celebrated by white (isotropic) noise. We will also assume that we have some prior information that $P \in \mathcal{P}_0$, which we will take to control the complexity of P (eg, low Hankel norm). With these constraints, we know have

$$\begin{aligned} &\inf_{P\in\mathcal{P}} \sup_{n\in\mathcal{N}, d\in\mathcal{D}} Q_{T,y}(r, y) \\ &\text{subject to:} \quad e = r - (y + n), \quad u = Ke, \quad y = P(u + d) \\ &P \in \mathcal{P}_0, \quad w \in \mathcal{W}, \quad v^{(j)} = Ps^{(j)} + w^{(j)}, j = 1, \ldots, N \end{aligned} \tag{1.2}$$

By elimiating for e and u, we get

$$\begin{aligned}
&\inf_{P\in\mathcal{P}} \sup_{n\in\mathcal{N}, d\in\mathcal{D}} Q_{T,y}(r,y) \\
&\text{subject to:} (I+PK)y = PKr - PKn + Pd \\
&P\in\mathcal{P}_0, \quad w\in\mathcal{W}, \quad v^{(j)} = Ps^{(j)} + w^{(j)}, j = 1,\dots,N
\end{aligned} \tag{1.3}$$

1.2 Convexity of Equation 1.4

In what follows, we assume that $Q_{T,y}$ is a convex quadratic, and the sets $\mathcal{N},\mathcal{D},\mathcal{P}_0,\mathcal{W}$ are all convex. Thus, Equation 1.4 suffers from only three sources of non-convexity, all in the iequality $(I+PK)y = PKr - PKn + Pd$:

1. PKy is bi-linear in P and y
2. PKn is bi-linear in P and n, both of which are unkown
3. Pd is bilinear in P and d

The latter two sources of non-convexity can be circumvented in two ways:

1. Use an "Outer Approximation" or relaxation to the set of admissible PKn and Pd
2. Sample K rollouts of "typical" noises and disturbances $\{n^{(i)}, d^{(i)}\}$, and solve the optimization

$$\begin{aligned}
&\inf_{P,u^{(i)},y^{(i)}\in\mathcal{P}} \frac{1}{M}\sum_{i=1}^{M} Q(r,y^{(i)}) \\
&\text{subject to:} (I+PK)y^{(i)} = PKr - PKn^{(i)} + Pd^{(i)} \\
&P\in\mathcal{P}_0, \quad w\in\mathcal{W}, \quad v^{(j)} = Ps^{(j)} + w^{(j)}, j = 1,\dots,N
\end{aligned} \tag{1.4}$$

   Note that whereas the parameter N controls the number of experiments (i.e. sample complexity), K controls the number of simulations (i.e. computational complexity)

We can also rewrite the previous optimization problem in the following form:

$$\begin{aligned}
&\inf_{P\in\mathcal{P},y^{(i)},u^{(i)}} \frac{1}{M}\sum_{i=1}^{M} Q(r,y^{(i)}) \\
&\text{subject to:} \quad y^{(i)} = Pu^{(i)}, u^{(i)} = K(r - (y^{(i)} + n^{(i)})) + d^{(i)}
\end{aligned} \tag{1.5}$$

1.3

Considering the second