controller validation for uncertain systems

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1 Setup

1.1 Basic idea

Given:

- a discrete set of candidate controllers $\mathcal{K} = \{K_1, \ldots, K_M\}$.
- P, the part of plant that is known a priori.
- an *initial* description of Δ, the unknown part of the plant, in the form of integral quadratic constraints $\mathcal{Q}_\Delta = \{Q_1, \ldots, Q_L\}$.
- $\mathcal{W}$, a bounded set for the unobserved exogenous noise w, in the form of IQCs $\mathcal{Q}_W$.

Goal: learn about the system so that we can find a controller $K \in \mathcal{K}$ which stabilizes the system and optimizes the ℓ_2 gain $\|T_{w \to y}\|_\infty$.

The basic idea of "controller invalidation" is as follows.

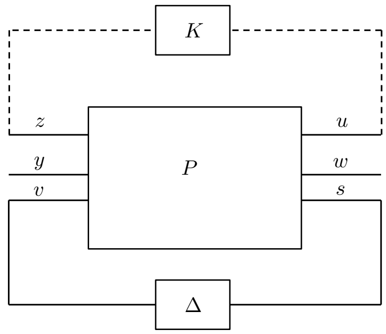
Consider closed-loop id without a candidate controller K in the system. (note that open loop id does not make sense here.) Given the observation of length T sequence $(u_t, y_t, z_t)_{t=1}^T$ generated by the system, try to find $(w_t, v_t, s_t)_{t=1}^T$ such that

- $(z, y, v) = P(u, w, s)$ and $u = Kz$.
- the IQCs $\mathcal{Q}_\Delta$ for (v, s) are satisfied.
- the IQCs $\mathcal{Q}_W$ for w are satisfied.
- Performance in terms of l_2 gain of $T_{w \to y}$ and the stability is achieved.

If we cannot find such (w, v, s), we should rule out K from $\mathcal{K}$, otherwise keep it.

We need to do this test for every K in the discrete set $\mathcal{K}$ to determine whether to keep it or throw it out. If more than one K is left, we can try:

- Get new observations for each K with the same setup. Run the same invalidation procedure.
- With the same observation, search for better performance so that sub-optimal controllers would be ruled out.
- Impose more IQCs (obtained from observation data) to shrink the uncertainty set of Δ.



2 Invalidation procedure

For a particular K, given the $\mathcal{K}, P, \mathcal{Q}_\Delta, \mathcal{Q}_W$, given length T observation $(u_t, y_t, z_t)_{t=1}^T$, try to invalidate K.

Let's get a bit more concrete for the invalidation procedure and write it out here.

3 Discussions

Naive questions

1. how does this approach compare to controller synthesis with the IQC for Δ? – we can use the observation sequences without id + control.

After studying the 0-th order question.

1. Start with a compact set for $\mathcal{K}$, we want to test one instance of K and be able to rule out a measurable subset of $\mathcal{K}$.
2. Incorporate the information in the observation sequences to get a finer description of the uncertainty set Δ in terms of more IQCs in $\mathcal{Q}_\Delta$, which would give us more invalidation power.
3. Experiment design. Note that in the closed loop id, choosing a particular controller K determines the u input sequence. We want to select a controller $K \in \mathcal{K}$ so that we can shave of a large portion of $\mathcal{K}$ with the invalidation procedure.