1 PID control

Our goal is to track a constant signal $r(t) = r_0$, using a PID controller, in the SISO setting. Assume our system has impulse-response function $g(t)$, so that starting from an zero initial state, $y(t) = u(t) * g(t) + d(t)$, where $d(t)$ is some disturbance.

2 Point-Estimating g

How to choose u depends on our goal. Suppose we just want to learn the coefficients of g. Let $H(u)$ denote the hankel matrix associated with U, so that $y(t) = H(u)g + d(t)$. Hence, for one roll out, the optimal robust experiment design to measure g is

$$\max_{u \in \mathcal{U}} \sup_{d \in \mathcal{D}} \|H^\dagger(u)y - g\|_2 = \max_{u \in \mathcal{U}} \sup_{d \in \mathcal{D}} \|H^\dagger(u)d(t)\|_2 \tag{2.1}$$

Other objectives would consider different norms. For example, we could try to pick u to minimize the $\|\|_{H_2}$ or $\| \cdot \|_\infty$ norm of g.

3 PID - Control usinig state feedback

We want to lear a controler F so that the system to minimize $\|y - r\|$, using input $y - r$. Thus, the new input to the system is $y(t+1) = (A + BF)y - BFr + BFd(t)$, and our doal is to minimize $\|C(y - r)\|_2^2$. Let's assume that even though our measurements have disturbances, but the true system has no disturbance (we don't need noise rejection). Then, our goal is to choose u and estimate $\hat{B}, \hat{A}$ so as to minimize

$$\tag{3.2}$$

In an order-one system g is of the form $g(t) = ca^t$. Rather than estimate g directly, we want to learn a controller $k = f(y - r)$ such that to minimize

$$\|y - r\| \tag{3.3}$$

This corresponds to the system $y(t+1) = y$