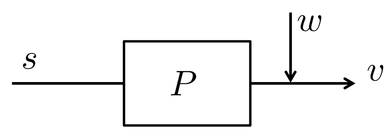
1 ID-Control Problem

The goal is to use IQC constraints to replace the clear-cut between sys id and robust controller synthesis.

To do list

1. simulation of the simple setting to see how it works (with IQC synthesis)
2. sample complexity of controller invalidation
3. sample complexity of robust controller synthesis

Step 1, sys id Open loop id for unknown plant P. Run experiment to obtain (input, output) sequence (s, v).

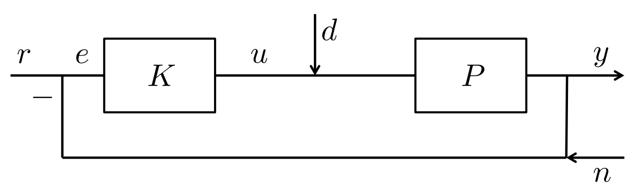


Assume the measurement noise w in a bounded set with IQC description $\mathcal{Q}_w$. Convert the observation (s, v) to IQCs about the plant.

For example, assume that P is FIR and the length of the observation is larger, and assume that w lies in bounded ℓ_2 ball. We parameterize the plant by its impulse response p. The observation (s, v) gives the following constraint on p:

$$\|v - p * s\|_2 \leq \gamma_w.$$

Step 2, controller invalidation Place the candidate controller K in the closed loop. Run experiment to obtain (reference, output) sequence (r, y).



Given the constraints for the uncertain plant from Step 1, the disturbance d and the measurement noise n as $\mathcal{Q}_d$, $\mathcal{Q}_n$. Let $\mathcal{Q}_{T,\gamma}$ denote the achieved quadratic performance of the closed loop channel $r \to y$ with parameter γ.

The invalidation of controller K is to certify that there does not exist noise signal (d, n) and impulse response of the plant p such that:

$$\begin{aligned}
&(r, y) \in \mathcal{Q}_{T,\gamma}, \quad \text{(tracking performance achieved)} \\
&e = r - (y + n), \quad u = Ke, \quad y = p * (u + d), \quad \text{(signals)} \\
&d \in \mathcal{Q}_d, \quad n \in \mathcal{Q}_n, \quad (v - p * s) \in \mathcal{Q}_w. \quad \text{(constraints satisfied)}
\end{aligned}$$

Set $\mathcal{Q}_d$ and $\mathcal{Q}_n$ to be ℓ_2 ball, and parameterize K by a FIR k. We invalidate K if there does not exist signal (d, n, p) such that

$$
\begin{aligned}
& e = r - (y + n), \quad u = k * e, \quad y = p * (u + d), \quad \text{(signals)} \\
& \|y - r\|_2 \leq \gamma, \quad \|d\|_2 \leq \gamma_d, \quad \|n\|_2 \leq \gamma_n, \quad \|v - p * s\|_2 \leq \gamma_w.
\end{aligned}
$$

(convex?)

Sample complexity question. Suppose K can be invalidated. Assume that w, d, n are Gaussian white noise. How many roll-outs in Step 1 do we need in order to invalidate K with high probability.

Step 3, robust controller synthesis The synthesis problem of K is as follows:

$$
\min_{K \in \mathcal{K}} \gamma, \quad s.t. \text{ the above inequalities hold.}
$$

Note that this is nonconvex.

Sample complexity question. Assume that w, d, n are Gaussian white noise. How many roll-outs in Step 1 do we need in order to find K sufficiently close to the optimal controller with respect to the underlying true plant.

2 Controller invalidation for synthesis

Robust synthesis is non-convex. If we can do experiments to test different controllers, can we "solve" the synthesis problem by ruling out controllers which do not achieve the robust performance? How to design the experiments to "solve" the synthesis problem in a more efficient way?

To do list

1. Find a simple parameterization of controller such that it works.
2. Sample complexity of invalidation?
3. Iterative id and invalidation?

Setting Given:

- The part of plant that is known a priori, denoted by P.
- Support of the of the unknown part of the plant Δ: an *initial* description in the form of integral quadratic constraints on its input-output pair $(v, s) \in \mathcal{Q}_\Delta = \{Q_1, \ldots, Q_L\}$.
- Support of the controller K: a (discrete) set of candidate controllers $K \in \mathcal{K} = \{K_1, \ldots, K_M\}$.
- Support of the unobserved exogenous noise w: a bounded set in the form of IQCs $w \in \mathcal{Q}_w$.

Goal: find a controller $K \in \mathcal{K}$ that achieves robust stability and performance.

Basic idea The basic idea of "controller invalidation experiments for synthesis" is as follows.

Consider closed-loop system with a candidate controller $K \in \mathcal{K}$ in place. Generate length T observation $(u_t, y_t, z_t)_{t=1}^T$ generated by the closed loop system.

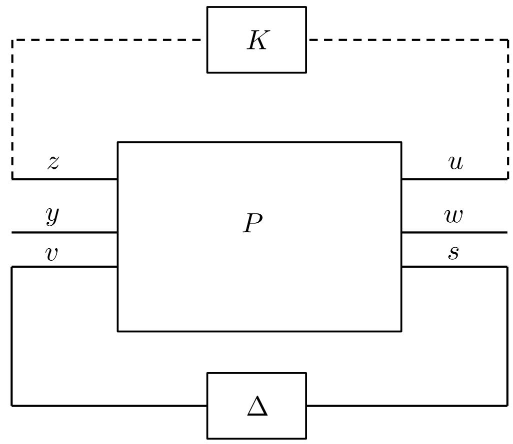
Given P, $\mathcal{Q}_\Delta$, $\mathcal{Q}_W$, invalidate K if there does not exist signal $(w_t, v_t, s_t)_{t=1}^T$ such that the following holds:

1. $$(z, y, v) = P(u, w, s), \quad u = Kz.$$
   Let $F_u(P, K)$ denote the linear fractional transformation, the two time domain equalities are equivalent to
   $$(y, v) = F_u(P, K)(w, s).$$
2. IQCs for system uncertainties are satisfied:
   $$w \in \mathcal{Q}_w, \quad (v, s) \in \mathcal{Q}_\Delta$$
3. Performance achieved.
   $$(w, y) \in \mathcal{Q}_{T,\gamma}.$$

(convex?)

We need to do this test for every K in the discrete set $\mathcal{K}$ to determine whether to keep it or throw it out. If more than one K is left, try:

- Get new observations for each K with the same setup. Run the same invalidation procedure.
- With the same observation, search for better performance so that sub-optimal controllers would be ruled out.



Discussions

1. Start with a compact set for $\mathcal{K}$, we want to test one instance of K and be able to rule out a measurable subset of $\mathcal{K}$.

   Experiment design. Note that in the closed loop id, choosing a particular controller K determines the u input sequence. We want to select a controller $K \in \mathcal{K}$ so that we can shave of a large portion of $\mathcal{K}$ with the invalidation procedure.

2. Incorporate the information in the observed sequences to get a finer description of the uncertainty set Δ, which would give us more invalidation power.

   For each $K_i \in \mathcal{K}$, we can potentially identify a set of Δ that are consistent with the observation for some $w \in \mathcal{Q}_w$. Denote it by Δ_{K_i}.
   Find the tightest IQC description for the set $\cap_i \Delta_{K_i}$ and denote it by $\mathcal{Q}'_\Delta$. Can we confidently rule out the subset of Δ in $\mathcal{Q}_\Delta \backslash \mathcal{Q}'_\Delta$? No system in this subset can generate the observed sequences.