

Pattern Recognition and Machine Learning: Homework 2

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Problem 1

Task1

1.

Take the object function in Equation(2) into three parts:

$$\begin{aligned}f_1 &= \frac{1}{l} \text{Tr}((Y - JKW)^T(Y - JKW)) \\f_2 &= \gamma_A \text{Tr}(W^T KW) \\f_3 &= \frac{\gamma_I}{(u + l)^2} \text{Tr}(W^T K L K W)\end{aligned}$$

Take the derivative of f_1 :

$$\begin{aligned}df_1 &= \frac{1}{l} \text{Tr}(d(Y - JKW)^T(Y - JKW) + (Y - JKW)^T d(Y - JKW)) \\&= \frac{2}{l} \text{Tr}((Y - JKW)^T d(Y - JKW)) \\&= \frac{2}{l} \text{Tr}((Y - JKW)^T (-JK) dW) \\ \frac{df_1}{dW} &= -\frac{2}{l} (JK)^T (Y - JKW)\end{aligned}$$

Take the derivative of f_2 :

$$\begin{aligned}df_2 &= \gamma_A \text{Tr}(dW^T KW + W^T K dW) \\&= 2\gamma_A \text{Tr}(W^T K dW) \\ \frac{df_2}{dW} &= 2\gamma_A KW\end{aligned}$$

Take the derivative of f_3 :

$$\begin{aligned}df_3 &= \frac{\gamma_I}{(u + l)^2} \text{Tr}((dW^T) K L K W + W^T K L K dW) \\&= \frac{2\gamma_I}{(u + l)^2} \text{Tr}(W^T K L K dW) \\ \frac{df_3}{dW} &= \frac{2\gamma_I}{(u + l)^2} K L K W\end{aligned}$$

2.

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1 self.W = np.linalg.inv(J.dot(K) + self.gamma_A*1*np.identity(1+u)
2      + (self.gamma_I*1)/(u+1)**2*L.dot(K)).dot(Y)
```

Task2