Pattern Recognition and Machine Learning: Homework 2

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April 24, 2023

Problem 1

Task1

1.

Take the object function in Equation(2) into three parts:

$$
\begin{aligned}
f_1 &= \frac{1}{l}\mathrm{Tr}((Y-JKW)^T(Y-JKW)) \\
f_2 &= \gamma_A \mathrm{Tr}(W^T K W) \\
f_3 &= \frac{\gamma_I}{(u+l)^2}\mathrm{Tr}(W^T K L K W)
\end{aligned}
$$

Take the derivative of f_1:

$$
\begin{aligned}
\mathrm{d}f_1 &= \frac{1}{l}\mathrm{Tr}(\mathrm{d}(Y-JKW)^T(Y-JKW)+(Y-JKW)^T\mathrm{d}(Y-JKW)) \\
&= \frac{2}{l}\mathrm{Tr}((Y-JKW)^T\mathrm{d}(Y-JKW)) \\
&= \frac{2}{l}\mathrm{Tr}((Y-JKW)^T(-JK)\mathrm{d}W) \\
\frac{\mathrm{d}f_1}{\mathrm{d}W} &= -\frac{2}{l}(JK)^T(Y-JKW)
\end{aligned}
$$

Take the derivative of f_2:

$$
\begin{aligned}
\mathrm{d}f_2 &= \gamma_A \mathrm{Tr}(\mathrm{d}W^T K W + W^T K \mathrm{d}W) \\
&= 2\gamma_A \mathrm{Tr}(W^T K \mathrm{d}W) \\
\frac{\mathrm{d}f_2}{\mathrm{d}W} &= 2\gamma_A K W
\end{aligned}
$$

Take the derivative of f_3:

$$
\begin{aligned}
\mathrm{d}f_3 &= \frac{\gamma_I}{(u+l)^2}\mathrm{Tr}((\mathrm{d}W^T)KLKW + W^T KLK\mathrm{d}W) \\
&= \frac{2\gamma_I}{(u+l)^2}\mathrm{Tr}(W^T KLK\mathrm{d}W) \\
\frac{\mathrm{d}f_3}{\mathrm{d}W} &= \frac{2\gamma_I}{(u+l)^2}KLKW
\end{aligned}
$$

2.

```
  # define Y
  l = len(X_l)
  u = len(X_u)
  X = np.concatenate([X_l, X_u])
  self.X = X
  Y_u = np.zeros([u, Y_l.shape[1]])
  Y = np.concatenate([Y_l, Y_u])

  self.W = np.linalg.inv(J.dot(K) + self.gamma_A*l*np.identity(l+u)
            + (self.gamma_I*l)/(u+l)**2*L.dot(K)).dot(Y)
```

Task2

Task3

1.

```
from sklearn.decomposition import PCA
from sklearn.discriminant_analysis import LinearDiscriminantAnalysis as LDA
from sklearn.manifold import MDS
from sklearn.manifold import Isomap
from sklearn.manifold import LocallyLinearEmbedding as LLE
from sklearn.manifold import TSNE

methods = {
  'PCA': PCA(n_components=2),
  'LDA':LDA(),
  'MDS':MDS(n_components=2),
  'Isomap':Isomap(n_components=2),
  'LLE':LLE(n_components=2),
  't-SNE':TSNE(n_components=2)
}
```

2.

Use the digits dataset for visualization. Visualization results are shown in Fig.1 and Fig.2.

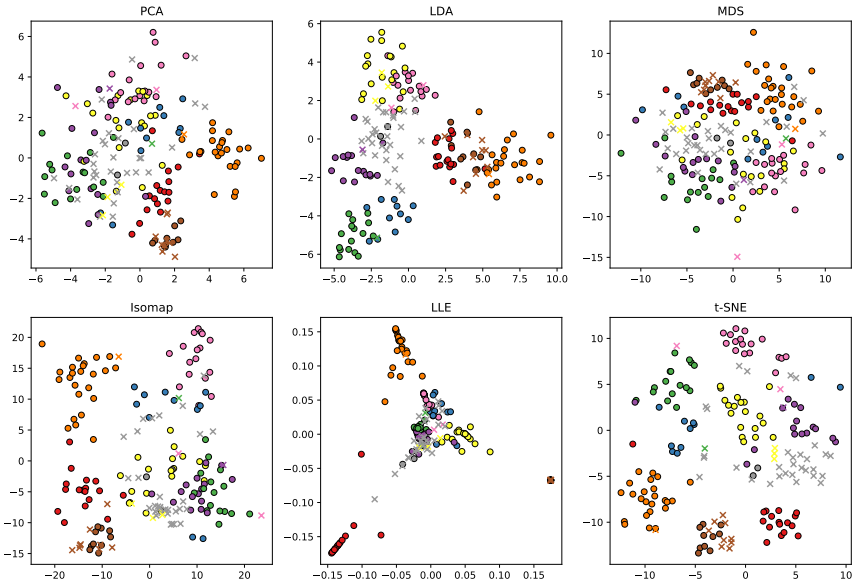

Figure 1: The visualization for RLS

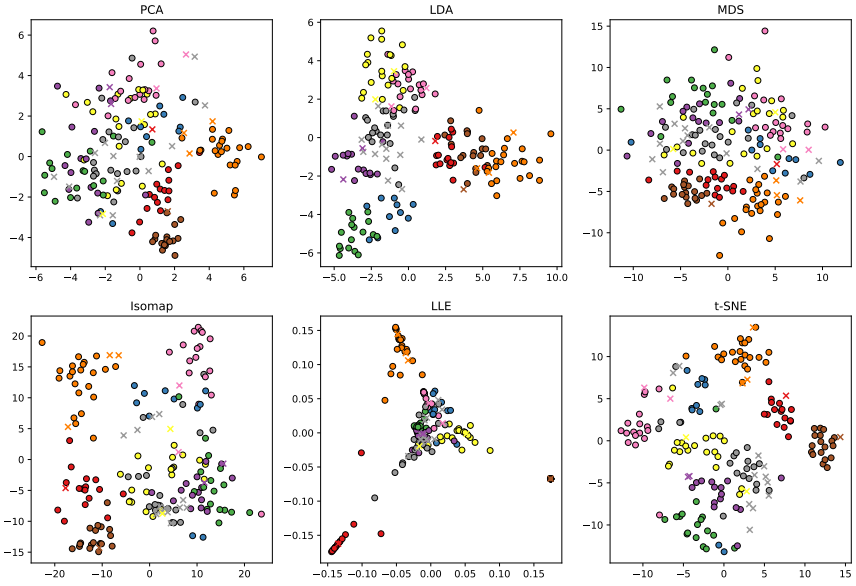

Figure 2: The visualization for LapRLS