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Reversing type II migration: resonance trapping of a lighter giant protoplanet

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ABSTRACT

We present a mechanism related to the migration of giant protoplanets embedded in a protoplanetary disc whereby a giant protoplanet is caught up, before having migrated all the way to the central star, by a lighter outer giant protoplanet. This outer protoplanet may get captured into the 2:3 resonance with the more massive one, in which case the gaps that the two planets open in the disc overlap. Two effects arise, namely a squared mass-weighted torque imbalance and an increased mass flow through the overlapping gaps from the outer disc to the inner disc, which both play in favour of an outwards migration. Indeed, under the conditions presented here, which describe the evolution of a pair of protoplanets respectively Jupiter- and Saturn-sized, the migration is reversed, while the semimajor axis ratio of the planets is constant and the eccentricities are confined to small values by the disc material. The long-term behaviour of the system is briefly discussed, and could account for the high eccentricities observed for the extrasolar planets with semimajor axis $a > 0.2$ au.

Key words: accretion, accretion discs – hydrodynamics – Solar system: formation – planetary systems: formation – planetary systems: protoplanetary discs.

1 INTRODUCTION

In the past few years a number of extrasolar giant planets have been discovered around nearby solar-type stars. These objects masses range from 0.17–11 M_J (where M_J is Jupiter's mass) and their orbital semimajor axis ranges from 0.038–3.3 au (Marcy, Cochran & Mayor 1999). Although many uncertainties remain about planet formation, it is now commonly accepted that planets have formed in and from protoplanetary discs. Necessarily, there must be some time interval over which a giant planet and the surrounding gaseous disc material coexist. The planet and the disc exchange angular momentum through tidal interactions which generally make the planet lose angular momentum. This mechanism is called migration. It can roughly be divided into two regimes.

(i) If the planet mass is small enough, the disc response is linear. The migration rate, in that regime, is proportional to the planet and disc masses, independent of the viscosity and weakly dependent of the disc surface density and temperature profiles. This is the so-called type I migration (Ward 1997).

(ii) When the protoplanet mass is above a certain threshold, the torques acting locally on the surrounding disc material open a gap (Papaloizou & Lin 1984), the width and depth of which are controlled by the balance between the tidal torques, which tend to open the gap, and the viscous torques, which tend to close it. The disc response is significantly non-linear, and most of the protoplanet Lindblad resonances fall in the gap and therefore cannot contribute to the planet–disc angular momentum exchange. The migration rate slows down dramatically compared with type I migration. Furthermore, the tidal truncation splits the disc into two parts and the planet is locked to the disc viscous evolution (Nelson et al. 2000). This is the type II migration, which describes the orbital evolution of giant protoplanets.

In this Letter we consider the coupled evolution of a system of giant protoplanets consisting of two non-accreting cores with masses 1 and 0.29 M_J, which we are going to call from now on respectively 'Jupiter' and 'Saturn'. Attempts have already been made to describe the behaviour of a system of planets embedded in a disc. Melita & Woolfson (1996) and Haghighipour (1999) considered an embedded Jupiter and Saturn system orbiting a solar-mass star, and showed how resonance trapping would affect their evolution. However, the dissipative force in these works was caused by the dynamical friction with a uniform density interplanetary medium, hence type II migration effects were not taken into account. Resonance trapping of planetesimals by a fixed-orbit Jupiter-sized protoplanet has also been investigated by Beaugé, Aarseth & Ferraz-Mello (1994), and shown to be able to build up a single planetary core with orbital characteristics close to Saturn's ones. Kley (2000) studied the orbital evolution of two maximally accreting giant cores embedded in a minimal mass protosolar disc,

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and showed how the migration of the inner core could be halted by the presence of the outer one, and how the eccentricity of the inner core is pumped up by the outer one.

2 RESULTS

2.1 Numerical code description

In order to investigate the long-term behaviour of the embedded Jupiter and Saturn system, we have used two independent hydrocodes, which have been described elsewhere in full detail (Nelson et al. 2000). These two codes are fixed Eulerian grid-based codes; one of them is NIRVANA (Ziegler & Yorke 1997) and the other one has been written by one of us (FM). Both have been endowed with the fast advection FARGO algorithm (Masset 2000), and can run either with this algorithm or with a standard advection algorithm. They gave very similar results. They consist of a pure *N*-body kernel based on either a fourth- (NIRVANA) or fifth-order adaptive time-step Runge–Kutta solver (sufficient for the short time-scales involved in this dissipative problem) embedded in a hydrocode that provides a tidal interaction with a 2D non-self-gravitating gaseous disc. The simulations are performed in the non-inertial non-rotating frame centred on the primary. The grid outer boundary does not allow inflow or outflow and is chosen sufficiently far from the planets in order for the spiral density waves that they launch to be damped before they reach it, while the grid inner boundary only allows outflow (inwards), so that the disc material can be accreted on to the primary. Failing to do so may lead us to overestimate the inner disc density and artificially favours an outwards migration. In the following our length unit is 5.2 au, the mass unit is one solar mass, and the time unit is the initial orbital period of Jupiter (the actual period may vary as Jupiter migrates). The disc aspect ratio H/R is uniform and constant. In the run presented here the grid resolution adopted is $N_r = 122$ and $N_\theta = 300$ with a geometric spacing of the interzone radii such that all the zones are 'as square as possible', i.e. $N_r \log(1 + 2\pi/N_\theta) = \log(R_{\max}/R_{\min})$. The grid outer boundary is at $R_{\max} = 5$ and its inner boundary is at $R_{\min} = 0.4$. The geometric spacing is the most natural one because the disc thickness scales as *r*. On the other hand, a constant spacing leads to an oversampling of the outer disc and an undersampling of the inner one, and therefore is likely to favour an inwards migration.

2.2 Initial setup

The cores we consider are embedded in a gaseous minimal mass protosolar nebula around a unit mass central object, and we assume they start their evolution with semimajor axis $a_{\mathrm{j}} = 1$ for Jupiter and $a_{\mathrm{s}} = 2$ for Saturn. The disc surface density is uniform and corresponds to two Jupiter masses inside Jupiter's orbit. The effective viscosity ν, the nature of which remains unclear and is usually thought to arise from turbulence generated by magneto-hydrodynamic (MHD) instabilities (Balbus & Hawley 1991), is assumed to be uniform through the disc and corresponds to a value of $\alpha \simeq 6 \times 10^{-3}$ in the vicinity of Jupiter's orbit. The disc aspect ratio is $H/r = 0.04$.

The mass of Jupiter is sufficient to open a deep gap and hence it settles in a type II migration (Nelson et al. 2000), whereas Saturn is unable to fully empty its co-orbital region because (i) its mass is smaller and (ii) the planet is in a regime known as the inertial limit (Ward & Hourigan 1989) where the inwards migration speed is so high that it makes the planet pass through what would be the gap inner edge before it had time to actually open it.

Therefore Saturn does not clear a deep gap initially, and its migration rate is typical of type I migration, because all its Lindblad resonances can still contribute to the angular momentum exchange with the disc.

2.3 Run results

We present in Fig. 1 the central star–planet distance curves as a function of time. We see how initially Jupiter migrates as if it were the only planet in the disc (see test run). In the meantime, Saturn starts a much faster migration (the obvious initial acceleration of its migration will be discussed elsewhere), and reaches the 1:2 resonance with Jupiter at time $t \simeq 110$. The eccentricities at that time are small (see Fig. 2), and in particular Saturn's eccentricity is much smaller than the eccentricity threshold below which the capture into resonance is certain if the 'adiabatic' condition on

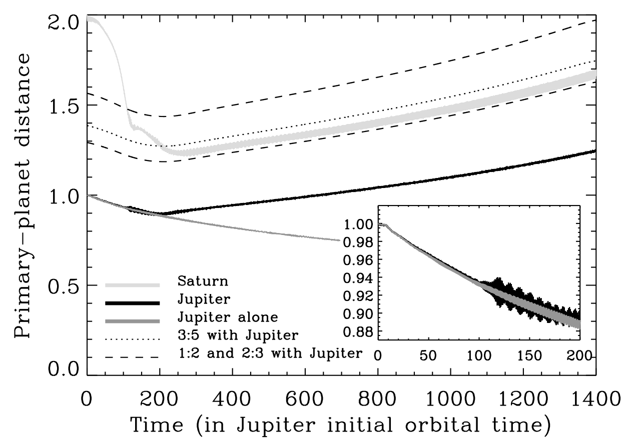

Figure 1. Primary–planet distances as a function of time. The outer dashed curve represents the nominal position of the 1:2 resonance with Jupiter, while the inner dashed curve is the nominal position of the 2:3 resonance. The zoomed plot enables one to compare Jupiter's orbital evolution closely against a test run without Saturn.

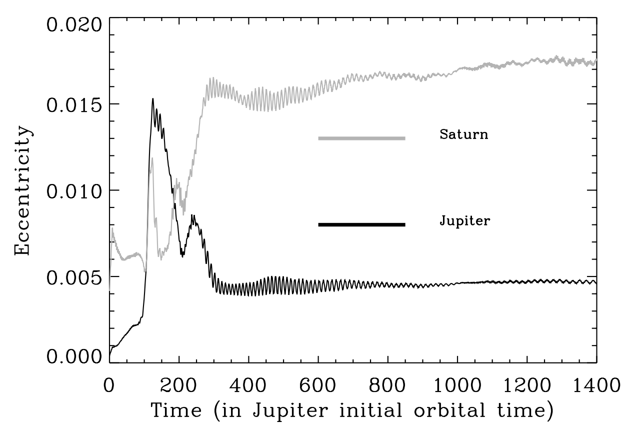

Figure 2. In this figure we see the eccentricities of the planets as a function of time. They simultaneously increase as Saturn passes through the 1:2 and 3:5 resonances with Jupiter. Once Saturn is trapped into the 2:3 resonance with Jupiter, both eccentricities settle at a roughly constant level, which results in a balance between the migration rate that pumps them up and the eccentricity damping by the disc co-orbital material.

the migration rate is satisfied (Malhotra 1993): $|\dot{a}_s|/(a_s\Omega_s) \leqslant 0.5j(j+1)\mu_J e_s$ for the $j:j+1$ resonance, where μ_J is the mass ratio of Jupiter to the central object, and where e_s is Saturn's eccentricity. This condition is not satisfied when Saturn reaches the 1:2 resonance, and it passes through.

The planets then obtain higher eccentricities, and Saturn's migration rate is reduced. This will appear later, and has to do with an increased inwards mass flow. Saturn's eccentricity increases again rapidly as it passes through the 3:5 resonance with Jupiter at $t \simeq 220$. Eventually the adiabatic condition on the migration rate is satisfied for the 2:3 resonance and Saturn's eccentricity is still below the corresponding critical threshold, so it gets trapped into the 2:3 resonance with Jupiter (both e and e' resonances, because the two critical angles $\phi = 3\lambda_S - 2\lambda_J - \tilde{\omega}_S$ and $\phi' = 3\lambda_S - 2\lambda_J - \tilde{\omega}_J$ librate, where λ is the mean longitude and $\tilde{\omega}$ the longitude of perihelion). At that time both planets steadily migrate outwards.

2.4 Interpretation

We define the system of interest as the system composed of the two planets. This resonance-locked system interacts with the inner disc through torques proportional to M_J^2, at Jupiter's inner Lindblad resonances (ILR), whereas it interacts with the outer disc through torques proportional to M_S^2 at Saturn's outer Lindblad resonances (OLR), as indicated in Fig. 3. It can be seen that Saturn's ILR fall in Jupiter's gap and Jupiter's OLR fall in Saturn's gap, so their effect is weakened compared with the situation where Jupiter is alone. As $M_J^2/M_S^2 \sim 10$, the torque imbalance does not favour an inwards migration as strongly as in a one-planet case, and may even lead to a positive differential Lindblad torque on the two-planet system. Actually one can estimate what the maximum mass ratio of the outer planet to the inner one should be to get a migration reversal, if one neglects the inner Lindblad torque on the outer planet and the outer Lindblad torque on the inner planet. The inner Lindblad torque on the inner planet reads as

$$T_{\rm ILR} = C_{\rm ILR}\mu_J^2\Sigma_0 a_J^2(a_J\Omega_J)^2 h'^{-3}, \quad (1)$$

where $C_{\rm ILR}$ is a dimensionless coefficient which is a sizable fraction of unity (Ward 1997), and where h' is the disc aspect ratio. There is a similar formula for the outer Lindblad torque on the outer planet (obtained by substituting the ILR and J indices in equation (1) respectively with OLR and S). The resulting torque imbalance will be positive if: $T_{\rm ILR} > T_{\rm OLR}$, which reads here as

$$\frac{\mu_S}{\mu_J} < \left(\frac{C_{\rm ILR}}{C_{\rm OLR}}\right)^{1/2}\left(\frac{2}{3}\right)^{1/3}. \quad (2)$$

If we assume that $C_{\rm ILR} = C_{\rm OLR}$ then we get $\mu_S/\mu_J < 0.87$, whereas if we make the conservative assumption that $C_{\rm ILR} = 0.5C_{\rm OLR}$, we have $\mu_S/\mu_J < 0.62$. This threshold is much bigger than the actual ratio, therefore if the common gap is deep enough to shut off Jupiter's OLR torques (and Saturn's ILR torques) then the net Lindblad torque on the two-planet system is positive. As the two planet system proceeds outwards in the disc, it does not act on the gas as a snow-plough, but rather it allows the material from the outer disc to travel across the common gap and eventually feed the inner disc. We can find the gap 'permeability' condition by requiring that the rate of angular momentum change of the ring of material lying immediately outside Saturn's gap that is required to expand accordingly to Saturn's orbit (snow-plough effect) is greater than the torque available from Saturn (at most the sum of its outer Lindblad torques, in which case we need to assume that the waves excited at their OLR are damped locally). Here, this turns out to be the case and most of the outer disc material flows through the common gap to the inner disc. We find that in all our runs it is possible to check that the rate of mass flow through the common gap (see Fig. 4) can be expressed as

$$\dot{M} \simeq 3\pi\nu\Sigma_0 + 2\pi r_s\dot{r}_s\Sigma_0 \quad (3)$$

with a reasonable precision (10–20 per cent). Furthermore we

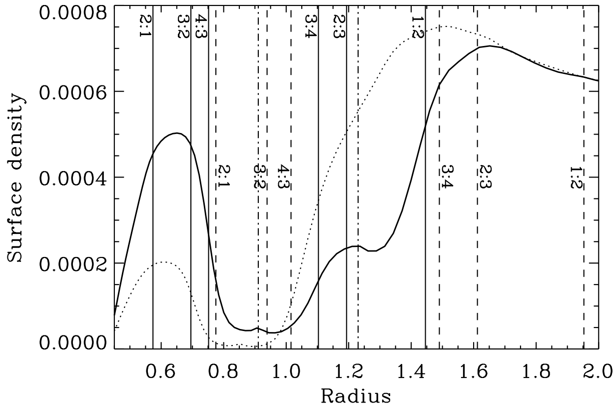

Figure 3. Azimuthally averaged surface density as a function of radius, for the two-planet run (solid curve) and for the test run with Jupiter only (dotted curve), at time $t = 286$ orbits. The solid vertical lines show Jupiter's circular Lindblad resonances, and the dashed lines Saturn's circular Lindblad resonances. The dot–dashed lines at $r = 0.91$ and $r = 1.23$ show respectively the positions of Jupiter (in the two-planet run) and Saturn. As can be seen also in Fig. 1, the Jupiter to Saturn orbital ratio is slightly larger than 3/2. This is a result of the fast precession of the perihelions.

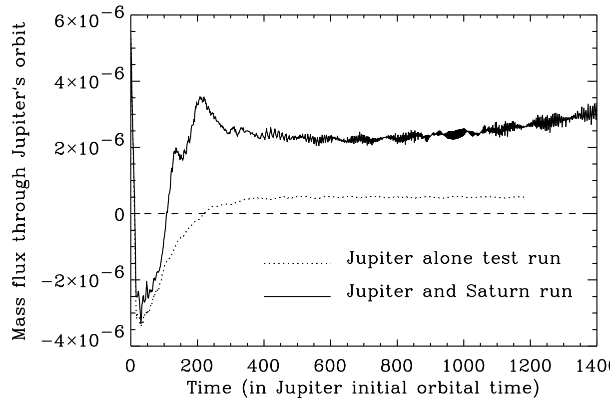

Figure 4. Mass flux crossing Jupiter's orbit (in mass units per orbital time), where positive is for an inwards flow. This quantity can be estimated from the total amount of mass located outside Jupiter's orbit (more precisely outside a circle having a radius equal to Jupiter's semimajor axis, in order to smooth out the short-period variations linked to the eccentricity), because the outer boundary is closed ($v_{\rm rad} \equiv 0$). The negative value at the early stages is caused by a relatively fast inwards migration; it reverses for both runs, even for the Jupiter-only test run (where the inner disc is rapidly depleted). Note that the mass flux is reversed *before* the migration reverses, when Saturn passes through the 1:2 resonance, which more or less corresponds to the time at which the co-orbital regions of both planets merge.

have performed many 'restart runs', which consist of restarting a run once Jupiter and Saturn are locked into resonance, and then by varying one parameter at one time, e.g. the viscosity or the aspect ratio (which changes the Lindblad torques and therefore the migration rate). The mass flux through the gap rapidly switches (in a few tens of orbits) to a new value after the restart, so that equation (3) remains fulfilled.

From the considerations above we can conclude that the presence of Saturn unlocks Jupiter from the disc evolution: the two-planet system evolution (outwards) and the disc viscous evolution (inwards) are basically decoupled. This decoupling and the corresponding mass flow through the common gap has two consequences.

(i) A refilling of the inner disc, which is too depleted for the torques at Jupiter's ILR to have any sizable effect in the one-planet case (the inner disc is accreted on to the primary on its short viscous time-scale and maintaining its surface density at not too low a value implies a permanent flow of material from the outer disc to the inner one).

(ii) The angular momentum lost by the material which flows from the outer disc to the inner one is gained by the planets. The exchange of angular momentum between a planet and a gas fluid element occurs during a 'close encounter' between these two, the one-planet version of which corresponds to the angular momentum exchange at each end of a horseshoe orbit of the fluid element. The resulting torque is the so-called co-orbital corotation torque (Goldreich & Tremaine 1979; Ward 1991, 1992). To the best of our knowledge, an analytical evaluation of the corotation torque in the case of a non-vanishing net mass flow through the orbit (either because of viscous accretion on to the primary or radial migration or both) has not been performed yet. Obviously even the one-planet case deserves a large amount of work on this specific topic, therefore the estimate of the corotation torque in this two-planet problem is far beyond the scope of this paper. We will just comment that the corotation torque in our case might not be negligible compared with the differential Lindblad torque at some stage.

3 DISCUSSION

We have performed a series of restart runs (see Section 2.4) in order to check for a variety of behaviours.

3.1 Differential Lindblad torque sign

The one-sided Lindblad torque has been shown to be proportional to h'^{-3} (Ward 1997). We have performed two restart runs ($h' = 0.04 \rightarrow 0.03$ and $h' = 0.04 \rightarrow 0.05$) in order to check that the migration rate variation is consistent with this dependence. This is indeed the case. We note in passing that the migration rate varies as h'^{-3}, and not as h'^{-2} as would be the case in a one-planet problem, because the outer/inner Lindblad torque asymmetry does not vanish as the disc thickness tends to zero (the OLRs would pile up at Saturn's orbit, whereas the ILRs would pile up at Jupiter's orbit). These results confirm that the behaviour we observe occurs mainly as a result of the differential Lindblad torque and also show that this latter quantity is positive, as expected from equation (2).

3.2 α viscosity versus uniform viscosity

So far we have only considered a uniform viscosity. Switching to a uniform α viscosity of the form $\nu = \alpha c_s H$ makes ν scale here as $r^{1/2}$, so the viscosity at the outer edge of the common gap is higher, whereas it is smaller in the inner disc. This has the following effect, which plays in favour of enhancing the migration reversal mechanism: the viscous time-scale of the inner disc is higher and therefore its surface density increases accordingly, because the material brought through the gap piles up in the inner disc for a longer time before being accreted on to the primary. This has been checked with a restart run.

3.3 Accretion on to the planets

The cores considered above do not accrete gas from the disc. One can wonder what would be the effects of accretion. We have performed a number of restart runs in order to investigate the effect of accretion on the mechanism presented here. We have only considered accretion on to Jupiter, as it is likely that the accretion rate on to Saturn can be regarded as being negligible (i.e. its mass doubling time is much longer than the time-scale of the outwards migration, see e.g. Pollack et al. 1996). The prescription we used to model accretion on to Jupiter consists of removing a proportion of the material which lies in the inner Roche lobe (i.e. a sphere with a radius of half the Hill radius). The amount which is removed in one time-step is calculated from the half-emptying time of the inner Roche lobe $\tau_{1/2}$. We have performed four different restart runs, corresponding to the following values of $\tau_{1/2}$: $\tau_{1/2} = T_0$ (maximally accreting core, see Kley 1999), $3T_0$, $10T_0$ and $30T_0$, where $T_0 = 2\pi/\Omega_J$ is Jupiter's orbital time. In each of these cases, turning on accretion had no impact on the system migration rate, at least in the early stages: in the first case, the mass doubling time for Jupiter is relatively short, and when Jupiter's mass is significantly larger than its initial mass some additional effects, which will be presented in much greater detail elsewhere, affect the migration rate, which then differs from the non-accreting case.

3.4 Smoothing

The smoothing parameter of the potential can have a dramatic impact on Saturn's initial migration rate. This rate is controlled by a subtle balance between outer disc and inner disc torques. In the case of Saturn, all the Lindblad resonances play a role, as there is no gap. Many prescriptions for the smoothing are unable to give trustworthy results for the balance between the outer and inner torques since, depending on the prescription, these two quantities are affected in a different way. On the other hand Jupiter's migration rate is much more robust, because the presence of the gap prevents high-m Lindblad resonances playing a role in the migration, which is therefore controlled only by remote, low m resonances and thus almost insensitive to the smoothing parameter. For this reason we have adopted an approach which involves choosing a smoothing prescription that endows Saturn with a migration velocity of the order of magnitude of the linear analytical predictions (type I migration), which is needed to give correct results for the capture into resonance. Once Saturn is trapped into resonance with Jupiter, it is dynamically slaved by the latter and the system evolution is only very weakly affected by the exact value of the outer disc torque exerted on Saturn. We have found that using either of the two prescriptions below satisfactorily preserves the analytical torque imbalance on Saturn and therefore gives it a type I migration rate.

(i) The potential of a planet acting on the disc is smoothed over the length $\varepsilon = 0.4R_{\mathrm{H}}$ where R_{H} is the Hill radius of the planet under consideration, whereas the potential of the disc acting back on the planet is smoothed over $\varepsilon' = \sqrt{H^2 + d^2}$ where H and d are respectively the local disc thickness and zone diagonal. As $\varepsilon' \neq \varepsilon$, the action–reaction law is not fulfilled and the numerical biases that arise favour an inwards migration, as can be easily checked.

(ii) The potential of a planet acting on the disc and the potential of the disc acting on the same planet are smoothed over $\varepsilon = 0.4R_{\mathrm{H}}$. This prescription does fulfil the action–reaction law.In both these two cases, as in any other which gives Saturn a type I migration rate, including runs performed with a uniform radial spacing, the migration gets reversed. The run presented here corresponds to the first prescription.

3.5 Impact of mass ratio and long-term behaviour

One can wonder about the size of the interval of Saturn's mass that causes the migration to be significantly slowed down or reversed. If 'Saturn' is not massive enough it will not significantly affect Jupiter's evolution (the common 'gap' will be too full on Saturn's side, and therefore Jupiter's OLR torques will not be shut off), whereas if it is too massive, the torque imbalance will be negative again. Work is in progress to determine accurately which range of parameters leads to a migration reversal. It should be noted that the results presented here depend on the artificial initial conditions. We have performed other runs in which Saturn is initially very close either to the 1:2 or 3:5 resonance, and it turns out that neither of these resonances is able to struggle against the strong Lindblad torques on Saturn: no resonance angle can be found that provides a resonant torque on Saturn to counteract the tide. Therefore a trapping into the 2:3 resonance is the most likely outcome when the system is still embedded in a massive disc, whatever the initial conditions: catching-up of 'Saturn' or in situ assembling from smaller, type I migrating bodies.

The long-term behaviour of the system is twofold

(i) The system is locked into resonance as long as the following conditions hold.

(a) The two-planet system can adjust its resonance angle in order to prevent the planets being 'pushed' towards each other by the Lindblad torques exerted by the disc on each of them. In all our runs we have never observed this behaviour. Now, given the small eccentricities involved here, and given the fact that the adiabatic criterion threshold increases as $j(j+1)$, the most probable outcome is that Saturn would then be captured in the next order resonance, that is to say 3:4, and all the physics exposed in this paper would still be valid (presence of a common gap, sharing of the co-orbital material by the two planets, mass-weighted torque imbalance, etc.)

(b) The planets are not pulled apart by any other torques. Now we have mentioned the possibly important role of the co-orbital corotation torque in this problem, which may be sufficient to move the planets apart at some stage, in which case we may ultimately get a low eccentricity double giant planet system when the disc disappears. This will be presented in greater detail elsewhere.

(ii) If the planets happen to be locked into resonance at the time that the gas effects become negligible, then the system is likely to be unstable (we mentioned already that at least two angles librate simultaneously, which strongly suggests a possible chaotic behaviour; see also Kley 2000), and the most likely outcome is that one planet will be ejected whereas the other planet will end up on an eccentric orbit. This could account for the observed eccentricities of the extrasolar planets that are not orbiting close to their host star, i.e. that have not migrated all the way to the star.

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