Problem Set V – Atmospheres

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Hand in HW-problems before the announced date

Hand in before the announced deadline, either before the start of class or by uploading your solutions to the WebLearning system (pdf format only!). Write down your answers in English and state any additional assumptions that you have made, e.g., in case when you think that the phrasing is unclear. Collaboration is allowed, but you should hand in your own solution and be able to reproduce your answer independently if asked to (e.g, in class!). Students whom typeset their answers (with, e.g., LaTeX) or present an excellent layout will be awarded a 10% bonus, if all layout standards have been met. Late returns are penalized at a rate of 10% per day.

Exercise V.1 Solar photosphere

(a) The density of the photosphere is approximately $\rho = 3 \times 10^{-7}\,\mathrm{g\,cm^{-3}}$. Using the Saha equation, show that for the conditions in the solar photosphere hydrogen is mostly neutral.

(b) The dissociation energy of molecular hydrogen is $E_D = 4.48\,\mathrm{eV}$. The photospheric temperature is therefore much lower than the dissociation energy, $kT_\mathrm{eff} < E_D$. On this ground one would (naively) expect a large fraction of molecular hydrogen. Still, the Sun's photosphere contains virtually no molecular hydrogen. Why?

(c) Sunspots are cooler regions on the Sun's photosphere. How cold should they become before molecular hydrogen will make up more than 1% (by number) of the hydrogen? (For simplicity, forget about all internal partition functions, $Z_\mathrm{int} = 1$.)

Exercise V.2 Rotational emission with CO

Angie wants to know the physical conditions – gas density and temperature – of her favorite dark cloud in the Interstellar medium (ISM). To do so, she proposes to look at CO rotational emission lines. Assume that the cloud she wants to observe is characterized by a fixed density and temperature.

(a) Approximating the CO molecule as a rigid rotor with bond length $2a_0$ where a_0 is the Bohr radius, give an expression for the moment of inertia of the CO molecule I, and of the rotational constant B_e where

$$E_J = B_e J(J+1). \tag{1}$$

Give the value of B_e in units of temperature (K).

The *Einstein*-A coefficient A_ul gives the rate at which an excited ("upper" or u) state falls back to the lower ("l") state. In other words A_ul^{-1} is the expectation time of the exited state. You can use in the below that $A_\mathrm{ul}/J_u^3 \approx 8 \times 10^{-8}\,\mathrm{s^{-1}}$ for the CO lines and that the partition function for the CO molecule is $Z_\mathrm{rot} \approx 0.36(T + 0.88)$.

(b) Getting excited from her preparations, Angie argues that the (frequency-integrated) flux at which the cloud radiates in the CO $J \to J-1$ line is

$$F_{\rm ul} = \frac{h\nu_{\rm ul} N_u A_{\rm ul}}{4\pi} \tag{2}$$

where N_u is the *total number* of CO molecules in the excited state. Can you motivate this expression?

(c) With the information you have obtained from the above parts, Plot $\log N_u/N g_u$ (y-axis) *vs.* E_u (x-axis) for $J_u = 1, 2 \dots 10$ and temperatures $T = 10$, 20, and 30 K. Connect your points by lines. Explain how Angie can obtain the temperature of the gas from this rotational line diagram.

(d) For a precise temperature determination, Angie should measures many lines. What is the risk when Angie:

- proposes to observe only low level J lines (say with $J_u = 2$ and $J_u = 1$);
- proposes to observe only high-level J lines (say, $J_u = 8$ and $J_u = 7$)

Exercise V.3 Mars and Titan

The atmosphere of Mars is mostly carbon dioxide (CO_2) and the typical surface pressure is 0.00628 that of the Earth.

(a) Mars has an albedo of 0.25. What is its surface temperature? Ignore the greenhouse effect.

(b) The pressure scaleheight of the atmosphere is defined

$$H = -\frac{P}{dP/dz} \tag{3}$$

using hydrostatic balance and the ideal gas law, write down the expression for the scaleheight? What is the atmosphere scaleheight for Mars?

(c) What is the total mass of the CO_2 atmosphere?

(d) Saturn's moon Titan has a surface pressure of 1.45 atm and is made mostly of N_2. Therefore, it has been argued, that with the brightening of the Sun, Titan may become a good place for human settlement. Argue why this is not an option. What happens to Titan's atmosphere when the Sun ascends the RGB?

Exercise V.4 Habitable Zone

We consider as the Habitable zone (HZ) the region where water exists in a liquid state on the surface of the Earth. If Earth were placed closer to the Sun, it would become too hot; but if it would orbit at distances much further than 1 au, it would become too cold.

(a) At present, the surface temperature of the Earth is (on average) $T_s = 15$ deg C. Kasting et al. (1993)'s model for the HZ gives a range of 0.95 au $\leq d \leq$ 1.37 au for the Earth. Assuming the simple greenhouse model discussed in class, to what range in surface temperature T_s do Kasting et al. (1993)'s values correspond?

(b) Venus has an albedo of $a = 0.77$ a surface temperature of $T_s = 737\,\mathrm{K}$ and orbits at a distance of 0.72 au. Using the greenhouse model, obtain the IR optical depth of Venus atmosphere.

(c) What is the range of the habitable zone for Venus according to the Kasting et al. (1993) model? (Assume again that τ_{ir} and the albedo do not change for Venus).

References

Kasting, J. F., Whitmire, D. P., & Reynolds, R. T. 1993, Icarus, 101, 108