Stars and Planets Problem Set6

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Exercise VI.1 Synodical timescale

(a)

The t_{syn} is the least common multiple of P_1 and P_2:

$$t_{syn} = [P_1, P_2]$$

(b)

how to calculate?

Exercise VI.2 Epicycle approximation

(a)

how to prove?

(b)

how to prove?

(c)

$$\phi'_{\text{eff}}(r_o) = Anr_o^{n-1} - \frac{l_z^2}{2}\frac{2}{r_o^3} = 0$$

$$r_o = (\frac{l_z^2}{An})^{\frac{1}{n+2}}$$

(d)

Expanding the potential around $r = r_o$:

$$\phi_{\text{eff}} = \phi_{\text{eff}}(r_o) + \phi'_{\text{eff}}(r_o)x + \frac{1}{2}\phi''_{\text{eff}}(r_o)x^2$$

where $x = r - r_o$ and $\phi'_{\text{eff}}(r_o) = 0$. Therefore the equation of motion (3) becomes:

$$\ddot{x} = -x\phi''_{\text{eff}}(r_o) = -x(An(n-1)r_o^{n-2} + 3l_z^2 r_o^{-4}) = -(n+2)l_z^2 r_o^{-4}x$$

Compared to $\ddot{x} = -\kappa^2 x$, we have:

$$\kappa = \frac{\sqrt{n+2} l_z}{r_o^2} = \sqrt{n+2}(\frac{l_z^{\frac{2-n}{2}}}{An})^{-\frac{2}{n+2}}$$

In a Keplerian potential (n=-1):

$$\kappa = A^2/l_z^3 = \Omega$$

where Ω is the orbital frequency. why the orbital frequency

(e)

For which values of n does the circular orbit solution become unstable? What is the physical reason?

Exercise VI.3 The Trojans

Exercise VI.4 Tides

(a)

For the Earth-Moon system, the value for n (the mean motion) is $n = \frac{2\pi}{28\text{days}} = 0.22\text{day}^{-1}$.

(b)

The spin-down timescale is:

$$t^{-1}_{\text{de- spin, p}} = \frac{\dot{\Omega}_p}{\Omega_p} = -\frac{\Gamma}{\Omega_p I_p} = -\frac{3k_{2p}}{2QC_I}\frac{m_s^2}{(m_s+m_p)\,m_p}\left(\frac{R_p}{d}\right)^3\frac{n}{\Omega_p}n$$

For the Moon to spin-down the Earth, the love number $k_{2p} =$, the Quality factor $Q =$, the inertia factor $C_I =$, $m_s =$, $m_p =$, $R_p =$, $d =$, $n =$, and $\Omega_P =$. Therefore the timescale for the Moon to spin down the Earth is: .

For the Sun to spin-down the Earth, the love number $k_{2p} =$, the Quality factor $Q =$, the inertia factor $C_I =$, $m_s =$, $m_p =$, $R_p =$, $d =$, $n =$, and $\Omega_P =$. Therefore the timescale for the Sun to spin down the Earth is: .