



# Robust and Intelligent Time Series Analysis and Applications

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Invited Talk at CS Dept. of Tsinghua University

Credits to our group members:

Tian Zhou, Ziqing Ma, Weiqi Chen, Chaoli Zhang, Zhiqiang Zhou, Linxiao Yang, Yingying Zhang, Liang Sun, Wotao Yin, Rong Jin, etc.



# Who we are

- Alibaba DAMO Academy - Decision Intelligence Lab
  - Research Focus:
    - AI for Time Series (AI4TS)
    - Explainable Artificial Intelligence (XAI)
    - Optimization
    - Others: ML, RL, ...
  - Products and Applications:
    - AIOps: Cloud Autoscaling AHPA
    - Green Energy AI: eForecaster
    - Optimization Solver: MindOpt
    - Others: ...
  - More Information:
    - <https://damo.alibaba.com/labs/decision-intelligence>

The screenshot shows the Alibaba DAMO Academy website. At the top, there is a navigation bar with links for 首页 (HOME), 实验室 (LABORATORIES), and 合作生态 (COLLABORATION). The LABORATORIES link is highlighted in red. To the right is the DAMO logo with the text "ALIBABA DAMO ACADEMY". Below the navigation, there are three main categories: Machine Intelligence, Data Computing, and Robotics. Under Machine Intelligence, there are Speech Lab, Vision Lab, Language Technology Lab, and Decision Intelligence Lab (which is also highlighted with a red border). Under Data Computing, there are Computing Technology Lab, Data Analytics and Intelligence Lab, Database and Storage Lab, and OS Lab. Under Robotics, there is Autonomous D. At the bottom of the page, there is a large graphic featuring a hand pointing at a chart with the words "Success", "Solution", "Business Strategy", and "Data Analysis". The chart has various data points and lines. The overall theme is cutting-edge machine learning research.



# Our R&D in *Time Series* Area

- **Publication (2019~now):**
  - 30+ in NeurIPS, ICML, AAAI, KDD, SIGMOD, ICDE, TKDE, PIEEE etc.
  - 20+ Patents: US, China
- **Open Source:** Paper code, Benchmark, Dataset, etc.
- **Other Academic:** Workshop, Tutorial, Competition, Journal SI, AIR with Prof.
- **Products**
  - Time Series Analysis: MindAnalytics (Internal, for 10+ BU, millions of calls per day)
  - Autoscaling: AHPA (at Alibaba Cloud)
  - Energy Forecasting: eForecaster (at Alibaba Cloud)
- **Competition/Awards:**
  - 2023 AAAI/IAAI Innovative Application Award (Autoscaling)
  - 2022 GDEC, Digital Economy Industrial Innovation Achievement
  - 2022 ICASSP Grand Challenge (AIOps in Network), First Place
  - 2021 State Grid New Energy Power Forecasting, Runner-up
  - 2021 International AIOps Challenge, Runner-up
  - ...

DAMO-DI-ML  
AI for Time Series (AI4TS) and XAI

Overview Repositories Projects Packages RI Teams People Settings

Popular repositories

- ICML2022-FEDformer
- AAAII2022-HCM
- NeurIPS2022-FILM
- KDD2022-TFAD
- AI-for-Time-Series-Papers-Tutorials-Surveys
- KDD22-Tutorial-Time-Series
- CIKM22-TFAD

CERTIFICATE OF GOLD PRIZE  
This certificate is presented to  
MindOps  
Qingsong Wan, Chuan Zhang, Chaoqun Zhu,  
Yan Wang, Yuxin Chen, Zhenyu Li, Jiaxin Tang  
For their outstanding contribution to the  
Development and Application of Four-Component Load and  
Forecasting Model for Short-term Power Consumption  
In the 2022 Global Digital Economy Conference  
Issued by Alibaba DAMO Academy  
Date: 22 May 2022

2022全球数字经济大会  
Digital Economy Industrial Innovation Achievement  
阿里云达摩院(北京)科技有限公司  
电力负荷与新能源功率预测产品  
Load and Renewable Energy Forecast System  
在2022全球数字经济大会“新品发布”活动中,经评议,获得  
“数字经济产业创新成果”荣誉称号,特此证书。  
Has won the honorary title of Digital Economy Industrial Innovation Achievement through evaluation in the “New Product Launch” event of 2022 Global Digital Economy Conference and is honored with this certification.  
2022全球数字经济大会组委会办公室  
2022年7月30日

【金秋云创季】 云上聚·创未来

阿里云 最新活动 产品 解决方案 云市场 合作伙伴 支持与服务 开发者 了解阿里云

精准可信电力预测解决方案

提高电力海量数据中潜在的规律和价值，方案深度融合台达摩研究院即时特征提取及可信AI  
提高精度可信的预测服务。

立即咨询





# Outline

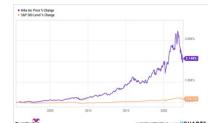
- *Introduction*
- Robust Time Series Processing Blocks
- Robust Time Series Applications



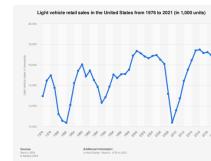
# Time Series Data is Ubiquitous

- A wide range of time series data
  - AIOps
  - IoT
  - Business data, e.g., sales volume, stock price
  - Many others

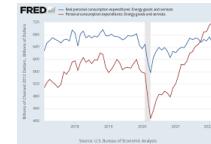
stocks



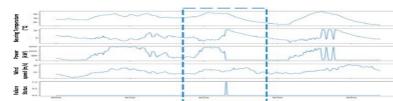
sales



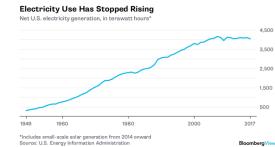
goods consumption



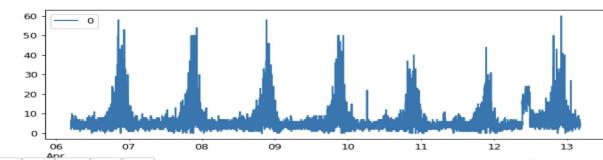
sensor



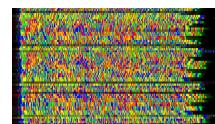
power demand



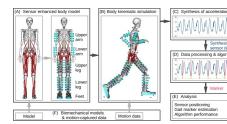
Cloud service monitoring



DNA sequence



motion detect



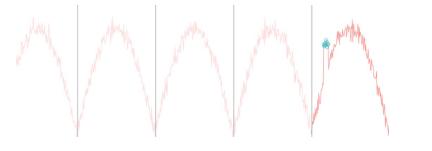
ECG



# Typical Application: Time Series Anomaly Detection

## Different Types of Anomalies

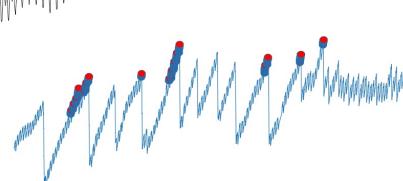
Spike & dip



Change of mean

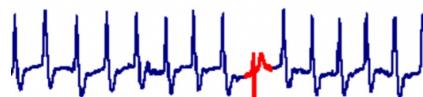


Change of variance



Long-term trend change

Subsequence anomaly



- Business service
- Software service
- Container and VM
- Server and components
- Network
- Data center infrastructure

In typical applications of Elastic Computing Service at Alibaba Cloud (2020):

- 5M+ servers monitored
  - 50M+ metrics monitored
  - In 2020 some typical root identification time is ~ 1 person-month
  - Some cases incur 1M+ USD loss
- 
- **Huge amount of data calls for automatic and accurate anomaly detection algorithm**



# Typical Application: Time Series Forecasting



Energy Forecasting



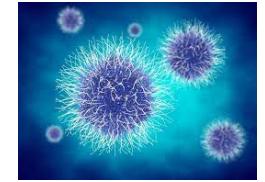
Traffic Forecasting



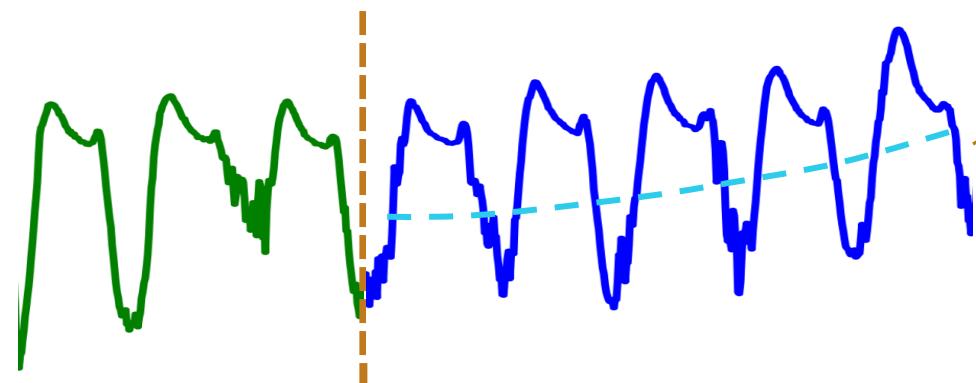
Economics Forecasting



Weather Forecasting



Disease Forecasting



Previous Observations

Future Forecast

**Long-term forecasting**  
for decision and planning

- Modeling complicated temporal dependencies (periodicity and trend) for accurate prediction
- Less computational complexity



# Time Series in Industry

- Time series with big data
  - Hundreds of millions metrics or even more
  - High frequency and real-time processing
  - Limited labels in many applications
  - Low deployment cost
- Time series data with complex patterns
  - Noises and outliers
  - Periodicity/Seasonality: no/single/multiple/variable periodic components
  - Fully automatic solution is challenging but highly preferred
- Connected with other applications
  - Anomaly detection is closely related to root cause analysis
  - Forecasting is closely related to decision-making, e.g., autoscaling

*Our solution: **robust and intelligent** algorithm family for time series analysis*



# Outline

## ❑ Introduction

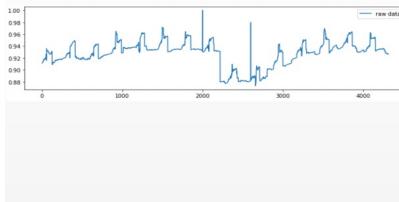
### ➤ ***Robust Time Series Processing Blocks***

- Time Series Periodicity Detection
- Time Series Trend Filtering
- Time Series Seasonal-Trend Decomposition

## ❑ Robust Time Series Applications

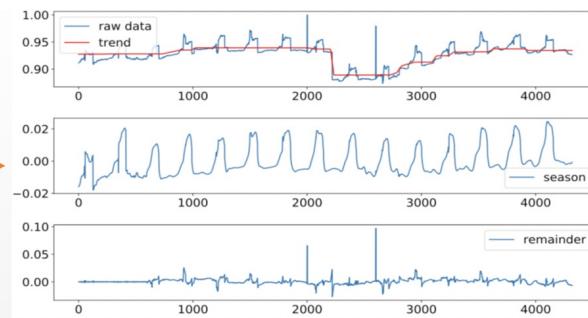
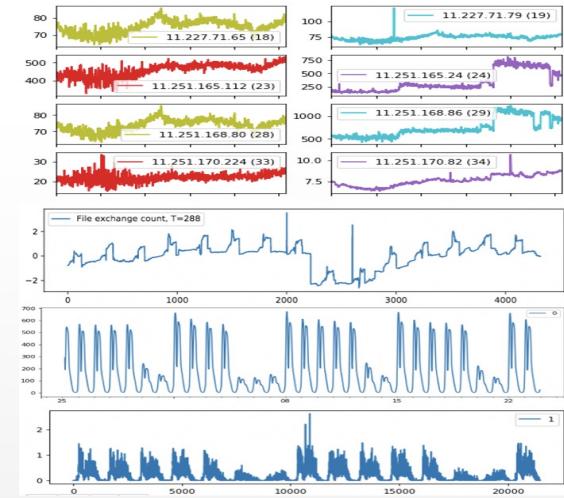
# Complex Periodic Time Series

- Time series
  - Periodicity/seasonality, trend, and noises are common
  - Periodicity detection, decomposition, similarity are crucial for downstream tasks (forecasting, anomaly detection, etc.)
- Challenges of robust time series processing blocks
  - Noise, outliers, missing data, abrupt trend changes
  - None/single/multiple periodicity
  - Need automatic and accurate processing for large-scale industrial time series



Periodicity  
Detection

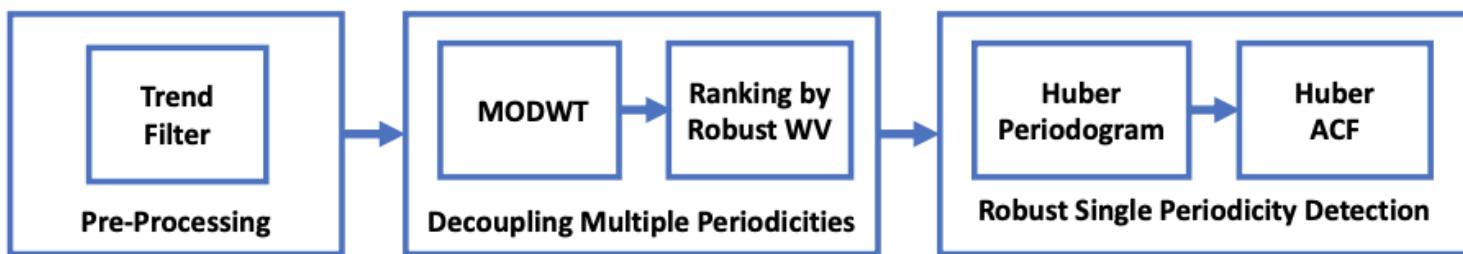
Time Series  
decomposition





# RobustPeriod Algorithm

- Robust and general periodicity detection: RobustPeriod
  - Key idea: *first isolate periodic components*, then detect each one robustly (“divide and conquer”)
  - Three blocks: Pre-processing, Decoupling Multiple Periodicities, Robust Single Periodicity Detection



$$y_t = \tau_t + \sum_{i=1}^m s_{i,t} + r_t$$

For periodic time series, period lengths are denoted as  $T_i, i = 1, \dots, m$ ,  $m$  is the number of periodic components



# Decouple Multiple Periodicities

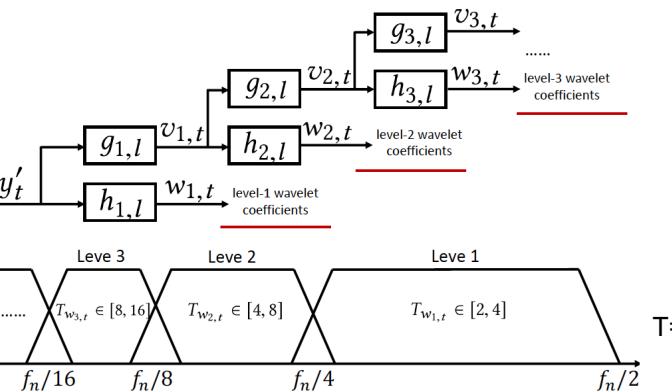
- MODWT for decoupling multiple periodicities
  - MODWT: maximal overlap discrete wavelet transform

*j*th level wavelet coefficients

$$w_{j,t} = \sum_{l=0}^{L_j-1} h_{j,l} y'_{t-l \bmod N}$$

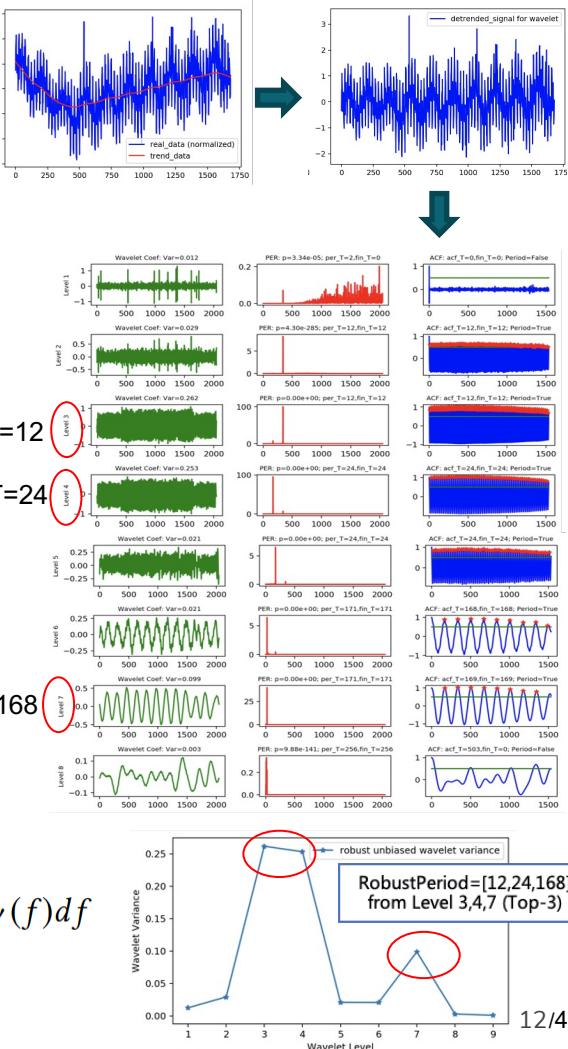
*j*th level scaling coefficients

$$v_{j,t} = \sum_{l=0}^{L_j-1} g_{j,l} y'_{t-l \bmod N}$$



- Wavelet variance ranking for speedup

- Wavelet variance decomposition:  $\hat{\sigma}_{y'}^2 = \sum_{j=1}^{J_0} \hat{\sigma}_{\mathbf{w}_j}^2 + \hat{\sigma}_{\mathbf{v}_{J_0}}^2$
- Relationship to power spectral density (PSD):  $\hat{\sigma}_{\mathbf{w}_j}^2 \approx \int_{1/2^{j+1} \leq |f| \leq 1/2^j} S_{y'}(f) df$
- If there is a periodic component in the  $j$ th level wavelet coefficient, a large wavelet variance would be expected



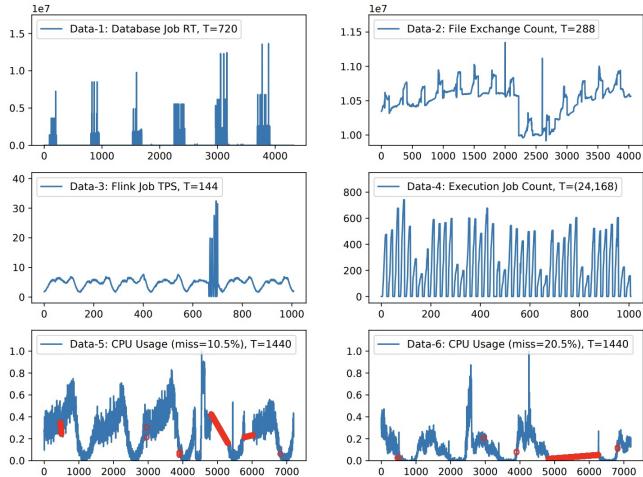


# Periodicity Detection Comparisons

- Real-world datasets:

- Alibaba Cloud monitoring data with outliers, noise, trend, missing data (**single/multiple** periodicity)

Monitoring datasets from Alibaba Cloud



Algorithms	Data-1, T=720 Database RT	Data-2, T=288 File Exchange	Data-3, T=144 Flink TPS
Siegel	(655,769,...)	(288,576,...)	(141,144)
AUTOPERIOD	(353,241,9)	(288,439,...)	(68,141)
Wavelet-Fisher	(372,745,...)	(282,585,...)	(73,146)
RobustPeriod	721	288	144

Algorithms	Data-4, T=(24,168) Job Count	Data-5, T=1440 CPU Usage	Data-6, T=1440 CPU Usage
Siegel	(24,168)	(1459,2597,...)	(1575,1063,...)
AUTOPERIOD	(24,26)	(1488,739,...)	(366,2880,...)
Wavelet-Fisher	(12,24,...)	(1489,712,...)	(1489,364,...)
RobustPeriod	(24,168)	1431	1426

Algorithms	Robust to outliers	Robust to amplitude changes	Robust to trend changes	Support multiple periods	Not need priors for number of periods
ACF	✗	✗	✗	✗	✗
FFT-Fisher	✗	✗	✗	✗	✗
FFT-Siegel	✗	✗	✗	✓	✓
Bandpass+ACF	✗	✓	✗	✓	✗
Wavelet-Fisher	✗	✗	✗	✓	✓
AUTOPERIOD	✗	✗	✗	✓	✓
<b>RobustPeriod</b>	✓	✓	✓	✓	✓



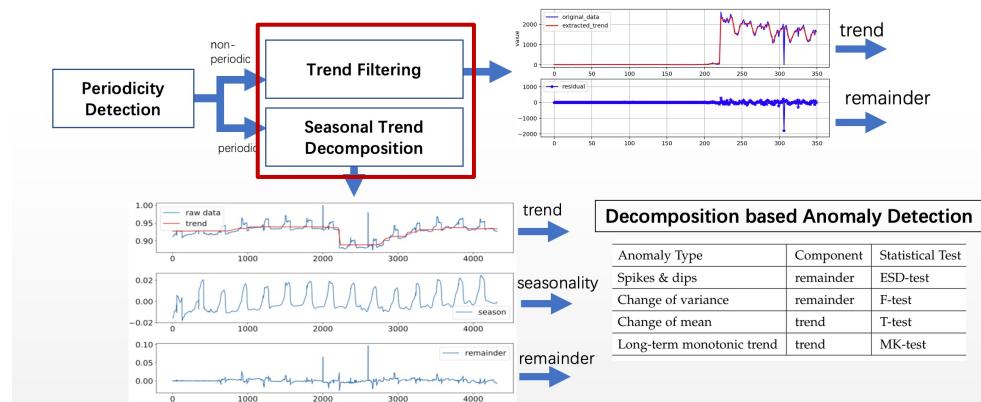
# Time Series Decomposition

- Trend filtering, seasonal-trend decomposition

$$y_t = \tau_t + r_t \quad y_t = \tau_t + \sum_{i=1}^m s_{i,t} + r_t$$

- Why need decomposition

- Insights and interpretability from different components
- Different utilization by components
  - e.g.: anomaly detection

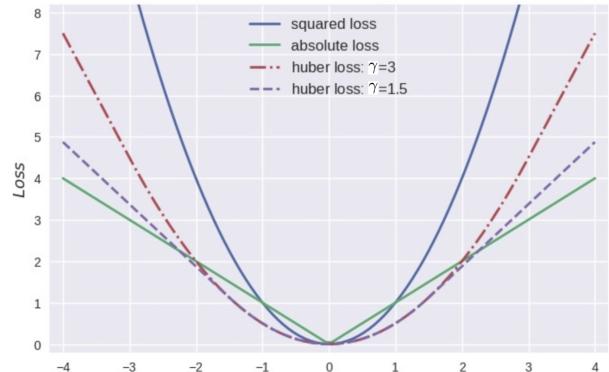
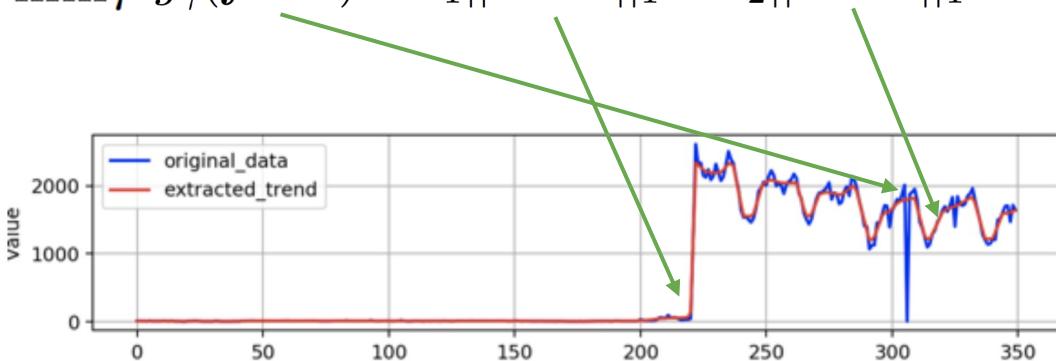




# RobustTrend Filter

- Huber loss: robust to outliers
- 1st order L1 regularization: abrupt trend changes
- 2nd order L1 regularization: slow trend changes and can effectively reduce staircasing effect

$$\min_{\tau} g_\gamma(\mathbf{y} - \boldsymbol{\tau}) + \lambda_1 \|\mathbf{D}^{(1)} \boldsymbol{\tau}\|_1 + \lambda_2 \|\mathbf{D}^{(2)} \boldsymbol{\tau}\|_1 \quad \text{where}$$

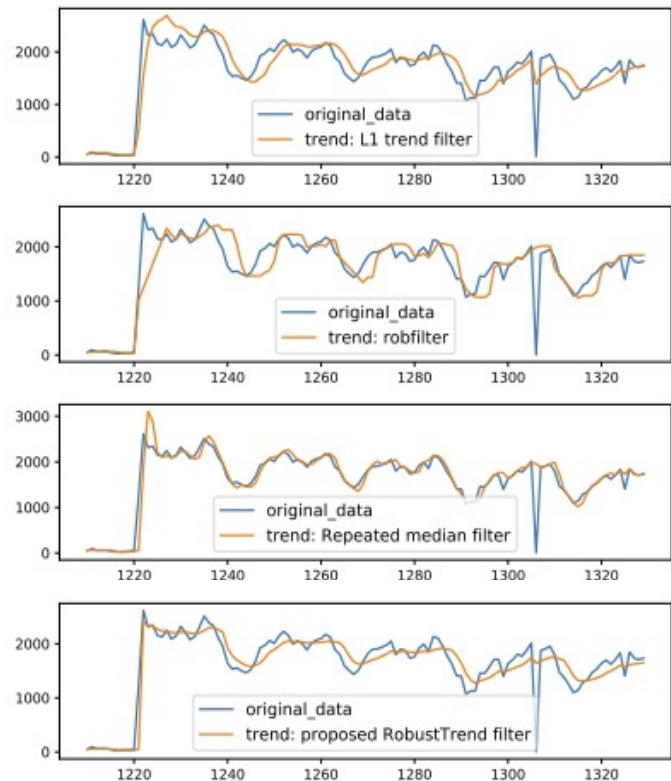


$$g_\gamma(x_i) = \begin{cases} \frac{1}{2}x_i^2, & |x_i| \leq \gamma \\ \gamma|x_i| - \frac{1}{2}\gamma^2, & |x_i| > \gamma \end{cases}$$

$$\mathbf{D}^{(1)} = \begin{bmatrix} 1 & -1 & & \\ & 1 & -1 & \\ & & \ddots & \\ & & & 1 & -1 \end{bmatrix}, \quad \mathbf{D}^{(2)} = \begin{bmatrix} 1 & -2 & 1 & & \\ & 1 & 1 & -2 & 1 & \\ & & \ddots & & \ddots & \\ & & & 1 & -2 & 1 \end{bmatrix}$$

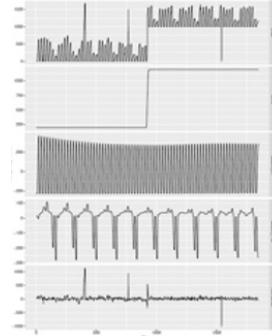
# Experiments on Real-World Data

- Compare trend filters of SOTA models
- Performance highlights
  - L1 trend filter: sensitive to the outliers
  - robfilter: some delay when trend changes
  - Repeated median filter: overshoots trend estimation
  - **RobustTrend filter**: best tradeoff under outliers and abrupt trend changes





# RobustSTL Algorithm



- **Fast and robust seasonal-trend decomposition:**  $y_t = \tau_t + \sum_{i=1}^m s_{i,t} + r_t$ 
  - Key idea: *sequentially and robustly* extract components in time series
  - Four blocks: noise removal, trend extraction, seasonality extraction, multi-season decomposition
  - *Efficient GADMM* for trend extraction and multi-season decomposition:  $O(N/\log N)$



Qingsong Wen, Jingkun Gao, Xiaomin Song, Liang Sun, Huan Xu, Shenghuo Zhu, "RobustSTL: A Robust Seasonal-Trend Decomposition Algorithm for Long Time Series," in Proc. 33th AAAI Conference on Artificial Intelligence (AAAI 2019), Honolulu, Hawaii, Jan. 2019. ([single periodicity version](#))

Qingsong Wen, Zhe Zhang, Yan Li, Liang Sun, "Fast RobustSTL: Efficient and Robust Seasonal-Trend Decomposition for Time Series with Complex Patterns," in Proc. 26th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining (KDD 2020), San Diego, CA, Aug. 2020. ([multiple periodicities and high-speed version](#))

Linxiao Yang, Qingsong Wen, Bo Yang, Liang Sun, "A Robust and Efficient Multi-Scale Seasonal-Trend Decomposition," in Proc. IEEE 46th International Conference on Acoustics, Speech, and Signal Processing (ICASSP 2021), Toronto, Canada, Jun. 2021. ([multiple-scale version](#))



# Trend Extraction

- Robust sparse model: LAD loss with two L1 regularizations

- Mitigate season effect by *seasonal difference*

$$g_t = \nabla_T y'_t = y'_t - y'_{t-T} = \nabla_T \tau_t + \nabla_T s_t + \nabla_T r'_t$$

$$T = \max T_i, i = 1, 2, \dots, m$$

- Estimate the *trend difference robustly*: LAD loss, two L1 regs

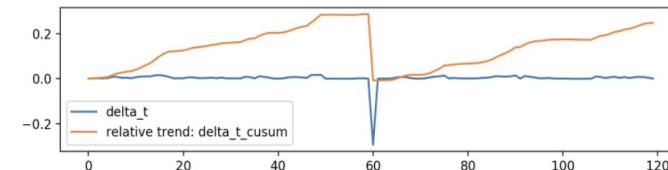
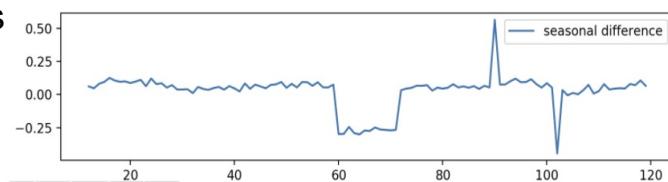
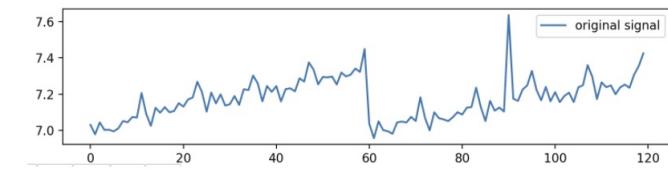
$$\sum_{t=T+1}^N |g_t - \sum_{i=0}^{T-1} \nabla \tau_{t-i}| + \lambda_1 \sum_{t=2}^N |\nabla \tau_t| + \lambda_2 \sum_{t=3}^N |\nabla^2 \tau_t|$$

To capture abrupt change:  $\nabla \tau_t = \tau_t - \tau_{t-1}$

To capture slow change:  $\nabla^2 \tau_t = \tau_t - 2\tau_{t-1} + \tau_{t-2}$

- Finally, recover trend by *cumulative sum*

$$\tilde{\tau}_t^r = \tilde{\tau}_t - \tau_1 = \tilde{\tau}_t - \tilde{\tau}_1 = \begin{cases} 0, & t = 1 \\ \sum_{i=2}^t \nabla \tilde{\tau}_i, & t \geq 2 \end{cases}$$





# Seasonality Extraction

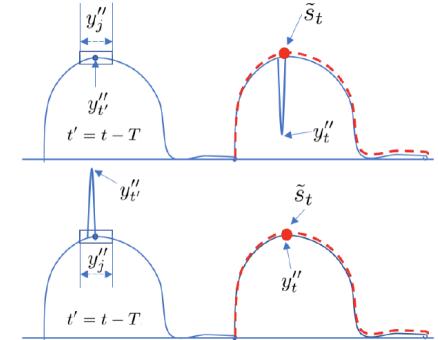
- Weighted non-local seasonal filtering
  - Consider  $K$  neighbourhoods for each season
  - Filter weights based on both value and position difference, similar to Bilateral filtering
  - Robust to *outlier* and adaptive to *seasonal shift*

$$\tilde{s}_t = \frac{1}{z} \sum_{i=1}^m \alpha_i \sum_{(t'_i, j) \in \Omega} w_{(t'_i, j)}^t y''_j$$

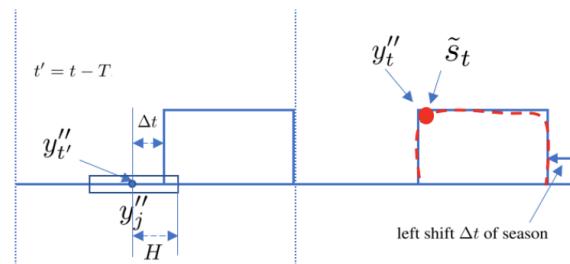
$$w_{(t'_i, j)}^t = e^{-\frac{|j-t'_i|^2}{2\delta_d^2} - \frac{|y''_j - y''_{t'_i}|^2}{2\delta_i^2}}$$

$$\Omega = \{(t'_i, j) | (t'_i = t \pm k \times T_i, j = t'_i \pm h)\}$$

$$k = 1, 2, \dots, K; h = 0, 1, \dots, H$$



(a) Outlier robustness



(b) Season shift adaptation



# GADMM for Efficient Computation: $O(N \log N)$

- ADMM → GADMM for trend extraction

ADMM

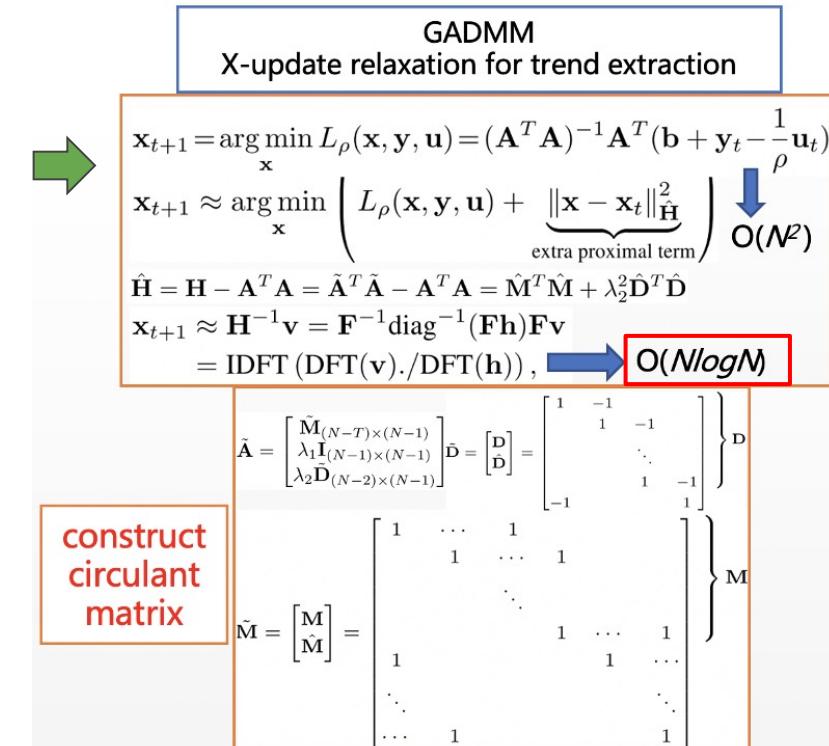
$$\mathbf{x}_{t+1} = \arg \min_{\mathbf{x}} L_\rho(\mathbf{x}, \mathbf{y}, \mathbf{u}) = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T (\mathbf{b} + \mathbf{y}_t - \frac{1}{\rho} \mathbf{u}_t) \quad O(N^2)$$

$$\mathbf{y}_{t+1} = \arg \min_{\mathbf{y}} L_\rho(\mathbf{x}, \mathbf{y}, \mathbf{u}) = S_{1/\rho}(\mathbf{A}\mathbf{x}_{t+1} - \mathbf{b} + \frac{1}{\rho} \mathbf{u}_t) \quad O(N)$$

$$\mathbf{u}_{t+1} = \arg \min_{\mathbf{u}} L_\rho(\mathbf{x}, \mathbf{y}, \mathbf{u}) = \mathbf{u}_t + \rho(\mathbf{A}\mathbf{x}_{t+1} - \mathbf{y}_{t+1} - \mathbf{b}) \quad O(N)$$

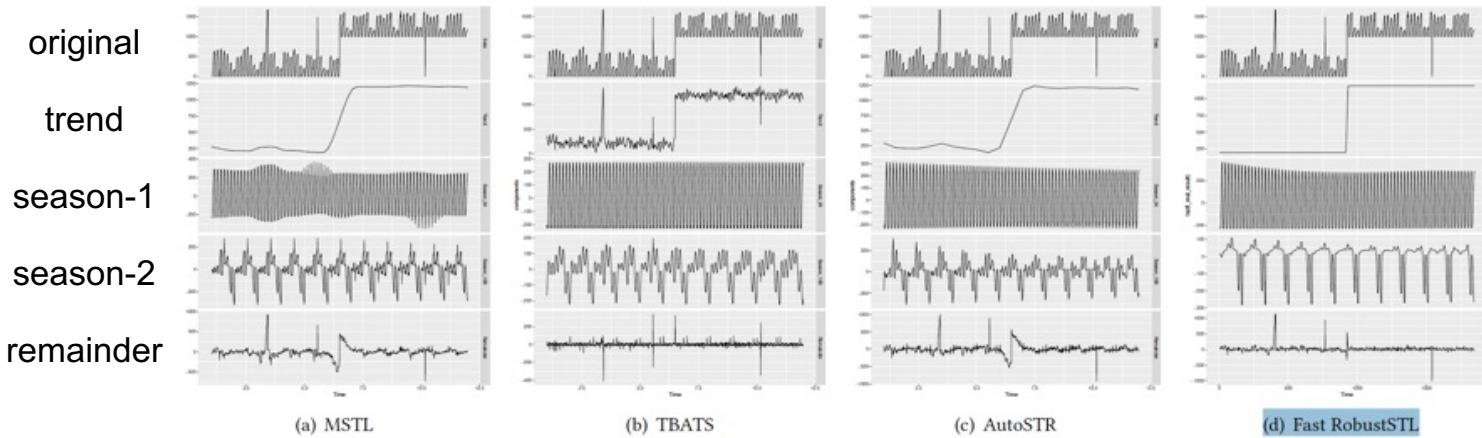
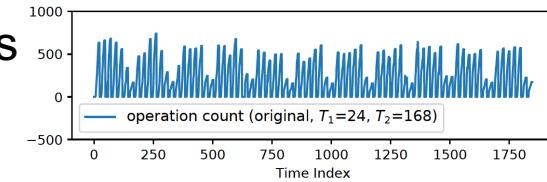
$$\mathbf{A} = \begin{bmatrix} \mathbf{M}_{(N-T) \times (N-1)} \\ \lambda_1 \mathbf{I}_{(N-1) \times (N-1)} \\ \lambda_2 \mathbf{D}_{(N-2) \times (N-1)} \end{bmatrix}, \mathbf{b} = \begin{bmatrix} \mathbf{g}_{(N-T) \times 1} \\ \mathbf{0}_{(2N-3) \times 1} \end{bmatrix}$$

- Similar GADMM can be formulated for multi-season decomposition



# Decomposition on Real-World Data

- Real-world data with daily and weekly periodic components
  - adding abrupt trend change
  - adding 2 single-point outliers, 1 pattern outlier spanning 1 day



RobustSTL is robust to abrupt trend changes and outliers

# Comparisons

- Speed experiments

N	Metrics	RobustSTL-ADMM	RobustSTL-PDHG	<b>Fast RobustSTL</b>
N=1080	Iter	41	310	37
	Time(Sec)	0.142	0.0319	0.0109
	SpeedUp		4.45x	<b>13.0x</b>
N=2160	Iter	48	319	40
	Time(Sec)	1.11	0.0571	0.0295
	SpeedUp		19.4x	<b>37.6x</b>
N=4320	Iter	68	602	37
	Time(Sec)	5.98	0.191	0.0988
	SpeedUp		31.3x	<b>60.5x</b>
N=8640	Iter	92	1377	53
	Time(Sec)	36.7	0.62	0.254
	SpeedUp		59.1x	<b>144x</b>

Algorithm	Time (N=4320)
AutoSTR	4441 seconds
TBATS	60 seconds
(M)STL	0.2 second
<b>RobustSTL</b>	<b>0.1 second</b>

- Algorithm comparisons

Algorithm	Robust to outlier	Robust to seasonal shift	Robust to trend changes	Support multiple periods	Efficient computation
ARIMA/SEATS	✗	✗	✗	✗	✗
SSA	✗	✗	✗	✗	✗
TBATS	✗	✗	✓	✓	✗
STL	✗	✗	✗	✗	✓
MSTL	✗	✗	✗	✓	✗
STR	✓	✓	✗	✓	✗
<b>RobustSTL</b>	✓	✓	✓	✓	✓

→ RobustSTL is more efficient than others



# Outline

## ❑ Introduction

## ❑ Robust Time Series Processing Blocks

### ➤ *Robust Time Series Applications*

- Time Series Anomaly Detection
- Time Series Forecasting
- Other Time Series Applications



# Time Series Anomaly Detection: Background

- Time-series anomalies

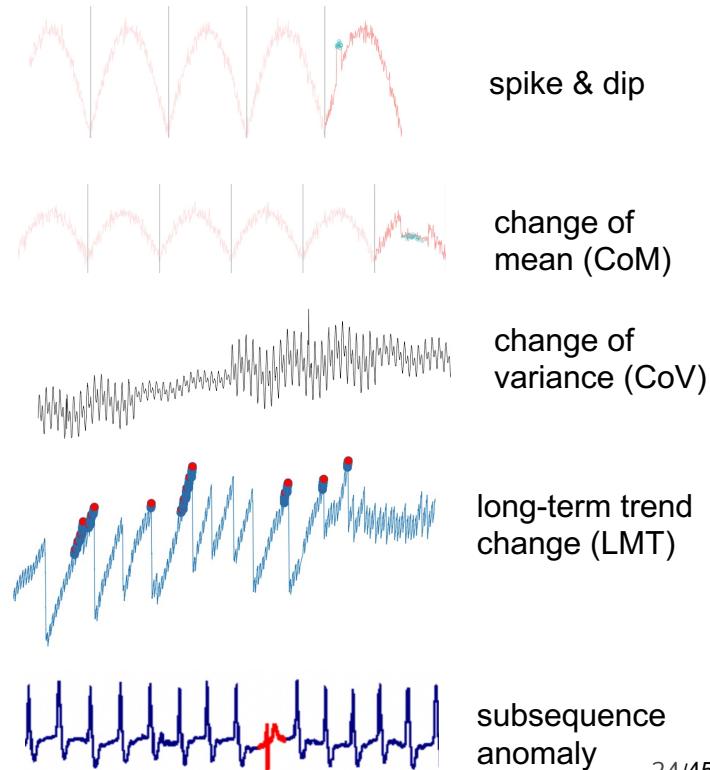
- Points/sequences that significantly deviate from the normal pattern
- **Differences** from static data anomaly detection:
  - Anomalies have temporal context
  - Noise is non-stationary (i.e., time-varying noise)
  - Lack of anomaly labels

- Types of anomalies

- Point: Spikes/Dips, CoM, CoV, LMT
- Subsequence anomaly: identifying anomalous subsequences (sequences of points)

- Models

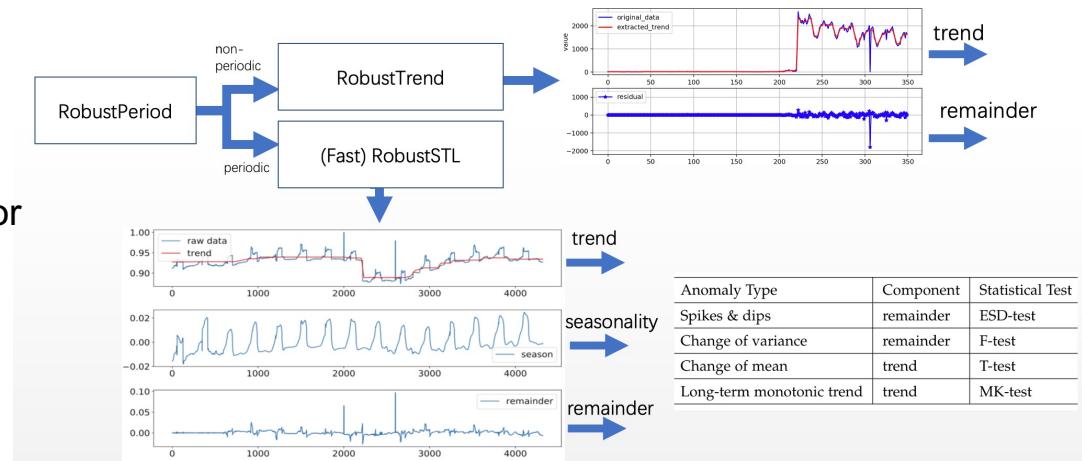
- *Traditional methods, deep models*



# Anomaly Detection: Decomposition based Model

- Robust Decomposition + Statistics

- Robust time series decompositions  
reduce complexity and bring  
explainability
- Robust statistical tests on each  
components lead to high accuracy for  
*different types of anomalies*

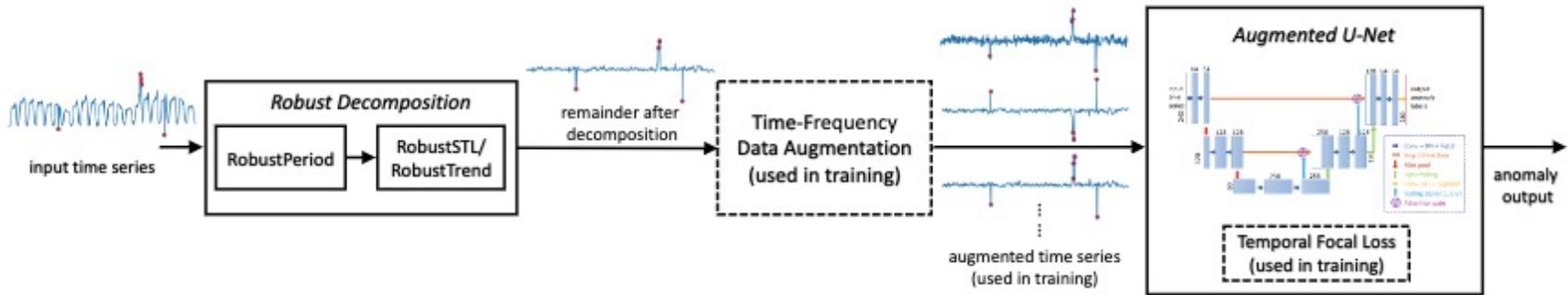




# Anomaly Detection: RobustTAD

- **Decomposition + Data Augmentation + U-Net (deep model)**

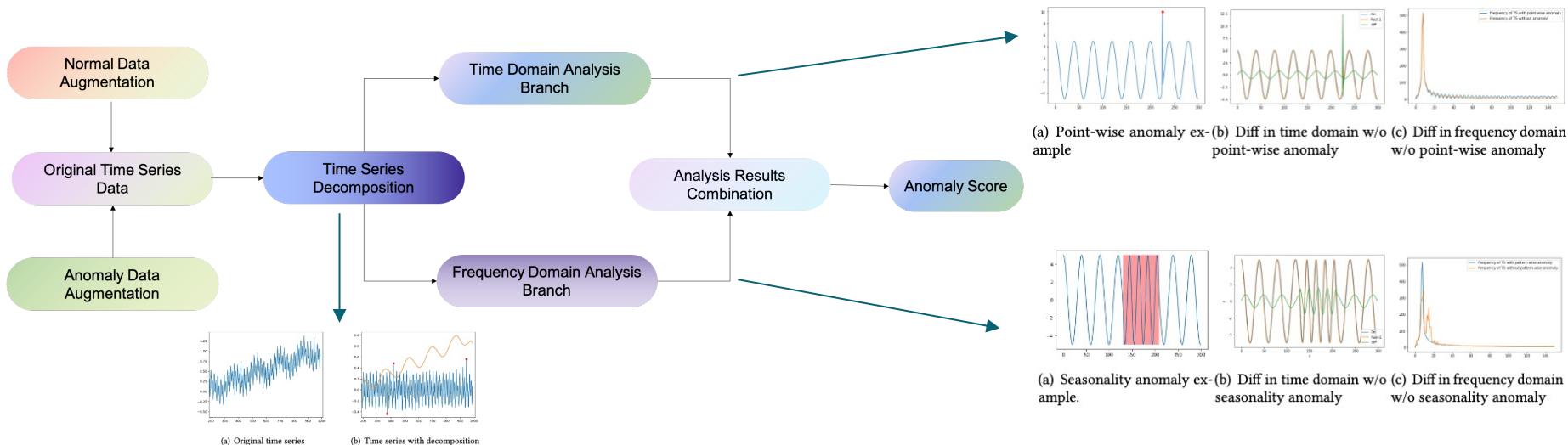
- Robust decomposition: handle complicated patterns, and simplify neural network
- Data augmentation: mitigate the effects of limited labeled data
- U-Net: capture multi-scale information of time series
- Temporal focal loss: label-based weight and value-based weight for unbalanced label





# Anomaly Detection: TFAD

- Data Augmentation + Decomposition + TCN + *Time-Frequency Processing*
  - Anomalies types: seasonal anomaly, trend anomaly, global/context point anomaly
  - Point anomaly easier to detect in **Time domain**; seasonal anomaly easier to detect in **Frequency domain**



## Dataset characteristics

data set	#Curves/Dims	#Points	%Anomaly
KPI	58	5922913	2.26
Yahoo	367	572966	0.68
SMAP	55	429735	12.8
MSL	27	66709	10.5

# Anomaly Detection: TFAD Experiments

- Univariate time series

- Datasets: Yahoo, KPI
- Algs: SPOT, DSPOT, DONUT, SR, SR-CNN, SR-DNN, NCAD
- Main metric: F1 score

### Results of univariate time series anomaly detection

Model	Yahoo (un.)	KPI (un.)	KPI (sup.)
SPOT	33.8	21.7	-
DSPOT	31.6	52.1	-
DONUT	2.6	34.7	-
SR	56.3	62.2	-
SR-CNN	65.2	77.1	-
SR-DNN	-	-	81.1
NCAD	<b>81.16 ± 0.43</b>	76.64 ± 0.89	79.20 ± 0.92
<b>TFAD</b>	<b>81.13 ± 0.52</b>	<b>79.80 ± 0.74</b>	<b>82.10 ± 0.42</b>

- Multivariate time series

- Datasets: SMAP, MSL
- Algs: AnoGAN, DeepSVDD, DAGMM, LSTM-VAE, MSCRED, OmniAnomaly, MTAD-GAT, THOC, NCAD
- Main metric: F1 score

### Results of multivariate time series anomaly detection

Model	SMAP (un.)	MSL (un.)
AnoGAN	74.59	86.39
DeepSVDD	71.71	88.12
DAGMM	82.04	86.08
LSTM-VAE	75.73	73.79
MSCRED	77.45	85.97
OmniAnomaly	84.34	89.89
MTAD-GAT	90.13	90.84
THOC	95.18	93.67
NCAD	<b>94.45 ± 0.68</b>	95.60 ± 0.59
<b>TFAD</b>	<b>96.32 ± 1.57</b>	<b>96.41 ± 0.34</b>

# Anomaly Detection: TFAD Experiments

- Ablation studies

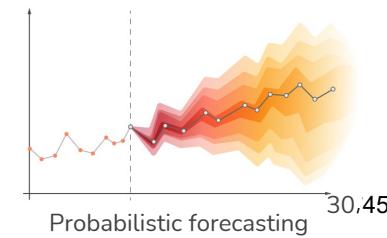
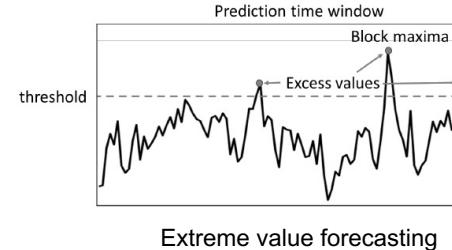
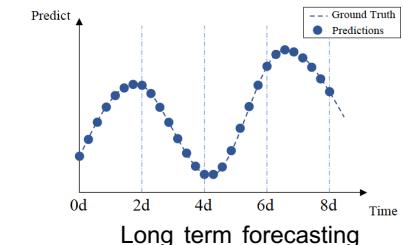
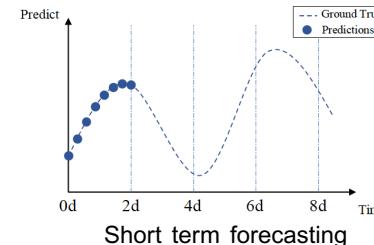
- Decomposition module contributes 30%↑
- Single domain analysis performs not well
- $F(\text{NormalAug} + \text{AbnormalAug}) > F(\text{NormalAug}) + F(\text{AbnormalAug})$
- F1 score increases + variance decreases with frequency branch added

case	TCN	Dec	NorAug	TimeAnAug	FreqAnAug	FreqBran	Precision	Recall	F1 score
Freq Branch									
Time Branch	✓								
(a)	✓	✓					13.81 ± 3.08	35.97 ± 4.40	19.59 ± 2.81
(b)	✓	✓	✓				44.888 ± 0.095	57.227 ± 0.0381	50.312 ± 0.0569
(c)	✓	✓					57.557 ± 5.374	81.111 ± 5.339	66.968 ± 2.58
(d)	✓	✓	✓	✓			57.869 ± 4.65	80.49 ± 4.85	67.099 ± 3.075
(e)	✓	✓	✓	✓			62.569 ± 7.059	<b>89.45 ± 7.84</b>	72.942 ± 2.644
TFAD	✓	✓	✓	✓	✓	✓	68.528 ± 9.412	85.949 ± 11.3698	74.934 ± 2.908
							69.444 ± 7.133	86.638 ± 8.044	76.385 ± 2.387
							<b>79.176 ± 1.875</b>	85.231 ± 1.464	<b>82.058 ± 0.4199</b>



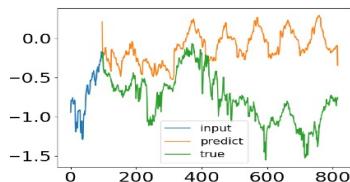
# Time Series Forecasting: Background

- Different forecasting types
  - **Short-term** forecasting: predict the near future
  - **Long-term** forecasting: predict the future with an extended period
  - **Extreme value** forecasting: predict the extreme values
  - **Point or Probabilistic** forecasting: predict point value or interval/probability distribution
- Challenges:
  - Accuracy, robustness
- Models:
  - Traditional: Statistical (ARIMA, ETS, Prophet)
  - Ensemble: Tree, MLP
  - Deep Models: CNN, RNN, Transformers



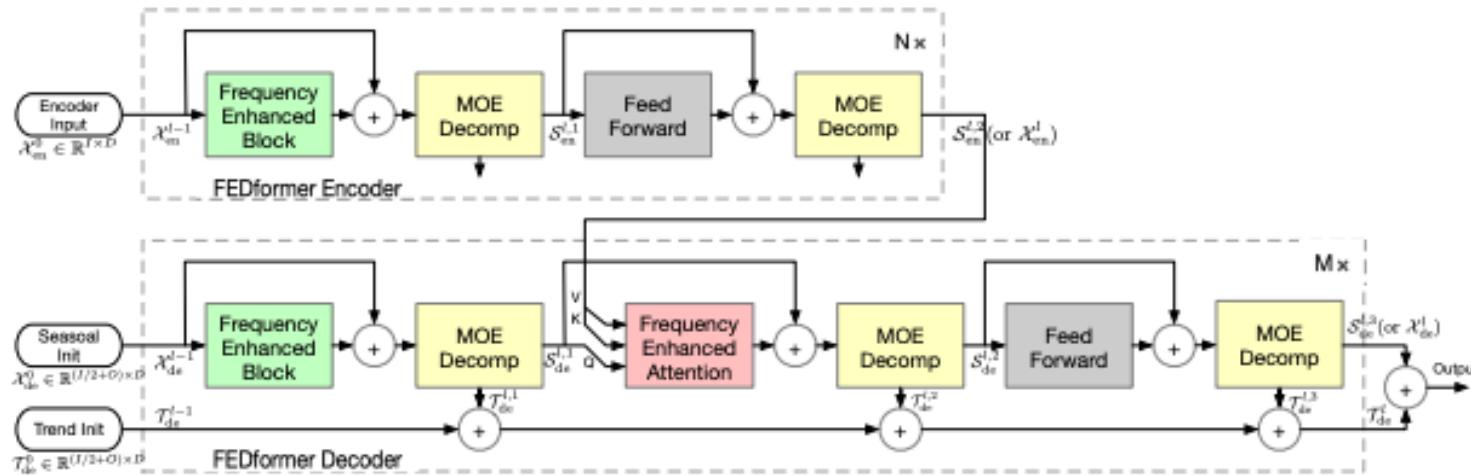


# Forecasting with Transformer: FEDformer



Trend and seasonality discrepancy  
of the conventional Transformer model

- **FEDformer: frequency enhanced decomposed Transformer**
  - Efficient and robust *frequency domain processing*: to capture key structures in time series
  - Mixture of experts *seasonal-trend decomposition*: to better capture global properties in time series





# Forecasting with Transformer: FEDformer

- FEDformer: frequency enhanced decomposed Transformer
  - Frequency enhanced block: substitute self-attention
  - Frequency enhanced attention: substitute cross-attention
  - Mixture of experts STL: to better capture global properties in time series

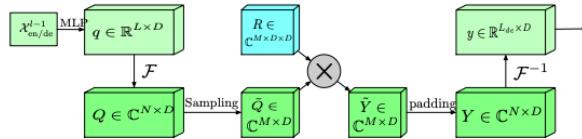


Figure 3. Frequency Enhanced Block with Fourier transform (FEB-f) structure.

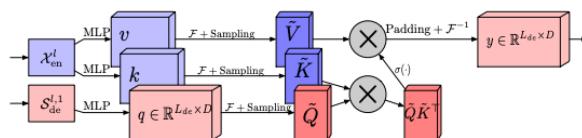
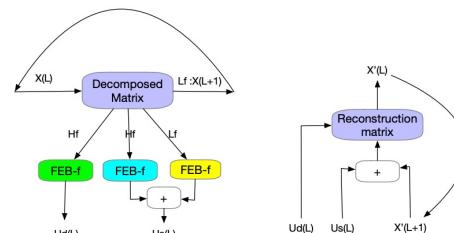


Figure 4. Frequency Enhanced Attention with Fourier transform (FEA-f) structure,  $\sigma(\cdot)$  is the activation function.



**MOE Seasonal-Trend Decomposition**

$$\mathbf{X}_{\text{trend}} = \text{Softmax}(L(x)) * (F(x)), \quad (10)$$

where  $F(\cdot)$  is a set of average pooling filters and  $\text{Softmax}(L(x))$  is the weights for mixing these extracted trends.

Figure 5. Top Left: Wavelet frequency enhanced block decomposition stage. Top Right: Wavelet block reconstruction stage shared by FEB-w and FEA-w. Bottom: Wavelet frequency enhanced cross attention decomposition stage.

# Forecasting with Transformer: FEDformer

Empirical comparison of FEDformer on six benchmark datasets

Table 2. Multivariate long-term series forecasting results on six datasets with input length  $I = 96$  and prediction length  $O \in \{96, 192, 336, 720\}$  (For ILI dataset, we use input length  $I = 36$  and prediction length  $O \in \{24, 36, 48, 60\}$ ). A lower MSE indicates better performance, and the best results are highlighted in bold.

Methods	Metric	ETTm2				Electricity				Exchange				Traffic				Weather				ILI			
		96	192	336	720	96	192	336	720	96	192	336	720	96	192	336	720	96	192	336	720	24	36	48	60
FEDformer-f	MSE	<b>0.203</b>	<b>0.269</b>	<b>0.325</b>	<b>0.421</b>	0.193	0.201	0.214	0.246	0.148	0.271	0.460	1.195	0.587	0.604	0.621	0.626	<b>0.217</b>	<b>0.276</b>	<b>0.339</b>	<b>0.403</b>	3.228	2.679	2.622	2.857
	MAE	<b>0.287</b>	<b>0.328</b>	<b>0.366</b>	<b>0.415</b>	0.308	0.315	0.329	0.355	0.278	0.380	0.500	0.841	0.366	0.373	0.383	0.382	<b>0.296</b>	<b>0.336</b>	<b>0.380</b>	<b>0.428</b>	1.260	1.080	1.078	1.157
FEDformer-w	MSE	0.204	0.316	0.359	0.433	<b>0.183</b>	<b>0.195</b>	<b>0.212</b>	<b>0.231</b>	<b>0.139</b>	<b>0.256</b>	<b>0.426</b>	<b>1.090</b>	<b>0.562</b>	<b>0.562</b>	<b>0.570</b>	<b>0.596</b>	0.227	0.295	0.381	0.424	<b>2.203</b>	<b>2.272</b>	<b>2.209</b>	<b>2.545</b>
	MAE	0.288	0.363	0.387	0.432	<b>0.297</b>	<b>0.308</b>	<b>0.313</b>	<b>0.343</b>	<b>0.276</b>	<b>0.369</b>	<b>0.464</b>	<b>0.800</b>	<b>0.349</b>	<b>0.346</b>	<b>0.323</b>	<b>0.368</b>	0.304	0.363	0.416	0.434	<b>0.963</b>	<b>0.976</b>	<b>0.981</b>	<b>1.061</b>
Autoformer	MSE	0.255	0.281	0.339	0.422	0.201	0.222	0.231	0.254	0.197	0.300	0.509	1.447	0.613	0.616	0.622	0.660	0.266	0.307	0.359	0.419	3.483	3.103	2.669	2.770
	MAE	0.339	0.340	0.372	0.419	0.317	0.334	0.338	0.361	0.323	0.369	0.524	0.941	0.388	0.382	0.337	0.408	0.336	0.367	0.395	0.428	1.287	1.148	1.085	1.125
Informer	MSE	0.365	0.533	1.363	3.379	0.274	0.296	0.300	0.373	0.847	1.204	1.672	2.478	0.719	0.696	0.777	0.864	0.300	0.598	0.578	1.059	5.764	4.755	4.763	5.264
	MAE	0.453	0.563	0.887	1.338	0.368	0.386	0.394	0.439	0.752	0.895	1.036	1.310	0.391	0.379	0.420	0.472	0.384	0.544	0.523	0.741	1.677	1.467	1.469	1.564
LogTrans	MSE	0.768	0.989	1.334	3.048	0.258	0.266	0.280	0.283	0.968	1.040	1.659	1.941	0.684	0.685	0.7337	0.717	0.458	0.658	0.797	0.869	4.480	4.799	4.800	5.278
	MAE	0.642	0.757	0.872	1.328	0.357	0.368	0.380	0.376	0.812	0.851	1.081	1.127	0.384	0.390	0.408	0.396	0.490	0.589	0.652	0.675	1.444	1.467	1.468	1.560
Reformer	MSE	0.658	1.078	1.549	2.631	0.312	0.348	0.350	0.340	1.065	1.188	1.357	1.510	0.732	0.733	0.742	0.755	0.689	0.752	0.639	1.130	4.400	4.783	4.832	4.882
	MAE	0.619	0.827	0.972	1.242	0.402	0.433	0.433	0.420	0.829	0.906	1.016	0.423	0.420	0.420	0.420	0.423	0.596	0.638	0.596	0.792	1.382	1.448	1.465	1.483

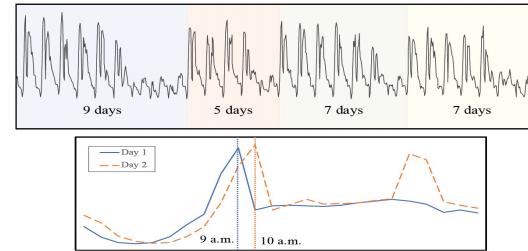
Linear complexity of FEDformer

Table 1. Complexity analysis of different forecasting models.

Methods	Training		Testing
	Time	Memory	
FEDformer	$\mathcal{O}(L)$	$\mathcal{O}(L)$	1
Autoformer	$\mathcal{O}(L \log L)$	$\mathcal{O}(L \log L)$	1
Informer	$\mathcal{O}(L \log L)$	$\mathcal{O}(L \log L)$	1
Transformer	$\mathcal{O}(L^2)$	$\mathcal{O}(L^2)$	L
LogTrans	$\mathcal{O}(L \log L)$	$\mathcal{O}(L^2)$	1
Reformer	$\mathcal{O}(L \log L)$	$\mathcal{O}(L \log L)$	L
LSTM	$\mathcal{O}(L)$	$\mathcal{O}(L)$	L

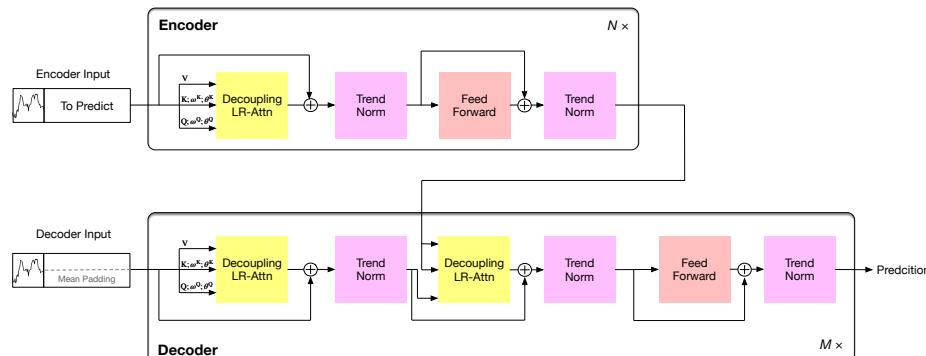


# Forecasting with Transformer: Quatformer

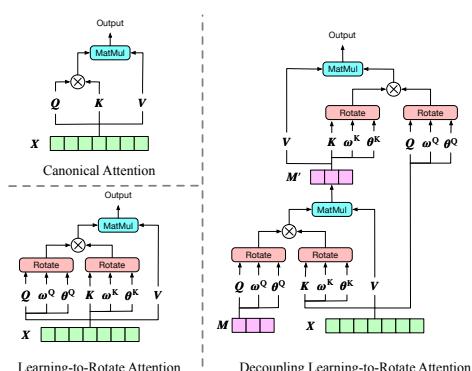


Multiple/variable periods and phase shift

- **Quatformer: Transformer with quaternions for periodic time series**
  - Learning-to-rotate attention (LR-Attn): modeling multiple *periods and periodic changes* by quaternions
  - Trend normalization: modeling slowly varying trend



Quatformer architecture



Learning-to-rotate attention

$$\begin{aligned} \gamma & \odot (\mathcal{X} - \text{MovingAvg}(\mathcal{X})) + \mathcal{T}, \\ \sigma & = \sqrt{\frac{1}{N} \sum_{i=1}^N (\mathcal{X}_i - \mu)^2}, \quad \mu = \frac{1}{N} \sum_{i=1}^N \mathcal{X}_i, \\ \mathcal{T} & = \sum_{i=0}^p \beta_i \text{pos}^i, \quad \text{pos} = [0, 1, 2, \dots, N-1]^\top / N. \end{aligned}$$

Trend normalization



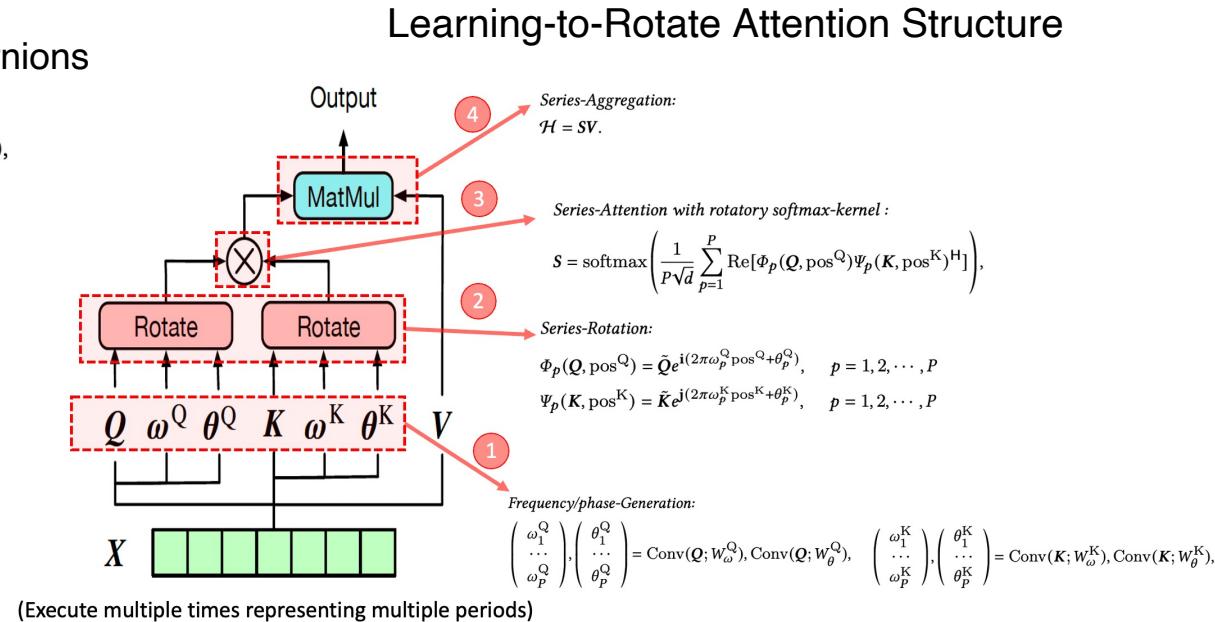
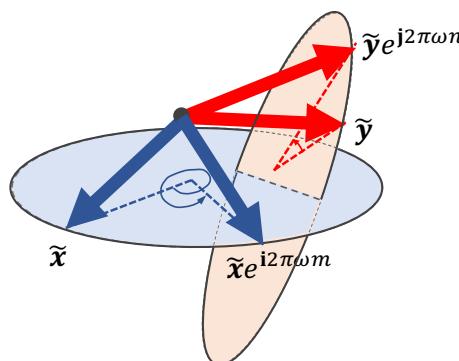
# Forecasting with Transformer: Quatformer

- Quatformer: LR Attention

## Rotatory Softmax-kernel with Quaternions

$$\text{SM}^{\text{rot}}(\phi(\mathbf{x}, m), \psi(\mathbf{y}, n)) = \exp(\text{Re}[\phi(\mathbf{x}, m)^H \psi(\mathbf{y}, n)]),$$

$$\phi(\mathbf{x}, m) = \tilde{\mathbf{x}} e^{i2\pi\omega m}, \quad \psi(\mathbf{y}, n) = \tilde{\mathbf{y}} e^{i2\pi\omega n},$$

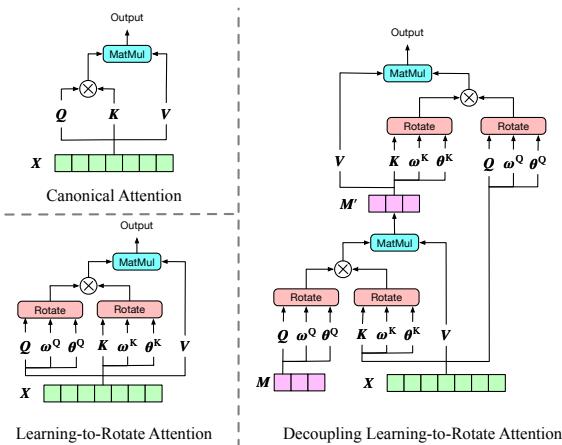


(Execute multiple times representing multiple periods)



# Forecasting with Transformer: Quatformer

- Quatformer: Decoupling LR-Attn, Performance
  - LR-Attn's complexity is  $O(N^2)$
  - Decoupling LR-Attn introduces a momentum-updated  $c$ -length latent series  $\mathcal{M} \in \mathbb{R}^{c \times d}$ , and decouple  $\mathcal{H} = \text{LR-Attn}(\mathcal{X}, \mathcal{Y})$  into



$$\begin{aligned}\mathcal{H} &= \text{LR-Attn}(\mathcal{X}, \mathcal{M}') \in \mathbb{R}^{N \times d}, \\ \mathcal{M}' &= \text{LR-Attn}(\mathcal{M}, \mathcal{Y}) \in \mathbb{R}^{c \times d}.\end{aligned}$$

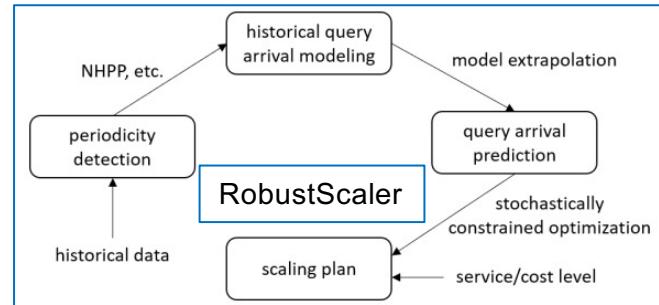
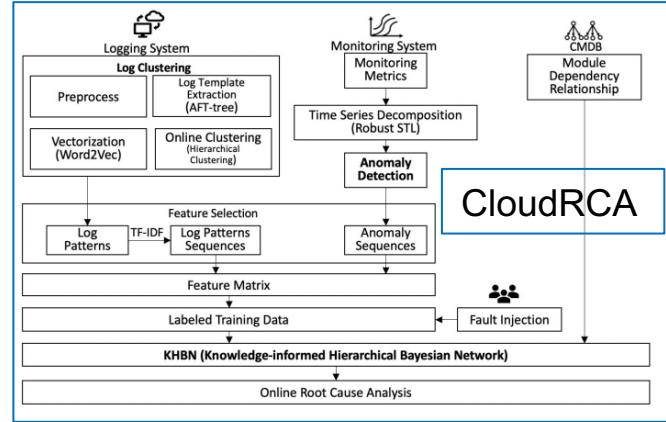
Complexity is decreased to  $O(2cN)$ .

Performance of Quatformer

Models	Quatformer		Quatformer <sup>†</sup>		Autoformer	
	MSE	MAE	MSE	MAE	MSE	MAE
ETTh	<b>0.403</b>	<b>0.434</b>	0.426	0.450	0.442	0.451
	<b>0.444</b>	<b>0.463</b>	0.453	0.466	0.500	0.482
	<b>0.452</b>	<b>0.455</b>	0.464	0.474	0.512	0.492
	0.477	0.490	<b>0.474</b>	<b>0.487</b>	0.514	0.512
ETThm	0.220	0.301	<b>0.217</b>	<b>0.297</b>	0.247	0.325
	0.279	0.333	<b>0.269</b>	<b>0.329</b>	0.278	0.335
	0.331	0.354	<b>0.330</b>	0.366	0.336	0.370
	0.422	0.413	0.433	0.428	0.439	0.435
Weather	<b>0.211</b>	<b>0.279</b>	0.213	0.287	0.259	0.332
	<b>0.263</b>	<b>0.325</b>	0.265	0.326	0.300	0.359
	<b>0.310</b>	<b>0.344</b>	0.315	0.354	0.364	0.401
	<b>0.381</b>	<b>0.374</b>	0.382	0.378	0.439	0.440
Exchange	<b>0.147</b>	<b>0.274</b>	0.148	0.276	0.154	0.284
	<b>0.254</b>	<b>0.364</b>	0.255	0.365	0.272	0.381
	0.427	0.481	<b>0.425</b>	<b>0.480</b>	0.461	0.509
	<b>0.974</b>	<b>0.751</b>	1.095	0.800	1.100	0.813
Traffic	0.618	<b>0.384</b>	<b>0.617</b>	0.387	0.636	0.397
	0.619	<b>0.384</b>	<b>0.600</b>	<b>0.367</b>	0.618	0.381
	0.622	<b>0.384</b>	<b>0.618</b>	0.385	0.626	0.388
	0.629	0.383	<b>0.616</b>	<b>0.379</b>	0.653	0.400
Electricity	<b>0.197</b>	<b>0.308</b>	0.200	0.311	0.203	0.318
	<b>0.205</b>	<b>0.302</b>	0.216	0.330	0.233	0.338
	<b>0.220</b>	<b>0.329</b>	0.228	0.343	0.259	0.359
	0.245	0.350	<b>0.242</b>	<b>0.349</b>	0.255	0.361

# Other Time Series Applications

- Fault Cause Localization
  - *From Anomaly Detection to Localization*
  - CloudRCA<sup>[1]</sup>, NetRCA<sup>[2]</sup>, Rule Set<sup>[3]</sup>
- Autoscaling
  - *from Forecasting to Decision-Making*
  - RobustScalar<sup>[4]</sup>, AHPA<sup>[5]</sup>
- More...



[1] Yingying Zhang, Zhengxiong Guan, Huajie Qian, Leili Xu, Hengbo Liu, Qingsong Wen, Liang Sun, Junwei Jiang, Lunting Fan, Min Ke, "CloudRCA: A Root Cause Analysis Framework for Cloud Computing Platforms," CIKM 2021.

[2] Chaoli Zhang\*, Zhiqiang Zhou\*, Yingying Zhang\*, Linxiao Yang\*, Kai He\*, Qingsong Wen\*, Liang Sun\* (\*Equally Contributed), "NetRCA: An Effective Network Fault Cause Localization Algorithm," ICASSP 2022. (**ICASSP'22 AIOps Challenge, First Place (1/382)**)

[3] Fan Yang, Kai He, Linxiao Yang, Hongxia Du, Jingbang Yang, Bo Yang, Liang Sun, "Learning Interpretable Decision Rule Sets: A Submodular Optimization Approach," **NeurIPS 2021 Spotlight**.

[4] Huajie Qian, Qingsong Wen, Liang Sun, Jing Gu, Qiulin Niu, Zhimin Tang, "RobustScaler: QoS-Aware Autoscaling for Complex Workloads," ICDE 2022.

[5] Zhiqiang Zhou, Chaoli Zhang, Lingna Ma, Jing Gu, Huajie Qian, Qingsong Wen, Liang Sun, Peng Li, Zhimin Tang, "AHPA: Adaptive Horizontal Pod Autoscaling Systems on Alibaba Cloud Container Service for Kubernetes," AAAI 2023. (**AAAI/IAAI 2023 Innovative Application Award**)



# Talk Summary

- ❑ **Introduction:** Challenges and Needs for Robust and Intelligent Time Series Analysis
- ❑ **Robust Time Series Processing Blocks**
  - Time Series Periodicity Detection
  - Time Series Trend Filtering
  - Time Series Seasonal-Trend Decomposition
- ❑ **Robust Time Series Applications**
  - Time Series Anomaly Detection
  - Time Series Forecasting
  - Other Time Series Applications



# Related Resource

## ➤ Source Code

- Time Series Forecasting
  - FEDformer(ICML'22): <https://github.com/DAMO-DI-ML/ICML2022-FEDformer>
  - Quatformer(KDD'22): <https://github.com/DAMO-DI-ML/KDD2022-Quatformer>
  - FiLM (NeurIPS'22): <https://github.com/DAMO-DI-ML/NeurIPS2022-FiLM>
- Time Series Anomaly Detection
  - TFAD (CIKM'22): <https://github.com/DAMO-DI-ML/CIKM22-TFAD>
- More
  - Group's Open Source: <https://github.com/DAMO-DI-ML>

## ➤ More Related Time Series Talk

- Tutorial @ KDD'22, “**Robust Time Series Analysis and Applications: An Industrial Perspective**”
  - <https://qingsongedu.github.io/timeseries-tutorial-kdd-2022/>
- Keynote @ CIKM'22 Time Series Workshop, “**Customized Transformers for Time Series Forecasting**”
  - <https://drive.google.com/file/d/1Bq-NLscGfs5Cqw8q93Fcys0Gdj6uxqSf/view?usp=sharing>

# Thanks!

## Q&A

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## Hiring: AI for Time Series, XAI

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