计 算 流 体 力 学

作 业 一

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$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0,$$

初始条件:

$$u(0, x) = \max\{1 - |x|, 0\}, \ x \in [-2, 2],$$

边界条件:

$$u(t, -2) = \mathrm{NaN}, \ t \in (0, 4].$$

注：此边界条件，是为求原方程在初始条件下的解;
同时看到差分格式对解的影响.

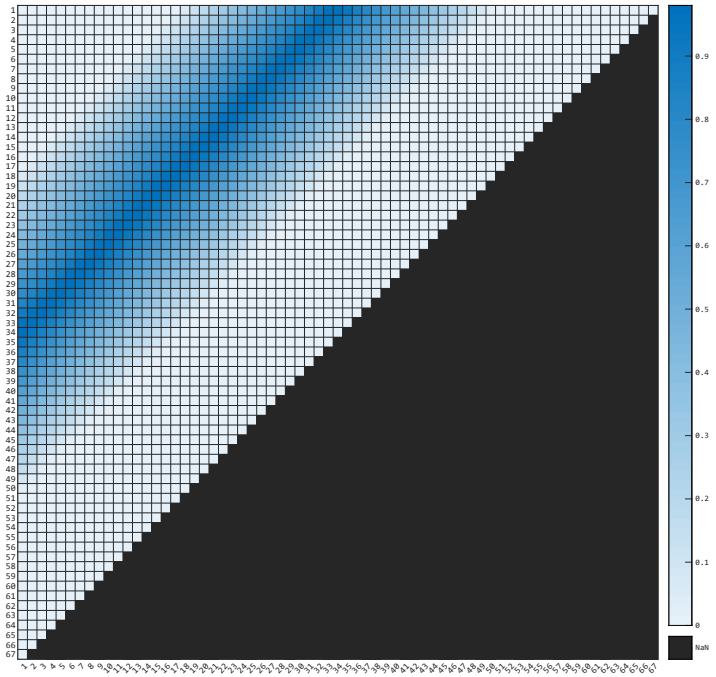
显式一阶迎风格式:

$$u_j^{i+1} = u_j^i - \frac{\Delta t}{\Delta x}\left(u_j^i - u_{j-1}^i\right),$$

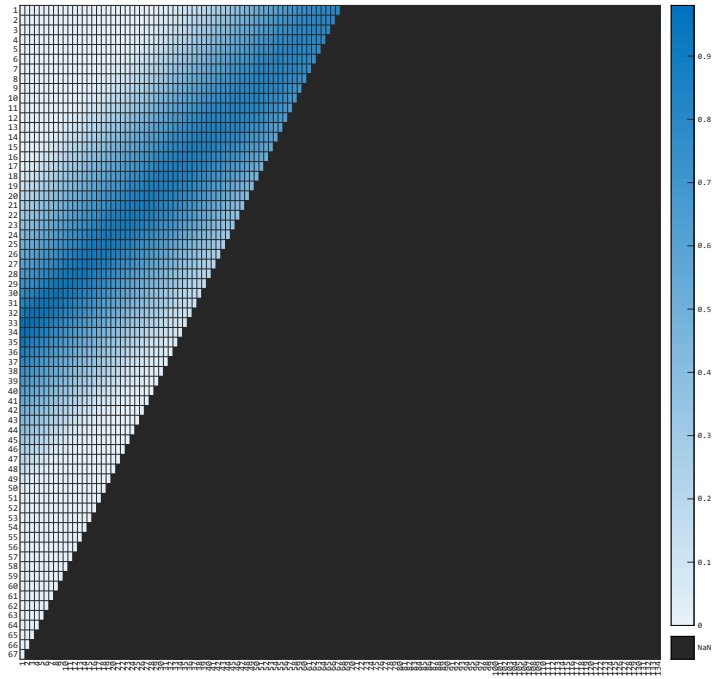
稳定条件 $\frac{\Delta t}{\Delta x} \leqslant 1$，精度 $O(\Delta t, \Delta x)$.

```
dx = 0.06; dt = 0.03; % dt = 0.03;
x = (-2:dx:2)'; t = 0:dt:4;
U = [max(1-abs(x),0), ...
     zeros(length(x),length(t)-1)];
U(1,2:end) = NaN; % U(1,2:end) = 1;
v = [1-dt/dx, dt/dx];
for idx = 2:length(t)
    U(2:end,idx) = conv(U(:,idx-1),v,'valid');
end
```

```
U(1,2:end)= NaN;
```

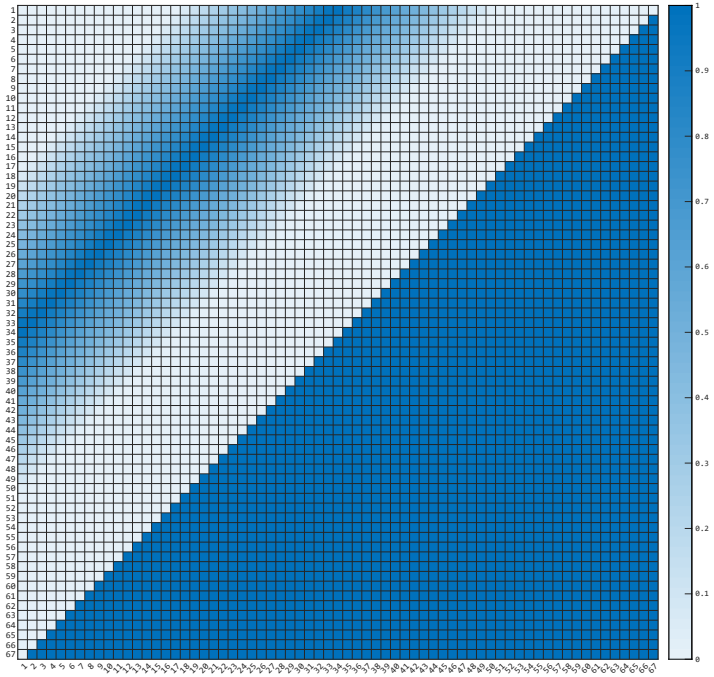


$\Delta x = 0.06, \Delta t = 0.06$

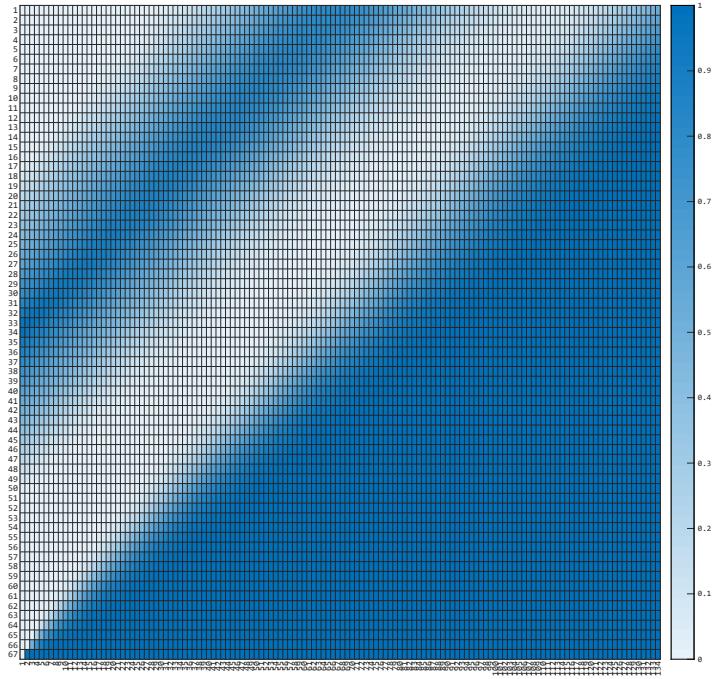


$\Delta x = 0.06, \Delta t = 0.03$

```
U(1,2:end)= 1;
```

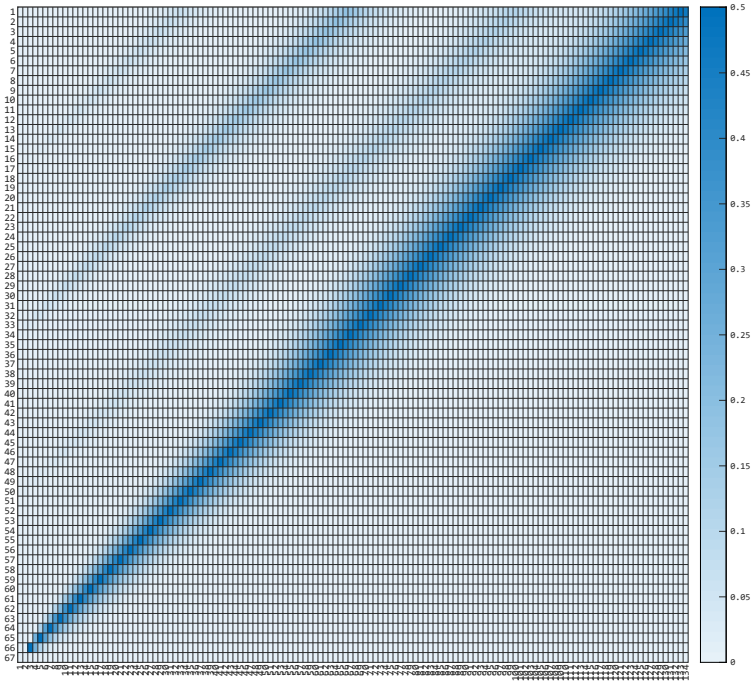


$\Delta x = 0.06, \Delta t = 0.06$

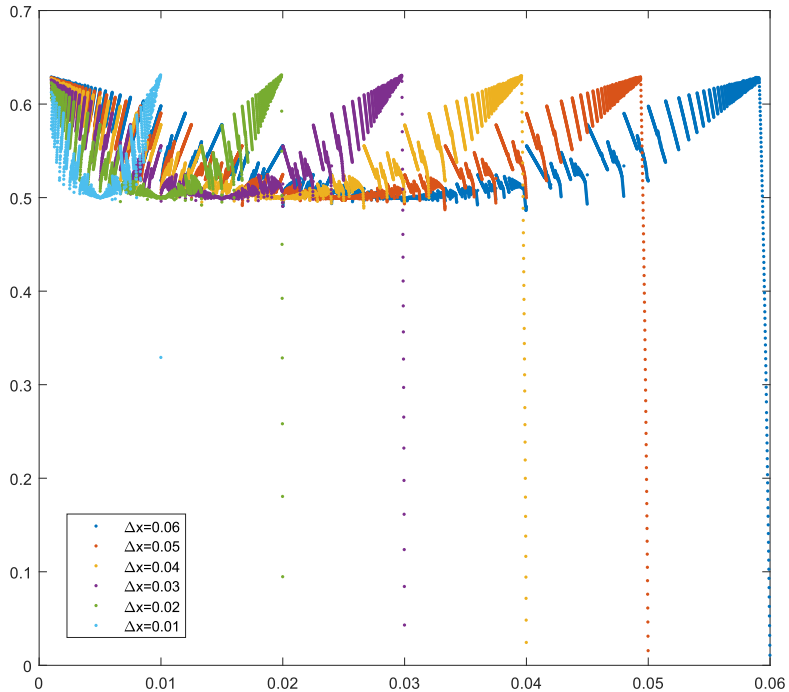


$\Delta x = 0.06, \Delta t = 0.03$

与解析解的误差



$\Delta x = 0.06, \Delta t = 0.03$



最大绝对误差

$$\Delta u = 0,$$

九点格式:

$$\mathring{U} \leftarrow U * \frac{1}{20}\begin{pmatrix} 1 & 2\frac{5\Delta x^2-\Delta y^2}{\Delta x^2+\Delta y^2} & 1 \\ 2\frac{5\Delta y^2-\Delta x^2}{\Delta x^2+\Delta y^2} & 0 & 2\frac{5\Delta y^2-\Delta x^2}{\Delta x^2+\Delta y^2} \\ 1 & 2\frac{5\Delta x^2-\Delta y^2}{\Delta x^2+\Delta y^2} & 1 \end{pmatrix}.$$