计算流体力学 （作业二）

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In[•]:= $Assumptions = And @@ {u ∈ ℝ, p > 0, ρ > 0, γ > 1};

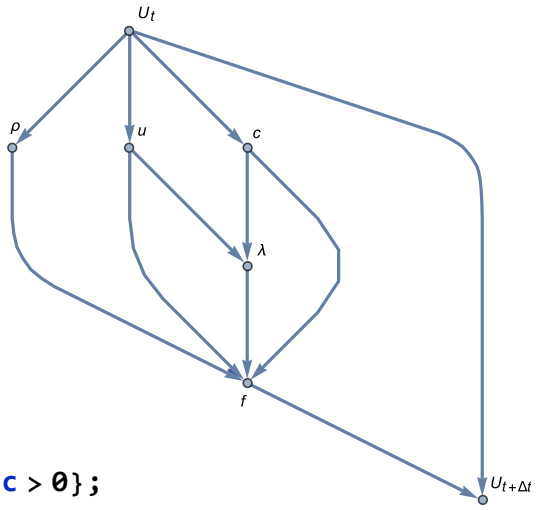
In[•]:= RuleE =
          Solve[Eliminate[E == ρ (u^2/2 + T c_v) && p == T ρ/(γ M_∞^2) && c_v == 1/((-1 + γ) γ M_∞^2), {T, c_v, M_∞}],
            E] // Simplify // Flatten

Out[•]= {E → p/(-1 + γ) + u^2 ρ/2}

In[•]:= U = {ρ, ρ u, E} /. RuleE;
        F = {ρ u, ρ u^2 + p, u (E + p)} /. RuleE;
        A = ∂{{ρ,u,p}} F.Inverse[∂{{ρ,u,p}} U] // Simplify;
        TraditionalForm@A

Out[•]//TraditionalForm=
```

$$\begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{2}u^2(\gamma-3) & -u(\gamma-3) & \gamma-1 \\ \frac{u^3(\gamma^2-3\gamma+2)\rho-2pu\gamma}{2(\gamma-1)\rho} & \frac{(-2\gamma^2+5\gamma-3)\rho u^2+2p\gamma}{2(\gamma-1)\rho} & u\gamma \end{pmatrix}$$

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In[•]:= Λ = Eigenvalues@A // Simplify

Out[•]= {u, u - Sqrt[p γ/ρ], u + Sqrt[p γ/ρ]}

In[•]:= $Assumptions = And @@ {u ∈ ℝ, p > 0, ρ > 0, γ > 1, c > 0};
        Rulep = Flatten@Solve[c == Sqrt[p γ/ρ], p];

In[•]:= S = Transpose@Inverse@Eigenvectors@A /. Rulep // Simplify;

In[•]:= f = Inverse[S].DiagonalMatrix@{λ1, λ2, λ3}.S.U /. Rulep // Simplify

Out[•]= {ρ (2 (-1 + γ) λ1 + λ2 + λ3)/(2 γ),
         ρ (2 u (-1 + γ) λ1 + (-c + u) λ2 + (c + u) λ3)/(2 γ),
         1/(4 (-1 + γ) γ) ρ (2 u^2 (-1 + γ)^2 λ1 +
           (2 c^2 - 2 c u (-1 + γ) + u^2 (-1 + γ)) λ2 + (2 c^2 + 2 c u (-1 + γ) + u^2 (-1 + γ)) λ3)}
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```
r = 1.4; e = 1e-14;
f = @(rho,u,c,Lambda) [ ...      % Steger-Waiming分裂法通用f函数
    [2*(r-1),1,1]*Lambda.*rho/2/r;                              ...
    (2*(r-1)*Lambda(1,:).*u+(u-c).*Lambda(2,:)+                 ...
        (u+c).*Lambda(3,:)).*rho/2/r;                           ...
    (2*(r-1)^2*Lambda(1,:).*u.^2+                               ...
        (2*c.^2-2*(r-1)*c.*u+(r-1)*u.^2).*Lambda(2,:)+          ...
        (2*c.^2+2*(r-1)*c.*u+(r-1)*u.^2).*Lambda(3,:)).*rho/4/r/(r-1)];
dx=1e-3; dt=1e-4; x = -5:dx:5; t = 0:dt:2;      % 初值(隐含边值)
U = [(x<0)+(x≥0)*0.125;0*x;((x<0)+(x≥0)*0.1)/(r-1)];
for idx = 1:length(t)-1                                 % 按图依次计算
    u = U(2,:)./U(1,:);
    c = sqrt(r*(r-1)*(U(3,:)./U(1,:)-u.^2/2));
    Lambda = [u;u-c;u+c];
    fp = f(U(1,:),u,c,(Lambda+sqrt(Lambda.^2+e))/2);
    fm = f(U(1,:),u,c,(Lambda-sqrt(Lambda.^2+e))/2);
    U(:,2:end-1)=U(:,2:end-1)-dt/dx*(fp(:,2:end-1)-fp(:,1:end-2)+...
                                     fm(:,3:end)-fm(:,2:end-1));
end
u = U(2,:)./U(1,:);
plot(x,[U(1,:);u;(r-1)*(U(3,:)-U(1,:).*u.^2/2)])
```