MS-CS Master Course Note (Non-Credit)

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This is my course note on "Essential Linear Algebra for Data Science" provided by Colorado University of Boulder. This is a non-credit prep course for an MS-CS degree.

Contents

1 Linear Systems and Gaussian Elimination

1.1 Linear Systems

In a linear system, we normally have constants and variables. The constants are the coefficients of the variables. The variables are the unknowns.

For example, $a_1 * x_1 + a_2 * x_2 + \cdots + a_n * x_n = b$ **is a linear system. For this to work the variables and constants must be linear. That is, the variables must be raised to the power of 1.**

Neither the product of variables nor the square root of them is allowed.

The linear system can be represented in matrix form as $Ax = b$**.**

Where A is the matrix of coefficients, x is the vector of variables, and b is the vector of constants.

For example, the following is a linear system:

$2x_1 + 3x_2 = 5$

$4x_1 + 5x_2 = 6$

This can be represented as:

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

When we have a linear system, we can have three possibilities.

1. The system has a unique solution.

2. The system has no solution.

3. The system has infinite solutions.

For example, the following system has a unique solution:

$2x_1 + 3x_2 = 5$

$4x_1 + 5x_2 = 6$

This system has a unique solution of $x_1 = 1$ and $x_2 = 1$.

Second, the following system has no solution:

$2x_1 + 3x_2 = 5$

$4x_1 + 6x_2 = 6$

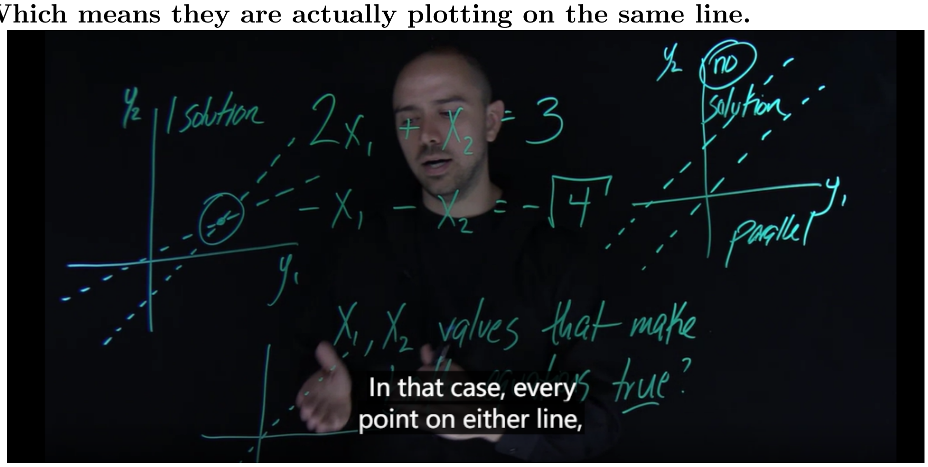
Which means there's no cross in the coordinate visualization system.

And the following system has infinite solutions:

$2x_1 + 3x_2 = 5$

$4x_1 + 6x_2 = 10$

Which means they are actually plotting on the same line.



1.2 Matrices and Gaussian Elimination

As displayed above, we can convert a linear system into a matrix form.

The matrix form is $Ax = b$.

Where A is the matrix of coefficients, x is the vector of variables, and b is the vector of constants.

If we extract all the coefficient, a.k.a. the constant values from the matrix, we get the Coefficient Matrix.

For example, the following is a linear system:

$2x_1 + 3x_2 = 5$

$4x_1 + 5x_2 = 6$

The Coefficient Matrix can be represented as:

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

Besides, we can also add the result of the linear system to the matrix, which make a coefficient matrix into an augmented matrix.

Here is the augmented matrix of the above linear system:

$$\begin{bmatrix} 2 & 3 & | & 5 \\ 4 & 5 & | & 6 \end{bmatrix}$$

1.3 Rules of Gaussian Elimination and Single Solution

Gaussian Elimination is a method to solve linear systems.

The rules of G.E. are as follows:

1. You can interchange two rows.

2. You can multiply a row by a non-zero constant.

3. You can add a multiple of one row to another row.

The fundamental idea of Gaussian elimination is to add multiples of one equation to the others to eliminate a variable. This process continues until only one variable remains. Once the final variable is determined, its value is substituted back into the other equations to evaluate the remaining unknowns.

The three steps involved in Gaussian elimination are:

1. Convert the system of equations to an augmented matrix.

2. Put the matrix in upper triangular form.

3. Solve for the variables starting with the last row and working your

way up.

Gaussian elimination is commonly used to solve linear systems, but can result in infinite solutions if there are more unknowns than equations.