Assignment 4

In this assignment, we will explore countmin sketches and bloom filters. We will use two text files `great-gatsby-fitzgerald.txt` and `war-and-peace-tolstoy.txt` to load up the text of two famous novels courtesy of Project Guttenberg.

We will explore two tasks:

- Counting the frequency of words of length 5 or more in both novels using a count-min sketch
- Using a bloom filter to approximately count how many words in the War and Peace novel already appears in the Great Gatsby.

Step 1: Making a Universal Hash Family (Already Done For You)

We will use a family of hash function that first starts by (a) generating a random prime number p (we will use the Miller-Rabin primality test for this purpopse); (b) generating random numbers a, b between 2 and p-1.

The hash function $h_{a,b,p}(n) = (a\,n + b)\,mod\,p$.

Note that this function will be between 0 and p-1. We will need to also make sure to take the hash value modulo m where m is the size of the hashtable.

To hash strings, we will first use python's inbuilt hash function and then use $h_{a,b,p}$ on the result.

As a first step, we will generate a random prime number.

(A) Generate Random Prime Numbers

```
# Python3 program Miller-Rabin randomized primality test
# Copied from geeksforgeeks: https://www.geeksforgeeks.org/primality-test-set-3-miller-rabin/
import random

# Utility function to do
# modular exponentiation.
# It returns (x^y) % p
def power(x, y, p):

      # Initialize result
      res = 1;

      # Update x if it is more than or
      # equal to p
      x = x % p;
```

```
    while (y > 0):

        # If y is odd, multiply
        # x with result
        if (y & 1):
            res = (res * x) % p;

        # y must be even now
        y = y>>1; # y = y/2
        x = (x * x) % p;

    return res;

# This function is called
# for all k trials. It returns
# false if n is composite and
# returns false if n is
# probably prime. d is an odd
# number such that d*2<sup>r</sup> = n-1
# for some r >= 1
def miillerTest(d, n):

    # Pick a random number in [2..n-2]
    # Corner cases make sure that n > 4
    a = 2 + random.randint(1, n - 4);

    # Compute a^d % n
    x = power(a, d, n);

    if (x == 1 or x == n - 1):
        return True;

    # Keep squaring x while one
    # of the following doesn't
    # happen
    # (i) d does not reach n-1
    # (ii) (x^2) % n is not 1
    # (iii) (x^2) % n is not n-1
    while (d != n - 1):
        x = (x * x) % n;
        d *= 2;

        if (x == 1):
            return False;
        if (x == n - 1):
            return True;

    # Return composite
    return False;
```

```python
# It returns false if n is
# composite and returns true if n
# is probably prime. k is an
# input parameter that determines
# accuracy level. Higher value of
# k indicates more accuracy.
def isPrime( n, k):

    # Corner cases
    if (n <= 1 or n == 4):
        return False;
    if (n <= 3):
        return True;

    # Find r such that n =
    # 2^d * r + 1 for some r >= 1
    d = n - 1;
    while (d % 2 == 0):
        d //= 2;

    # Iterate given nber of 'k' times
    for i in range(k):
        if (miillerTest(d, n) == False):
            return False;

    return True;

# Driver Code
# Number of iterations
k = 4;

print("All primes smaller than 100: ");
for n in range(1,100):
    if (isPrime(n, k)):
        print(n , end=" ");

# This code is contributed by mits (see citation above)
```

```
All primes smaller than 100:
2 3 5 7 11 13 17 19 23 29 31 37 41 43 47 53 59 61 67 71 73 79 83 89 97
```

Step 2: Universal Hash Families

We will provide three useful functions for you:

- `get_random_hash_function`: Generate triple of numbers `(p, a, b)` at random, where p is prime, a and b are numbers between 2 and p-1. The hash function $h_{p,a,b}(n)$ is given by $ (an + b) \mod p$.
- `hashfun`: apply the random hash function on a number `num`.

- `hash_string`: apply the hash function on a string `hstr`. Note that the result is between 0 and p-1. If your hash table has size `m`, you should take a `mod m` on this result where you call `hash_string`.

Please use these functions in your code below.

```
# Get a random triple (p, a, b) where p is prime and a,b are numbers
betweeen 2 and p-1
def get_random_hash_function():
    n = random.getrandbits(64)
    if n < 0:
        n = -n
    if n % 2 == 0:
        n = n + 1
    while not isPrime(n, 20):
        n = n + 1
    a = random.randint(2, n-1)
    b = random.randint(2, n-1)
    return (n, a, b)

# hash function fora number
def hashfun(hfun_rep, num):
    (p, a, b) = hfun_rep
    return (a * num + b) % p

# hash function for a string.
def hash_string(hfun_rep, hstr):
    n = hash(hstr)
    return hashfun(hfun_rep, n)
```

Step 3: Loading Data

We are going to load two files `great-gatsby-fitzgerald.txt` and `war-and-peace-tolstoy.txt` to load up the text of two famous novels courtesy of Project Guttenberg. We will filter all wordsd of length >= 5 and also count the frequency of each word in a dictionary. This will be fast because it is going to use highly optimized hashtable (dictionaries) built into python.

```
# Let us load the "Great Gatsby" novel and extract all words of length
5 or more
filename = 'great-gatsby-fitzgerald.txt'
file = open (filename,'r')
txt = file.read()
txt = txt.replace('\n',' ')
words= txt.split(' ')
longer_words_gg = list(filter(lambda s: len(s) >= 5, words))
print(len(longer_words_gg))
# Let us count the precise word frequencies
word_freq_gg = {}
for elt in longer_words_gg:
```

```
    if elt in word_freq_gg:
        word_freq_gg[elt] += 1
    else:
        word_freq_gg[elt] = 1
print(len(word_freq_gg))
```

```
21342
8849
```

```
# Let us load the "War and Peace" novel by Tolstoy translation and
extract all words of length 5 or more
filename = 'war-and-peace-tolstoy.txt'
file = open (filename,'r')
txt = file.read()
txt = txt.replace('\n',' ')
words= txt.split(' ')
longer_words_wp = list(filter(lambda s: len(s) >= 5, words))
print(len(longer_words_wp))
word_freq_wp = {}
for elt in longer_words_wp:
    if elt in word_freq_wp:
        word_freq_wp[elt] += 1
    else:
        word_freq_wp[elt] = 1
print(len(word_freq_wp))
```

```
237611
38777
```

Problem 1: Implement count-min sketch

Implement `CountMinSketch` class below where `num_counters` is the number of counters. You are given the constructor that already generates a random representative of a hash function family. Implement the functions:

- `increment`
- `approximateCount`.

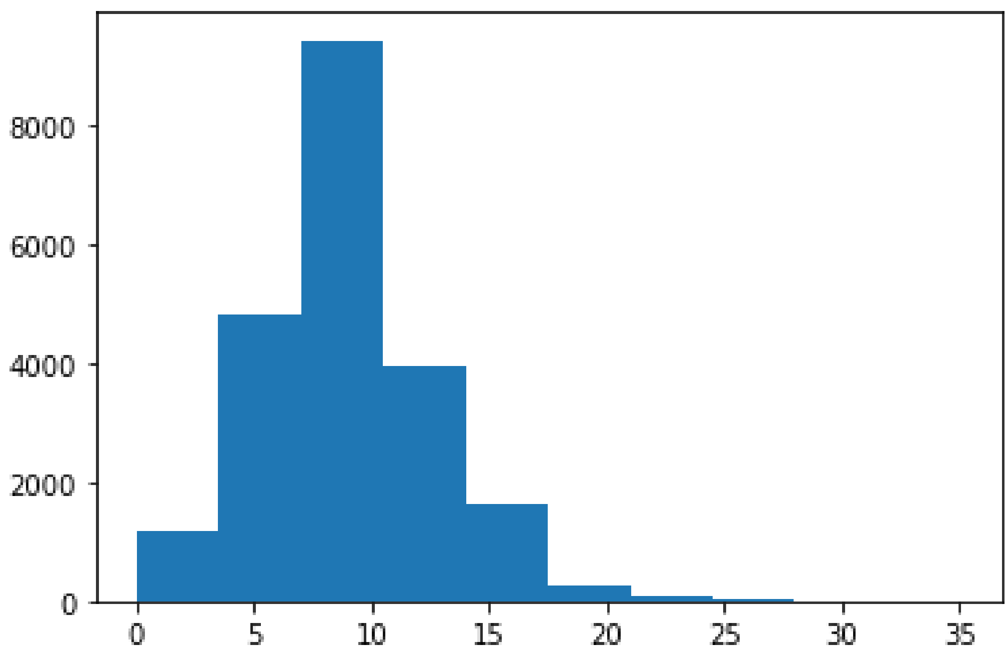
Please read the constructor carefully: it initializes the counters and generates the hash function for you. Also, when you call `hash_string` function defined previously, do not forget to take result modulo m.

```
# Class for implementing a count min sketch "single bank" of counters
class CountMinSketch:
    # Initialize with `num_counters`
    def __init__ (self, num_counters):
        self.m = num_counters
        self.hash_fun_rep = get_random_hash_function()
        self.counters = [0]*self.m
```

```python
    # your code here


    # Function: increment
    # Given a word, increment its count in the count-min sketch
    def increment(self, word):
        # Hash the word to get the index in the counters array
        index = hash_string(self.hash_fun_rep, word) % self.m
        # Increment the counter at the hashed index
        self.counters[index] += 1

    # Function: approximateCount
    # Given a word, get its approximate count
    def approximateCount(self, word):
        # Hash the word to get the index in the counters array
        index = hash_string(self.hash_fun_rep, word) % self.m
        # Return the value of the counter at the hashed index
        return self.counters[index]


# We will now implement the algorithm for a bank of k counters

# Initialize k different counters
def initialize_k_counters(k, m):
    return [CountMinSketch(m) for i in range(k)]

# Function: increment_counters
# Increment each of the individual counters with the word
def increment_counters(count_min_sketches, word):
    for cms in count_min_sketches:
        cms.increment(word)


# Function: approximate_count
# Get the approximate count by querying each counter bank and taking
the minimum
def approximate_count(count_min_sketches, word):
    return min([cms.approximateCount(word) for cms in
count_min_sketches])


%matplotlib inline
from matplotlib import pyplot as plt

# Let's see how well your solution performs for the Great Gatsby words
cms_list = initialize_k_counters(5, 1000)
for word in longer_words_gg:
    increment_counters(cms_list, word)
```

```
discrepencies = []
for word in longer_words_gg:
    l = approximate_count(cms_list, word)
    r = word_freq_gg[word]
    assert ( l >= r)
    discrepencies.append( l-r )

plt.hist(discrepencies)

assert(max(discrepencies) <= 200), 'The largest discrepency must be
definitely less than 200 with high probability. Please check your
implementation'
print('Passed all tests: 10 points')
```

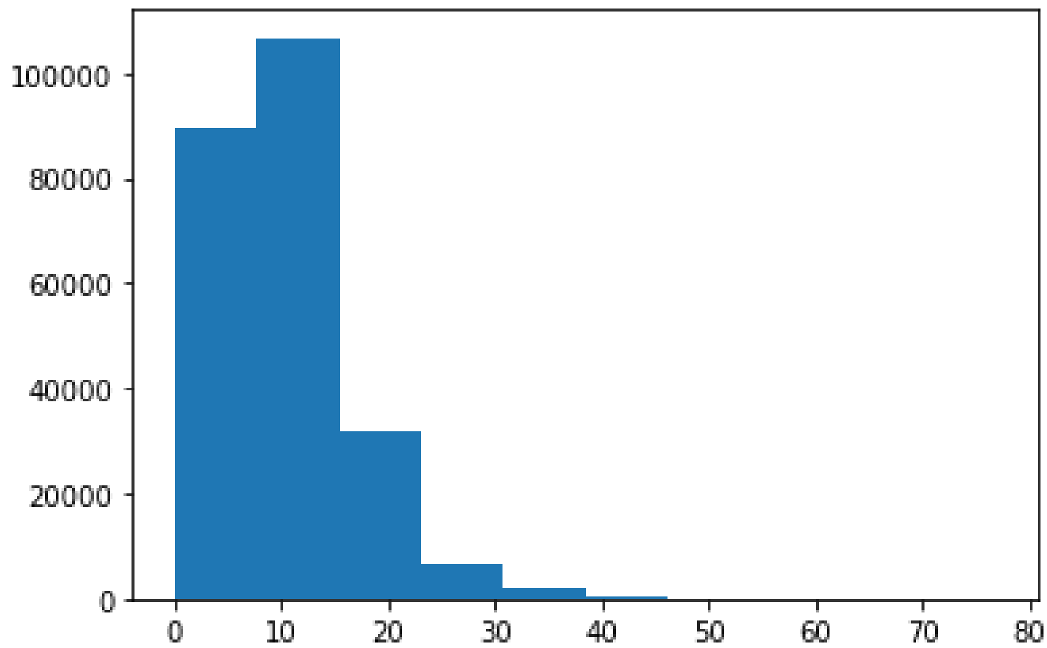
```
Passed all tests: 10 points
```




```
# Let's see how well your solution performs for the War and Peace
cms_list = initialize_k_counters(5, 5000)
for word in longer_words_wp:
    increment_counters(cms_list, word)

discrepencies = []
for word in longer_words_wp:
    l = approximate_count(cms_list, word)
    r = word_freq_wp[word]
    assert ( l >= r)
    discrepencies.append( l-r )
```

```
plt.hist(discrepencies)
print('Passed all tests: 5 points')
```

```
Passed all tests: 5 points
```



Problem 1B

Check the data obtained above with calculations along the lines of what was done in class. If we had 5 banks of counters with 5000 counters each and a uniform hash function family, what is the probability that when counting a total of $N = 2.5 \times 10^5$ words, we have a discrepency by 80 or more.

This problem will not be graded but simply for you to understand the calculations involved.

YOUR ANSWER HERE

Problem 2: Using a Bloom Filter to Count Common Words.

In this problem, we will implement a Bloom filter to count how many elements of `longer_words_wp` (the words of length 5 or more in War and Peace) appear in the Great-Gatsby novel. To do so, we will do the following:

- Instantiate a Bloom filter with number of bits `n` and number of hash functions `k`.
- Insert all words from great-gatsby into the filter.
- For each word from war and peace, check membership in the Bloom filter and count the number of yes answers.

```
class BloomFilter:
    def __init__(self, nbits, nhash):
```

```
        self.bits = [False] * nbits  # Initialize all bits to False
        self.m = nbits
        self.k = nhash
        # Get k random hash functions
        self.hash_fun_reps = [get_random_hash_function() for _ in
range(self.k)]

    # Function to insert a word in a Bloom filter
    def insert(self, word):
        for hash_fun_rep in self.hash_fun_reps:
            index = hash_string(hash_fun_rep, word) % self.m
            self.bits[index] = True

    # Check if a word belongs to the Bloom Filter
    def member(self, word):
        for hash_fun_rep in self.hash_fun_reps:
            index = hash_string(hash_fun_rep, word) % self.m
            if not self.bits[index]:
                return False
        return True

#do the exact count
# it is a measure of how optimized python data structures are under
the hood that
# this operation finishes very quickly.
all_words_gg = set(longer_words_gg)
exact_common_wc = 0
for word in longer_words_wp:
    if word in all_words_gg:
        exact_common_wc = exact_common_wc + 1
print(f'Exact common word count = {exact_common_wc}')
```

```
Exact common word count = 124595
```

```
# Try to use the same using a bloom filter.
bf = BloomFilter(100000, 5)
for word in longer_words_gg:
    bf.insert(word)

for word in longer_words_gg:
    assert (bf.member(word)), f'Word: {word} should be a member'

common_word_count = 0
for word in longer_words_wp:
    if bf.member(word):
        common_word_count= common_word_count + 1
print(f'Number of common words of length >= 5 equals :
{common_word_count}')
assert ( common_word_count >= exact_common_wc)
print('All Tests Passed: 10 points')
```

```
Number of common words of length >= 5 equals : 125625
All Tests Passed: 10 points
```

Problem 2 B

Given a Bloom filter with $m = 100000$ bits and $k = 5$ hash functions that map each key uniformly at random to one of the bits (assumption), estimate the probability that k bits $i_1, \dots, i_k$ are simultaneously set when $n = 10000$ words are inserted. Assume that whether or not a particular bit is set is independent of another.

YOUR ANSWER HERE

Manually Graded Solutions

Problem 1 B

Note that for each word we have $E(\text{approxCount}(word) - \text{count}(word)) \leq \frac{N}{m}$. The probability that for some word, the approximate count differs from the real one by at least 80 for one of the counter banks is bounded by Markov Inequality as:

$$P(\text{approxCount}(word) - \text{count}(word) \geq 80) \leq \frac{E(\text{approxCount}(word) - \text{count}(word))}{80} \leq \frac{2.5 \times 10^5}{80 \times 5000} \approx \frac{5}{8}$$

.

The probability that this happens for all five counter banks is bounded by $\left(\frac{5}{8}\right)^5 \approx 0.095$.

However, this bound happens to be not so tight. Empirically, we see that this happens for roughly one word out of the nearly quarter million words in the corpus.

Problem 2 B

The probability that any given bit is not set when n words are inserted is $\left(1 - \frac{1}{m}\right)^{kn} = \left(1 - \frac{1}{100000}\right)^{5 \times 10000} = e^{-0.5}$.

The probability that all five bits are simultaneously set is $(1 - e^{-0.5})^5 = 0.009$.

Therefore, we will expect the false positive rate to be roughly 1%.

That's All Folks!